Task2 - ELLIPSE FITTING

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September 18,2015

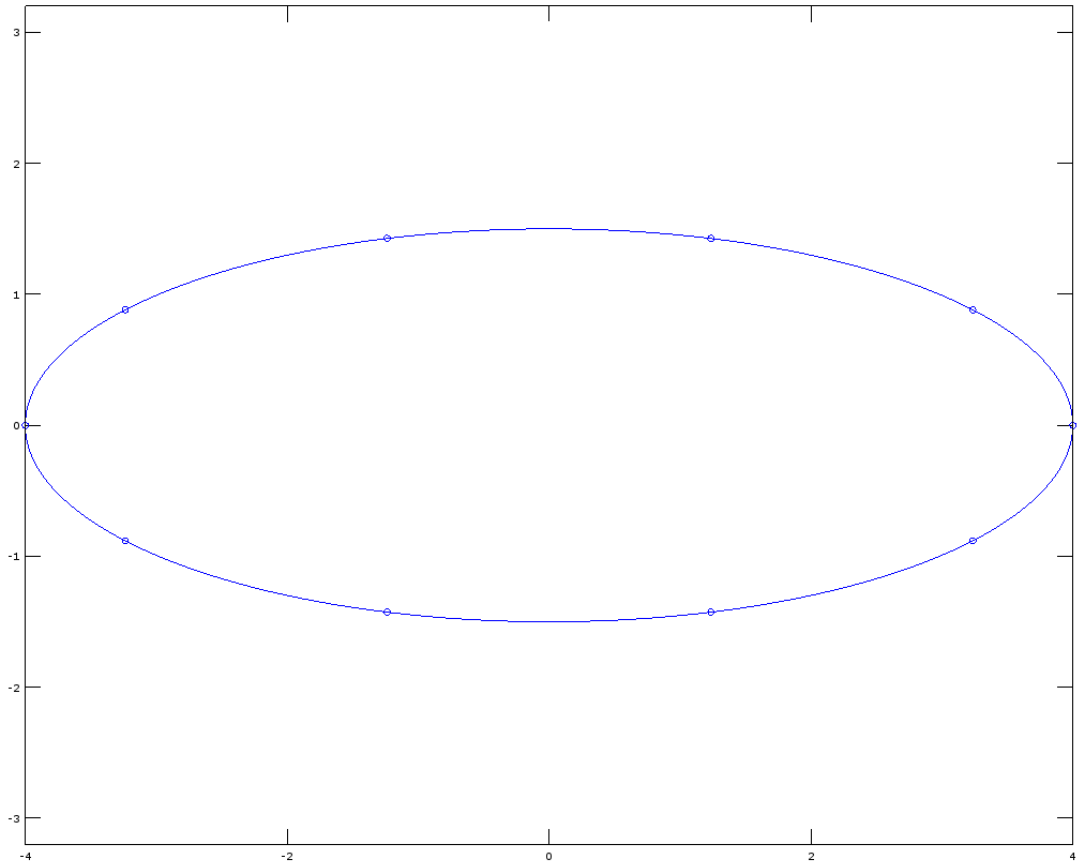
1 Question 5

Our Program is using cholesky Decomposition Method of Scatter Matrix to find Matrix A .
Following are condition where err =-1

- If Scatter(S) is not positive Definite error return is -1
- No negative eigenvalue corresponds to C*
- if alpha(α) value is not real (ie α = $\sqrt{-1/\mu^T C \mu}$)

Observation for different Tests DATA
Running FindEllipse_Basic on:

EDATA1:
Output:
axis1 = 8.0000
axis2 = 3.0000
center_x = -6.4146e-017
center_y = 3.7007e-018
theta_radian = -3.5886e-017
theta_degree = -2.0561e-015
err = 0
Cofficient_matrix =
-1.8750e-001 -8.2239e-017 -1.3333e+000 -2.4055e-017 9.8686e-018
3.0000e+000

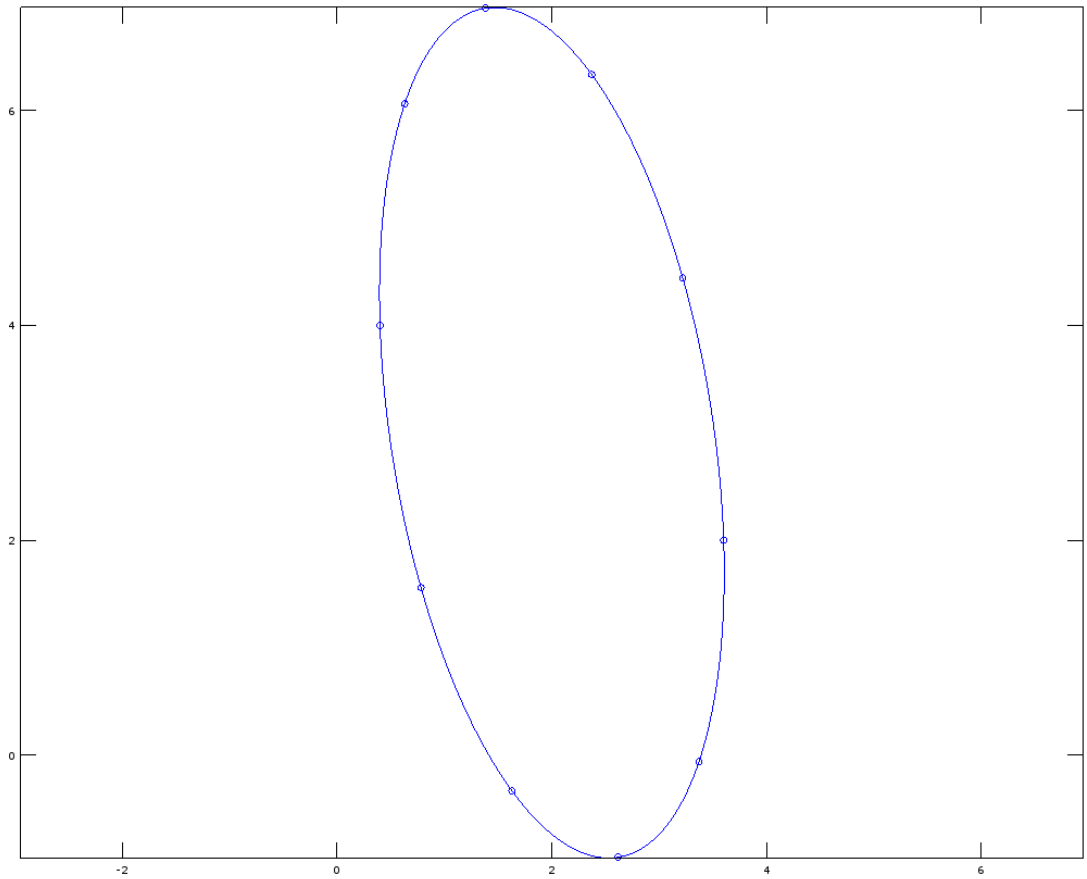


Observation : Found a exact fit for EDATA1

EDATA2:

Output:

axis1 = 8.0000
axis2 = 3.0000
center_x = 2.0000
center_y = 3.0000
theta_radian = -1.4159
theta_degree = -81.127
err = 0
Cofficient_matrix =
-1.30607 -0.34926 -0.21476 6.27207 1.98710 -6.25272



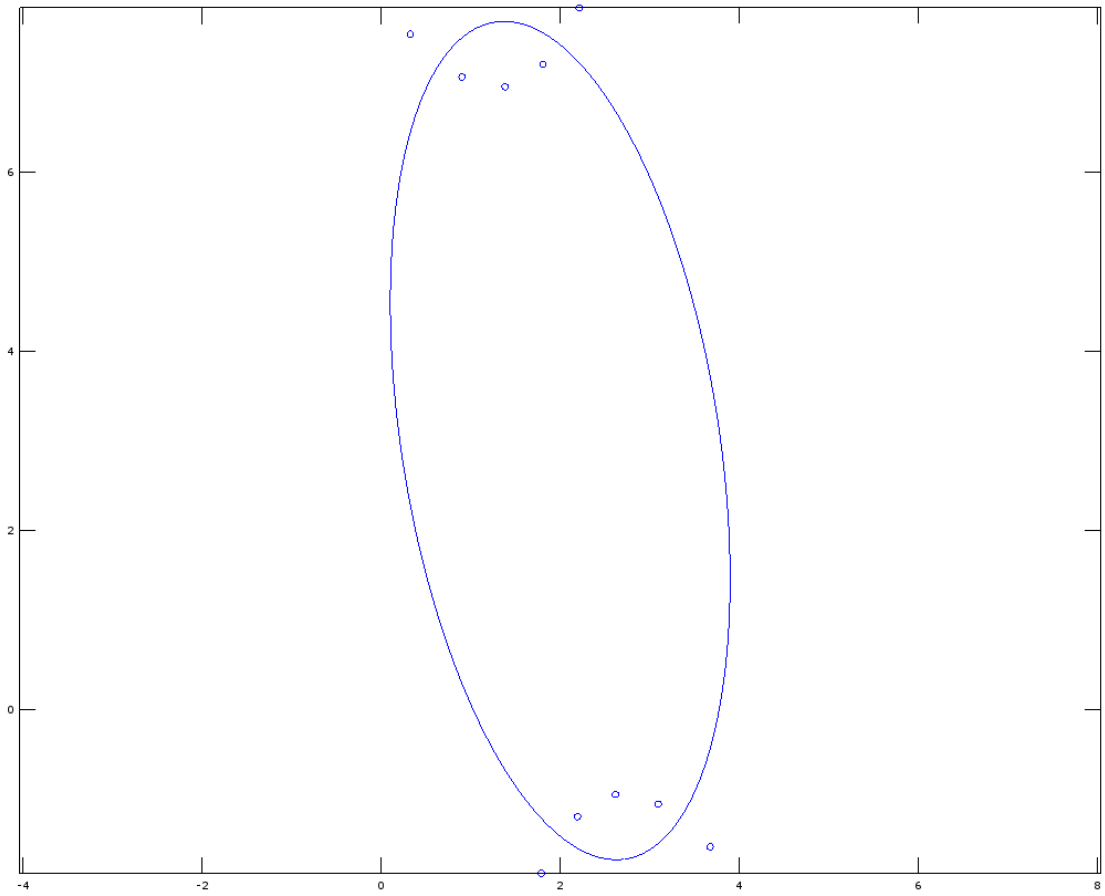
Observation : Found a exact fit for EDATA2

HDATA3:

Scatter matrix is not positive definite
err = -1
Can not fit ellipse

HDATA4:

axis1 = 9.4605
axis2 = 3.5477
center_x = 2.0000
center_y = 3.0000
theta_radian = -1.4159
theta_degree = -81.127
err = 0
Cofficient_matrix =
  -1.30607 -0.34926 -0.21476   6.27207   1.98710 -5.05738

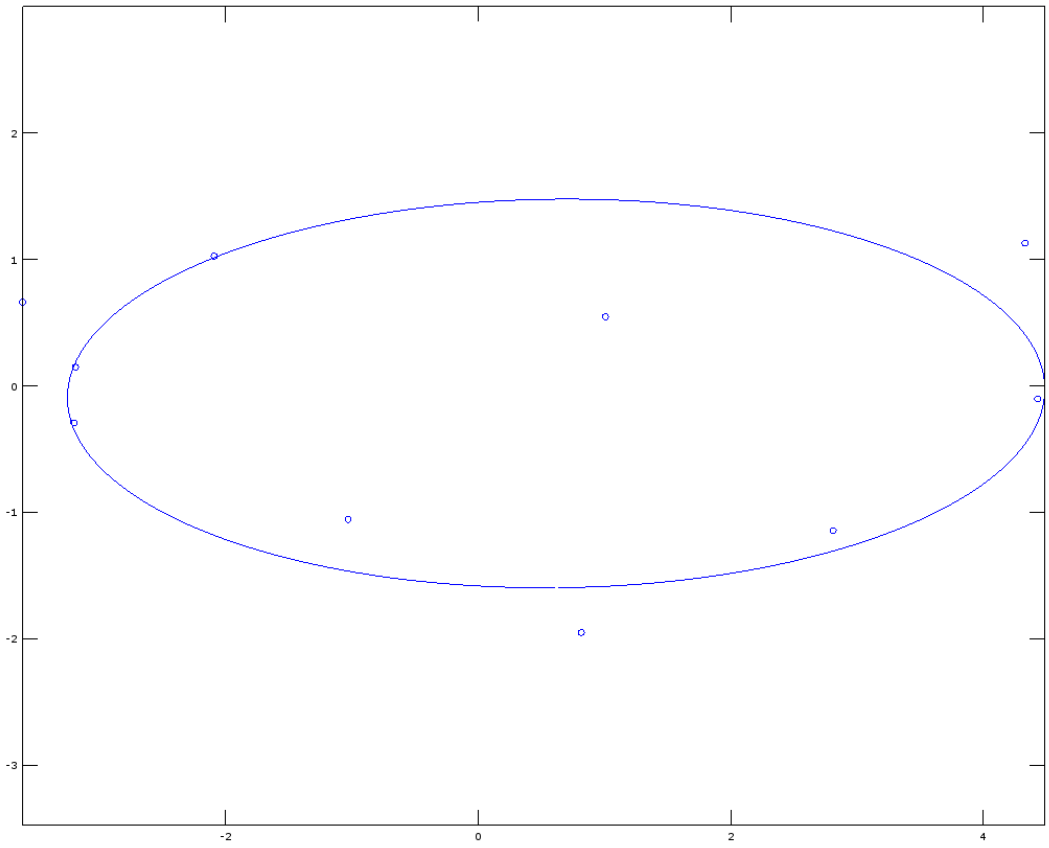


Observation : Algorithm Try to fit a approximate ellipse to Hyperbola

EDATA1_noisy

axis1 = 3.0741
axis2 = 7.7372
center_x = 0.61712
center_y = -0.060161
theta_radian = -1.5602
theta_degree = -89.394
err = 0
Cofficient_matrix =

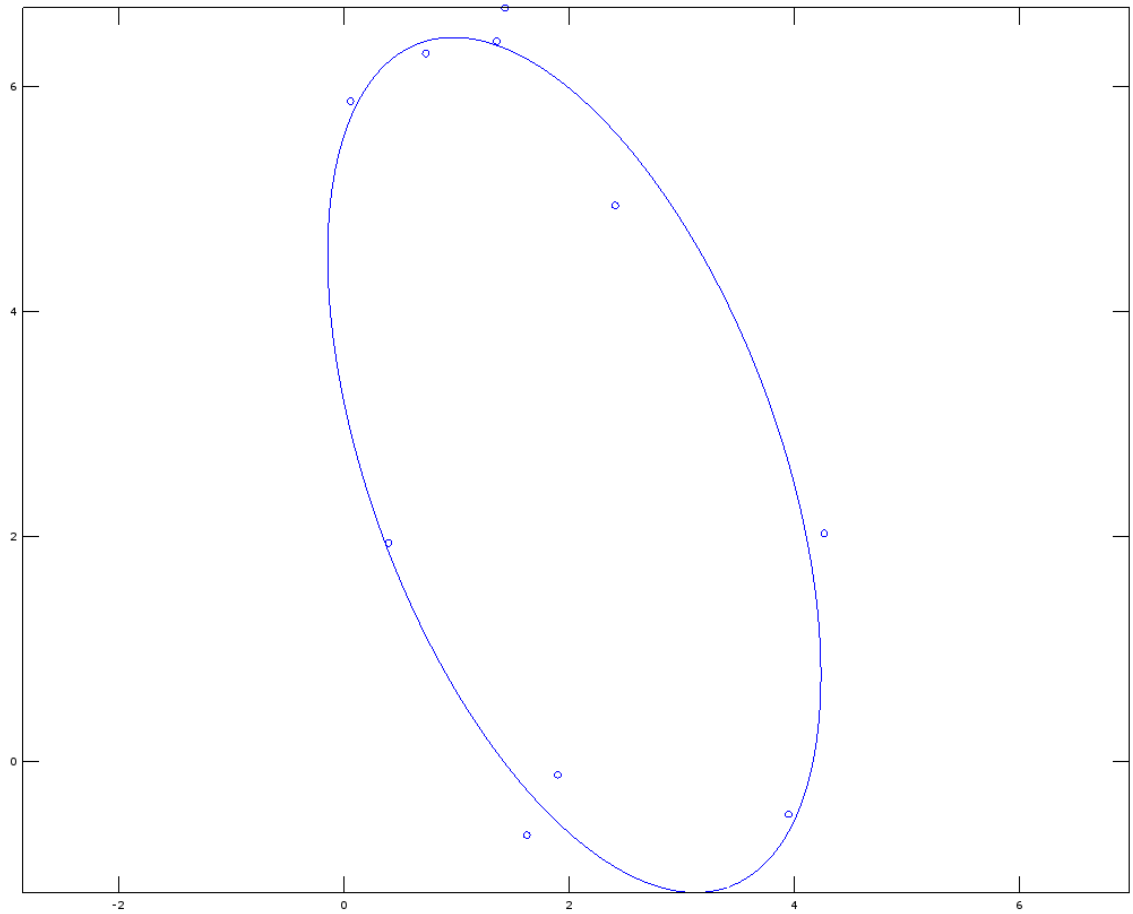
0.198776	-0.022416	1.258329	-0.246685	0.165239	-2.892066



Observation : A good fit is found for a noisy ellipse(1)

EDATA2_Noisy

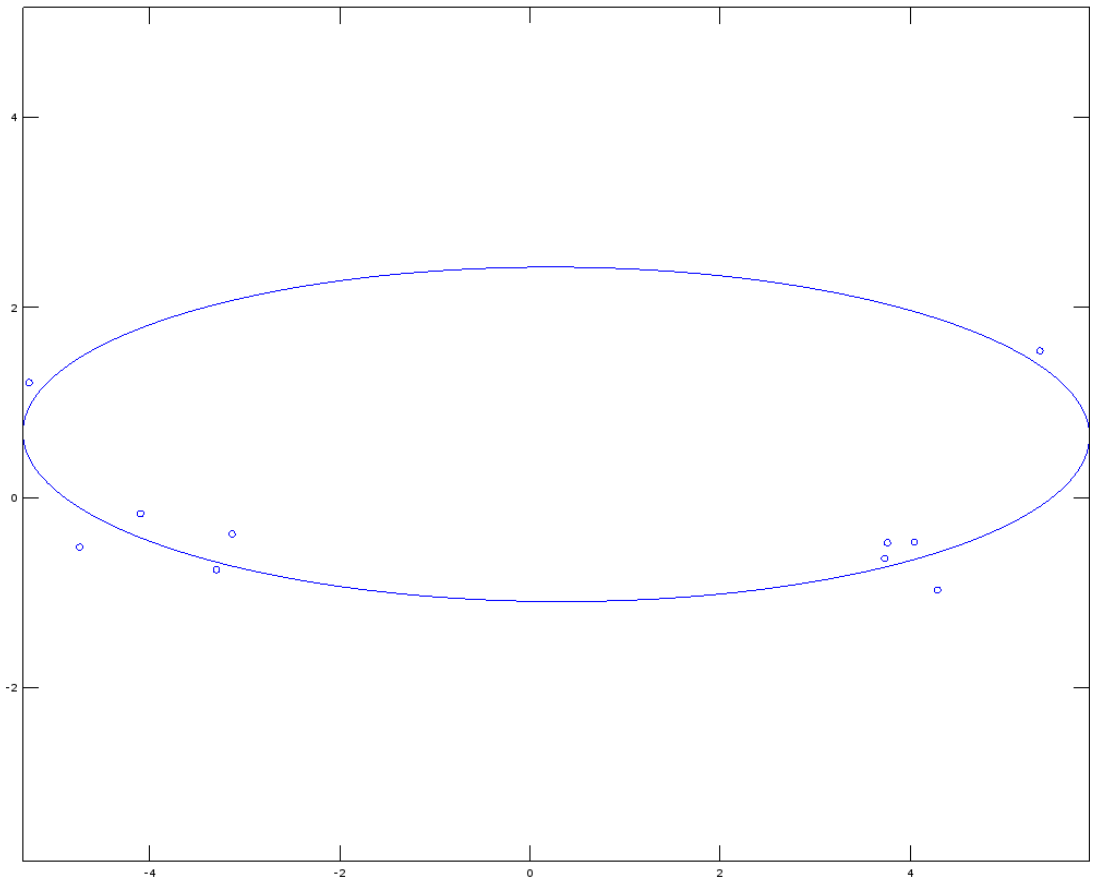
axis1 = 7.9862
axis2 = 3.6323
center_x = 2.0501
center_y = 2.6319
theta_radian = -1.2193
theta_degree = -69.860
err = 0
Cofficient_matrix =
-0.99597 -0.56371 -0.33077 5.56728 2.89679 -5.89276



Observation : A good fit is found for a noisy ellipse(2)

HDATA3_Noisy

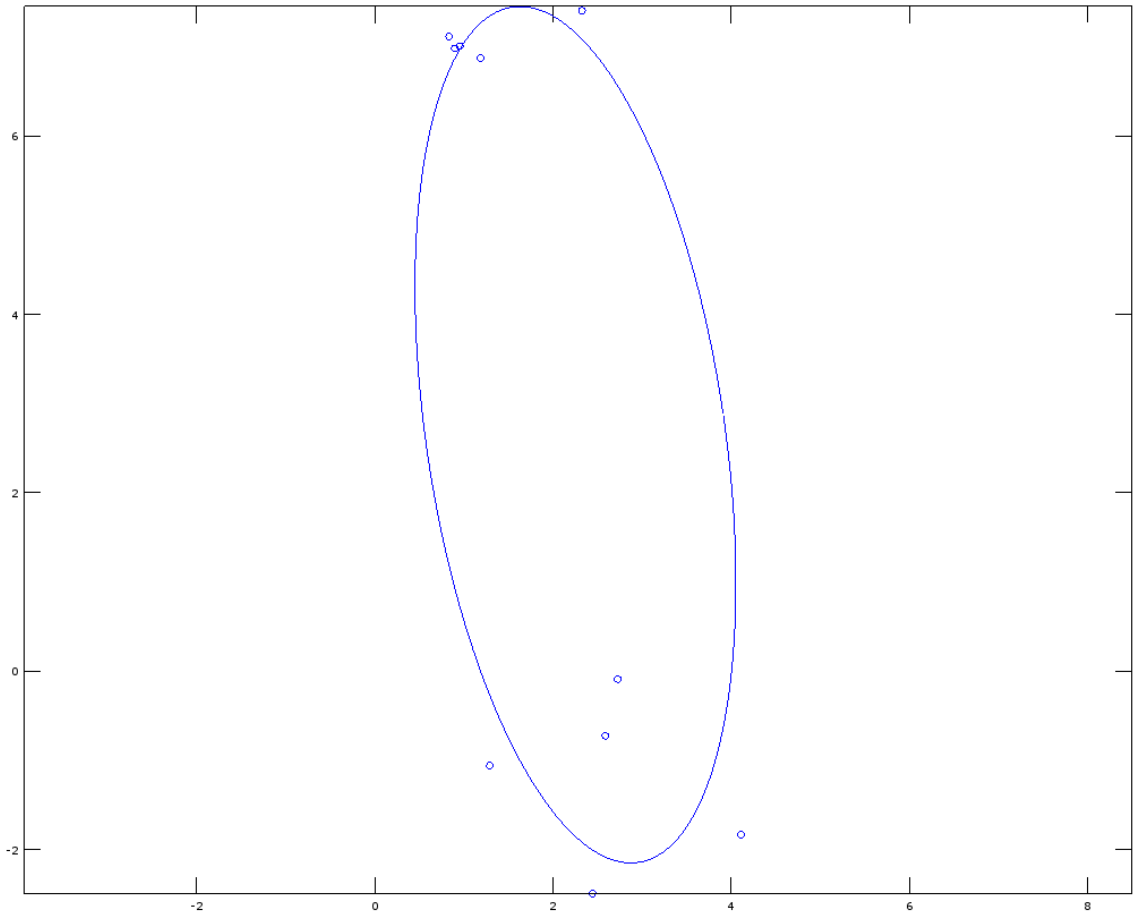
axis1 = 11.220
axis2 = 3.5185
center_x = 0.27647
center_y = 0.66338
theta_radian = -0.0037116
theta_degree = -0.21266
err = 0
Cofficient_matrix =
-0.156809 -0.010672 -1.594473 0.093787 2.118437 4.219192



Observation : Algorithm Try to fit a approximate ellipse to Hyperbola(1)

HDATA4_Noisy

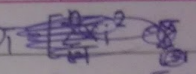
axis1 = 3.3476
axis2 = 9.6935
center_x = 2.2516
center_y = 2.6516
theta_radian = 0.14649
theta_degree = 8.3932
err = 0
Cofficient_matrix =
1.42065 0.36827 0.19984 -7.37407 -1.88901 6.74997



Observation : Algorithm Try to fit a approximate ellipse to Hyperbola(2)

1 Question 6

Alternate Method

We Have $D \doteq (D_1 | D_2)$ 

& Matrices $S = \left(\begin{array}{c|c} S_1 & S_2 \\ \hline S_2^T & S_3 \end{array}\right)$ where $S_1 = D_1^T D_1$, $S_2 = D_1^T D_2$, $S_3 = D_2^T D_2$

&

$$C = \left(\begin{array}{c|c} C_1 & 0 \\ \hline 0 & 0 \end{array}\right) \text{ where } C_1 = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

& Efficient Vector a into

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad a_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad a_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

We have ~~equation~~ generalized Eigen Value System.

$$S_1 a_1 + S_2 a_2 = \lambda' C_1 a_1 \quad (5)$$

$$S_2^T a_1 + S_3 a_2 = 0 \quad (6)$$

$$a_1^T C_1 a_1 = -1 \quad (7)$$

Our Solution

from (6) we have $a_2 = -S_3^{-1} S_2^T a_1$ (8)

[let $Z = -S_3^{-1} S_2^T$]

putting a_2 in equation (5) we have.

$$a_1^T C_1 a_1 = -1 \quad (7)$$

Our Solution

from (6) we have $a_2 = -S_3^{-1} S_2^T a_1$ (8)

$[$let $Z = -S_3^{-1} S_2^T]$

putting a_2 in equation (5) we have.

$$S_1 a_1 - S_2 S_3^{-1} S_2^T a_1 = \lambda' C_1 a_1$$

$$\Rightarrow (S_1 - S_2 S_3^{-1} S_2^T) a_1 = \lambda' C_1 a_1 \quad [\text{let } M = S_1 - S_2 S_3^{-1} S_2^T]$$

$$\Rightarrow M a_1 = \lambda' C_1 a_1$$

Since C_1 is invertible

$$\Rightarrow (C_1^{-1} M) a_1 = \lambda' a_1 \quad - (9) \quad [$$

Solution to this equation are eigen vector of $C_1^{-1} M$ which satisfyes the condition

$$4ac - b^2 > 0$$

Now

a_2 will be from (8)

$$a_2 = Z a_1 = -S_3^{-1} S_2^T a_1$$

final solution is

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

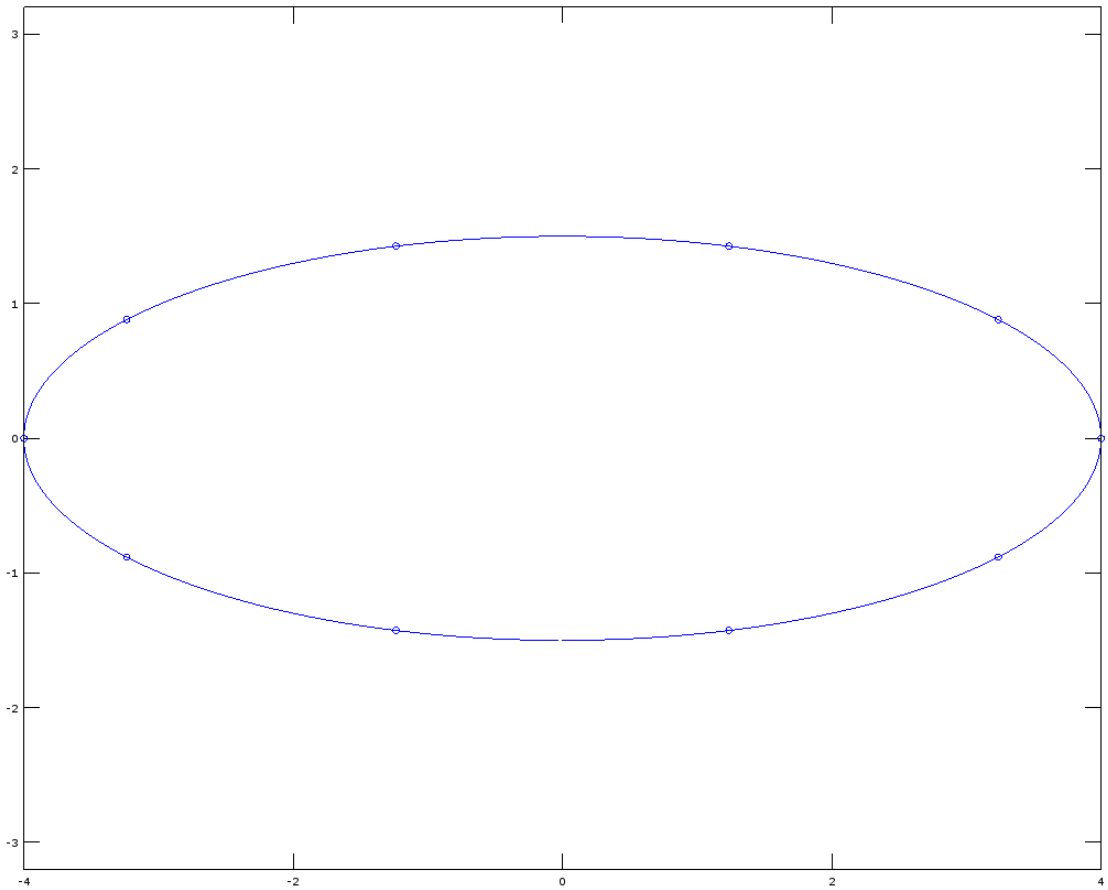
1 Question 7

Our Program is uses above Algorithm to fit Ellipse
gives error condition if

- if there exist no eigen vector of $C^{-1}M$ such that 4*a*c- b.^2 >0 condition is satisfied.

Observation for different Tests DATA
Running FindEllipse_Alternate on:

EDATA1:
Output:
axis1 = 3
axis2 = 8.0000
center_x = -1.9737e-017
center_y = 3.7007e-018
theta_radian = -1.5708
theta_degree = -90
err = 0
Cofficient_matrix =
1.3925e-001 -1.0994e-016 9.9026e-001 5.4970e-018 -7.3294e-018
-2.2281e+000

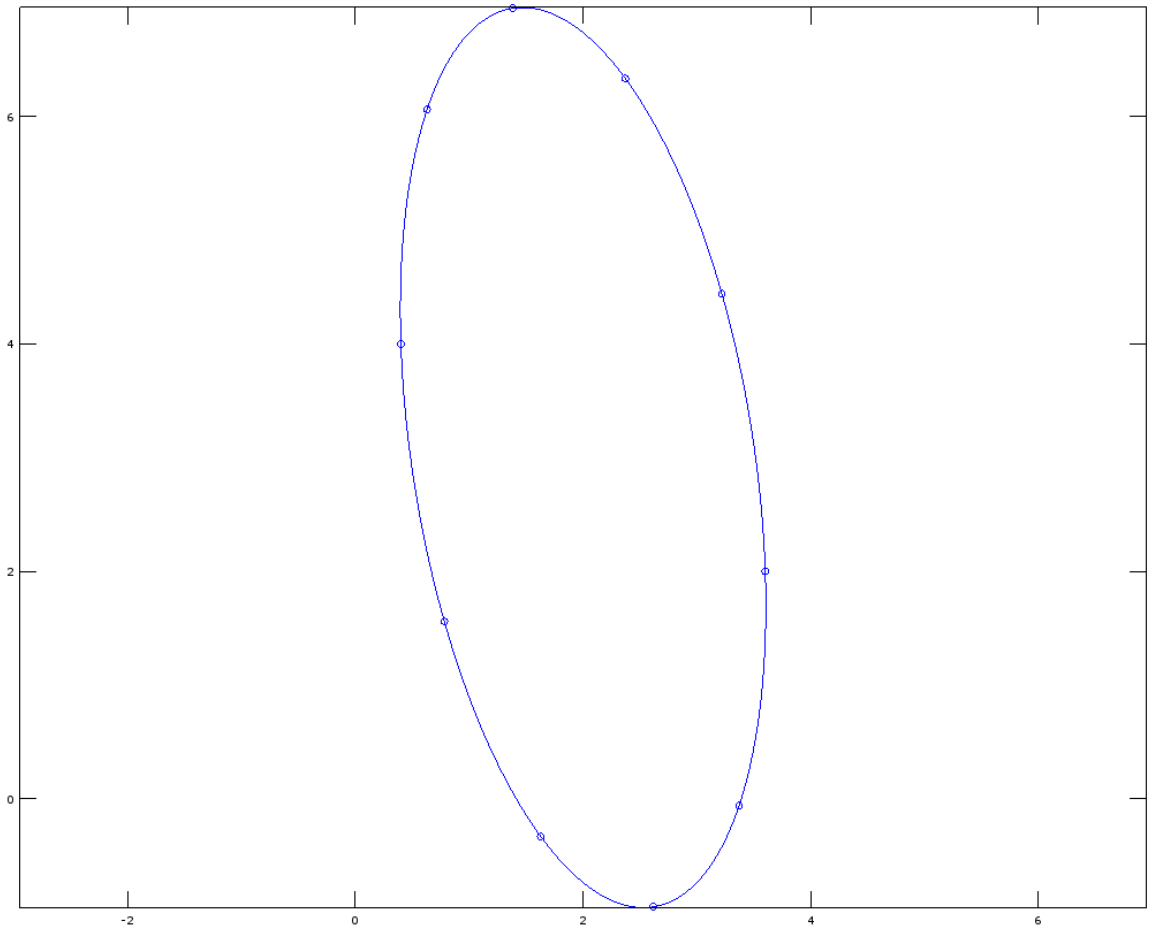


Observation :Found a exact fit for EDATA1 (ellipse(1))

EDATA2:

Output:

axis1 = 8.0000
axis2 = 3.0000
center_x = 2.0000
center_y = 3.0000
theta_radian = -1.4159
theta_degree = -81.127
err = 0
Cofficient_matrix =
-0.95409 -0.25514 -0.15689 4.58178 1.45159 -4.56765



Observation :Found a exact fit for EDATA2(ellipse(2))

HDATA3:

Output:

axis1 = 9.4605

axis2 = 3.5477

center_x = -1.6458e-017

center_y = 0

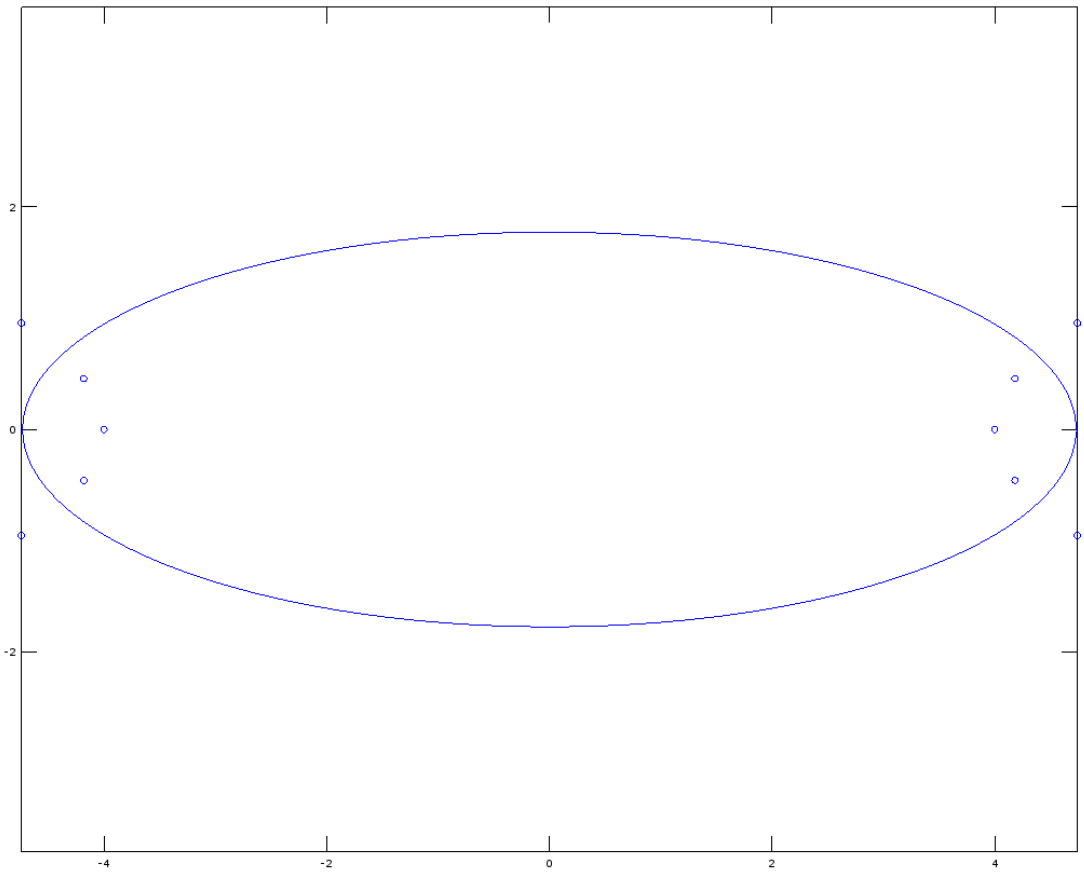
theta_radian = 0

theta_degree = 0

err = 0

Cofficient_matrix =

-0.13925 0.00000 -0.99026 -0.00000 0.00000 3.11585

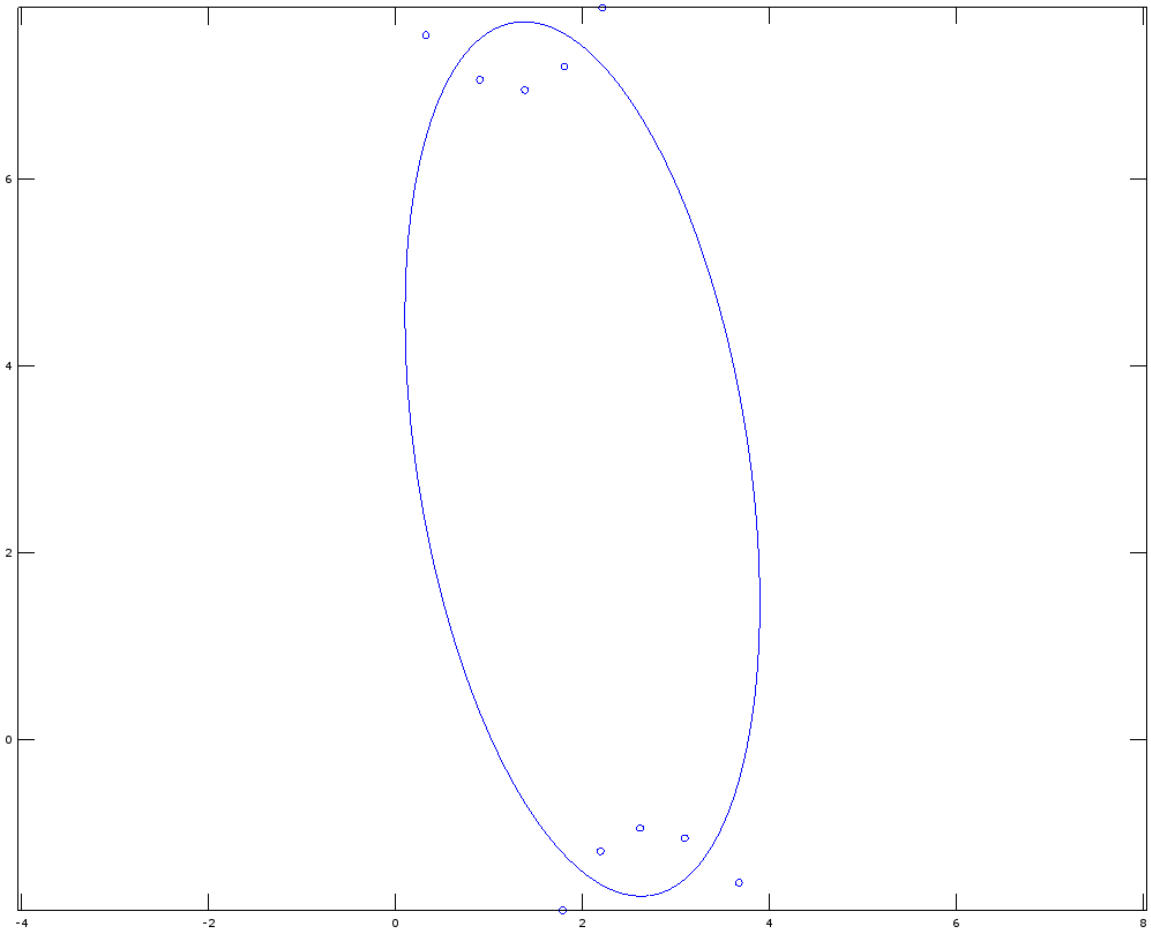


Observation : Try to fit a approximate fit to hyperbola(1)

HDATA4:

Output:

axis1 = 9.4605
axis2 = 3.5477
center_x = 2.0000
center_y = 3.0000
theta_radian = -1.4159
theta_degree = -81.127
err = 0
Cofficient_matrix =
-0.95409 -0.25514 -0.15689 4.58178 1.45159 -3.69445

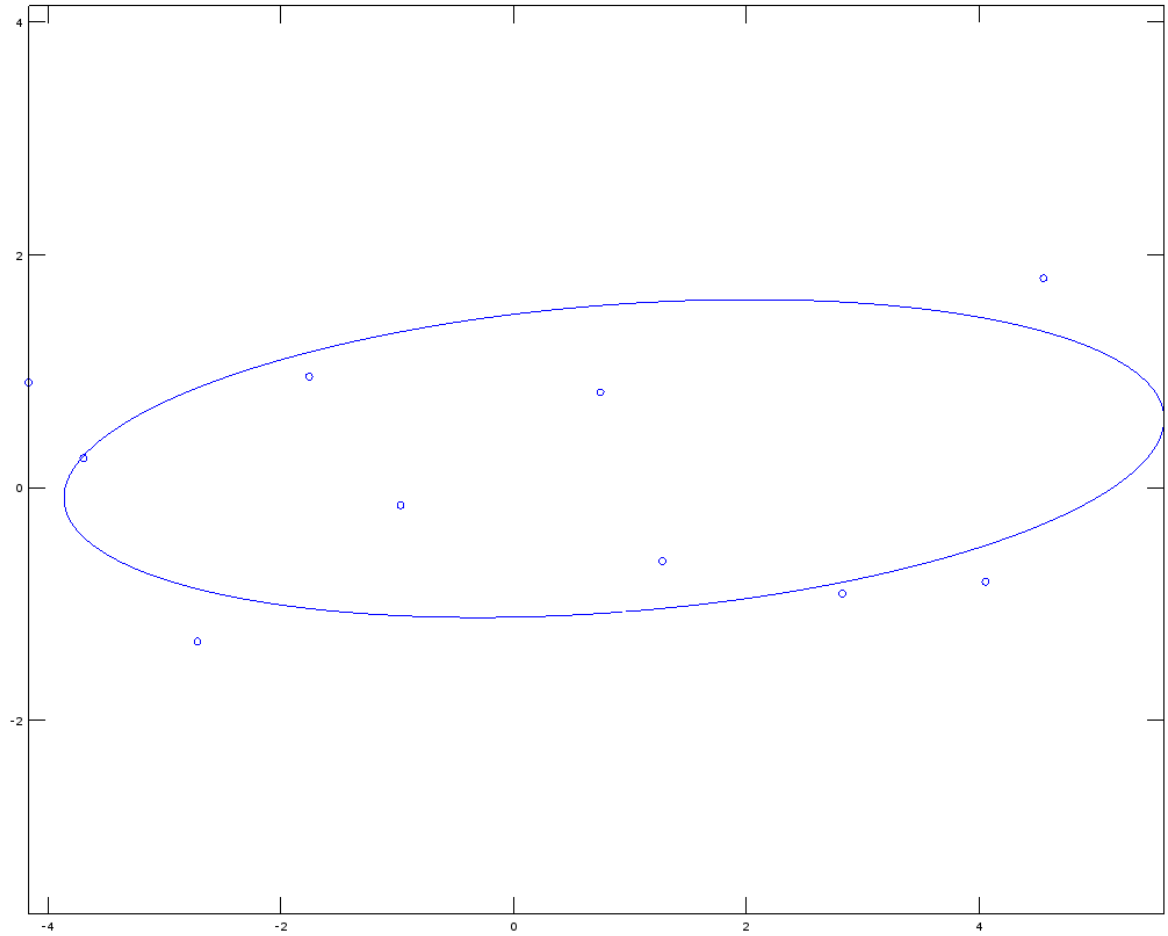


Observation : Try to fit a approximate fit to hyperbola(2)

EDATA1_noisy:

Output:

axis1 = 2.6398
axis2 = 9.4734
center_x = 0.86229
center_y = 0.25066
theta_radian = -1.4932
theta_degree = -85.551
err = 0
Cofficient_matrix =
0.082529 -0.141517 0.986490 -0.106855 -0.372520 -1.635389

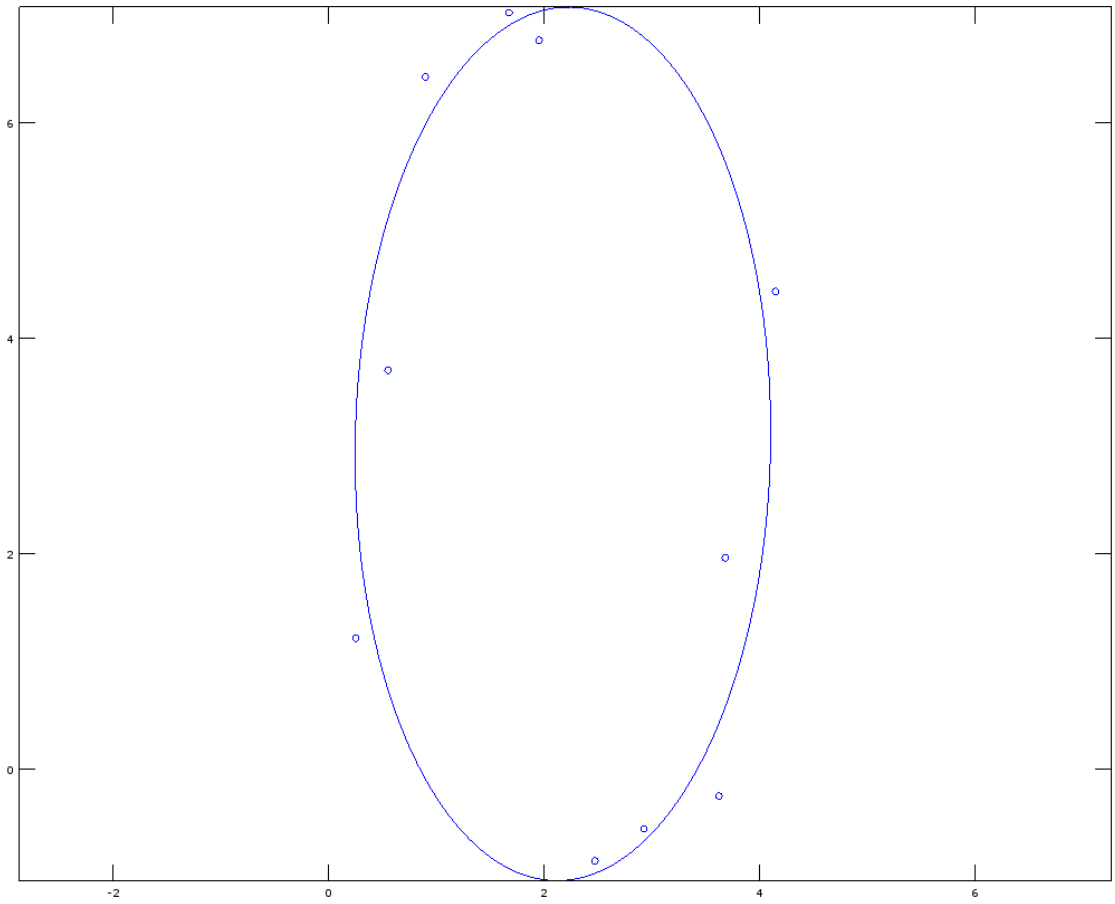


Observation : a good fit for noisy ellipse (1)

EDATA2_noisy:

Output:

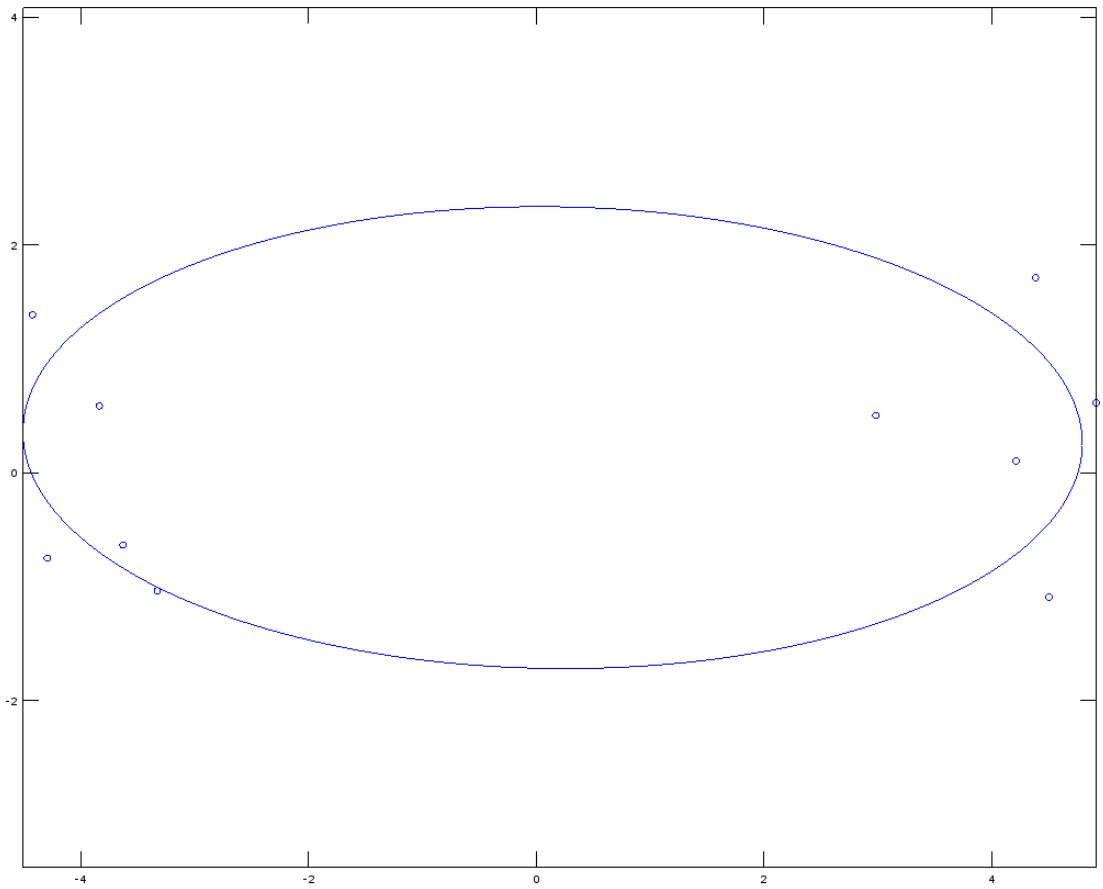
axis1 = 8.1111
axis2 = 3.8553
center_x = 2.1719
center_y = 3.0197
theta_radian = 1.5540
theta_degree = 89.040
err = 0
Cofficient_matrix =
-0.975049 0.025286 -0.220546 4.158990 1.277038 -2.820528



Observation : a good fit for noisy ellipse (2)

HDATA3_noisy:
Output:
axis1 = 9.3036
axis2 = 4.0535
center_x = 0.14555
center_y = 0.30692
theta_radian = -0.012892
theta_degree = -0.73866
err = 0
Cofficient_matrix =

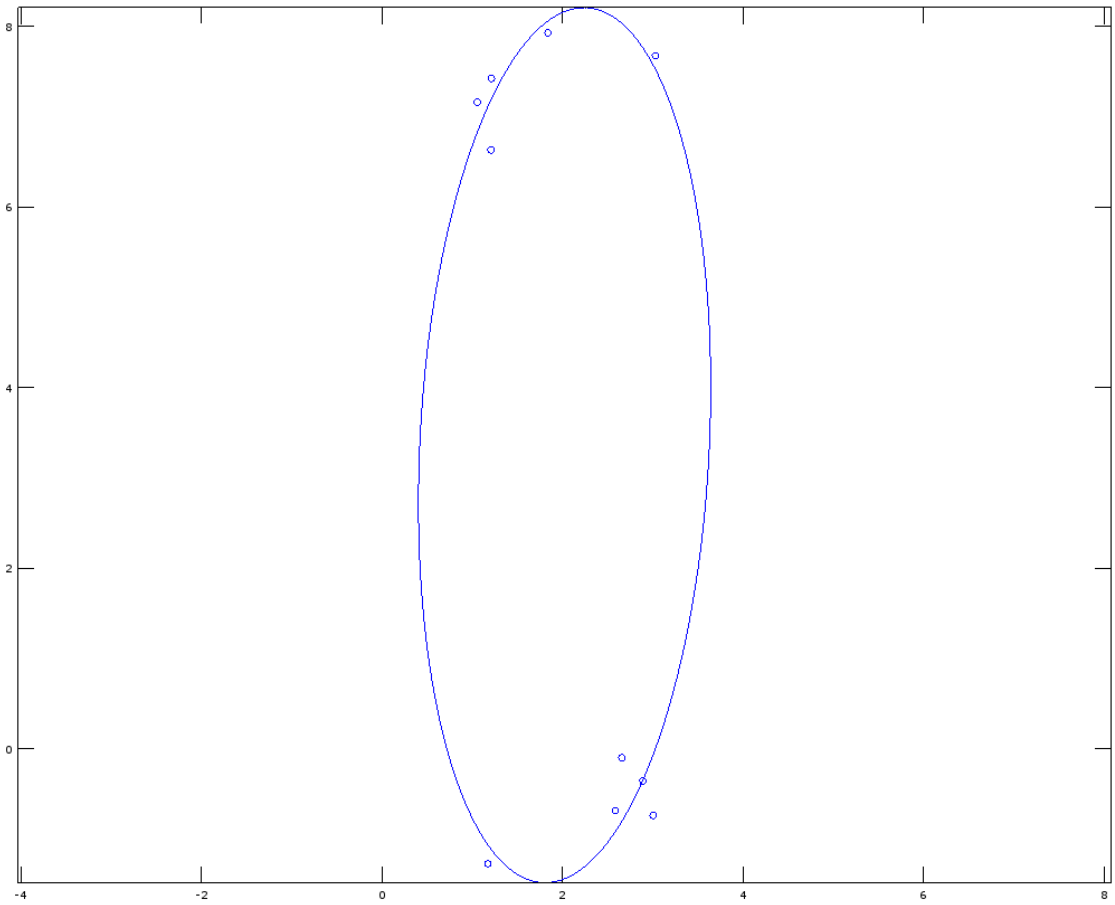
-0.186611 -0.020519 -0.982220   0.060618   0.605906   3.937843



Observation : try to fit a noisy hyperbola(1) to a ellipse

HDATA4_noisy:
Output:
axis1 = 9.7009
axis2 = 3.2059
center_x = 2.0251
center_y = 3.3616
theta_radian = 1.5209
theta_degree = 87.142
err = 0
Cofficient_matrix =

-0.989963 0.088032 -0.110557 3.713710 0.565023 -2.160717



Observation : try to fit a noisy hyperbola(1) to a ellipse

Comparison of two Methods

1. In Alternative method it is easy to get solution a1 as the Matrix $C^{-1}M$ is invertible thus easy to calculate eigenvalues which satisfy $4ac - b^2 > 0$ property, while in the Basic Method. We needed to Decompose the Scatter(**S**) Matrix by cholesky Decomposition for which the basic need to positive definite(which can be false even for some Datas for a ellipse).
2. Alternative method try to fit more DataSet as compare to Basic Method (Basic method can not fit HDATA3 while Alternative Method does).