Experiment: 3

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1. Aim/Overview of the practical:

2D rigid-body transformations; Yaw; Pitch; Roll

2. Task to be done:

Discuss about the 2D rigid-body transformations in detail. In Addition, implement as well as write down the steps to perform the Yaw, Pitch and Roll transformations.

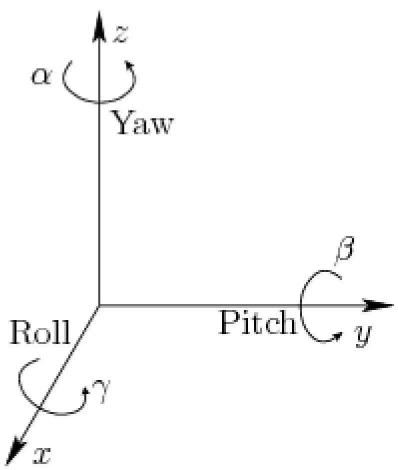
3. Theory:

Transformation Matrix:
The real-world Euclidean distance between any two coordinate positions must stay unaffected by the transformation in the 2D rigid body model. Because the AIR package allows for anisotropic voxel sizes inside a single file as well as varied voxel sizes between files, several considerations must be made while performing a 2D rigid body transformation. The 2D rigid body model in the AIR package is parameterized as a rotation around the z-axis and translations along the x- and y-coordinate axes. To make these parameters more intuitive, the rigid body transformation's rotations are defined as occurring around the centers of the files rather than the internal coordinate system's origin.

The product of a series of homogeneous transformation matrices is best described as the 2D rigid body transformation for transforming from an internal coordinate in the standard file to the matching internal coordinate in the reslice file:

(reslice file internal coordinates)=Zr*Cr*T*R*P*Cs*Zs*(standard file internal coordinates)

Yaw, Pitch and Roll Rotations:



A three-dimensional body can be rotated around three orthogonal axes, as indicated in the diagram above. These rotations will be referred to as yaw, pitch, and roll in aviation terminology.

Yaw:
A yaw is a counter-clockwise rotation of about the -axis. The rotation matrix is given by:

$$R_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Pitch:
A pitch is a counterclockwise rotation of about the -axis. The rotation matrix is given by:

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}.$$

Roll:
A roll is a counterclockwise rotation of about the -axis. The rotation matrix is given by:

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}.$$

A 3D body can be positioned in any orientation using the yaw, pitch, and roll rotations. By multiplying the yaw, pitch, and roll rotation matrices, a single rotation matrix may be obtained.

$$R(\alpha,\beta,\gamma) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) =$$

$$\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}.$$

Determining yaw, pitch, and roll from a rotation matrix:
It is often convenient to determine the α, ß and Γ parameters directly from a given rotation matrix.

For an arbitrary rotation matrix:

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Solving for each angle yields:

$$\alpha = \tan^{-1}(r_{21}/r_{11}),$$

$$\beta = \tan^{-1}\left(-r_{31}/\sqrt{r_{32}^2 + r_{33}^2}\right),$$

$$\gamma = \tan^{-1}(r_{32}/r_{33}).$$

For the inverse tangent functions, there are four quadrants to choose from. The numerator and denominator of the argument should be used to select each quadrant. The denominator determines whether the direction is to the left or right of the y-axis, while the numerator determines whether the direction is above or below the x-axis.

Also, this expands the range of arctangent to (0, 2π)
This can be applied to express as:

$$\alpha = 2(r_{21}, r_{11}),$$

$$\beta = 2\left(-r_{31}, \sqrt{r_{32}^2 + r_{33}^2}\right),$$

$$\gamma = 2(r_{32}, r_{33}).$$