Appendix 1: Action space

Table S1: Management sub-actions and their effects on the population model

Sub-action	**Effect on system model**	**Management parameters**
Crop choice: a_k, levels 1 – 3		
wheat	higher survival rate for black grass and herbicide less effective due to spraying occurring earlier	Highest profit in the absence of black grass
alt	Black grass survival reduced under alternative crop due to competition or broad spectrum herbicide use before sowing	lower income than wheat
fallow	no crop planted. All above ground plants killed as we assume used alongside plowing or broad spectrum herbicide	small negative income, no production, costs for killing above ground plants
Herbicide: a_h, levels 1 – 4		
no herb	no effect	no cost
herb 1	reduced survival for emerged individuals with low g_1, high g_2 level may provide some protection if there is cross resistance.	small application cost.
herb 2	reduced survival for emerged individuals with low g_2, high g_1 level may provide some protection if there is cross resistance.	small application cost.
both	reduced survival for emerged individuals with low g_2 or g_1.	larger application cost.
Seed bank management: a_b, levels 1 – 2		
1	Moves seeds from one level of the seed bank to the other	fixed cost
0	no effect	no cost

spot control: a_s, levels 1 – 2

1	No effect of above ground population	No cost
0	Kills all remaining above ground plants	Cost scales with above ground post control populations N'

These sub-actions are combined to create a single action a_q that could be taken in a time step. However some sub-action combinations do not make sense, for example applying herbicide to the population when a_k = 'fallow', since we assume all above ground plants are destroyed under this crop choice. The list of all allowed sub-action combinations is the action space **A**

Table S2: Action space (**A**) with all eligable combinations of sub actions

$\mathbf{a_j}$	$\mathbf{a_k}$	$\mathbf{a_h}$	$\mathbf{a_b}$	$\mathbf{a_s}$
a_1	wheat	no herb	0	1
a_2	wheat	no herb	0	0
a_3	wheat	no herb	1	1
a_4	wheat	no herb	1	0
a_5	wheat	herb 1	0	1
a_6	wheat	herb 1	0	0
a_7	wheat	herb 1	1	1
a_8	wheat	herb 1	1	0
a_9	wheat	herb 2	0	1
a_{10}	wheat	herb 2	0	0
a_{11}	wheat	herb 2	1	1
a_{12}	wheat	herb 2	1	0
a_{13}	wheat	both	0	1
a_{14}	wheat	both	0	0
a_{15}	wheat	both	1	1
a_{16}	wheat	both	1	0
a_{17}	alt	no herb	0	1
a_{18}	alt	no herb	0	0
a_{19}	alt	no herb	1	1
a_{20}	alt	no herb	1	0
a_{21}	alt	herb 1	0	1
a_{22}	alt	herb 1	0	0

a_{23}	alt	herb 1	1	1
a_{24}	alt	herb 1	1	0
a_{25}	alt	herb 2	0	1
a_{26}	alt	herb 2	0	0
a_{27}	alt	herb 2	1	1
a_{28}	alt	herb 2	1	0
a_{29}	alt	both	0	1
a_{30}	alt	both	0	0
a_{31}	alt	both	1	1
a_{32}	alt	both	1	0
a_{33}	fallow	no herb	0	1
a_{34}	fallow	no herb	1	1