Experiment: WS3

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1. Aim/Overview of the practical:

2D Rigid-Body Transformations; Yaw, Pitch, Roll

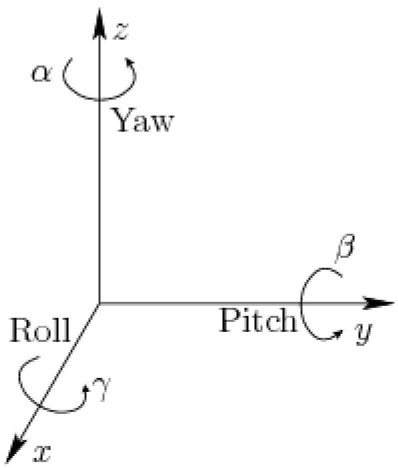
2. Task to be done:

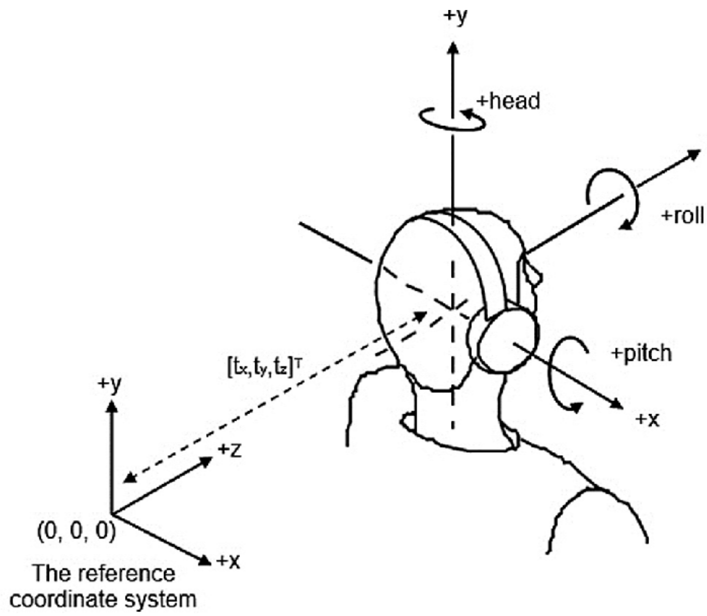
Discuss the 2D rigid-body transformations in detail. In Addition, implement as well as write down the steps to perform the Yaw, Pitch and Roll transformations.

3. Theory:

Rigid Body Transformation:
A rigid body transformation is mapping from one set of points to another subset of Euclidean space such that the Euclidean distance between the points is preserved. Any such transformations can be represented by either translation or rotation or both together.

Yaw, Pitch and Roll Transformations:





A three-dimensional body can be rotated around three orthogonal axes. These rotations will be referred to as yaw, pitch, and roll in aviation terminology. Yaw, Pitch and Roll all have their own rotational matrices that are as follows:

Yaw: A yaw is a counter-clockwise rotation of about the z-axis. The rotation matrix is given by:

$$R_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Pitch: A pitch is a counterclockwise rotation of about the y-axis. The rotation matrix is given by:

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}.$$

Roll: A roll is a counterclockwise rotation of about the x-axis. The rotation matrix is given by:

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}.$$

A 3D body can be positioned in any orientation using the yaw, pitch, and roll rotations. By multiplying the yaw, pitch, and roll rotation matrices, a single rotation matrix may be obtained.

$$R(\alpha,\beta,\gamma) = R_z(\alpha)\,R_y(\beta)\,R_x(\gamma) = \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}.$$

Determining yaw, pitch, and roll from a rotation matrix:

It is often convenient to determine the α, ß and Γ parameters directly from a given rotation matrix.

For an arbitrary rotation matrix such as:

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Solving for each angle yields:

$$\alpha = \tan^{-1}(r_{21}/r_{11}), \qquad \gamma = \tan^{-1}(r_{32}/r_{33}).$$

$$\beta = \tan^{-1}\left(-r_{31}/\sqrt{r_{32}^2 + r_{33}^2}\right),$$

For the inverse tangent functions, there are four quadrants to choose from. The numerator and denominator of the argument should be used to select each quadrant. The denominator determines whether the direction is to the left or right of the y-axis, while the numerator determines whether the direction is above or below the x-axis.

Also, this expands the range of arctangent to (0, 2π).

This can be applied to express as:

$$\alpha = 2(r_{21}, r_{11}),$$

$$\beta = 2\left(-r_{31}, \sqrt{r_{32}^2 + r_{33}^2}\right),$$

$$\gamma = 2(r_{32}, r_{33}).$$