Explanatory Notes for 6.390

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Overfitting with order

It's difficult to know how many terms to include in our polynomial, but we run into two problems if our order is **too high**:

- It becomes time-consuming to calculate, with little benefit
- We start overfitting more and more.

The first part makes sense: with more terms, we have to do more multiplications, more additions, etc.

Concept 1

More **complex models** tend to be more **expensive** to train, and slower to use. This is a trade-off for more **accuracy**.

Usually, there's a point where cost **outweights** benefits. A problem is rarely perfectly solved, even by an excellent model, so you can't just continue until it's "perfect".

But what about the second part? Why do we increase overfitting?

With a higher order, our polynomial becomes more complex: it can take on more shapes, which are increasingly complex and perfectly fit to the data.

This can cause our data to overlook obvious patterns, and instead create a very precise shape that is paying attention to the noise in our model.

Concept 2

High-order polynomials are very vulnerable to **overfitting**.

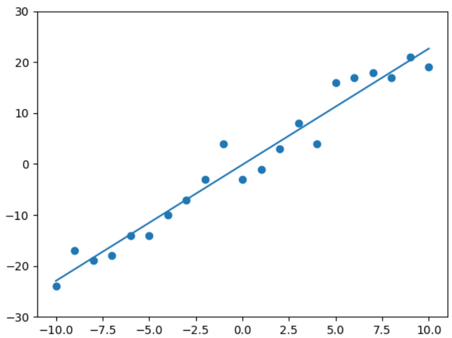
Because they can take on so many different, **complex** functions, they can very very closely **match** the original data set.

This can cause the model to "learn" noise, and **miss** broader and simpler patterns that actually exist. In may fail to learn something broad and useful, while **memorizing** the dataset with its expressiveness.

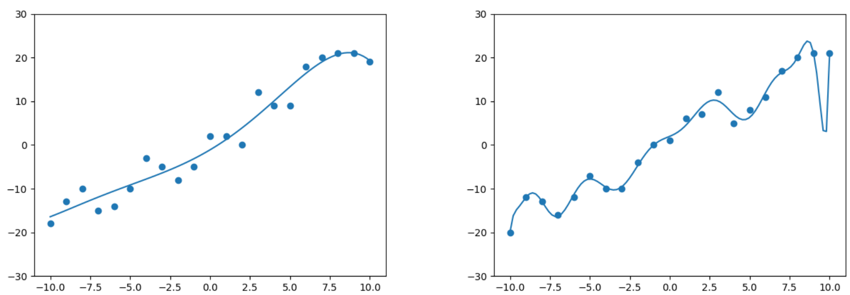
Let's see this in action: we'll generate some data based on $2x + 1$, while applying some random noise to it. We'll see the optimized linear regression model for each.

For ease, we'll exclude regularization: it does help mitigate this problem, but it doesn't totally solve it.

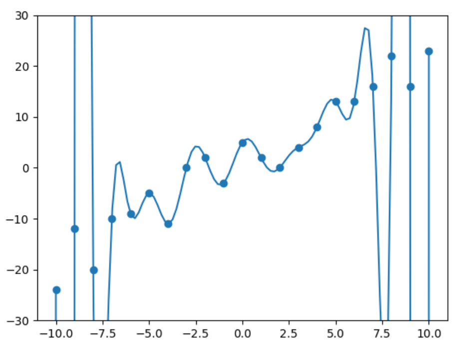
Rather than transform the data, we'll transform the separator: this really highlights the overfitting effect.



Here's the 1st order solution: in this case, correct for the underlying distribution. It fits our data fine.



5th and 15th order. The left model looks suspicious, and the right is way overfit. It's very unlikely that we know such an intricate pattern, from so little data.



20th order. We have one order for each data point: now, our model is capable of doing regression going through every single data point: as overfit as physically possible, perfectly matching the data.