Explanatory Notes for 6.390

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Fall 2022

X.17   Dimensions (Optional)

Here's a quick aside to clear up possible confusion from the last section: our definition of axes and "dimensions".

We said a vector has 1 axis, or "dimension" of movement. But, can't a vector have **multiple** dimensions?

Clarification 1

We have two competing definition of **dimension**: this explains why we can say seemingly conflicting things about derivatives.

$\approx\!\infty\approx$

So far, by "**dimension**", we mean, "a separate **value** we can **adjust**".

- Under this definition, a $(k \times 1)$ column **vector** has k dimensions: it contains k different scalars we can **adjust**.

$$\left.\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}\right\} \text{We can adjust each of our } k \text{ scalars.}$$

- You might say a $(k \times r)$ **matrix** has k dimensions, too: based on the **dimensionality** of its column vectors.

    - Since we prioritize the size of the vectors, we could say this is a very "vector-centric" definition.

$\approx\!\infty\approx$

In this section, by "dimension", we mean, "an **index** we can **adjust** (move along) to find another scalar.

- Under this definition, a $(k \times 1)$ column **vector** has 1 dimension: we only have 1 axis of **movement**.

- You might say a $(k \times r)$ **matrix** has 2 dimensions: a **horizontal** one, and a **vertical** one.

    - This **definition** is the kind we use in the following sections.