Experiment: 2

Student Name: Shaurya Gulati **UID: 18BCS6092**
Branch: AIML **Lab Group: A**
Semester: 8th **Date of Performance: February 17**
Subject Name: AR- Lab **Subject Code: CSF- 484**

1. Aim/Overview of the practical:

Geometric modeling; linear transformations

2. Task to be done:

Discuss about Geometric modeling in detail. In Addition, implement as well as write down the steps to perform the linear transformations.

3. Theory:
Geometric modeling is a branch of applied mathematics and computational geometry that studies methods and algorithms for the mathematical description of shapes.
Geometric modeling is nothing but representing the world on a piece of board by the three coordinates, i.e. we have x-axis going horizontally, y-axis going upward and we have z-axis coming out of a board.
Each point in R^3,$(x,y,z)\ \varepsilon\ W$

On geometric modeling, we study two kinds of models of the world:
1. Stationary Models: The things which do not move. For example, the buildings, the trees or anything that is still i.e. not in motion comes under stationary models. They remain fixed all the time. This model has no space of possible transformations.
2. Non-Stationary Models: The kind of model which consists of the movable objects and this model has a space of possible transformations.

We commonly have modeling choices; while we're making geometric fashions of those our bodies which are withinside the world, whether or not they're desk bound or moveable. 1, the selection is to have what's known as a Solid illustration and the 2d one is what's known as a Boundary illustration
In the case of Solid illustration, we've got 3-dimensional primitives and so, the maximum fundamental description unit we've got represents a few three-d bite of the sector or of the body. And withinside the Boundary illustration case, we've got 2D primitives.

Linear Transformations:

A linear transformation is a feature from one vector area to some other that respects the underlying (linear) shape of every vector area. A linear transformation is likewise referred to as a linear operator or map. The variety of the transformation can be similar to the domain, and whilst that happens, the transformation is referred to as an endomorphism or, if invertible, an automorphism. The vector areas ought to have the equal underlying field.

The defining characteristic of a linear transformation $T:V\to W$ is that, for any vectors v_1 and v_2 in V and scalars aa and bb of the underlying field,

$$T(av_1 + bv_2) = aT(v_1) + bT(v_2)$$

Linear variations are beneficial due to the fact they keep the shape of a vector area. So, many qualitative exams of a vector area this is the area of a linear transformation may, beneath neath sure conditions, routinely keep withinside the photograph of the linear transformation. For instance, the shape straight away offers that the kernel and photograph are each subspaces (now no longer simply subsets) of the variety of the linear transformation.

Let $M = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Expression for T_M.

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y \\ 3x+7y \end{bmatrix} = (x+2y, 3x+7y)$$

$T_M(1,0) = (1,3)$

$T_M(0,1) = (2,7)$

Rigid Body Transformations:

By definition, a rigid body transform is a mapping from this set to another subset of the Euclidean space, such that the Euclidean distances between points are preserved. Any such mapping can be represented as a composition of one translation and one rotation. We are interested in compact and convenient mathematical descriptions of rigid body transforms.

It has 3 cases:

1. Easy- Translation
2. Hard- Rotation
3. Hardest- Rotation and Translation

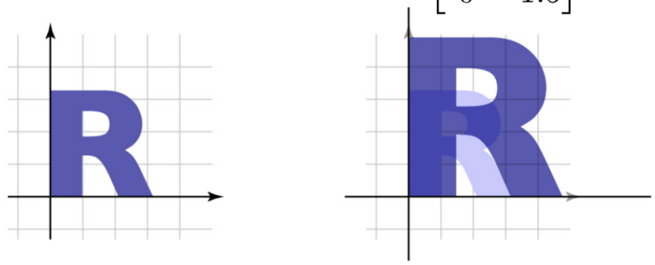
2D Linear Transformations:

2x2 matrices have simple geometric interpretations

- uniform scale

• **Uniform scale** $\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$

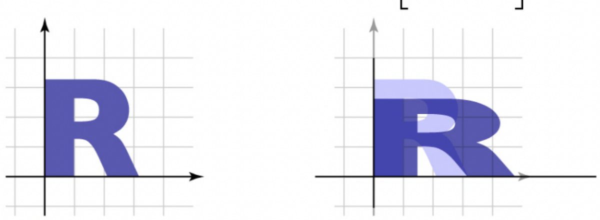
$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$



- non-uniform scale

• **Nonuniform scale** $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$

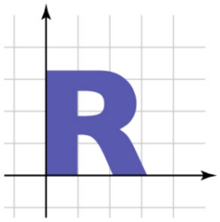
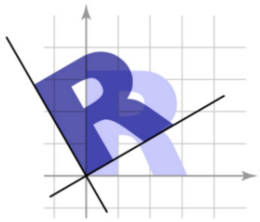
$$\begin{bmatrix} 1.5 & 0 \\ 0 & 0.8 \end{bmatrix}$$



- rotation

- **Rotation** $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$

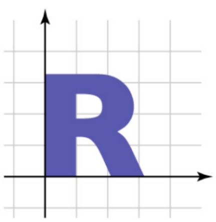
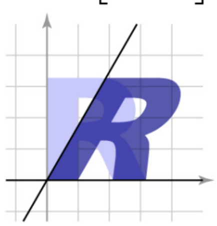
$$\begin{bmatrix} 0.866 & -.05 \\ 0.5 & 0.866 \end{bmatrix}$$

- shear

- **Shear** $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$

- reflection

- **Reflection**
  - can consider it a special case of nonuniform scale

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$