Explanatory Notes for 6.390

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Convergence

If you do this procedure with the above equation, though, you'll often run into **problems**. Why is that?

Well, because each of your steps is too **big** or too **small**: we won't be able to find a **stable** answer, i.e. **converge**!

What does it mean to **converge**?

It means we get a **single answer** after repeated steps: given enough time, we'll get **close as we want** to one number, and **stay there**.

Definition 1

If a sequence **converges**, then our result gets as **close as we want** to a **single number**, without going **further away**.

Example: The numbers $1/n$: $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ converges to 0.

If our answer **doesn't** converge, then it **diverges**. We can see why this might be bad: if we never **approach** a single answer, how do we know what value to **pick**?

Convergence: A little more formally (Optional)

Let's be more specific. Our sequence S will converge to r.

$$S = \{s_1, s_2, s_3, s_4, \dots\} \tag{1}$$

"As close as we want": let's say we want the maximum distance to be ϵ. That means, no matter what $\epsilon > 0$, we'll get closer at some point: $|m - s_i| < \epsilon$

$$|m - s_i| < \epsilon \text{ for some } i \tag{2}$$

"And stay there": at some time k, we never move further away again:

Definition 2

If a sequence S **converges** to m, then for all $\epsilon > 0$, we can say

$$|m - s_i| < \epsilon \text{ for all } i > k \tag{3}$$

This is a "formal" definition of convergence.