Worksheet WTW 164

1. (a) $-\sin(\tan x)^4 \cdot \sec^2 x$

   (b) $\dfrac{5}{25x^2+10x-4}$

   (c) $\dfrac{-t}{t+1}\cdot\dfrac{1}{2\sqrt{t}} + \dfrac{t^2}{t^2+1}$

2. (a) -12

   (b) $\frac{1}{2}$

3. (i) $\frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + c$

   (ii) $\sin(\ln x) + c$

   (iii) $10/3$

   (iv) $\frac{1}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}x^2\tan^{-1}x + c$

   (v) $\dfrac{-2(2-x)}{2}\cdot\dfrac{\sqrt{4x-x^2}}{2} - 2\sin^{-1}\dfrac{2-x}{2} + c$

   (vi) $2\ln(x+2) - \ln(x-1) - \dfrac{3}{x-1} + c$

   (vii) $\frac{1}{16}(1+2x) - \frac{3}{16}\ln(1+2x) - \frac{3}{16}\,\dfrac{1}{1+2x} + \frac{1}{32}\,\dfrac{1}{(1+2x)^2} + c$

   (viii) $\frac{2}{3}\sqrt{x+2}\,(2(x+2)-3) + c$

   (ix) $2\sqrt{x} - 2\ln(\sqrt{x}+1) + c$

   (x) $2(1-\cos\sqrt{3})$

   (xi) $10/3$

   (xii) $2 - \pi/2$

   (xiii) $\sin^{-1}\left(\frac{x}{3}\right) + c$

   (xiv) $-7\sin^{-1}\left(\frac{x}{3}\right) - \sqrt{9-x^2} + c$

   (xv) $5\ln(x+4) - 2\ln(x+2) + c$

<u>OR</u>

3(i) $(x-1)^{3/2}\left[\frac{2}{5}(x-1) + \frac{2}{3}\right] + c$

   (xvi) $\dfrac{-3}{\sqrt{x^2-9}} + c$

   (xvii) $-\frac{1}{2}e^{-x^2} + c$

   (xviii) $-\frac{1}{2}\ \dfrac{1}{x^2+2x+5} + c$

   (xix) $e^{-x}(-x-1) + c$

   (xx) $\frac{1}{3}(9+x^2)^{3/2} - 9\sqrt{9+x^2} + c$

   (xxi) $-4\ln|x| + \frac{2}{x} - \frac{1}{x^2} + 2\ln|1+2x^2| + \frac{4}{\sqrt{2}}\tan^{-1}\sqrt{2}\,x + c$

   (xxii) $\ln\left|\dfrac{\sqrt{(x+1)^2+4}}{2} + \dfrac{x+1}{2}\right| + c$

   (xxiii) $\frac{1}{2}\left(\dfrac{x}{1+x^2}\right) + \frac{1}{2}\tan^{-1}x + c$

   (xxiv) $x - \frac{3}{2}\ln|x| + \frac{3}{4}\ln|x^2+4| - 5/2\tan^{-1}\frac{x}{2} + c$

   (xxv) $\dfrac{1}{e^{2x}-4e^x+4} = \dfrac{1}{(e^x-2)^2}$

   Let $u = e^x - 2 \Rightarrow e^x = u+2$ and $du = e^x\,dx$

   Answer:

   $-\frac{1}{4}\ln(e^x-2) - \dfrac{1}{2(e^x-2)} + \dfrac{x}{4} + c$

4 (i) $\frac{1}{3}\left(\frac{\pi}{2} - \tan^{-1}\left(-\frac{1}{3}\right)\right)$

(ii) Divergent

(iii) 0

(iv) Divergent

(v) $\frac{1}{36}$

(vi) Divergent

(vii) Divergent → longdivision and comparison test

(viii) $\frac{1}{3}$

(ix) Use comparison test – convergent.

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6 (a) $\ln 2 - \frac{1}{2}$

(b) $\int_{-3}^{3} \left[5 - (y^2 - 4)\right] dy$

(c) $2\sqrt{3} + \frac{\pi}{3}$

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7 (a) 64π

(b) 8π

(c) $\frac{\pi}{2}$

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8. $\int_0^\infty \frac{1}{\sqrt{x+1}} dx$

$= \lim_{t \to \infty} \int_0^t \frac{1}{\sqrt{x+1}} dx$

$= \lim_{t \to \infty} \left[2\sqrt{x+1}\right]_0^t$

$= \lim_{t \to \infty} \left[2\sqrt{t+1} - 2\right] = \infty$

Area is infinite.

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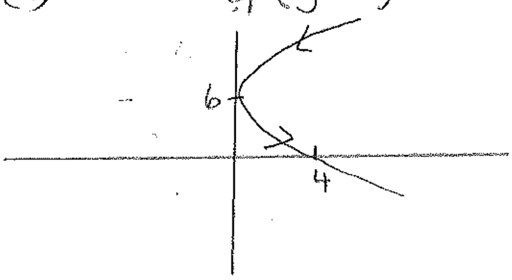
9. (a) $x = \frac{1}{9}(y-6)^2$



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5. (a) $1 + \sqrt{x} > 1$ for all $x \geq 1$

$\therefore \sqrt{1+\sqrt{x}} > 1$

$\Rightarrow \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} > \frac{1}{\sqrt{x}}$

Since $\int_1^\infty \frac{1}{\sqrt{x}} dx$ divergent (p-test, $p<1$) by comparison $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$ is divergent.

(b) $0 \leq |\sin 3x| \leq 1$

$\therefore 0 \leq \frac{|\sin 3x|}{x^2} \leq \frac{1}{x^2}$

$\int_1^\infty \frac{1}{x^2} dx$ convergent (p-test, $p>1$) by comparison $\int_1^\infty \frac{|\sin 3x|}{x^2} dx$ convergent.

(c) Use $-\frac{\pi}{2} \leq \arctan x \leq \frac{\pi}{2}$

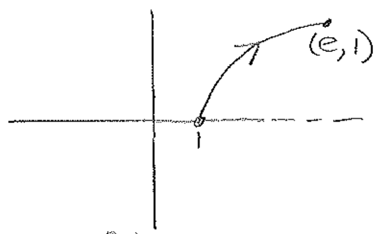
$\Rightarrow \frac{-\frac{\pi}{2}}{x^3} > \frac{\arctan x}{x^3} > \frac{\frac{\pi}{2}}{x^3}$

since $-\infty < x \leq -1$

Show that $\int_{-\infty}^{-1} \frac{-\frac{\pi}{2}}{x^3} dx$ converges, so by comparison $\int_{-\infty}^{-1} \frac{\arctan x}{x^3} dx$ converges.

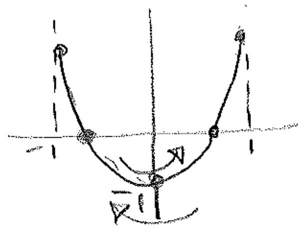
(b) $x = e^{y^2}$

$y \geq 0$



(c) $y = \cos 2t = 2\cos^2 t - 1$

$= 2x^2 - 1$



10. (a) $\frac{dx}{dt} = 2t+1, \quad \frac{dy}{dt} = 2t$

$\Rightarrow \frac{dy}{dx} = \frac{2t}{2t+1}$ and

$\frac{d^2y}{dx^2} = \frac{2}{(2t+1)^3}$

(b) $\frac{dy}{dx} = \frac{-2\sin 2t}{\sec^2 t}$

$\frac{d^2y}{dx^2} = \frac{-4\cos 2t + 2\sin 2t \cdot \tan t}{\sec^4 t}$

13. (a) $(x-\frac{3}{2})^2 + (y-\frac{1}{2})^2 = \frac{10}{4}$

circle $r = \frac{\sqrt{10}}{2}$ and centre $(\frac{3}{2}, \frac{1}{2})$

(b) Two circles radii 2 and 1 respectively.

(c) $x^2 + (y-\frac{1}{2})^2 = \frac{1}{4}$

Upperhalf of circle.

(d) $y = 2$

(e) $y^2 = 1 + 2x$

15 (a) $\frac{1}{\sqrt{3}}$

(b) $-\frac{3\sqrt{3}+4}{11}$

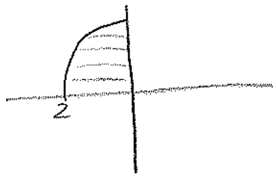
17. (a) $\frac{dy}{dx} = \frac{2\sin\theta\cos^2\theta - \sin^3\theta}{\cos^3\theta - 2\cos\theta\sin^2\theta}$

(b) -1

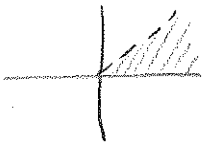
(c)* $(0,0)$ $(0{,}272; \pm 0{,}385)$

11. $y = -\frac{3}{4}x + 6$

12. (a)



(b)



14. (a) $r^2 = 3$

(b) $r = \frac{\sin\theta}{\cos^2\theta}$ $\quad \cos\theta \neq 0$

(c) $\theta = \tan^{-1}(5)$ $\quad$ (f) $\frac{1}{2\cos\theta - \sin\theta}$

(d) $r = 2\sin\theta$

(e) $r = 10\cos\theta - 6\sin\theta$

16. (a) $r \leq 2, \quad \frac{\pi}{4} \leq \theta \leq \frac{5\pi}{4}$

(b) $2 \leq r \leq \frac{6}{\cos\theta}$ and $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

18. $\frac{1}{3\sqrt{3}}$

19. (a) $D_F = \{t / t \in \mathbb{R}\}$

(b) $\langle 0, 1, e^{\pi/2} \rangle$

(c) $\langle -2, 0, 2e^{\pi/2} \rangle$

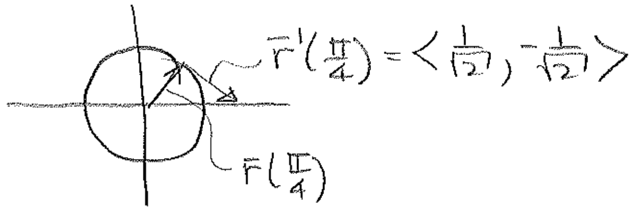
(d) f, g, h continuous $\Rightarrow$ $\bar{r}(t)$ continuous at $\frac{\pi}{2}$

(e) $\langle 2, 1, 2e^{\pi/2} - 2 \rangle$

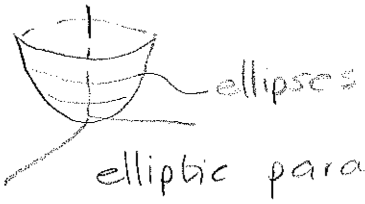
20. (a) $\left\langle \frac{12}{\sqrt{292}}, \frac{12}{\sqrt{292}}, \frac{2}{\sqrt{292}} \right\rangle$

(b) $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$

21. $\left.\begin{array}{l} x = \sin t \\ y = \cos t \end{array}\right\} \Rightarrow x^2 + y^2 = 1$

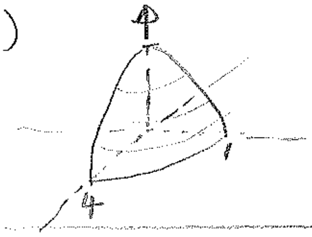


25. (a)



elliptic paraboloid

(b)



Top half of ellipsoid.

27. (a) $f_x(x,y) = \frac{2x}{x^2+y^2}$

$f_y(x,y) = \frac{2y}{x^2+y^2}$

(b) $f_u(u,v) = \frac{v}{u^2+v^2}$

$f_v(u,v) = \frac{-u}{u^2+v^2}$

28. $f_{xx} = 2y$

$f_{xy} = 2x + \frac{1}{2\sqrt{y}}$

$f_{yx} = $ "

$f_{yy} = -\frac{x}{4y^{3/2}}$

22. $x = 2 - 4t$

$y = t$

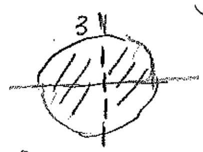
$z = 5 - 4t$

23. (a) 1

(b) $\mathbb{R}^2$

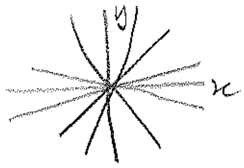
(c) $\{z / z > 0\}$

24. (a) $\{(x,y) / x^2 + y^2 \leq 9,\ y \neq 0\}$

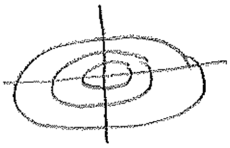


(b) $\{(x,y) / xy > 1\}$

26. (a)



(b)



(c)

29. $2x - y + 2 = z$

Answers

30(a) $(2,3,4)$

(b) $\bar{r} = \langle 1,1,1\rangle + t\langle 2,2,-2\rangle,\ t \in R$

31. $-x + 2y + 4z = 7$

32. $< x,y,z,w > = < -12,2,9,0 > + t < 0,0,1,1 >$

33. $(A-B)(A^2+AB+B^2) = A^3 + A^2B + AB^2 - BA^2 - BAB + B^3 \neq A^3 - B^3$ in general.

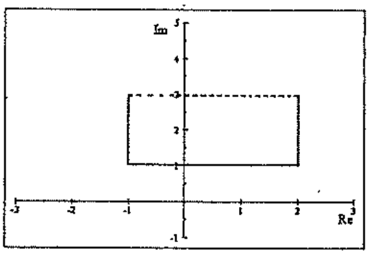
34. (i) $k = 2$ (ii) $k = -1$ or $k = 0$ (iii) $\mathbb{R}\backslash\{-1,0,2\}$

35. $X = \begin{bmatrix} -16 & -7 \\ 9 & 5 \end{bmatrix}$

36. $\bar{x} = \begin{bmatrix} \frac{3}{2} & 1 & \frac{1}{14} \end{bmatrix}^T$.

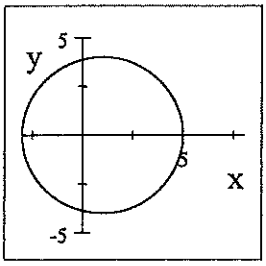
37. $z = -\frac{3}{13} + \frac{2}{13}i$

38. (i)



(i)

(ii)



(ii)

39. $2^{\frac{3}{8}}cis(\frac{3\pi}{16}), 2^{\frac{3}{8}}cis(\frac{11\pi}{16}), 2^{\frac{3}{8}}cis(\frac{19\pi}{16}), 2^{\frac{3}{8}}cis(\frac{27\pi}{16})$

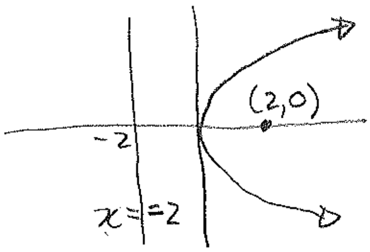
40. $e^2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

41. (a) $x^4 + 16 = (x-(\sqrt{2}+\sqrt{2}i))(x-(-\sqrt{2}+\sqrt{2}i))(x-(\sqrt{2}-\sqrt{2}i))(x-(-\sqrt{2}-\sqrt{2}i))$

(b) $x^3 - 64 = (x-4)(x-(-2-2\sqrt{3}i))(x-(-2+2\sqrt{3}i))$

(c) $x^4 - 2x^3 + 2x^2 - 2x + 1 = (x-i)(x+i)(x-1)(x-1)$.

42 (a)



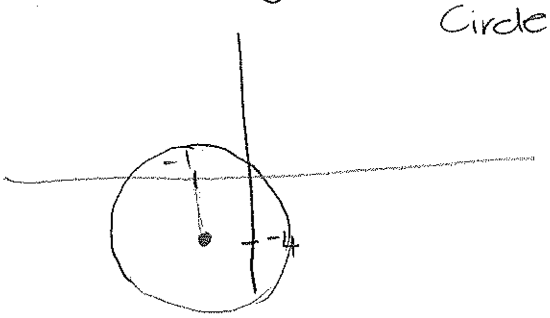
$y^2 = 8x$

$= 4(2)x$

$\Rightarrow p = 2$

(b) $(x+1)^2 + (y+4)^2 = 17$

Circle with centre $(-1,-4)$ and radius $\sqrt{17}$



43 a) $\sqrt{x+4} \approx M_3(x) = 2 + \frac{1}{4}x - \frac{1}{2^6}x^2 + \frac{1}{2^9}x^3$

b) $\sqrt{x+4} \approx T_3(x)$

$= f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2 + \frac{f'''(4)}{3!}(x-4)^3$

$= \sqrt{8} + \frac{1}{2\sqrt{8}}(x-4) - \left(\frac{1}{4}\right)(8)^{-3/2} \times \frac{1}{2}(x-4)^2$

$+ \left(\frac{3}{8}\right)(8)^{-5/2}\left(\frac{1}{6}\right)(x-4)^3$