MTH 441A: Lab 2

P 1. ***The Rocket Propellant Data:*** Consider the Rocket Propellant Data from Example 2.1 on page 15 of the book Introduction to Linear Regression Analysis by Montgomery et.al. Fit a simple linear regression model for the Rocket Propellant Data.

P 2. ***The Delivery Time Data:*** Consider the Delivery Time Data from Example 3.1 on page 74 of the book Introduction to Linear Regression Analysis by Montgomery et.al. Fit a multiple linear regression model for the Delivery Time Data.

Note: Save both data sets to **.xlsx** files and have a copy of it before coming to the lab. You must know how to import the data in R.

P 3. For the data in **P 1** compute the following:

(1) Value of the unbiased estimator of σ^2.

(2) $t_j = \widehat{\beta}_j / \sqrt{\mathrm{Var}(\widehat{\beta}_j)}$.

P 4. Generating sample from a Chi–square random variable:

(1) Generate $n = 5000$ samples using $W = \sum_{i=1}^{3} Z_i^2$, where $Z_i \sim \mathcal{N}(0,1)$ for $i = 1, 2, 3$.

(2) Plot the histogram of W. Compute the sample mean and variance of W.

(3) Compute the theoretical mean and variance of chi–square random variable with 3 degrees of freedom.

(4) Compare the results of (2) and (3).

P 5. Generating sample from a Chi–square random variable:

(1) Given a full rank 8×5 matrix X (create using *matrix* function) compute $\mathrm{P_X} = \mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}'$. Verify if $\mathrm{P_X}$ is idempotent.

(2) Generate 5000 samples using $u = Y'\mathrm{P_X}Y$ where $Y \sim \mathcal{N}(\boldsymbol{\mu}, \mathrm{I})$, $\boldsymbol{\mu} = (0,0,0,0,0,0,0,0)'$ and I is a 8×8 diagonal matrix with all ones as diagonal.

(3) Plot the histogram of u. Compute the sample mean and variance of u.

(4) Compute the theoretical mean and variance of chi–square random variable with 5 degrees of freedom.

(5) Compare the results of (3) and (4).

P 6. Generating sample from a F–distribution:

(2) Generate 5000 samples using $f = (Y'\mathrm{P_{X_2}}Y/df_1)/(Y'\mathrm{P_{X_1}}Y/df_2)$ where $Y \sim \mathcal{N}(\boldsymbol{\mu}, \mathrm{I}_8)$, $\boldsymbol{\mu} = (0,0,0,0,0,0,0,0)'$ and I_8 is a 8×8 diagonal matrix with all ones in diagonal, $\mathrm{P_{X_2}} = \mathrm{I}_8 - \mathrm{P_{X_1}}$, $df_1 = Rank(\mathrm{P_{X_2}})$, $df_2 = Rank(\mathrm{P_{X_1}})$ and X_1 is same as X from **P 5.** (1).

(3) Plot the histogram of f. Compute the sample mean and variance of f.

(4) Compute the theoretical mean and variance of F–random variable with (3,5) degrees of freedom.

(5) Compare the results of (3) and (4).