

Appendix 4: Action space

The sub-actions link management decisions to the population of *Alopecurus myosuroides*, and thus the yield of wheat and so the reward function. Each sub-action also has a cost which affects the reward function.

Table S2: Management sub-actions and their effects on the population model

Sub-action	Effect on system model	Management parameters
<i>Crop choice: a_k</i>		
wheat	higher survival rate for <i>Alopecurus myosuroides</i> without selective herbicides.	Highest profit in the absence of <i>Alopecurus myosuroides</i> .
alt	<i>Alopecurus myosuroides</i> survival reduced under alternative crop due to competition or broad spectrum herbicide use before sowing. We parameterize the alternative crop as spring barley, a common rotation used in the control of <i>Alopecurus myosuroides</i> .	Lower income than winter wheat.
fallow	no crop planted. All above ground plants killed as we assume used alongside plowing or broad spectrum herbicide.	No production, costs for killing above ground plants.
<i>Herbicide: a_h</i>		
no herb	No effect	No cost
herb R	Individuals with resistance alleles <i>RR</i> or <i>Rr</i> are protected, but all other individuals have greatly reduced survival.	Application cost.
herb Q	Individuals with resistance alleles <i>QQ</i> or <i>Qq</i> are protected, but all other individuals have greatly reduced survival.	Application cost.
both	Individuals with both resistance alleles (<i>RRQQ</i> , <i>RRQq</i> , <i>RrQQ</i> , <i>RrQq</i>) are protected, but all other individuals have greatly reduced survival.	Double application cost.
<i>Seed bank management: a_b</i>		
1	Moves seeds from one level of the seed bank to the other.	Fixed cost.
0	No effect	No cost.
<i>spot control: a_s</i>		
1	No effect of above ground population.	No cost.
0	Kills all remaining above ground plants.	Cost scales with above ground post control populations.

These sub-actions are combined to create a single action a_j^t that could be taken in a time step (Table S3). However, some sub-action combinations do not make sense, for example applying herbicide to the population when $a_k = \text{'fallow'}$, since we assume all above ground plants are destroyed under this crop choice. The list of all allowed sub-action combinations

is the action space \mathbf{A} .

Table S3: Action space (\mathbf{A}) with all eligible combinations of sub actions

$\mathbf{a_j}$	$\mathbf{a_k}$	$\mathbf{a_h}$	$\mathbf{a_b}$	$\mathbf{a_s}$
a_1	wheat	no herb	0	1
a_2	wheat	no herb	0	0
a_3	wheat	no herb	1	1
a_4	wheat	no herb	1	0
a_5	wheat	herb R	0	1
a_6	wheat	herb R	0	0
a_7	wheat	herb R	1	1
a_8	wheat	herb R	1	0
a_9	wheat	herb Q	0	1
a_{10}	wheat	herb Q	0	0
a_{11}	wheat	herb Q	1	1
a_{12}	wheat	herb Q	1	0
a_{13}	wheat	both	0	1
a_{14}	wheat	both	0	0
a_{15}	wheat	both	1	1
a_{16}	wheat	both	1	0
a_{17}	alt	no herb	0	1
a_{18}	alt	no herb	0	0
a_{19}	alt	no herb	1	1
a_{20}	alt	no herb	1	0
a_{21}	alt	herb R	0	1
a_{22}	alt	herb R	0	0
a_{23}	alt	herb R	1	1
a_{24}	alt	herb R	1	0
a_{25}	alt	herb Q	0	1
a_{26}	alt	herb Q	0	0
a_{27}	alt	herb Q	1	1
a_{28}	alt	herb Q	1	0
a_{29}	alt	both	0	1
a_{30}	alt	both	0	0
a_{31}	alt	both	1	1
a_{32}	alt	both	1	0
a_{33}	fallow	no herb	0	1
a_{34}	fallow	no herb	1	1