Assignment 2 – QT 207

All the problems are from the book

An Introduction to Quantum Computing by Kaye, Laflamme and Mosca

Exercise 3.3.1 Consider the 2-qubit state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle. \tag{3.3.2}$$

Show that this state is entangled by proving that there are no possible values $\alpha_0, \alpha_1, \beta_0$, β_1 such that

$$|\psi\rangle = \big(\alpha_0|0\rangle + \alpha_1|1\rangle\big)\big(\beta_0|0\rangle + \beta_1|1\rangle\big). \tag{3.3.3}$$

Exercise 3.4.1

(a) Prove that if the operators P_i satisfy $P_i^\dagger = P_i$ and $P_i^2 = P_i$, then $P_iP_j = 0$ for all $i \neq j$.

(b) Prove that any pure state $|\psi\rangle$ can be decomposed as $|\psi\rangle = \sum_i \alpha_i|\psi_i\rangle$ where $\alpha_i = \sqrt{p(i)}$, $p(i) = \langle\psi|P_i|\psi\rangle$, and $|\psi_i\rangle = \frac{P_i|\psi\rangle}{\sqrt{p(i)}}$.

Also prove that $\langle\psi_i|\psi_j\rangle = \delta_{i,j}$.

(c) Prove that any decomposition $I = \sum_i P_i$ of the identity operator on a Hilbert space of dimension N into a sum of nonzero projectors P_i can have at most N terms in the sum.

Exercise 3.4.2 Show that measuring the observable $|1\rangle\langle 1|$ is equivalent to measuring the observable Z up to a relabelling of the measurement outcomes.

Exercise 4.2.3

(a) Prove $XR_y(\theta)X = R_y(-\theta)$ and $XR_z(\theta)X = R_z(-\theta)$.

(b) Prove Corollary 4.2.1.

Hint: Using Theorem 4.2.1 we can write

$$U = e^{i\alpha}R_z(\beta)R_y(\gamma)R_z(\delta). \tag{4.2.12}$$

Then take $A \equiv R_z(\beta)R_y(\gamma/2)$, $B \equiv R_y(-\gamma/2)R_z(-(\delta+\beta)/2)$, and $C \equiv R_z((\delta-\beta)/2)$.

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Exercise 4.2.4 Describe the effect of the CNOT gate with respect to the following bases.

(a) $B_1 = \left\{ |0\rangle\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right),\ |0\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right),\ |1\rangle\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right),\ |1\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \right\}$

(b) $B_2 = \left\{ \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right),\ \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right),\ \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right),\ \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \right\}$

Express your answers both using Dirac notation, and also with matrix notation.

Exercise 5.2.1 Prove that

$$|\psi\rangle|\beta_{00}\rangle = \tfrac{1}{2}|\beta_{00}\rangle|\psi\rangle + \tfrac{1}{2}|\beta_{01}\rangle(X|\psi\rangle) + \tfrac{1}{2}|\beta_{10}\rangle(Z|\psi\rangle) + \tfrac{1}{2}|\beta_{11}\rangle(XZ|\psi\rangle). \qquad (5.2.8)$$

Exercise 6.4.1 Prove that

$$\left(\frac{|0\rangle+(-1)^{x_1}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle+(-1)^{x_2}|1\rangle}{\sqrt{2}}\right)\cdots\left(\frac{|0\rangle+(-1)^{x_n}|1\rangle}{\sqrt{2}}\right) \qquad (6.4.14)$$

$$= \frac{1}{\sqrt{2^n}} \sum_{z_1 z_2 \ldots z_n \in \{0,1\}^n} (-1)^{x_1 z_1 + x_2 z_2 + \cdots + x_n z_n}|z_1\rangle|z_2\rangle\cdots|z_n\rangle. \qquad (6.4.15)$$

Exercise 6.5.1 Let $\mathbf{x}, \mathbf{y} \in \{0,1\}^n$ and let $\mathbf{s} = \mathbf{x} \oplus \mathbf{y}$. Show that

$$H^{\otimes n}\left(\frac{1}{\sqrt{2}}|\mathbf{x}\rangle + \frac{1}{\sqrt{2}}|\mathbf{y}\rangle\right) = \frac{1}{\sqrt{2^{n-1}}} \sum_{\mathbf{z}\in\{\mathbf{s}\}^{\perp}} (-1)^{\mathbf{x}\cdot\mathbf{z}}|\mathbf{z}\rangle. \qquad (6.5.5)$$