ECE 600 Homework 3
COMER

1. A call occurs at time t, where t is a randomly selected point in the interval $(0, 10)$ (all points in the interval being equally likely).

   (a) Find $P(6 \le t \le 8)$.

   (b) Find $P(6 \le t \le 8 | t > 5)$.

2. Using the fact that there are $\binom{n}{k}$ ways for k successes to occur in n Bernoulli trials, show that

$$P(\text{k successes in n trials}) = \binom{n}{k} p^k (1-p)^{n-k},$$

   where p is the probability of success in a trial.

3. A box contains n identical balls labeled 1 through n. Suppose k balls are drawn (without replacement). For $m \in \{1, \dots, n\}$,

   (a) what is the probability that m is the largest number drawn?

   (b) what is the probability that the largest number drawn is less than or equal to m?

4. Find the conditional probability that the ith trial in n Bernoulli trials is a success given that there are a total of k successes.

5. The definition of a random variable defined on a probability space $(\mathcal{S}, \mathcal{F}, P)$ depends on $\mathcal{F}$. This means that, in general, a function $X : \mathcal{S} \to \mathbb{R}$ might be a valid random variable under some choices of $\mathcal{F}$, but not under others. Show that in the case where X is a constant function, it is a random variable under any event space $\mathcal{F}$.

6. Let $\mathcal{S} = \{1, 2, 3, 4\}$ and $\mathcal{F} = \{\emptyset, \mathcal{S}, \{1\}, \{2, 3, 4\}\}$. Is the function $X(\omega) = 1 + \omega$ a random variable with respect to $\mathcal{F}$? If not, give an example of a non-constant function that is.

7. Let X be a random variable with cumulative distribution function (cdf) $F_X(x)$. Show that if $b > a$, then $F_X(b) \ge F_X(a)$.

8. Which of the following are valid cumulative distribution functions?

   (a)

$$F_X(x) = \begin{cases} 1 & x > 1 \\ x^2 & 0 \le x \le 1 \\ 0 & x < 0 \end{cases}$$

   (b)

$$F_X(x) = \begin{cases} 0 & x \le c \\ 1 & x > c \end{cases}$$

   (c)

$$F_X(x) = \begin{cases} 1 & x \ge 3 \\ x - 1 & 2 \le x < 3 \\ 0 & x < 2 \end{cases}$$

(d)

$$F_X(x) = 1 - e^{-x}$$

(e)

$$F_X(x) = (1 - e^{-x})u(x)$$

where c is a constant and $u(x)$ is the unit step function.

9. When ten fair coins are flipped, the event of interest is the number of heads. Let this number be a random variable X. Assuming the ten flips are independent of each other, find the cumulative distribution function of X.