Assignment 1

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1 Stadium Boundary Equations

The boundary of the stadium consists of:

1. **Rectangle (centered at origin):**
$$|x| \leq a, \quad |y| \leq b$$
2. **Left ellipse cap (centered at $x = -a$):**
$$\frac{(x+a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$
3. **Right ellipse cap (centered at $x = +a$):**
$$\frac{(x-a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

2 Classical Motion (Newtonian Dynamics)

2.1 Collision Conditions

- **Horizontal walls:** Collision occurs when $y = +b$ or $y = -b$ and $|x| \leq a$.
- **Ellipse caps:** Collision occurs when the particle reaches the ellipse boundary at $x = \pm a$ and satisfies:
$$\frac{(x \mp a)^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$
- **Note:** Only collisions with horizontal walls and ellipse caps are considered; there are no vertical-wall reflections at $x = \pm a$.

2.2 Simulation, Checks and Results

Stadium parameters used for simulation: $a = 3.0$, $b = r_y = 1.0$, $r_x = 2.0$, mass: $m = 1$

2.2.1 Simulation

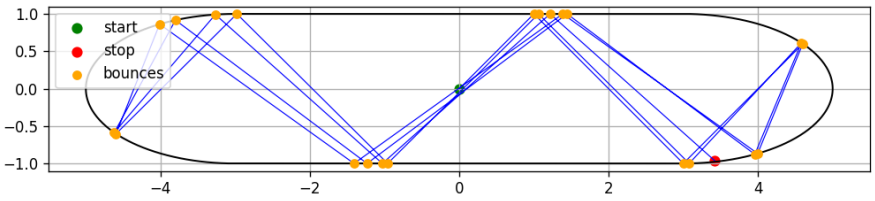

Figure 1: Stadium billiard trajectory and boundary