Explanatory Notes for 6.390

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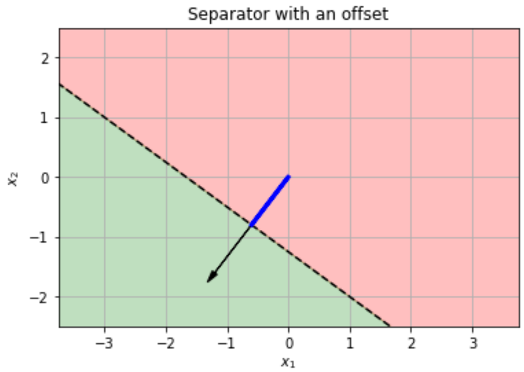
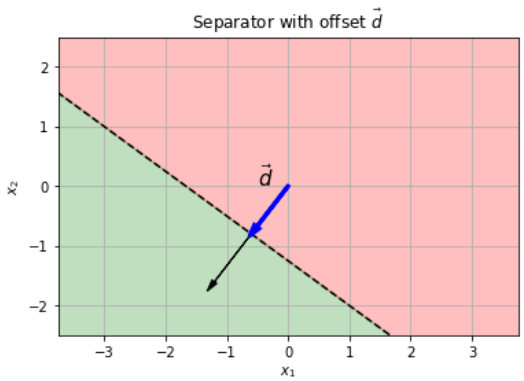
Distance from the Origin to the Plane



Notice that the **shortest** path from the origin to the line is **parallel** to θ!

So, we can think of our **line** as having been **pushed** in the θ direction. This **matches** what we did for 1-D separators: $x_1 > 3$ was moved in the x_1 direction.

So, we'll take the closest point on the line, $\vec{d}$. The **magnitude** d will give us the **distance** that the separator has been **shifted**.



Since $\vec{d}$ is in the direction of θ, the direction can be captured by the unit vector $\hat{\theta}$. Let's take a look at that:

Remember, a vector is direction (unit vector) times magnitude (scalar).

$$\theta = ||\theta||\hat{\theta} \tag{1}$$

$$\vec{d} = d\hat{\theta}$$

$\vec{d}$ is on the **line**, so it satisfies:

$$\theta^T \vec{d} + \theta_0 = 0 \tag{2}$$

Since θ and $\vec{d}$ are in the same direction, we can use that fact:

$$\|\theta\|\hat{\theta} \cdot d\hat{\theta} + \theta_0 = \|\theta\| \left(\hat{\theta} \cdot \hat{\theta}\right) d + \theta_0 \tag{3}$$

We know that $\hat{u} \cdot \hat{u} = 1$:

$$\|\theta\| d + \theta_0 = 0 \tag{4}$$

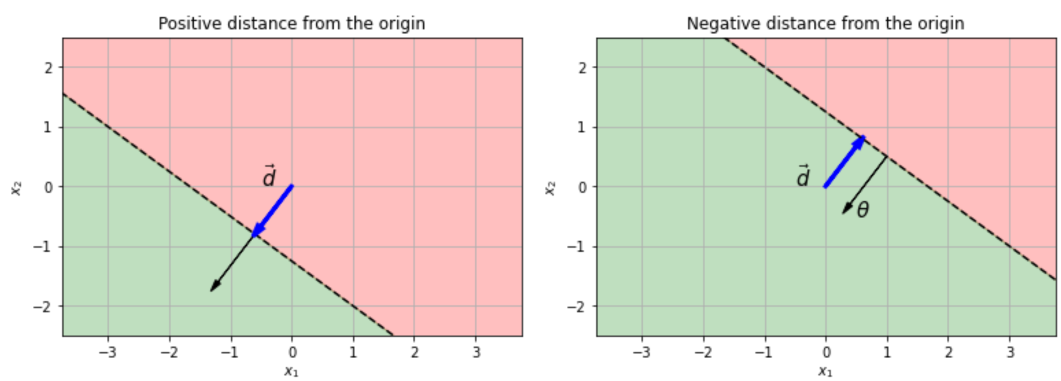
And now, we just solve for d:

Concept 1

The **distance** d from the **origin** to our **linear separator** is

$$d = \frac{-\theta_0}{\|\theta\|} \tag{5}$$

A "negative" distance means $\vec{d}$ (the vector from the origin to the line) is pointed in the opposite direction of θ.



Notice, again, that this agrees with our **earlier** thought: the sign of θ_0 is the opposite (-1) of the θ direction we move in.