

# Stadium Billiard Assignment — Analytic Simulation

## Assignment Question

A stadium billiard consists of a rectangle of width  $2a$  and height  $2b$ , capped at each end by a half-ellipse of semi-axes  $r_x$  (horizontal) and  $r_y$  (vertical). A point particle moves freely inside the stadium, bouncing elastically off the boundary.

### (a) Stadium Boundary Equations

The boundary of the stadium consists of:

1. **Rectangle (centered at origin):**

$$|x| \leq a, \quad |y| \leq b$$

2. **Left ellipse cap (centered at  $x = -a$ ):**

$$\frac{(x + a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

3. **Right ellipse cap (centered at  $x = +a$ ):**

$$\frac{(x - a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

### Collision Conditions

- **Horizontal walls:** Collision occurs when  $y = +b$  or  $y = -b$  and  $|x| \leq a$ .
- **Ellipse caps:** Collision occurs when the particle reaches the ellipse boundary at  $x = \pm a$  and satisfies:

$$\frac{(x \mp a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

- **Note:** Only collisions with horizontal walls and ellipse caps are considered; there are no vertical-wall reflections at  $x = \pm a$ .

## Simulation Results

- Stadium parameters:  $a = 1.0$ ,  $b = 1.0$ ,  $r_x = 2.0$ ,  $r_y = 1.0$
- Number of bounces recorded: 38
- Final velocity:  $v_x = 1.2828$ ,  $v_y = 0.5953$ , speed = 1.4093

Figure 1: Stadium billiard trajectory and boundary

## Bounce Data (first 10 shown)

Index	$x$	$y$	Kind
53	1.0000	1.0000	ellipse
137	2.6000	-0.6000	ellipse
196	1.9959	0.8672	ellipse
316	-0.6138	-1.0000	hwall
412	-2.7221	0.5085	ellipse
469	-2.0576	-0.8488	ellipse
609	1.2243	0.9937	ellipse
687	2.9403	-0.2425	ellipse
842	-1.0337	0.9999	ellipse
913	-2.8282	0.4055	ellipse