A probabilistic diagnostic tool to assess Laplace approximations: proof of concept and non-asymptotic experimentation

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Abstract

In many statistical models, we need to integrate functions that may be high-dimensional. Such integrals may be impossible to compute exactly, or too expensive to compute numerically. Instead, we can use the *Laplace approximation* for the integral. This approximation is exact if the function is proportional to the density of a normal distribution; therefore, its effectiveness may depend intimately on the true shape of the function. To assess the quality of the approximation, we use *probabilistic numerics*: recasting the approximation problem in the framework of probability theory. In this probabilistic approach, uncertainty and variability don't come from a frequentist notion of randomness, but rather from the fact that the function may only be partially known. We use this framework to develop a diagnostic tool for the Laplace approximation, modelling the function and its integral as a Gaussian process and devising a "test" by conditioning on a finite number of function values.We will discuss approaches for designing and optimizing such a tool and demonstrate it on known sample functions, highlighting in particular the challenges one may face in high dimensions.

1 Introduction

Coming soon. Some combination of abstract (above) and framework (below). Specifically mention:

1. That we are building on the work of Zhou [1]
2. That this is non-asymptotic and not intended as a substitute for full-on MC integration or BQ - rather as a "middle-ground" amount of effort.
3. The goal is to "test" the assumptions underlying the Laplace approximation (e.g. "how Gaussian is this function?"). The Laplace approximation may still hold for a non-Gaussian shape, but such a function should be

rejected by our diagnostic ("sufficiently non-Gaussian things warrant further attention), at which point a more involved integration would show that the approximation was fine after all.

2 Framework and notation

References

[1] Haoxuan Zhou. Bayesian Integration for Assessing the Quality of the Laplace Approximation. Master's thesis, Simon Fraser University, nov 2017. URL `http://summit.sfu.ca/item/17765`.