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# Quantum Optics Applied to Quantum Qubits

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# Abstract

We consider the steps towards building a functional photonic quantum computer. A photonic quantum computer uses the state of a photon as qubit. We first see how to prepare specific states of light i.e Gaussian and Squeeze states. Then we design gates that evolve these states unitarily so we can develop quantum algorithms. We discuss the method of Quantum Teleportation. Then, finally we see some measurement based quantum computation, where we show partial proof of universality.

*Keywords – Photonic computer, Gaussian state, Squeeze state,*

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## (1) Introduction

Quantum Computing is a rising field nowadays. A Quantum Computer uses a qubit that is a quantum state of a particle as single unit of information rather than using a bit. Any problem that can be solved by a classical computer can also be solved by a quantum computer. Generally, quantum algorithms are unreasonably faster than a classical algorithm. But the problem arises to create high quality qubits. Due to decoherence qubits easily gets destroyed and quantum behaviour can no longer be used.

One of solution given is to use photons as they are much more resistant to decoherence. But the state of photon is infinite dimensional space. To run a quantum computer we need to prepare the states, unitarily evolve and measuring(entangled) quantum states, is achieved in quantum optics utilizing continuous quadrature amplitudes of the quantized electromagnetic field. For example, the tools for measuring a quadrature with near-unit efficiency or for displacing an optical mode in phase space are provided by homodyne detection and feed-forward techniques, respectively. Continuous variable entanglement can be done through squeezed light which is produced by some non-linear interactions. But this entanglement and hence any entanglement based protocol are imperfect. Whereas the any discrete variable quantum operation is always perfect but we are guaranteed to give perfect result but only works sometimes.

We first discuss about classical(Gaussian) and non-classical states(Squeeze, Cluster) of light are prepared. Then, we talk about how to unitarily evolve the states through gates and finally measure those states.

## (2) Preparation of non-Classical states of light

Non classical states of light are extensively used in optical qubits and their applications in quantum information processing. Hence, it is important to first have a firm grasp of these different non-classical states of light to understand how they are applied in photonic qubits.

### (2.1) Optical Phase Space diagrams

In quantum optics, an optical phase space serves as a representation of all quantum states within an optical system. Every point within this phase space signifies a distinct state of the optical system. When considering such systems, a graphical depiction of the quadratures plotted against each other, which may vary over time, is termed as a phase diagram. By illustrating the quadratures as functions of time, the optical phase diagram can elucidate the progression of a quantum optical system over time.

When discussing the quantum theory of light, it is very common to use an electromagnetic oscillator as a model. Quantum oscillators are described using creation and annihilation operators  $a$  and  $a^\dagger$ .

In the quantum oscillator mode, most operators representing physical quantities are typically expressed in terms of the creation and annihilation operators. In this example, the electric field strength is given by:

$$\hat{E}_i = u_i^*(\mathbf{x}, t)\hat{a}^\dagger + u_i(\mathbf{x}, t)\hat{a}$$

The eigenstates of the annihilation operator are called coherent states. A **coherent state** is the

specific quantum state of the quantum harmonic oscillator often described as a state that has dynamics most closely resembling the oscillatory behaviour of a classical harmonic oscillator. It is the state that minimises the uncertainty relation with uncertainty equally distributed among  $x$  and  $p$ .

## (2.2) Glauber–Sudarshan P representation

The Glauber–Sudarshan P representation is a method for describing the phase space distribution of a quantum system. In this representation, observables are expressed in normal order, making it a quasi-probability distribution. Particularly in quantum optics, it is preferred over other representations because it naturally expresses typical optical observables, like the particle number operator, in normal order.

The density matrix of any light can be written as:

$$\hat{\rho} = \int P(\alpha) |\alpha\rangle \langle \alpha| d^2\alpha$$

Where  $|\alpha\rangle$  is a coherent state, A classical state is where  $P(a)$  is probability distribution, if not, the state is said to be non-classical.

Aspects of  $P(\alpha)$  that would make it non-classical are:

- Having negative value at any point
- Being more singular than the Dirac-delta function

## (2.3) Squeezed states of light

In quantum physics, light is in a squeezed state when its electric field strength for certain phases has a lower quantum uncertainty compared to a coherent

state. This reduction in uncertainty is referred to as squeezing. However, to satisfy Heisenberg's uncertainty principle, a squeezed state must also exhibit phases where the electric field uncertainty is increased, known as anti-squeezing, compared to a coherent state.

In quantum physics, the electric field strengths of a wave at phase angles are determined by the normalized quadrature operator  $X_v$ . This operator is defined in terms of annihilation and creation operators, representing the photon oscillator.  $X$  represents the amplitude quadrature (akin to position in optical phase space), while  $Y$  represents the phase quadrature (akin to momentum). These observables do not commute and satisfy an uncertainty relation.

$$\sigma_X \sigma_Y \geq \frac{1}{16}$$

For a light mode in its ground state  $|0\rangle$  (with zero average photon number), the uncertainty relation reaches its minimum, with variances  $\sigma_X = \sigma_Y = 1/4$ . Light is in a squeezed state if there exists a phase angle  $V$  where the uncertainty in  $X$  is less than the ground state uncertainty.

Squeezed light is utilized in optical high-precision measurements, particularly in laser interferometers, to diminish photon counting noise, also known as shot noise. Laser interferometers split a laser beam into two paths, recombining them later. Any changes in the relative optical path length cause variations in interference, altering the light power at the output port, detected by a photodiode as a continuous voltage signal. Vibrations in one mirror, for example, induce oscillating path length differences, resulting in amplitude modulation of the output light. Despite the presence of this classical signal, light always carries at least vacuum state uncertainty. While increasing light power

enhances the modulation signal, it also leads to technical issues such as mirror absorption, thermal deformation, and interference contrast reduction. Squeezed states of light mitigate these problems by improving the signal-to-noise ratio without increasing the light's power or signal, but by reducing noise instead. This makes them valuable in enhancing the performance of interferometric measurements, including those aimed at detecting gravitational waves.

## (2.4) Parametric down conversion

It is a non-linear optical process, where an incoming pump photon decays under energy and momentum conservation, into a photon pair. The creation of entangled photons allows for the implementation of a single-photon source by detecting one photon(trigger) to herald the presence of its partner (signal).

Ling-An Wu, H.J. Kimble, J.L Hall and Huifa Wu first employed the process of parametric down conversion in a sub threshold optical parametric oscillator to reduce the noise level in homo-dyne detection by more than a factor of 2 relative to the vacuum noise level.

## (3) Quantum Gates

Quantum gates are fundamental operations in quantum computing that manipulate qubits (quantum bits) to perform specific quantum transformations. Similar to classical logic gates in traditional computing, quantum gates are the building blocks of quantum circuits and algorithms.

## (3.1) Working of quantum gates

Quantum gates operate on qubits, which are quantum systems that can exist in superpositions of classical states (0 and 1) and exhibit quantum entanglement. Unlike classical bits, which can only be in one of two states (0 or 1), qubits can exist in a linear combination of both states simultaneously, thanks to the principles of superposition.

Quantum gates typically act on one or more qubits and perform transformations that change the state of the qubits according to specific rules dictated by quantum mechanics. These gates leverage quantum phenomena such as superposition and entanglement to perform computations.

For instance, a quantum gate might rotate the state of a qubit around different axes in the Bloch sphere representation, effectively changing the probability amplitudes of the qubit's states. By chaining together multiple quantum gates in a quantum circuit, complex computations can be performed.

## (3.2) Difference between optical and classical QUBIT

### 1. States

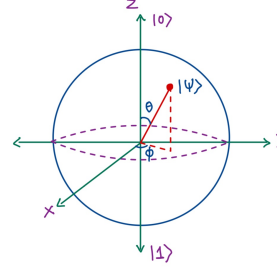
- Classical Bit: A classical bit can exist in one of two states: 0 or 1
- Qubit: A qubit can exist in a superposition of the classical states 0 and 1, meaning it can be in a combination of both states simultaneously. Mathematically, a qubit's state can be represented as  $\alpha|0\rangle + \beta|1\rangle$ , where  $|\alpha|^2$  and  $|\beta|^2$  are

complex probability amplitudes.

## 2. Information Encoding

- **Classical Bit:** Classical bits encode information in a straightforward binary manner, with each bit representing a distinct value of either 0 or 1
- **Qubit:** Qubits can encode and process quantum information in a more complex manner due to superposition. They can represent and manipulate multiple pieces of information simultaneously.

3. **Entanglement:** Qubits can be entangled, meaning the state of one qubit is dependent on the state of another qubit, even if they are physically separated. Entanglement enables non- local correlations and is a unique feature of quantum systems.



## (3.4) Single Qubit Gates

A single qubit gate is a unitary operation  $U$  that takes a single qubit  $\psi = \alpha|0\rangle + \beta|1\rangle$  as an input and transforms it into an output state  $\psi' = \alpha'|0\rangle + \beta'|1\rangle$  with:

$$|\psi'\rangle = U|\psi\rangle$$

In the circuit formalism, this transformation is depicted as

$$|\psi\rangle \text{ --- } \boxed{U} \text{ --- } |\psi'\rangle$$

## (3.3) Representation on the Bloch sphere

The state  $|\psi\rangle$  can be represented geometrically on a unit sphere in three dimensions, called the Bloch sphere. For this, the state  $|\psi\rangle$  can be rewritten in the following form.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

The description of quantum states as points on the Bloch sphere is useful for the visualization of single qubits and operations on single-qubits.

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

The gate changes the amplitude coefficients, which can be seen when the transformation is written in the form of matrix:

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

The unitary transformation follows from the fact that the norm must be preserved:

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|UU^\dagger|\psi\rangle = 1 \rightarrow U^\dagger U = I$$



**Important single-qubit gates in quantum computation are the Pauli operators**

$$\begin{aligned}\sigma_1 &= \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 &= \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 &= \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\end{aligned}$$

$$|0\rangle - \boxed{x} - |1\rangle$$

$$|1\rangle - \boxed{x} - |0\rangle$$

**Hadamard Gate :** It is used to construct superposition of state

$$|0\rangle - \boxed{H} - \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|1\rangle - \boxed{H} - \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

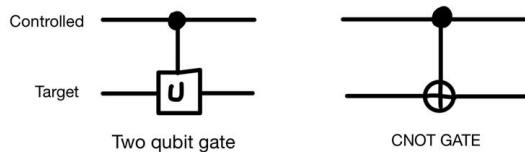
$$-\boxed{H}- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

### (3.5) Multi Qubit Gates

It takes multiple qubits as input and perform operations on them.

$$\begin{array}{c} |\psi\rangle_1 \\ |\psi\rangle_2 \end{array} \begin{array}{c} \text{---} \end{array} \boxed{U} \begin{array}{c} \text{---} \end{array} |\psi'\rangle_{1,2} .$$

$|\psi_1\rangle$  is used as control qubit(as input) and  $|\psi_2\rangle$  is called target qubit and is used for output.



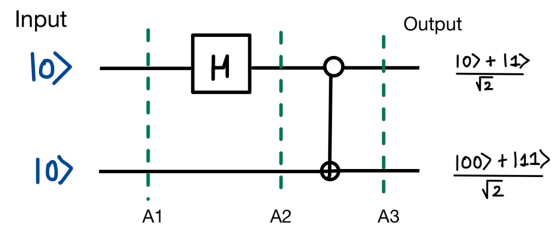
Important two-qubit gates for quantum computing are the controlled-NOT gate (CNOT)

$$CNOT |i\rangle|j\rangle = |i\rangle |i \oplus j\rangle$$

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

#### Preparation of entangled states

A quantum circuit consisting of a Hadamard gate and a CNOT gate can be used to generate entangled state (bell states):



Mathematically it can be written as

$$|\phi_+\rangle = U_{CNOT} (H \otimes I)(|0\rangle \otimes |0\rangle)$$

$$|\phi_+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad (\text{Entangled / EPR state})$$

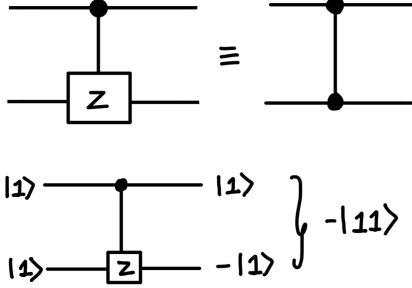
Points	Control qubits	Target qubits
at $A_1$	$ 0\rangle$	$ 0\rangle$
at $A_2$	$\frac{ 0\rangle +  1\rangle}{\sqrt{2}}$	$ 0\rangle$
at $A_3$	$\frac{ 0\rangle +  1\rangle}{\sqrt{2}}$	$\frac{ 00\rangle +  11\rangle}{\sqrt{2}}$

#### Controlled Z-gate

The Controlled-Z (CZ) gate is two-qubit gate used in quantum computing. It operates on a pair of qubits, with one qubit acting as the control and the other as the target. In layman's terms, the CZ gate

applies a phase flip (change in the relative phase)

to the target qubit only when the control qubit is in the state  $|1\rangle$ . If the control qubit is in the state  $|0\rangle$ , the CZ gate does not affect the target qubit.



$$U_{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

## (4) Manipulation and Teleportation of Quantum Systems

Quantum manipulation can provide the entangled level of desirable value, and particularly enhance a limited entangled degree limited by imperfect optical components. controlling the state of the qubit can be considered as manipulation of qubit. Usually, there are two types of approaches to manipulate of non-classical states.

- Theoretical and experimental investigations of phase-sensitive manipulation of degenerate optical parametric amplifier (DOPA) and non-degenerate optical parametric amplifier (NOPA).
- The feedback mechanism which is used to control noise from classical to quantum domain

## (4.1) Optical Parametric Amplifiers

**Optical Parametric Amplifiers** (OPAs) don't directly manipulate qubits themselves. Instead, they act as powerful light source engineers. They take a weak laser beam as input and amplify it into a much stronger beam, but crucially, in a non-classical state. These non-classical states exhibit properties that defy classical physics, making them ideal for encoding information onto qubits. Once the non-classical light is generated, OPAs allow for precise control over its properties like Intensity, Phase and mode.

## (4.2) Feedback Control

Qubit manipulation using feedback control involves controlling the state of a qubit to reach a desired state by monitoring its state and applying necessary operations. XThere are two methods to achieve this:

- **Measurement-Based Feedback Control** (MBFC), the monitoring is done directly by interacting with the qubit. This means that measurements are performed on the qubit to gather information about its state. Based on the measurement outcomes, appropriate operations can be applied to steer the qubit towards the desired state. However, this direct interaction may introduce noise and disturbances to the qubit.
- **Coherent Feedback Control** (CFC) takes an indirect approach to monitoring the qubit. Instead of directly interacting with the qubit, CFC uses ancillary systems or auxiliary qubits to indirectly gather information about

the qubit's state. By reducing the direct interaction with the qubit, CFC can help minimize the noise and disturbances that could affect the qubit's state.

### (4.3) Quantum Teleportation Protocol

Quantum teleportation is a quantum communication protocol that enables the transfer of quantum information from one location to another, typically between distant quantum systems, without physically transporting the quantum state itself. This process involves the entanglement of quantum particles, measurement of their properties, and classical communication of the measurement outcomes to reconstruct the quantum state at the destination, preserving its quantum properties.

The protocol is as follows:

1. A Bell state is generated with one qubit sent to location A and other to location B.
2. A Bell measurement of the Bell state qubit and the qubit to be teleported ( $|\phi\rangle$ ) is performed at location A. This yields one of four measurement outcomes which can be encoded in two classical bits of information. Both qubits at location A are then discarded.
3. Using the classical channel, the two bits are sent from A to B.
4. As a result of the measurement performed at location A, the Bell state qubit at location B is in one of four possible states. Of these four possible states, one is identical to the original quantum state  $|\phi\rangle$ , and the other three are closely related. The identity of the state actually obtained is encoded in two classical bits and sent to location B. The Bell state

qubit at location B is then modified in one of three ways, or not at all, which results in a qubit identical to  $|\phi\rangle$ , the state of the qubit that was chosen for teleportation.

The above protocol assumes that the qubits are distinguishable and physically labeled.

### (4.4) Quantum Teleportation Experiment

The quantum state being teleported in this experiment is  $|\chi\rangle_1 = \alpha|H\rangle_1 + \beta|V\rangle_1$ , where  $\alpha$  and  $\beta$  are unknown complex numbers,  $|H\rangle$  represents the horizontal polarization state, and  $|V\rangle$  represents the vertical polarization state. The qubit prepared in this state is generated in a laboratory in Ngari, Tibet. The goal was to teleport the quantum information of the qubit to the Micius satellite that was launched on August 16, 2016, at an altitude of around 500 km. When a Bell state measurement is conducted on photons 1 and 2 and the resulting state is  $|\phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2)$ , photon 3 carries this desired state. If the Bell state detected is  $|\phi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 - |V\rangle_1|V\rangle_2)$ , then a phase shift of  $\pi$  is applied to the state to get the desired quantum state. The distance between the ground station and the satellite changes from as little as 500 km to as large as 1,400 km. Because of the changing distance, the channel loss of the uplink varies between 41 dB and 52 dB. The average fidelity obtained from this experiment was 0.80 with a standard deviation of 0.01. Therefore, this experiment successfully established a ground-to-satellite uplink over a distance of 500–1,400 km using quantum teleportation. This was an essential step towards creating a global-scale quantum internet.

## (5) Measurement based quantum computing

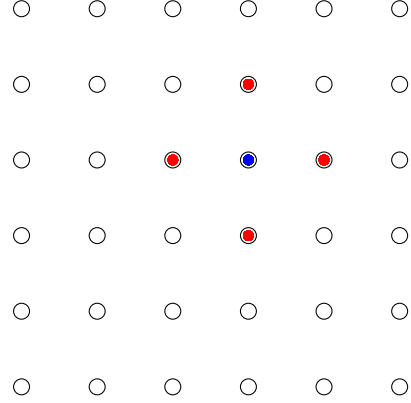
This is a particular scheme of Quantum computing where we prepare an entangled state, typically a cluster or graph state and measure it to evolve the states instead of unitary operations. We have  $k$  input qubits,  $m$  "ancillary qubits" in a  $n$  qubit computer. It is shown to be an universal quantum computer which means it can reproduce any unitary operations. Notice this way of evolving the states is irreversible whereas unitary transformation was reversible.

Pauli operators:

$$K_a = \sigma_x^a \prod_{b \in n(a)} \sigma_z^b \quad \forall a \in \mathcal{C}$$

here  $n(a)$  represents neighbourhood of  $a$

$$K_a|\mathcal{C}\rangle = |\mathcal{C}\rangle$$



### (5.1) General Method

We first entangle the qubits then measure the ancillary qubits. Which collapses the input qubits due to entanglement. To obtain the results in a deterministic way we need to introduce some correction operators called byproducts.

### (5.2) Cluster States

To prepare the cluster states:

- Consider a product state  $\bigotimes_{a \in \mathcal{C}} \frac{|0\rangle_a + |1\rangle_a}{\sqrt{2}}$  in a  $d$ -dimensional qubit lattice  $\mathcal{C}$
- Apply the Ising-interaction for fixed time  $T$ :

$$U_{Ising} = \exp -i \frac{gT}{\hbar} \sum_{\langle i,j \rangle} \sigma_z^{(i)} \sigma_z^{(j)}$$

with  $gt/\hbar = \pi/4$

Cluster states are entangled states in lattice. For example a bipartite entanglement like Bell states or multipartite like GHZ states.

Formally a cluster state  $|\mathcal{C}\rangle$  is defined as a stabilizer state. It is the simultaneous  $+1$  eigenstate of the

### (5.3) Bell-State

Bell state are two particle entangled state in **Fock space**. These are maximally entangled state i.e measuring the state of one particle completely collapses the state of the other particle. The Bell states are:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

These Bell state measurements are used in Quantum teleportation as discussed earlier. We can easily verify that these are cluster states.

### (5.4) Greenberger-Horner-Zeilinger state

This is a three particle entanglement state

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

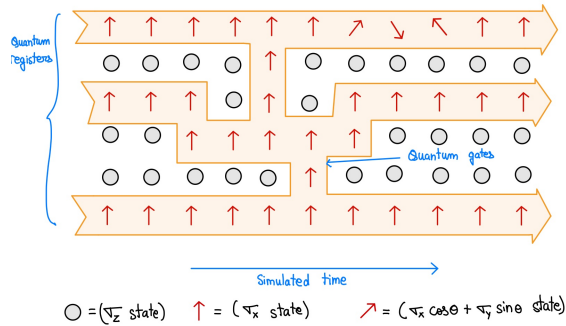
Generalization to  $d$ -dimensional  $M$  particle system is pretty intuitive:

$$|GHZ\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} \bigotimes_{j=1}^M |i\rangle$$

These are also Cluster states.

### (5.5) How does it work?

Here is a diagram to help us visualize how does it work.



As we can see from the diagram time is simulated by multiple qubits in the cluster states. We can see information is mediated through the transmission channels marked (identity gate). Quantum gates are there to mediate through registers, it is basically a CNOT gate as we will prove. Also we can rotate the states in x-y plane. Z-measurement cuts out the network. These are some statements to prove the universality of the Quantum Computer. The universality doesn't hold for 1-D cluster states.

### (5.6) Universality

Consider the gates  $\{\exp\{-i\frac{\theta}{2}\sigma_z^{(i)}\} = R_z(\theta)\}, \exp\{-i\frac{\theta}{2}\sigma_x^{(i)}\} = R_x(\theta)\}, \exp\{-i\frac{\theta}{2}\sigma_z^{(i)} \otimes \sigma_x^{(i+1)}\}$  with  $i$  and  $i+1$  adjacent qubits. It can be claimed that they form a universal set of gates. We know that  $SU(2)$  is generated by the  $R_x(\theta)$  and  $R_z(\theta)$  and hence any sequence of single qubit operation can be done through these gates. If we include the  $R_{zx}(\theta)$  gate it suffices to implement a CNOT gate between the nearest neighbours. Now we can swap data in a sequence of three CNOT gates. Now we can permute the logical qubits to implement CNOT gate among arbitrary qubits.

Now consider a cluster state  $|\mathcal{C}\rangle$  with an input state  $|I\rangle$  and with  $(X, Y)$  plane measurements  $\{|+\rangle, |-\rangle\}$  along the  $X$ -axis and unitary operation  $\hat{U}$  on the input state can be operated.

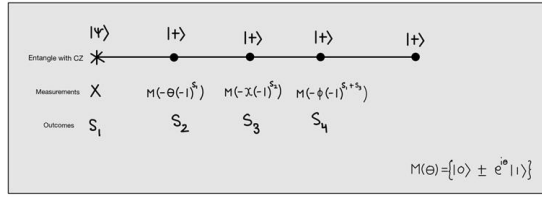
$$\hat{U} = \prod_{j=1}^{m-1} \left( \bigotimes_{i=1}^n \hat{H}_i \right)_j \left( \bigotimes_{i=1}^{n-1} \text{Ctrl} - Z_{i,i+1} \right)_j$$

We can see that by measuring in the  $\{|+\rangle, |-\rangle\}$  basis, the output is equal to  $\hat{U}$  from the one-bit teleportation scheme. With these two lemmas the universality of the MBQC can be proved.

So to apply a unitary operation  $U$  using MBQC method first we write  $U$  as:

$$U = R_x(\phi)R_y(\chi)R_x(\theta)$$

To apply  $U$  to  $|\psi\rangle$  we start with  $|\psi\rangle$  with 4  $|+\rangle$  after that in a line. Entangle the neighbouring states with CZ. Now measure the first 4 states in bases given in the below diagram and let their outcome be  $s_1, s_2, s_3, s_4$ .



Qubit 5 can be shown to be collapsed to  $X^{s_2+s_4}Y^{s_1+s_3}U|\psi\rangle$ . The extra factors are handled by by-product operators.

## (5.7) Application

This computer can be created using a cluster of photons. The problem is MBQC is very qubit expensive. Whatever amount of time steps we want to simulate we need that times number of photons required in a normal quantum computer using unitary transformation.

A method is proposed to reuse the used qubits. A conveyor belt will rotate the qubits while destroying the collapsed states of used qubits and reuse them.

Using Grover's algorithm MBQC is already demonstrated in a  $2 \times 2$  cluster of photon

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