Appendix 3:Reward Function

The reward function measures how good a given IPM strategy is given a initial starting condition and parameter set that the model is run under. The reward function encodes the goals of a manager. We assume farmers are primarily driven by economic returns. The economic return consists of two parts, the income made from the crop and the costs of producing that crop. We assume that usual farm costs, such as buildings and machinery as constant from year to year, so we focus on gross margin, i.e. income - variable costs (Redman, 2016, pp. 3).

Income from the crop in year t is

$$Y(N'', a_k^t, a_k^{t-1}) = \begin{cases} W(a_k^t, a_k^{t-1})(Y_0 - \beta_D N''\rho & \text{if } a_k^t = \text{wheat} \\ W(a_k^t, a_k^{t-1})\vartheta & \text{if } a_k^t = \text{alt} \\ 0 & \text{if } a_k^t = \text{fallow} \end{cases} \tag{S12}$$

where Y_0 is the yield of winter wheat (in t/ha) when the density of black grass after management (N'') is 0, β_D is the rate at which yield decreases with increasing black grass density. To estimate this yield function We use yield data from combine harvesters for 10 fields in England for which we also have estimates of *A. myosuroides* density (see Hicks *et al.* 2018 for survey methodology). Yield estimates are $Y_0 = 11.43$ (CI: 10.8–12.04) t/ha and $\beta_D = 0.0000223$ (CI: 0.00000145–0.0000425) t/plant/ha, see section 0.1 for parameter estimation. ρ is the price of winter wheat, taken from Redman (2016, pp. 9) as 146/t. Yield of a crop can be affected by planting the same crop two years in a row, largely due to species specific parasites and pathogens in the soil [REF Kwadjo]. We use the weighting function

$$W(a_k^t, a_k^{t-1}) = \begin{cases} \varpi & \text{if } a_k^t = a_k^{t-1} \\ 1 & \text{otherwise} \end{cases} \tag{S13}$$

to reduce yield if the same crop is used two times in succession, where $\varpi \in [0, 1]$ is the proportional yield achieved when the same crop is used, following [REF Kwadjo] we use $\varpi = 0.9$. We assume the yield of the alternative crop, φ, is not affected by black grass. The alternative crop is based on spring barley, a common spring crop used in the UK for the control of black grass. The average income from spring barley is £796/ha (Redman, 2016, pp. 12).

Costs depend on both the action chosen and non-weed control costs, such as fertilizer, seed and other sprays such as fungicides. We assume these other variable costs change with crop choice (a_k), but are constant from year to year within a crop choice. Thus the cost of action is a_q

$$C(a_q^t) = \sum_{\forall a_j \in a_q^t} c(a_j) \tag{S14}$$

where $c(a_j)$ is the cost of sub-action a_j.

The cost for herbicide action a_h is

$$c(a_h) = \begin{cases} 0 & \text{if } a_h = 0 \\ \eta_h & \text{if } a_h = 1 \bigvee a_h = 2 \\ 2\eta_h & \text{if } a_h = \text{both} \end{cases} \tag{S15}$$

Where η_h is the cost of a single herbicide application to control black grass. We assume that most herbicide costs in winter wheat are associated with black grass control, and so take $\eta_h = \pounds 96$ (Redman, 2016, pp. 9).

The cost function for crop choice is

$$c(a_k) = \eta_{a_k} \tag{S16}$$

where the parameter η_{a_k} are the constant costs associated with each crop choice. The cost for winter wheat excludes herbicide costs associated with black grass control, and is taken as η_{wheat} =£383/ha (Redman, 2016, pp. 9). We assume both a fallow rotation and the alternative crop are used, at least in part, to control black grass, and so we include all costs, including those associated with black grass control in η_{alt} =£273/ha (Redman, 2016, pp. 12) and η_{fallow} = £36/ha (Redman, 2016, pp. 202 and 284). The cost of the fallow rotation is based on two applications of glyphosate (a broad spectrum herbicide) to kill back grass after it has germinated.

The cost function for plowing is

$$c(a_b) = \eta_b a_b \tag{S17}$$

where θ_b = £73.96/ha is the contractor rates for deep plowing (which are assumed to combine all labour and capital costs) (Redman, 2016, pp. 202), and $a_b \in \{0, 1\}$.

Finally the cost function for spot control is assumed to increase proportionally with black grass density after other control actions have been taken so that

$$c(a_s, t) = a_s \left(\eta_s^0 + \eta_s \sum_{\forall G} n'(G, t) \right) \tag{S18}$$

where δ_0 combines the costs of spot control incurred even when there is no *A. myosuroides*) and δ controls how quickly the costs of spot control increase with black grass density. This functional form assumes the costs of spot control increase linearly with *A. myosuroides* density. $a_s \in \{0, 1\}$ is a switch, so that the cost is incurred only if the action is taken.

To explicitly link the above ground population to the reward function we define $N''(\mathbf{a}, n_0, t)$, the total above ground population after all control actions, at time t given an initial population n_0 and a sequence of actions

$$\mathbf{a} = \{a_j^1, a_j^2, \cdots, a_j^T\} \tag{S19}$$

where a_j^t is the action $a_j \in \mathbf{A}$ taken at time t and T is the time horizon over which management is run. We assume all returns after T are ignored. The reward function is

$$R(\mathbf{a}, n_0) = \sum_{t=0}^{T} \gamma^t \Big(Y(N''(\mathbf{a}, n_0, t)) - C(a_j^t) \Big) \tag{S20}$$

where $R(\mathbf{a}, n_0)$ is the time discounted reward for action sequence $\mathbf{a}$ given starting population n_0, $\gamma \in [0, 1]$ is the discount rate. When $\gamma = 0$ only the reward in the first time step is considered, when $\gamma = 1$ returns in all future time steps up to T are valued equally.

0.1 Parametrization of yield function

To fit the yield function we use data from 10 fields where harvesters recorded wheat yield for every 20m by 20m gird square of the field. We also have black grass density estimates for each grid square from state structured surveys (Hicks *et al.*, 2018). These density states were calibrated to plants/m^2 by Queenborough *et al.* (2011): the density states were absent (0 [0–0.1667] plants/m^2)(median[inter-quartile range]), low (0.5000 [0.1667–1.5000] plants/m^2), medium (2.6666 [1.3333–4.7500] plants/m^2), high (5.0833 [3.0000–7.7916] plants/m^2), and very high (9.6666 [7.1250–13.1666] plants/m^2). We fit a linear yield function to the median plants/m^2 of each density state, D.

$$Y_i \sim N(\widehat{Y}_i, \sigma_y) \tag{S21a}$$

$$\widehat{Y}_i = Y_0 + Y_0^j + (\beta_D + \beta_D^j)D \tag{S21b}$$

We assume the yield for grid square i (Y_i, in t/ha) is drawn from a normal distribution with standard deviation σ_y. Predicted yield ($\widehat{Y}_i$) is a linear function of black grass density, where Y_0 is the average winter wheat yield across all fields and $Y_0^j \sim N(0, \sigma_y^0)$ is the random effect of field j on the intercept of field, drawn from a normal distribution with mean of 0 and standard deviation of σ_y^0. β_D is the change in yield when black grass density (D) changes by 1 plant/m^2, averaged across all fields, and $\beta_D^j \sim N(0, \sigma_y^D)$ is the effect of field on the relationship between black grass density and yield. This model was fit with the 'lme4' package (Bates *et al.*, 2015) in the R statistical language (R Core Team, 2013). We assume our population exists in a 1ha field. Converting from plants/m^2 to plants/ha the yield loss function becomes

$$\widehat{Y} = 11.43 - 0.0000223N'' \tag{S22}$$

We use 95% likelihood profile prediction intervals to estimate uncertainty around these estimates. All costs are £, to put the yield and costs on the same scale we assume a winter wheat price of £146/t (Redman, 2016),

$$\widehat{Y} = 1668 - 0.00326N'' \tag{S23}$$

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