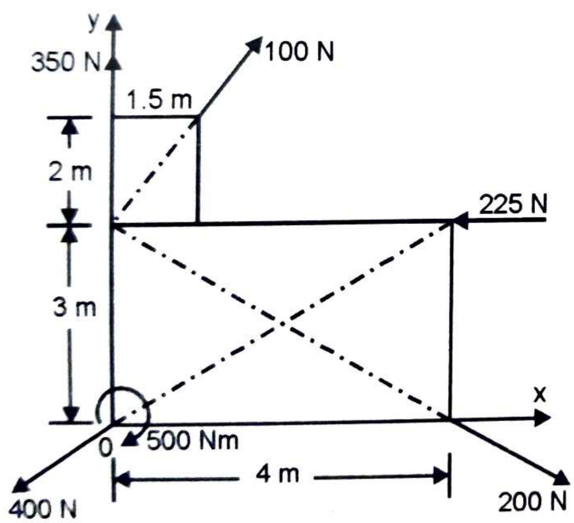
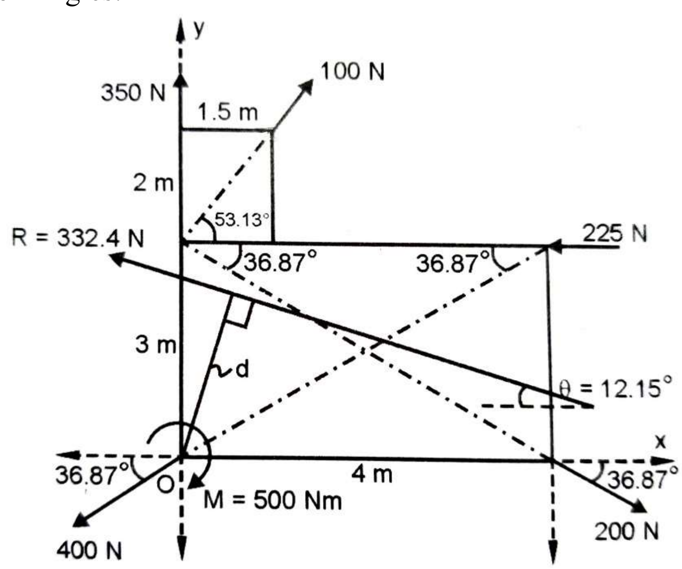
SOMAIYA VIDYAVIHAR UNIVERSITY

<table>
<tr><td colspan="4">Semester: Feb. 2023 – June 2023
Maximum Marks: 30   Examination: In-Semester Examination   Duration: 1 Hr 15 Min</td></tr>
<tr><td colspan="2">Programme code: Full Time UG
Programme: B.Tech</td><td>Class: FY</td><td>Semester: II (SVU 2020)</td></tr>
<tr><td colspan="2">Name of the Constituent College:
K. J. Somaiya College of Engineering</td><td colspan="2">Name of the department:
COMP/ETRX/EXTC/IT/MECH</td></tr>
<tr><td>Course Code: 116U06C104</td><td colspan="3">Name of the Course: Engineering Mechanics</td></tr>
</table>

<table>
<tr><th>Question No.</th><th>SOLUTION CUM MARKING SCHEME</th><th>Max. Marks</th></tr>
<tr><td>Q1.</td><td>Answer any ONE question.

(i) Determine the resultant of the given coplanar system of forces and a couple. Also locate the resultant on the x axis w.r.t. the origin. Reduce the system to a force couple system at O.


Solution:
Calculation of Angles:

</td><td>10

1</td></tr>
</table>

$\therefore \sum F_x = 200\cos 36.87° + 100\cos 53.13° - 400\cos 36.87° - 225$

$\Rightarrow \sum F_x = -325$ N or 325 N (←) **1**

$\therefore \sum F_y = -200\sin 36.87° + 100\sin 53.13° - 400\sin 36.87° + 350$

$\Rightarrow \sum F_y = 70$ N (↑) **1**

$\therefore R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{325^2 + 70^2} = 332.4$ N **1**

Also, $\tan\theta = \dfrac{\sum F_y}{\sum F_x} = \dfrac{70}{325} \Rightarrow \theta = 12.15°$ **1**

Since, $\sum F_x$ is in ← direction and $\sum F_y$ is in ↑ direction, the resultant R lies in the 2nd quadrant (↖). **1**

For the location of the resultant, let us assume that it is at a distance d to the right side of O. Using Varignon's theorem,

$\sum M_O^F = M_O^R$ (↺ +ve)

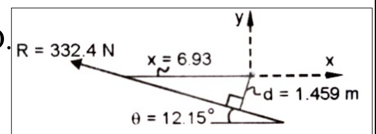
$-(200\sin 36.87° \times 3) - (100\cos 53.13° \times 5)$

$+(100\sin 53.13° \times 1.5) + (225 \times 3) - 500 = +(325 \times d)$

$\therefore d = -1.459$ m Or $d = 1.459$ m to the left of O **2**

Let resultant cut the x-axis at a distance x from O.

$\sin\theta = \dfrac{d}{x} \Rightarrow \sin 12.15° = \dfrac{1.459}{x} \Rightarrow x = 6.93$ m **1**



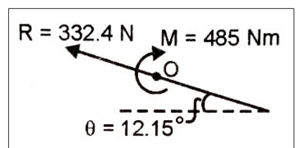
Resultant - Force

To replace the system by a force couple at point O, we shift the resultant to O and introduce a couple with force R and moment arm d.

$M_O = -R \times d$ (↺ +ve)

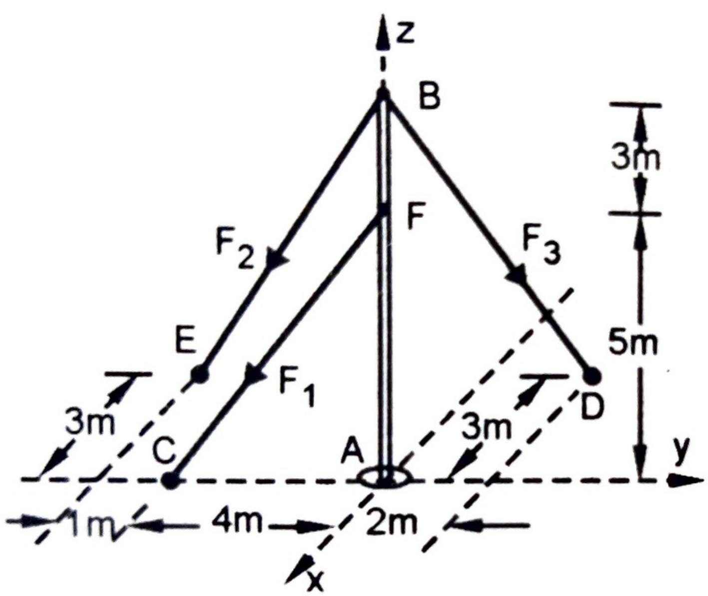
$M_O = -325 \times 1.459 = -485$ Nm

Or $M_O = 485$ Nm ↻ **1**

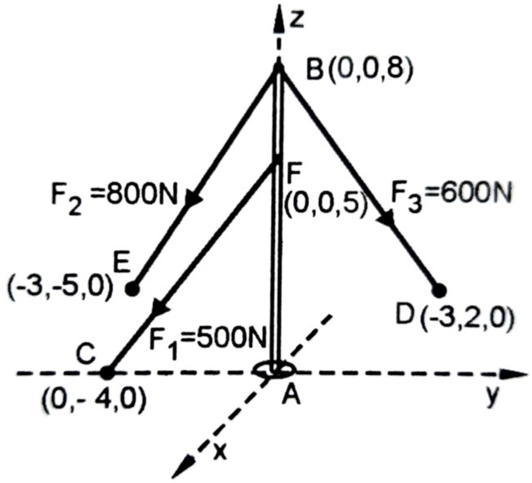


Resultant - Force Couple at O

(ii) Force F_1=500 N, F_2 = 800 N and F_3 = 600 N act on a vertical mast AB as shown. Find the resultant force and couple at the origin **10**



Solution:
Finding the co-ordinates:



2

$$\bar{F}_1 = F_1 \cdot \hat{e}_{FC} = 500\left[\frac{-4\hat{j} - 5\hat{k}}{\sqrt{4^2 + 5^2}}\right] = \left(-312.3\hat{j} - 390.4\hat{k}\right)\text{ N}$$ **1**

$$\bar{F}_2 = F_2 \cdot \hat{e}_{BE} = 800\left[\frac{-3\hat{i} - 5\hat{j} - 8\hat{k}}{\sqrt{3^2 + 5^2 + 8^2}}\right] = \left(-242.4\hat{i} - 404\hat{j} - 646.5\hat{k}\right)\text{ N}$$ **1**

$$\bar{F}_3 = F_3 \cdot \hat{e}_{BD} = 600\left[\frac{-3\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{3^2 + 2^2 + 8^2}}\right] = \left(-205.1\hat{i} + 136.7\hat{j} - 547\hat{k}\right)\text{ N}$$ **1**

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = \left(-312.3\hat{j} - 390.4\hat{k}\right) + \left(-242.4\hat{i} - 404\hat{j} - 646.5\hat{k}\right) + \left(-205.1\hat{i} + 136.7\hat{j} - 547\hat{k}\right)$$
$$\therefore \bar{R} = \left(-447.5\hat{i} - 579.6\hat{j} - 1583.9\hat{k}\right)\text{ N}$$ **1**

Taking moments of all forces about A (0, 0, 0)

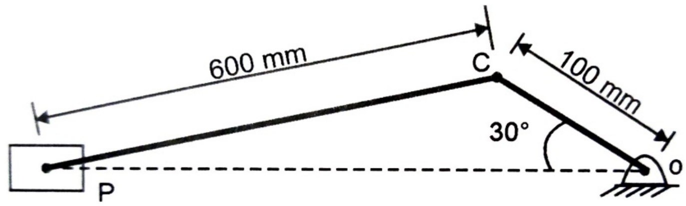
$$\bar{M}_A^{F_1} = \bar{r}_{AC} \times \bar{F}_1 = (-4\hat{j}) \times \left(-312.3\hat{j} - 390.4\hat{k}\right) = (1561.6\hat{i})\text{ Nm}$$ **1**

$$\bar{M}_A^{F_2} = \bar{r}_{AB} \times \bar{F}_2 = \left(8\hat{k}\right) \times \left(-242.4\hat{i} - 404\hat{j} - 646.5\hat{k}\right) = (3232\hat{i} - 1939.2\hat{j})\text{ Nm}$$ **1**

$$\bar{M}_A^{F_3} = \bar{r}_{AB} \times \bar{F}_3 = \left(8\hat{k}\right) \times \left(-205.1\hat{i} + 136.7\hat{j} - 547\hat{k}\right) = (-1093.6\hat{i} - 1640.8\hat{j})\text{ Nm}$$ **1**

$$\bar{M}_A = \bar{M}_A^{F_1} + \bar{M}_A^{F_2} + \bar{M}_A^{F_3}$$
$$\bar{M}_A = (1561.6\hat{i}) + (3232\hat{i} - 1939.2\hat{j}) + (-1093.6\hat{i} - 1640.8\hat{j})$$
$$\therefore \bar{M}_A = (3700\hat{i} - 3580\hat{j})\text{ Nm}$$ **1**

<table>
<tr><td>Q2</td><td>In a slider crank mechanism as shown in figure the crank is rotating at as constant speed of 120 rev/min. The connecting rod is 600 mm long and the crank is 100 mm long. For an angle of 30^0, determine the absolute velocity of the crosshead P. Crank OC performs clockwise rotation.

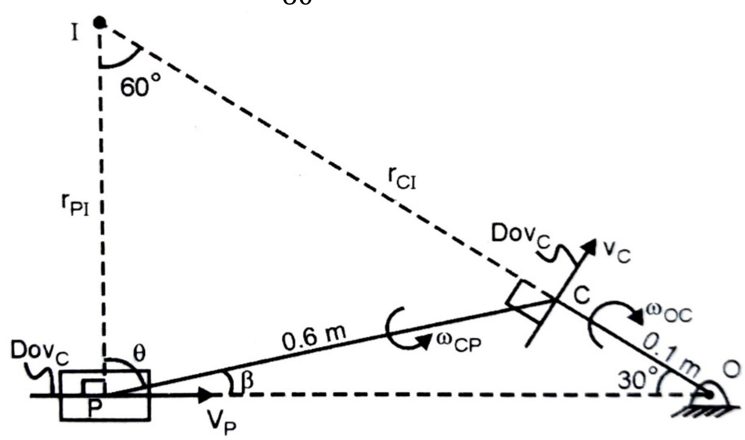


</td><td>10</td></tr>
<tr><td></td><td>

Solution:

$OC = 100\text{ mm} = 0.1\text{ m}, CP = 600\text{ mm} = 0.6\text{ m}, \angle COP = 45°$

$\omega_{OC} = 120\text{ rpm } ↻ = 120 \times \frac{2\pi}{60}\text{ rad/s} = 12.566\text{ rad/s}$
</td><td>1</td></tr>
<tr><td></td><td>



</td><td>2</td></tr>
<tr><td></td><td>

$v_C = r_{OC} \times \omega_{OC} = 0.1 \times 12.566 = 1.257\ \text{m/s}\ \nearrow$
</td><td>1</td></tr>
<tr><td></td><td>

Drawing perpendiculars from the velocity of A (↗) and the velocity of B (→), we can locate their intersection point I.

In $\Delta OCP, \angle COP = 30°\ \&\ \text{let } \angle CPO = \beta$

$\therefore$ using sine rule, $\frac{0.1}{\sin\beta} = \frac{0.6}{\sin 30°} \Rightarrow \beta = 4.8°$
</td><td>1</td></tr>
<tr><td></td><td>

In $\Delta OPI, \because \angle POI = 30°\ \&\ \angle IPO = 90° \Rightarrow \angle PIO = 180° - 90° - 30° = 60°$

$\therefore$ In $\Delta CPI, \quad \angle CPI = \theta = 90° - \beta = 90° - 4.8° = 85.2°$

$\Rightarrow \angle ICP = 180° - 85.2° - 30° = 34.8°$
</td><td>1</td></tr>
<tr><td></td><td>

Using sine rule in $\Delta CPI, \frac{0.6}{\sin 60°} = \frac{r_{CI}}{\sin 85.2°} = \frac{r_{PI}}{\sin 34.8°}$

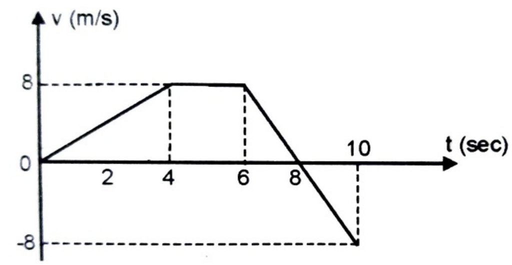
$\Longrightarrow r_{CI} = 0.69\text{ m}\ \ \&\ \ r_{PI} = 0.395\text{ m}$
</td><td>2</td></tr>
<tr><td></td><td>

$v_C = r_{CI} \times \omega_{CP} \Rightarrow 1.257 = 0.69 \times \omega_{CP} \Longrightarrow \omega_{CP} = 1.822\ \text{rad/s}\ ↺$
</td><td>1</td></tr>
<tr><td></td><td>

$v_P = r_{PI} \times \omega_{CP} = 0.395 \times 1.822 = 0.7196\ \text{m/s} \rightarrow$
</td><td>1</td></tr>
<tr><td>Q3</td><td>

Answer any ONE question.

(i) For a particle performing rectilinear motion, the v – t diagram is shown. Draw a – t and x – t diagrams for the motion if at t = 2 sec., x = 20 m. What is the displacement during 6 to 10 sec.? Also find the total distance traveled.
</td><td>10</td></tr>
</table>



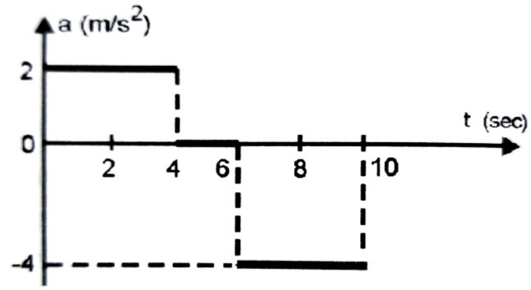
Solution:

Acceleration calculations: $a = [\text{slope v} - \text{t curve}]_{\text{at time} = t}$

$a_{0-4} = \left[\dfrac{v_4 - v_0}{\Delta t}\right]_{0-4} = \dfrac{8-0}{4-0} = 2\ \text{m/s}^2$ **1**

$a_{4-6} = \left[\dfrac{v_6 - v_4}{\Delta t}\right]_{4-6} = \dfrac{8-8}{6-4} = 0$ **1**

$a_{6-10} = \left[\dfrac{v_{10} - v_6}{\Delta t}\right]_{6-10} = \dfrac{-8-8}{10-6} = -4\ \text{m/s}^2$ **1**



1

Position calculations: $x_f = x_i + [\text{AUC v} - \text{t}]_{t_i - t_f}$

Given: $x_2 = 20$ m, at $t = 2$ sec

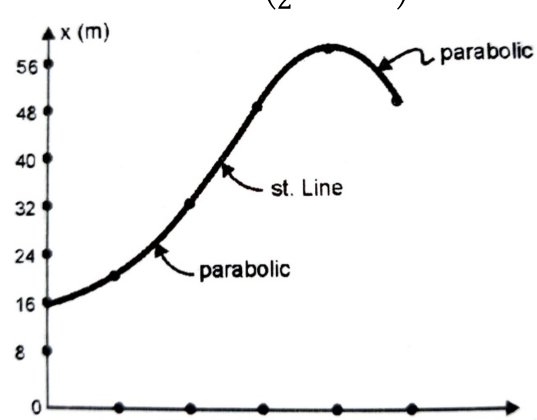
$x_2 = x_0 + [\text{AUC v} - \text{t}]_{0-2} \Longrightarrow 20 = x_0 + \dfrac{1}{2} \times 2 \times 4 \Longrightarrow x_0 = 16\ \text{m}$ **1**

$x_4 = x_0 + [\text{AUC v} - \text{t}]_{0-4} = 16 + \left(\dfrac{1}{2} \times 4 \times 8\right) = 32\ \text{m}$ **1**

$x_6 = x_4 + [\text{AUC v} - \text{t}]_{4-6} = 32 + (2 \times 8) = 48\ \text{m}$ **1**

$x_8 = x_6 + [\text{AUC v} - \text{t}]_{6-8} = 48 + \left(\dfrac{1}{2} \times 2 \times 8\right) = 56\ \text{m}$ **1**

$x_{10} = x_8 + [\text{AUC v} - \text{t}]_{8-10} = 56 - \left(\dfrac{1}{2} \times 2 \times 8\right) = 48\ \text{m}$ **1**



1

	Displacement from 6 to 10 seconds $= x_{10} - x_6 = 48 - 48 = 0$ Total distance $= \lvert x_{10} - x_8 \rvert + \lvert x_8 - x_0 \rvert = \lvert 48 - 56 \rvert + \lvert 56 - 16 \rvert = 48\ \text{m}$	**1**
	(ii) A particle performing rectilinear motion starts from rest from origin and has its acceleration defined by a = 25 – v^2 m/s^2. Determine the time spent and the particle's position, when v = 4 m/s. **Solution**: $a = 25 - v^2 \Rightarrow \dfrac{dv}{dt} = 25 - v^2 \Rightarrow \dfrac{dv}{25 - v^2} = dt$ $\text{At } t = 0, v = 0 \Rightarrow \int_0^v \dfrac{dv}{25 - v^2} = \int_0^t dt \Rightarrow t = \dfrac{1}{10} \ln\left(\dfrac{5 + v}{5 - v}\right)$ $\therefore \text{ at } v = 4\ \text{m/s}, t = \dfrac{1}{10} \ln\left(\dfrac{5 + 4}{5 - 4}\right) = 0.2197\ \text{s}$ $\text{Using } a = v\dfrac{dv}{dx} \Rightarrow v dv = (25 - v^2) dx \Rightarrow \dfrac{v dv}{25 - v^2} = dx$ $\text{At } v = 0, x = 0 \Rightarrow \int_0^v \dfrac{v dv}{25 - v^2} = \int_0^x dx \Rightarrow x = -\dfrac{1}{2} \int_0^v \dfrac{-2v dv}{25 - v^2}$ $x = -\dfrac{1}{2} [\ln(25 - v^2)]_0^v = -0.5 \left[\ln \dfrac{(25 - v^2)}{25}\right]$ $\therefore \text{ at } v = 4\ \text{m/s}, x = -0.5 \left[\ln \dfrac{(25 - 4^2)}{25}\right] = 0.5108\ \text{m}$	10 **1** **2** **1** **1** **2** **2** **1**