Explanatory Notes for 6.390

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Learning Algorithms

Evaluating Hypotheses

So, we know that we have these **two** types of **error**. But it's **difficult** to **measure** them separately.

So instead, we just want to measure the **overall performance** of our hypothesis.

We do this using our **testing error**: this tells us how good our hypothesis is **after** training.

$$\mathcal{E}(h) = \frac{1}{m} \sum_{i=n+1}^{n+m} \left(h(x^{(i)}) - y^{(i)} \right)^2 \tag{1}$$

Note that, before, we were using **regularization**. This is so we can **make** a more **general** model.

But here, we've **removed** it, because training is **done**: we're **not** going to make our hypothesis **better**. We just care about how **good** it came out.

We're already measuring the **generalizability** by using **new data**!

Clarification 1

When we **evaluate a hypothesis** using **testing error**, we are **done training**: our hypothesis will not change.

Because of this, we **do not** include the **regularizer** when **evaluating** our hypothesis.

λ's purpose: learning algorithms

Notice that we **removed** regularization when we were **evaluating** our hypothesis: regularization was used to **create** our hypothesis, but it is not **part** of that hypothesis.

Our hypothesis only includes the parameters Θ: not λ!

That's because λ is part of our **algorithm**: it determines how we run our algorithm. So, let's talk about that.

Definition 2

A **learning algorithm** is our procedure for **learning** from data. It uses that data to create a **hypothesis**. We can diagram this as:

$$\mathcal{D}_n \longrightarrow \boxed{\text{learning alg } (\mathcal{H})} \longrightarrow h$$

In a way, it's a function that takes in **data** $\mathcal{D}_n$, and outputs a **hypothesis** h.

We're choosing **one hypothesis** h from the hypothesis class $\mathcal{H}$: this is why $\mathcal{H}$ appears in the notation above.

We can write this as $h \in \mathcal{H}$

Comparing Hypotheses and Learning Algorithms

We can take our learning algorithm

$$\mathcal{D}_n \longrightarrow \boxed{\text{learning alg } (\mathcal{H})} \longrightarrow h$$

And compare it to our hypothesis h:

$$x \rightarrow \boxed{h} \rightarrow y$$

In a way, our learning algorithm is a function, that outputs another function!

This is similar to $\mathcal{E}_n$, which instead **takes in** a function!

Our **hypothesis** can be adjusted with our **parameter** Θ: if we change Θ, we change our **performance**.

Our **learning algorithm** depends on λ: so, it's like a **parameter**. But, it's different from Θ: Θ **is** our model, λ controls how we **choose** our model.

So, it's a parameter (λ) that affects other parameters (Θ). Because of that, we call it a **hyperparameter**.

It affects our hypothesis by pressuring it to have lower magnitude!

Definition 3

Parameters are **variables** that adjust the behavior of **our model**: our hypothesis.

A **hyperparameter** is a **variable** that can adjust **how we make models**: our learning algorithm.

The **only** hyperparameter we have for now is λ, but the **development** of hyperparameters is an ongoing area of **research**.

Concept 4

Lambda, or λ, is a **hyperparameter**: it controls our **learning algorithm.**

Evaluating our Learning Algorithm

So, while we can evaluate each **hypothesis**, it's also important to measure how our **learning algorithm** is performing.

How do we measure it? Well, the job of our **learning algorithm** is to **pick good hypotheses**.

Concept 5

We can **evaluate** the performance of a **learning algorithm** using **testing loss**: a good learning algorithm will create **hypotheses** with **low testing loss**.

You could think of this as measuring the **skill** of a **teacher** (the learning algorithm) by the **success** of their **student** (the hypothesis) on a **test** (testing loss).

Validation: Evaluating with lots of data

When we were creating hypotheses, **randomness** caused some problems: you might not get **training data** that matched the **testing data** very well.

The **same** can happen here, when **evaluating** your **algorithm**: maybe your model happened to create a bad (or unusually good!) hypothesis because of **luck**.

The easy solution to **randomness** is to add **more data**: we get more **consistency** that way.

So, we **repeatedly** get new training data and test data. For each, we **train** a different hypothesis. We can **average** their performance out, and use that to **estimate** the quality of our algorithm.

Definition 6

Validation is a way to **evaluate a learning algorithm** using **large amounts of data**.

We do this by **running** our algorithm **many times** with new data, and **averaging** the testing error of all the hypotheses.

This process is often requires having **lots of data** to train with, but is a **provably** good approach.

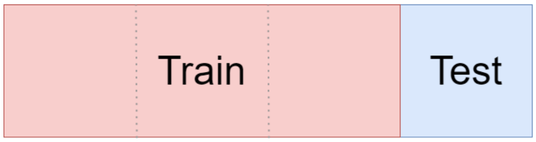
Our Problem: When data is less available

As mentioned, this takes up **lots of data**. What if we can't get as much: it's **expensive**, or not even possible? In this case, we have some **finite** data, $\mathcal{D}_n$. We **can't get more**.

We solved the **randomness** problem by using **more** training and testing **data**. So, we need some way to **get** more **distinct** hypotheses.

One set of data gives us one **hypothesis**. But, what if, rather than using **completely** new data, we used **slightly different** data each time?

First, need to break $\mathcal{D}_n$ into a chunk for training, and a chunk for testing.



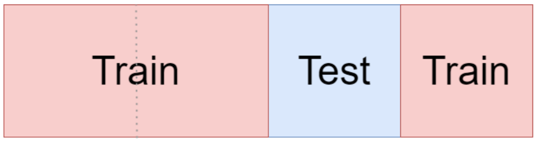
How do we get more hypotheses from this dataset?

Cross-Validation

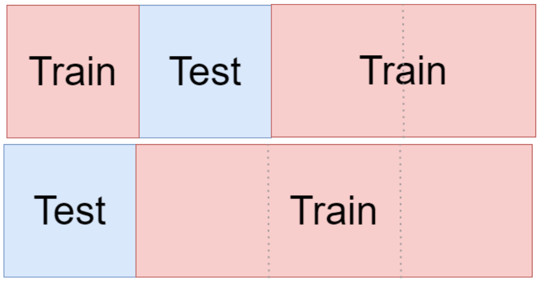
We mentioned that we just want **different** hypotheses. Our hypotheses depend on our **training data**. So we want to **change** our training data.

We can't **add** data to it, because then we **lose** testing data. We shouldn't **remove** data, because then we're just making a hypothesis that's **less well-informed**.

Instead, we'll **swap** some of the training data for testing data.



This will create a new hypothesis, and the data is partially different! In fact, we can do this for each of our chunks:



We now have **four different hypotheses** for the price of one!

Definition 7

Cross-validation is a way to **evaluate a learning algorithm** using **limited data**.

We do this by **breaking** our data it into **chunks** to create **multiple hypotheses** from one dataset.

For each **chunk**, we train one dataset on all the data **not in that chunk**. We get our **test error** using the chunk **we left out**.

For k chunks, we end up with k hypotheses. By **averaging** out their performance, we can **approximate** the quality of our algorithm.

This approach is much **less expensive**, and very common in machine learning! But, some of the theoretical **benefits** of validation are not **proven** to be true for cross-validation.

Clarification 8

Note that the goal of validation and cross-validation is **not** to evaluate **one hypothesis**.

Instead, it is instead meant to evaluate a **learning algorithm**. This is why we have to create **many** hypotheses: we want to see that our algorithm is **generally** good!