Explanatory Notes for 6.390

Shaunticlair Ruiz (Current TA)

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OLS Objective-Matrix Form

This section follows from the "Using Multiple Data Points" topics section.

Putting it together: Matrices

Now, we have shown both a way to express x_1, x_2, x_3 as a single $(d \times 1)$ matrix:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix} \tag{1}$$

We'll leave off the appended 1 for now.

And a way to express $x^{(1)}, x^{(2)}, x^{(3)}$ as a single $(1 \times n)$ matrix:

$$X = \begin{bmatrix} x^{(1)} & x^{(2)} & x^{(3)} & \dots & x^{(d)} \end{bmatrix} \tag{2}$$

Why not combine them into a single object?

Key Equation 1

X is our **input matrix** in the shape $(d \times n)$ that contains information about both **dimension** and **data points**.

$$X = \left.\overbrace{\begin{bmatrix} x_1^{(1)} & \cdots & x_1^{(n)} \\ \vdots & \ddots & \vdots \\ x_d^{(1)} & \cdots & x_d^{(n)} \end{bmatrix}}^{n \text{ data points}}\right\} d \text{ dimensions} \tag{3}$$

If we include the appended 1, we write this as the $((d + 1) \times n)$ matrix

$$X = \left.\overbrace{\begin{bmatrix} 1 & \cdots & 1 \\ x_1^{(1)} & \cdots & x_1^{(n)} \\ \vdots & \ddots & \vdots \\ x_d^{(1)} & \cdots & x_d^{(n)} \end{bmatrix}}^{n \text{ data points}}\right\} d + 1 \text{ dimensions} \tag{4}$$

Because each data point $y^{(i)}$ has only one dimension, it's the same as in the last section:

Key Equation 2

Y is our **output matrix** in the shape $(1 \times n)$ that contains all data points.

$$Y = \begin{bmatrix} y^{(1)} & \cdots & y^{(n)} \end{bmatrix}$$

All we have to do is combine our **equations**: We can use the one in the last section, but because θ is a matrix, we have to **transpose** it.

Key Equation 3

Using our **appended** matrix, we can write our **objective function** for **multiple** variables and **multiple** data points as

$$J = \frac{1}{n}\left(\theta^T X - Y\right)\left(\theta^T X - Y\right)^T$$

It is important to **remember** the **shape** of our objects, as well.

Concept 4

Our matrices have the shapes:

- X: $(d \times n)$ - matrix
- Y: $(1 \times n)$ - row vector

- θ: $(d \times 1)$ - column vector
- θ_0: (1×1) - scalar

- J: (1×1) - scalar

If we combine θ_0 into θ, replace every use of d with $d + 1$.

These shapes are worth **memorizing**.

Notice that these shapes make sense for our above equation! Try working through the matrix multiplication to verify this.

Alterate Notation

One side problem: some ML texts use the **transpose** of X and Y.

Notation 5

Some subjects use **different notation** for **matrices**. The main difference is that X and Y use their **transpose**, which we'll notate as

$$\tilde{X} = X^T \qquad \tilde{Y} = Y^T$$

Thus, our equation above becomes

$$J = \frac{1}{n}\left(\tilde{X}\theta - \tilde{Y}\right)^T \left(\tilde{X}\theta - \tilde{Y}\right)$$