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UCS2403: DESIGN & ANALYSIS OF ALGORITHMS

Assignment 7

1. Given two sequences $X = \langle x_1, \ldots, x_m \rangle$ and $Y = \langle y_1, \ldots, y_n \rangle$, the longest common sub-sequence problem (LCS) seeks to find a maximum length common sub-sequence of X and Y.
   For example, if $X = \langle A, B, C, B, D, A, B \rangle$ and $Y = \langle B, D, C, A, B, A \rangle$, then the sequences $\langle B, C, B, A \rangle$ and $\langle B, D, A, B \rangle$ are the longest common sub-sequences, since X and Y have no common sub-sequence of length 5 or greater.

   (a) Obtain a recursive formula for the LCS problem.

   (b) Design a dynamic programming algorithm to solve the LCS problem using the recursive formula in Q 1(a). Write the Python code to implement the same.

   (c) Populate the table using bottom-up approach, starting from the base case(s), for the below example: $X = \langle A, B, A, A, B, A \rangle$ and $Y = \langle B, A, B, B, A, B \rangle$. Find the answer using the data populated in the table.

   (d) Compare the output/answers obtained in Q 1(b) and Q 1(c) for the given example.

2. In Domino Solitaire, you have a grid with two rows and many columns. Each square in the grid contains an integer. You are given a supply of rectangular 2×1 tiles, each of which exactly covers two adjacent squares of the grid. You have to place tiles to cover all the squares in the grid such that each tile covers two squares and no pair of tiles overlap. The score for a tile is the difference between the bigger and the smaller number that are covered by the tile. The aim of the game is to maximize the sum of the scores of all the tiles. Here is an example of a grid, along with two different tilings and their scores.

Tiling 1   *Tiling 2*

8	6	2	3
9	7	1	2

8	6	2	3
9	7	1	2

Score 12

8	6	2	3
9	7	1	2

Score 6

The score for Tiling 1 is $12 = (9 - 8) + (6 - 2) + (7 - 1) + (3 - 2)$ while the score for Tiling 2 is $6 = (8 - 6) + (9 - 7) + (3 - 2) + (2 - 1)$. There are other tilings possible for this grid, but you can verify that Tiling 1 has the maximum score among all tilings. Your task is to read the grid of numbers and compute the maximum score that can be achieved by any tiling of the grid.