ECE 600 Homework 4
COMER

1. Consider a random variable X defined on $(\mathcal{S}, \mathcal{F}, P)$ as

$$X(\omega) = \begin{cases} 1 & \omega \in A \\ -2 & \omega \in B \\ 2 & \text{otherwise,} \end{cases}$$

   where A and B are disjoint events in $\mathcal{F}$. Find the cumulative distribution function of X if $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$.

2. For two real numbers a and b, where $a < b$, write the following probabilities in terms of the distribution function of X:

   (a) $P_X((a, b])$,

   (b) $P_X((a, b))$,

   (c) $P_X([a, b])$.

3. Find the cdf F_X if $\mathcal{S}$ is the unit square $[0, 1] \times [0, 1]$ with uniform measure (which means that the probability of a region in $\mathcal{S}$ is the area of the region), and $X(\omega)$ is the distance from the outcome ω to the nearest edge of the square.

4. The median of the random variable X is defined as the real number x_m such that P$(X \leq x_m) = 0.5$. Find the median of the random variable that has the probability density function

$$f_X(x) = \begin{cases} be^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

5. Let X be an exponential random variable with parameter λ. Segment the positive real line into three disjoint intervals such that the random variable X is equally likely to be in any of the three intervals.

6. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f_X(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

   What is the probability that of 6 such devices at least 3 will function for at least 15 hours? You may assume that the functioning of each device is independent of the functioning of the other devices.

7. A discrete random variable X is said to be a geometric random variable if

$$P(X = k) = pq^{k-1} \quad k = 1, 2, 3, ...$$

   where $p, q > 0$ and $p + q = 1$.

   (a) Show that for any natural numbers m and n,

$$P(X > m + n | X > m) = P(X > n)$$

   This is known as the memoryless property of a geometric random variable.

(b) Show that the converse of part a is also true, i.e., if X is a positive integer-valued random variable satisfying the memoryless property for any two natural numbers m and n, then X is in fact a geometric random variable.

8. Let the random variable X have a binomial distribution with parameters p and n. Show that the recursive equation

$$P(X = k) = \frac{p}{1-p} \frac{n-k+1}{k} P(X = k-1)$$

holds.