(Sobgula Math Important)

TE 4225

Mathematical Problems

Problem 1:

Let, a sewing machine runs daily 8 hr and produces 1000 goods. Daily failure rate is 2 and rejected goods produces 12 per day. If average time to repair the machine is 1 hr and machine works at 95% efficiency, Find out the OEE%.

Solution:

We know, OEE%= Uptime%* Speed%* Quality% ....(1)

Uptime%= (MTBF-MTTR)/MTBF

MTBF= (Running Time/No of Failures)

= (8/2) = 4

So, Uptime%= (4-1)/4= 0.75

And, Quality%= (Good Products/Total Products)

= (988/1000) = 0.988

So, OEE%= (0.75*0.95*0.988)*100%

=70.4% (Ans)

Problem 2:

Eco Energy Ltd. tests its product tube lights for 2000 hours. A sample of 100 lights were put through the test, during which two failed. If customers use the lights for an average of 4 hours per day and if 10000 tubes were sold, then

(a) In one year from their selling how many tubes will be expected to fail?

(b) What is the MTBF for the tubes?

Solution:

(a) Total time of testing= (100 Tubes)*2000 Hours= 200000 Tube-Hours

However, two tubes have failed (Assume each lasted for average of half test period)

So, Lost Hours= (2 Tubes)*(2000/2 Hours) = 2000 Tube-Hours

Two failures occurred during testing of = (200000-2000) = 198000 Tube-Hours

So,

During 365 days of the year (4 Hours a day) for 10000 tubes, the expected failures are= (2/198000 failures per Tube-Hours)*(10000 Tubes)*(365*4Hours)

= 147.47=148 Failures (Ans)

(b) Mean Time Between Failures= (198000 Tube-Hours/ 2 Failures)

= 99000 Tube-Hours/ Failures

= (99000/4*365) Tube-Years/ Failures

= 67.68 Tube-Years/ Failures (Ans)

Problem 3:

A dyeing factory has 25 identical dyeing machines used to dye a variety of fabrics that generate a profit of BDT1000 per machine per day. The machines fail according to Poisson distribution on an average of 2.2 machines down each day.

(a) What is the chance having exactly 03 machines down on a given day?

(b) What is the expected amount of lost profit per day due to this Poisson failure rate of 2.2/ day?

Solution:

(a) Since failure occurs according to Poisson distribution, the probability of x machines failure on a given day is:

$$\boxed{P(X) = \frac{e^{-\mu}\mu^{x}}{x!}}$$

Where, x= No of machine broken down= 3

μ= Mean failure rate= 2.2/day

e= 2.718

So, P(x=3) = 0.1966= 20% (Ans)

(b) Expected loss per day, E(X)= X * μ

Where, X= Amount of loss per machine per day= 1000

=1000*2.2 BDT/day

=2200 BDT/day (Ans)

Problem 4:

The following failure rates have been observed for a certain type of transistors in a computer

End of week	1	2	3	4	5	6	7
Probability of failure to date	0.07	0.18	0.30	0.48	0.69	0.89	1.00

The cost of replacing a failed transistor individually is BDT 9. If all the transistors are replaced simultaneously it would cost BDT 3 per transistor. Any one of the following two options can be followed to replace the transistors.

a) Replace the transistors individually when they fail (Individual replacement policy)

b) Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (Group replacement policy)

Assuming 100 transistors in use, find out the optimal replacement policy. If group replacement policy is optimal, then at what intervals should all the transistors be replaced?

Solution:

Let, P_i be the probability of a transistor which was new when placed in position for use, fails during the i^{th} week of its life,

So, P_1= 0.07, P_2= 0.11, P_3= 0.12, P_4= 0.18, P_5= 0.21, P_6= 0.20, P_7=0.11

Since the sum of P_i is equal to 1 at the end of the 7^{th} week, the transistors are sure to fail during the 7^{th} week.

Let, transistors that fail during a week are replaced just before the end of the week.

Now let, N_i is the number of transistors replaced at the end of the i^{th} week.

So, N_0 = the number of transistors replaced at the end of 0^{th} week (beginning of first week) =100

So, N_1= No of transistors replaced at the end of 1st week

= N_0 x P_1

= 100 x 0.07 =7

So, N_2 = N_0 x P_2 + N_1 x P_1

= 100x0.11+7x0.07 = 12

N_3 = N_0 x P_3 + N_1 x P_2 + N_2 x P_1 = 100x0.12+7x0.11+12x0.07 = 14

N_4 = N_0 x P_4 + N_1 x P_3 + N_2 x P_2+ N_3 x P_1 = 100x0.18+7x0.12+12x0.11+14x0.07 = 21

N_5 = N_0 x P_5 + N_1 x P_4 + N_2 x P_3+ N_3 x P_2+ N_4 x P_1

= 100x0.21+7x0.18+12x0.12+14x0.11+21x0.07 = 27

N_6 = N_0 x P_6 + N_1 x P_5 + N_2 x P_4+ N_3 x P_3+ N_4 x P_2+ N_5 x P_1

= 100x0.2+7x0.21+12x0.18+14x0.12+21x0.11+27x0.07 = 30

N_7 = N_0 x P_7 + N_1 x P_6 + N_2 x P_5+ N_3 x P_4+ N_4 x P_3+ N_5 x P_2+ N_6 x P_1

= 100x0.11+7x0.2+12x0.21+14x0.18+21x0.12+27x0.11+30x0.11 = 25

(a) Calculation of individual replacement cost:

Expected life of each transistor=$\sum_{i=1}^{7} iPi$
= (1*0.07+2*0.11+3*0.12+4*0.18+5*0.21+6*0.2+7*0.11)
= 4.39 weeks

So, Average no of failures per week= (100/4.39) = 23

Therefore, individual replacement cost per week= 23x 9 = 207

(b) Calculation of group replacement cost:

End of week (A)	Cost of replacing 100 at a time (B)	Cost of replacing during a given period (C)	Total cost (B+C)	Average cost per week (B+C/A)
1	100x3= 300	7x9= 63	363	363
2	300	(7+12)x9=171	471	235.5
3	300	297	597	199
4	300	486	786	196.5
5	300	729	1029	205.80
6	300	999	1299	216.50
7	300	1224	1524	217.71

So, it's clear that, the average cost/week is minimum for the 4^{th} week. Hence group replacement policy period is 4 weeks.

Problem 5:

A mechanical equipment contains 1000 gears. When any one of the gear fails, it is replaced. The cost of replacing a gear individually is BDT20 and if all the gears are replaced at a time, the cost per gear is BDT6. The present surviving, S_i at the end of i^{th} month is given bellow:

i	0	1	2	3	4	5	6
S_i (%)	100	96	89	68	37	13	0

Evaluate the optimal replacement policy.

Solution:

Self-Study