Probability

PROBABILITY

The word probability literally denotes ‘chance’ and the theory of probability deals with the laws governing the chances of occurrence of phenomena which are unpredictable in nature.

Before going to the actual definition of ‘probability’, let us define certain terms associated with probability.

(i) Random experiment- The word experiment is used to describe an act which can be repeated under some given conditions. Random experiments are those experiments whose results depend on chance (the word ‘random’ may be taken as equivalent to ‘depending on chance’).

For example: (a) tossing a coin (or several coins).

(b) Throwing a dice (or several dice).

(c) Drawing cards from a pack etc.

(ii) Out come- The result of a random experiment will be called an outcome. The possible outcomes of a random experiment may often be described in several ways.

For example:

(a) In tossing a coin once, there are two possible outcomes- head and tail or in symbols H and T.

(b) In random experiment of tossing a coin twice (or tossing two coins once simultaneously) there are four possible outcomes namely HH,HT,TH and TT.

where the two letters indicate the results of the first and second tosses(or coins) respectively. If we are interested in finding the number of heads turned up, then the possible outcomes may be stated as two heads, one head, and zero head.

(iii) Event-In the theory of probability, the term ‘event’ is used to denote any phenomenon which occurs in arandom experiment. One or more out-comes are said to constitute an event.

An event is said to be elementary if it cannot be decomposed into simpler events.

An event is said to be composite event if it is an aggregate of several elementary events.

For example:

(a) when a die is thrown, there are six possible outcomes 1,2,3,4,5,6. Now the aggregate of three outcomes 1,3,5 may be set to form an event ‘odd numbers’. Also the event ‘at least five’ is to aggregate of two outcomes 5,6 etc.

(b) When a coin is tossed we may speak of the event ‘Head’ and ‘Tail’ each of which is an elementary event.

When two coins are tossed, the event ‘both heads’ is an elementary event. But, ‘one head and one tail’ is a composite event consisting of elementary event HT and TH.

(iv)Mutually exclusive- Events are said to be ‘mutually exclusive’ when two or more of themcan’t occurs simultaneously. It means that mutually exclusive events can occur only one at a time and occurrence of any event signifies impossibility of the remaining events in any particular performance of the random experiment.

For example:

(a) When a coin is tossed, the elementary events are ‘H’ and ‘T’ which are mutually exclusive because ‘H’ implies non occurrence of ‘T’ i.e. they cannot appear simultaneously in any single toss.

(b) When two coins are tossed, let

A_1 denote the event HH

A_2 denote the event HT

A_3 denote the event TH

A_4 denote the event TT

These four events are mutually exclusive because the result of any toss yields either A_1 or A_2 or A_3 or A_4, because the result cannot be, for instance, HH as well as TH.

Similarly, in this experiment let

B_1 denote the event 'two heads'

B_2 denote the event 'one head'

B_3 denote the event 'no head'

Then these events are also mutually exclusive.

(v) Exhaustive- Several events are said to form and exhaustive set if at least one of them must necessarily occur. The complete group of all possible elementary events of a random experiment gives an exhaustive set of events.

For example:

(a) In tossing a single coin, the events Head and Tail form an exhaustive set, because one of these two must necessarily occurs.

(b) In the example cited above (i.e. (iv)(b)), the groups of events (A_1, A_2, A_3, A_4)and (B_1, B_2, B_3, B_4) form exhaustive set.

(c)When a dice is thrown, let

A denote the event 'odd no. of points'

B denote the event 'at least two points'

Here these two events form an exhaustive set because appearance of A or B describes all the possible outcomes. But they are not 'mutually exclusive' because occurrence of event A does not necessarily imply non-occurrence of event B. For instance occurrence of five of event A implies the occurrence of 'at least two points' of event B.

(vi) Trial- Any particular performance of a random experiment is usually called a trial.

(vii) Cases favourable to an event- Among all possible outcomes of arandom experiment, those cases which entail occurrence of an event A are called 'cases favourable to A'.

1,2,3,4,5,6. Among those three cases (i.e. 1,3,5) are favourable to the event 'odd number of points' and 3 cases (i.e. 2,4,6) are favourable to the event 'even number of points'.

(viii) Certain event-The certain event is the event that occurs whenever anyone of the outcomes occurs.

(ix) Equally Likely –The outcomes of a random experiment are said to be equally likely if after taking into consideration all relevant evidences, none of them can be expected in preference to another.

For example:

Out of a full pack of 52 cards, two cards may be drawn in $^{52}C_2 = \frac{52*51*50!}{2!*(52-2)!} = \frac{52*51*50!}{2!*50!} = 1326$ ways and the drawing of a pair of cards gives rise to an outcome. Since drawing of two cards will reveal any of these 1326 possible combinations, the outcomes equally likely.

(x) <u>Sample space</u>- The whole set of elementary events is called the sample space.

<u>Classical definition of probability</u>

If a random experiment has 'n' possible outcomes, which are mutually exclusive, exhaustive, 'equally likely' and 'm' of these are favourable to an event A, then the probability of the event A,represented by P(A), is defined as the ratio $\frac{m}{n}$.In symbol$P(A) = \frac{m}{n}$.

i.e. probability of an event=

$$\frac{no.of\ outcomes\ favourable\ to\ the\ event}{total\ no\ of\ mutually\ exclusive,\ \ exhaustive\ and\ \ equally\ likely\ outcomes\ of\ the\ event}$$

But here we are restricted in two respects :

(i) Sample space is finite so that the treatment is not applicable to the case where the no. of elementary event is infinite.

(ii) The experiment is s.t. all the elementary events are equally likely.

Examples:

1. Two cards are drawn from a full pack of 52 cards. Find the probability that

(i) both are red cards,

(ii) one is heart and the other is a diamond.

Solution:

(i) Total no. cards=52 and out of these 52 cards we can choose two cards (irrespective of the colour) in $^{52}C_2$=$\frac{52!}{2!*(52-2)!} = \frac{52*51}{2} = 1326$.

Again total no of red cards = 26 and from these 26 cards we can draw 2 red cards in $^{26}C_2$ ways.

i.e. $\frac{26!}{2!*(26-2)!} = \frac{26*25}{2} = 325$ ways

i.e. total no of possible outcomes (n)=1326 and no of outcomes favourable to the event 'both are red cards' (m) =325.

Therefore, Pr.(both red cards)= $\frac{325}{1326} = \frac{25}{102}$

(ii) One is heart (red) and the other is diamond (red).

Total no. of possible outcome (n)=1326.

No. of cards with symbol heart =13.

Therefore, out of these 13 heart cards we can draw one heart card in $^{13}C_1$ ways=13 ways.

Again no. of diamond cards is also 13.

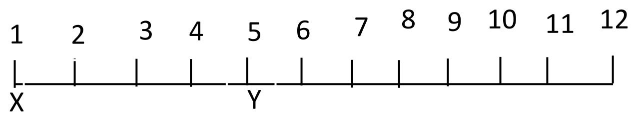
Therefore, one diamond card can be drawn from these 13 cards in $^{13}C_1$ways=13 ways.

No. of outcomes favourable to the event that one is heart and the other is diamond is (m)=13x13.

Therefore, Pr(one heart and other diamond)=$\frac{13*13}{1326} = \frac{13}{102}$.

2.X and Y stand in a line at random with 10 other people. What is the probability that there are 3 people between X and Y.

Solution:



Total no. of people=12.

X and Y are stand in such a manner that there are always 3 people between X and Y.

Hence total possible arrangements =12!

Now fixing X and Y say in position 1 and 5 so that three people stand between them, the remaining 10 people (except X and Y) can be arranged in 10! ways. Since X and Y can interchange among themselves, for this particular position at 1 and 5 of X and Y(or Y and X) we have 2x10! outcomes.

Again among the total no. of arrangements only those cases will be favourable to the event when X and Y occupy the following points of places: (1, 5), (2, 6), (3, 7), (4, 8), (5, 9), (6, 10), (7, 11), (8, 12).

Therefore, no. of arrangements favourable to the event is m=$\frac{8*(2*10!)}{12!} = \frac{4}{33}$.