Parameter Estimation

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① $$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$$

Now, taking ln of likelihood function we get,

$$\ln(\theta_1, \theta_2) = \sum_{i=1}^{n}\left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\right)$$

Partial derivatives wrt θ_1 θ_2

a) $$\frac{\partial}{\partial\theta_1}\left(\ln L(\theta_1,\theta_2)\right) = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

b) $$\frac{\partial}{\partial\theta_2}\left(\ln L(\theta_1,\theta_2)\right) = \sum_{i=1}^{n}\left(-\frac{(x_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n}\left(\frac{(x_i-\theta_1)^2}{\theta_2}\right) - \frac{n}{\theta_2} = 0$$

$$\Rightarrow \frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_i)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

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(2) MLE for binomial distribution $B(m, \theta)$

$m = +ve$ integer

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

By natural log,

$$\Rightarrow \ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + X_i \ln(\theta) + (m - X_i) \ln(1-\theta) \right)$$

$$\frac{\partial}{\partial \theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{X_i}{\theta} - \frac{m - X_i}{1-\theta} \right) = 0$$

Now, we have to solve for θ,

$$\sum_{i=1}^{n} \frac{X_i}{\theta} = \sum_{i=1}^{n} \frac{m - X_i}{1-\theta}$$

$$\Rightarrow \sum_{i=1}^{n} X_i (1-\theta) = \sum_{i=1}^{n} (m - X_i) \theta$$

$$\Rightarrow \theta \left(\sum_{i=1}^{n} X_i \right) = m \sum_{i=1}^{n} \theta$$

$$\Rightarrow \theta = \frac{1}{m} \sum_{i=1}^{n} X_i \rightarrow \text{sample mean}$$