

Explanatory Notes for 6.390

Shauntclair Ruiz (Current TA)

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7.X.17 Dimensions (Optional)

Here's a quick aside to clear up possible confusion from the last section: our definition of axes and "dimensions".

We said a vector has 1 axis, or "dimension" of movement. But, can't a vector have **multiple** dimensions?

Clarification 1

We have two competing definition of **dimension**: this explains why we can say seemingly conflicting things about derivatives.

So far, by "**dimension**", we mean, "a separate **value** we can **adjust**".

- Under this definition, a $(k \times 1)$ column **vector** has **k** dimensions: it contains **k** different scalars we can **adjust**.

$$\left[\begin{array}{c} v_1 \\ v_2 \\ \vdots \\ v_k \end{array} \right] \left. \vphantom{\begin{array}{c} v_1 \\ v_2 \\ \vdots \\ v_k \end{array}} \right\} \text{We can adjust each of our } k \text{ scalars.}$$

- You might say a $(k \times r)$ **matrix** has **k** dimensions, too: based on the **dimensionality** of its column vectors.
 - Since we prioritize the size of the vectors, we could say this is a very "vector-centric" definition.

In this section, by "dimension", we mean, "an **index** we can **adjust** (move along) to find another scalar."

- Under this definition, a $(k \times 1)$ column **vector** has **1** dimension: we only have **1** axis of **movement**.
- You might say a $(k \times r)$ **matrix** has **2** dimensions: a **horizontal** one, and a **vertical** one.
 - This **definition** is the kind we use in the following sections.