

A probabilistic diagnostic tool to assess Laplace approximations: proof of concept and non-asymptotic experimentation

Shaun McDonald, Dave Campbell, Haoxuan Zhou

June 3, 2020

Abstract

In many statistical models, we need to integrate functions that may be high-dimensional. Such integrals may be impossible to compute exactly, or too expensive to compute numerically. Instead, we can use the *Laplace approximation* for the integral. This approximation is exact if the function is proportional to the density of a normal distribution; therefore, its effectiveness may depend intimately on the true shape of the function. To assess the quality of the approximation, we use *probabilistic numerics*: recasting the approximation problem in the framework of probability theory. In this probabilistic approach, uncertainty and variability don't come from a frequentist notion of randomness, but rather from the fact that the function may only be partially known. We use this framework to develop a diagnostic tool for the Laplace approximation, modelling the function and its integral as a Gaussian process and devising a “test” by conditioning on a finite number of function values. We will discuss approaches for designing and optimizing such a tool and demonstrate it on known sample functions, highlighting in particular the challenges one may face in high dimensions.