CS 740: Task 2: Ellipse. Described 01/09. Posted 04/09 Due: 18/09 12 noon

- Please write (only if true) in your handwriting the honor code. If you used any source (person or thing) explicitly state it.

1. Write an Octave function to generate n points that lies on:
   (a) Ellipse with axis lengths (a,b), centered at (x,y), with major axis making an angle θ radians with the X-axis.
   (b) Hyperbola with axis lengths (a,b), centered at (x,y), with major axis making an angle θ radians with X-axis.

   In the ideal case, you want these points to be distributed on the curve, rather than lie in some small regions.

2. (a) Generate points (using the previous function) on the ellipse with the parameters
      i. $n = 10, a = 8, b = 3, x = 0, y = 0, \theta = 0$; Store them in `EData1` (see Fig.1)
      ii. $n = 10, a = 8, b = 3, x = 2, y = 3, \theta = 30$; Store them in `EData2`
   (b) Generate points (using the previous function) on the hyberbola with the parameters
      i. $n = 10, a = 8, b = 3, x = 0, y = 0, \theta = 0$; Store them in `HData3`
      ii. $n = 10, a = 8, b = 3, x = 2, y = 3, \theta = 30$; Store them in `HData4`

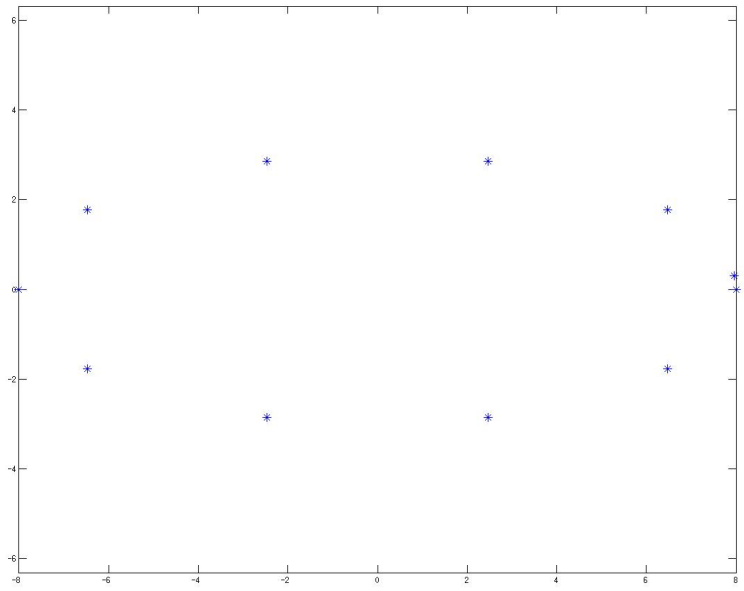

Figure 1: Noiseless Ellipse Data points

3. Add Gaussian noise $\mathbf{N}(\mathbf{0}, \left[\begin{smallmatrix} 0.3 & 0 \\ 0 & 0.3 \end{smallmatrix}\right])$ to all generated points `EData1`, `EData2`, `HData3`, `HData4`, and store the resulting points in `EData1_noisy`(see Fig.2), `EData2_noisy`, `HData3_noisy`, and `HData4_noisy` respectively.

4. Write an octave function with the following format:

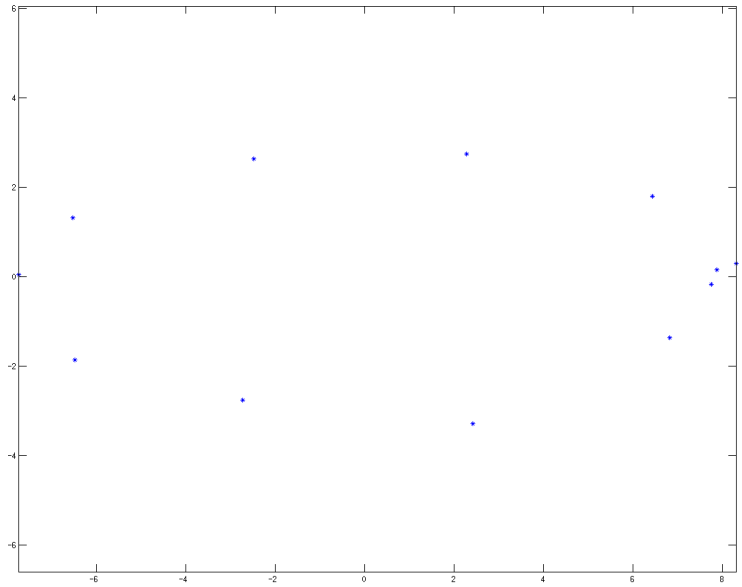

Figure 2: Noisy Ellipse Data points

```
function [a,err] = FindEllipse( X,Y )
%   Input: (X,Y) -- 2D pointset
%  Output: a -- coefficients of equation
%          err -- error code, 0 if no error , -1 if error
end
```

What kind of error conditions do you want to catch?

5. Run `FindEllipse` on all 8 inputs.

   (a) Note down your observations and error giving data variables, with explanations.

   (b) Plot estimated ellipse equations variables.(see Fig. 3)

   (c) Print ellipse parameters.

6. **Alternate method:** We decompose design matrix D into its quadratic and linear parts,

$$D = \left(\begin{array}{c|c} D_1 & D_2 \end{array} \right) \tag{1}$$

where, $D_1 = \begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 \\ x_2^2 & x_2 y_2 & y_2^2 \\ \vdots & \vdots & \vdots \\ x_N^2 & x_N y_N & y_N^2 \end{bmatrix}$ $D_2 = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_N & y_N & 1 \end{bmatrix}$

With this, the scatter matrix S can be split as follow:

$$S = \left(\begin{array}{c|c} S_1 & S_2 \\ \hline S_2^T & S_3 \end{array} \right), \text{ where } \begin{array}{l} S_1 = D_1^T D_1 \\ S_1 = D_1^T D_1 \\ S_1 = D_1^T D_1 \end{array} \tag{2}$$

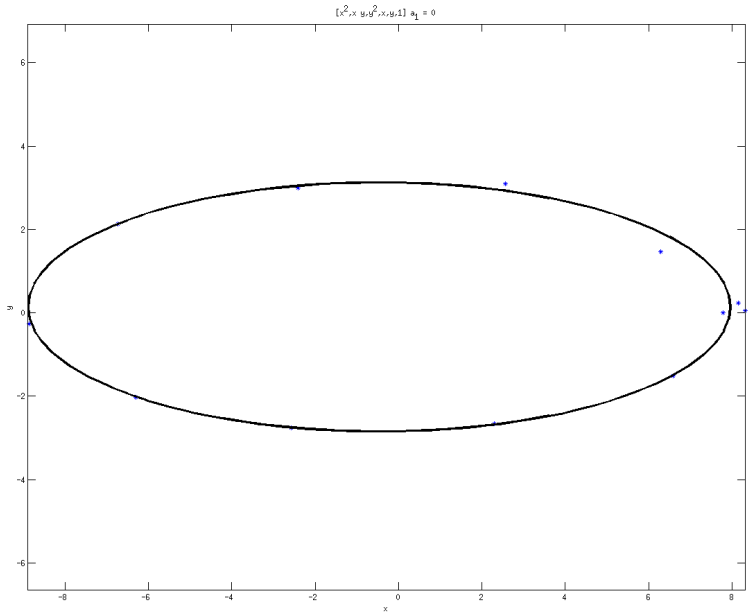

Figure 3: Ellipse Fit

Similarly, the constraint matrix C can be expressed as:

$$C = \left(\begin{array}{c|c} C_1 & 0 \\ \hline 0 & 0 \end{array}\right) \text{, where} \quad C_1 = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{bmatrix} \tag{3}$$

Finally, we split vector of coefficient $\mathbf{a}$ into

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_1 \\ \hline \mathbf{a}_2 \end{pmatrix} \text{,where} \quad \mathbf{a}_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \mathbf{a}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \tag{4}$$

Based on the given decomposition, the generalized eigenvalue system becomes

$$S_1\mathbf{a}_1 + S_2\mathbf{a}_2 = \lambda' C_1\mathbf{a}_1 \tag{5}$$
$$S_2^T\mathbf{a}_1 + S_3\mathbf{a}_2 = 0 \tag{6}$$
$$\mathbf{a}_1^T C_1\mathbf{a}_1 = -1 \tag{7}$$

Write the steps to solve this new system to find closed-form solution for $\mathbf{a}$. Turn in the paperwork for this.

7. Write Octave code using the alternate method for the input data
   (a) Compare the two methods
   (b) Plot estimated ellipse equations
   (c) Print ellipse parameters