Knowledge Representation and Reasoning (KRR-3)

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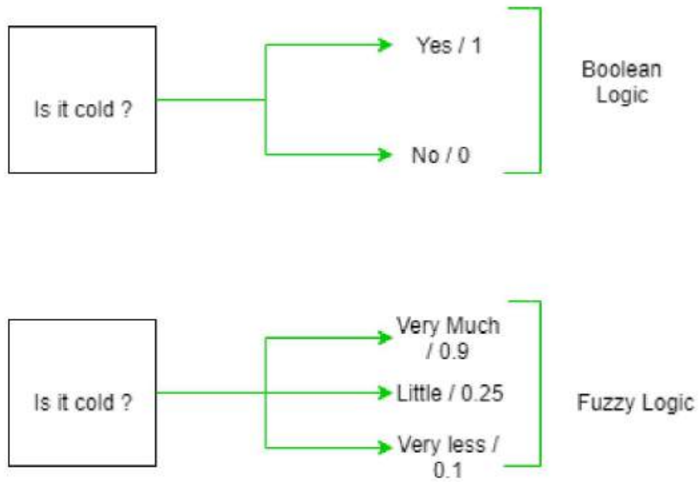
Knowledge Representation

- Propositional Logic
- Predicate Logic (chapter 8)
- Semantic Networks
- Frames
- **Fuzzy Logic (FL)**

Fuzzy Logic

- A logic based on the two truth values ***True*** *and* ***False*** is sometimes inadequate when describing human reasoning.
- Fuzzy logic uses the whole interval between 0 (False) and 1 (True) to describe human reasoning.
- A form of many-valued logic in which the truth values of variables may be any real number between 0 and 1 both inclusive.
- The concept was introduced by Lotfi Zadeh (the University of California) in 1965.



What is Fuzzy Logic?

- Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning.
- The approach of FL imitates the way of decision making in humans that involves all intermediate possibilities between digital values YES (1) and NO (0).
- Fuzzy logic is determined as a set of mathematical principles for **knowledge representation** based on degrees of membership and degrees of truth.
- **Fuzzy logic** is a method for reasoning with logical expressions describing membership in fuzzy sets. For example, the complex sentence **Tall(Tom) ∧ Heavy(Tom)** has a fuzzy truth value that **is a function** of the truth values of its components. The **standard rules** for evaluating the **fuzzy truth, T,** of a complex sentence are:

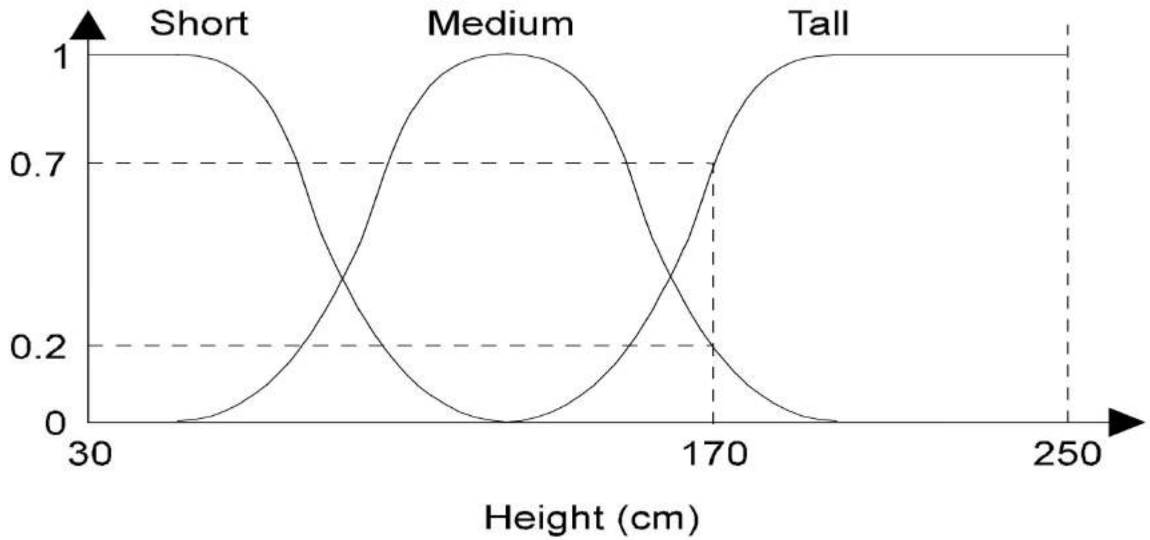
  $T(A \wedge B) = \mathbf{min}(T(A), T(B))$, $T(A \vee B) = \mathbf{max}(T(A), T(B))$, $T(\neg A) = 1 - T(A)$.

Fuzzy Logic…

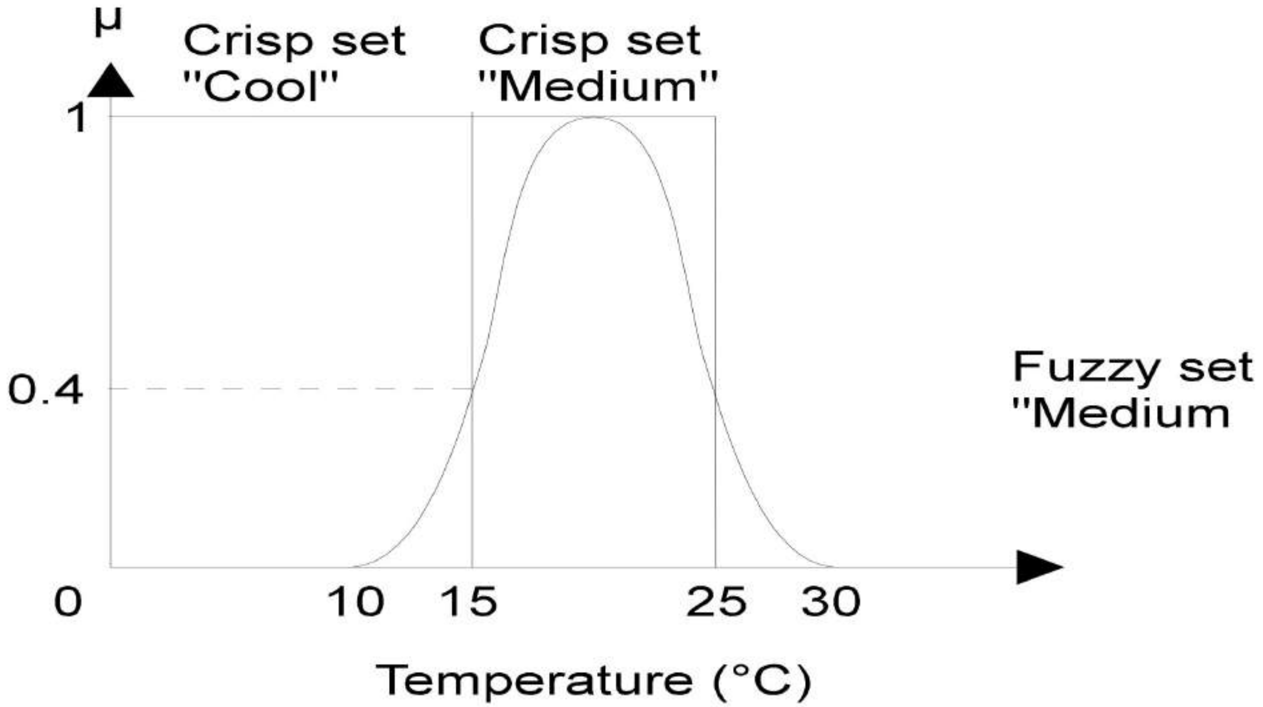
- *Fuzzy sets*
- *Fuzzy input and output variables*
- *Fuzzy rules*
- *Fuzzy inference mechanism*

Fuzzy Set

- A fuzzy set can be defined as a set with fuzzy boundaries.
- Example: For person's "height", the fuzzy set can be defined as: {'short', 'average', 'tall'}.
- To represent a fuzzy set in a computer, we express it as a function and then map the elements of the set to their degree of membership.

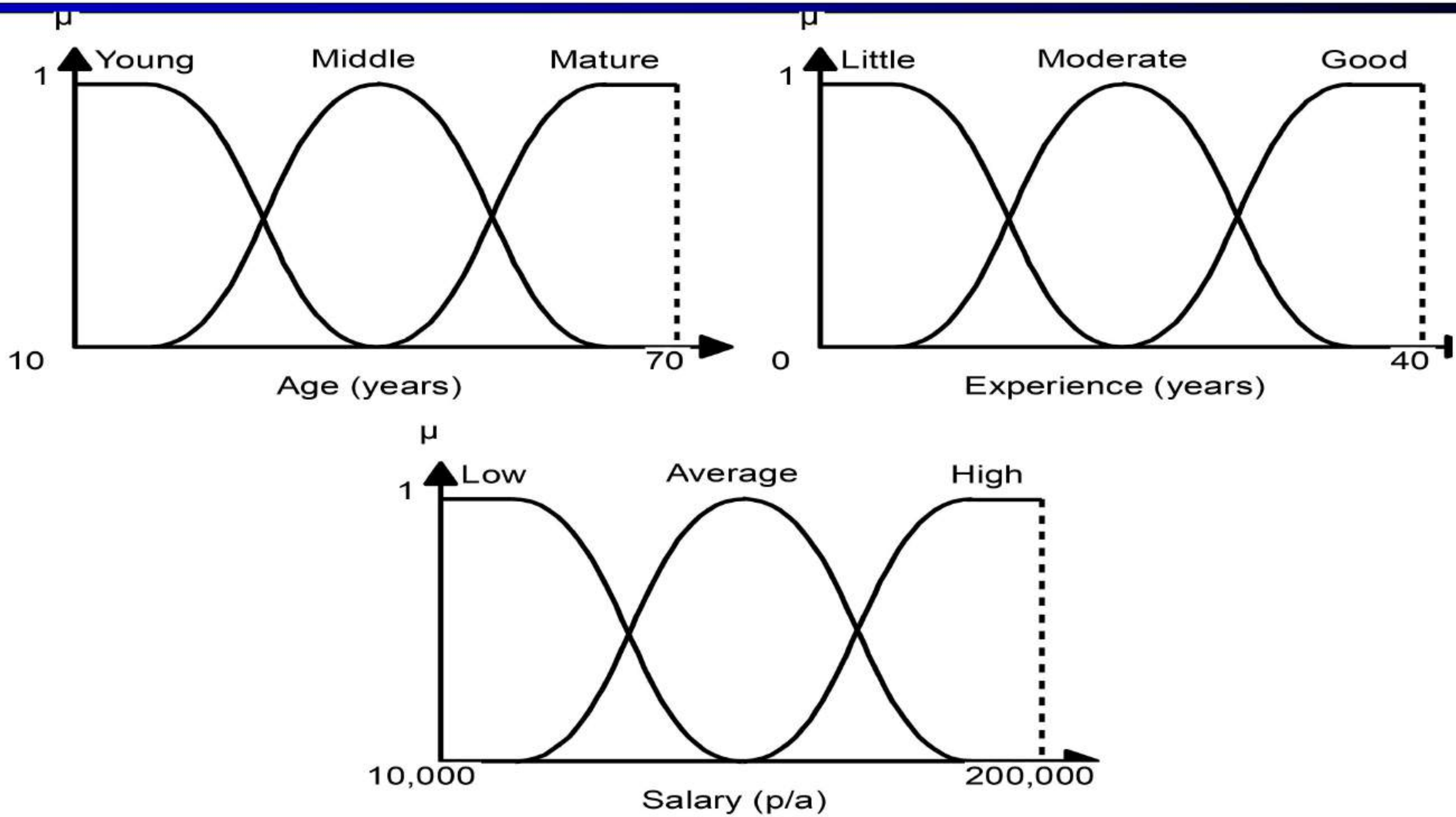


Fuzzy Systems

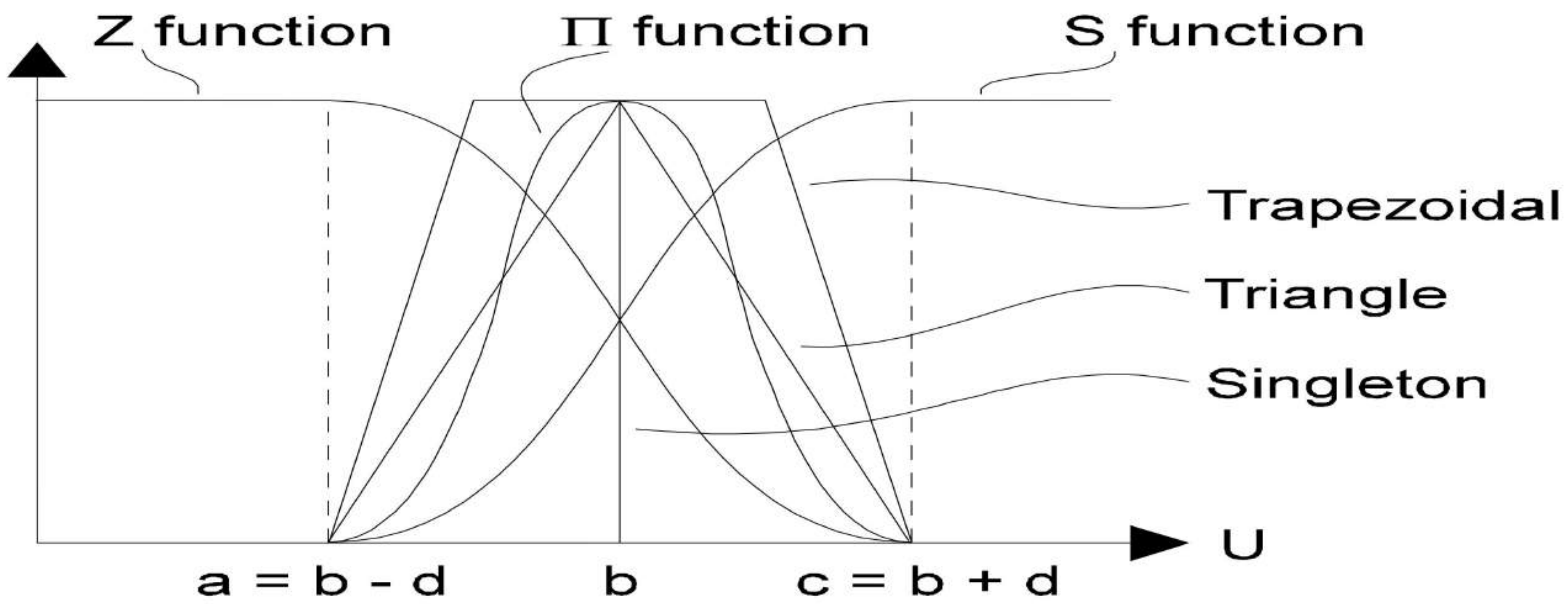


- *Representing crisp and fuzzy sets as subsets of a domain (universe of discourse) U.*

Membership Functions



Standard Membership Functions

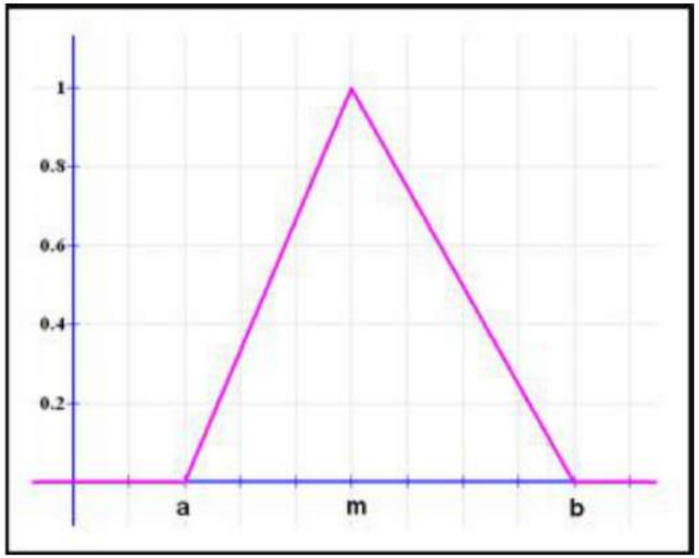


- ***Most commonly used shapes of membership functions:*** *triangular, trapezoidal, piecewise linear, and Gaussian.*

Standard Membership Functions

Triangular function: defined by a lower limit **a**, an upper limit **b**, and a value **m**, where **a < m < b**.

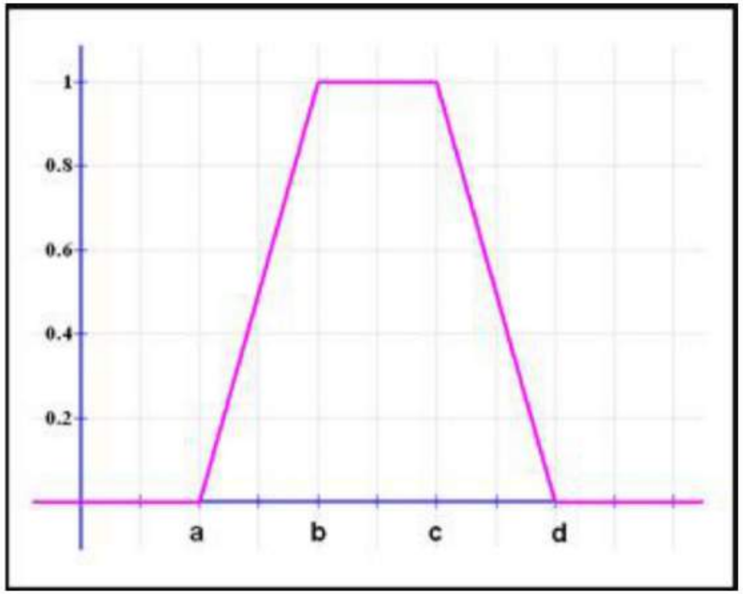
$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \dfrac{x-a}{m-a}, & a < x \leq m \\ \dfrac{b-x}{b-m}, & m < x < b \\ 0, & x \geq b \end{cases}$$



Standard Membership Functions

Trapezoidal function: defined by a lower limit **a**, an upper limit **d**, a lower support limit **b**, and an upper support limit **c**, where **a < b < c < d**.

$$\mu_A(x) = \begin{cases} 0, & (x < a) \text{ or } (x > d) \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \end{cases}$$

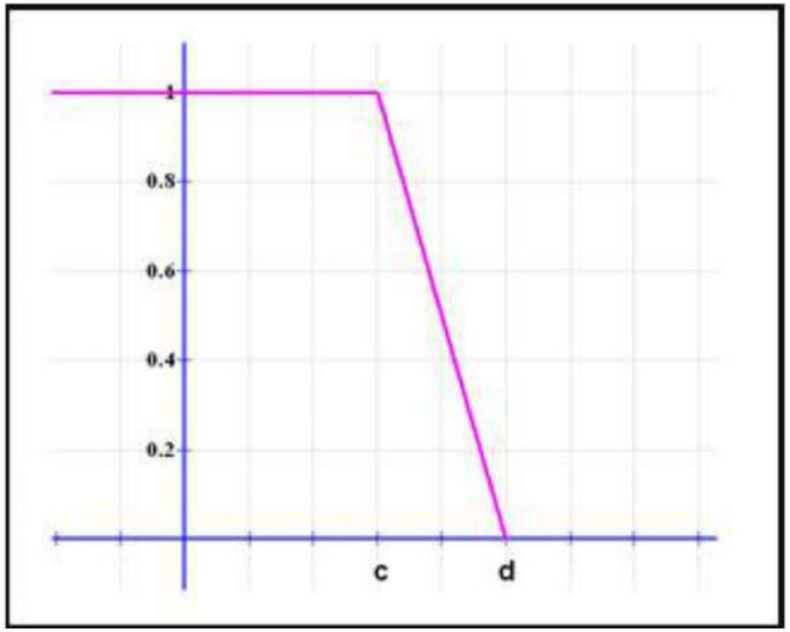


Standard Membership Functions

There are two special cases of a trapezoidal function, which are called R-functions and L-functions:

R-functions: with parameters a = b = - ∞

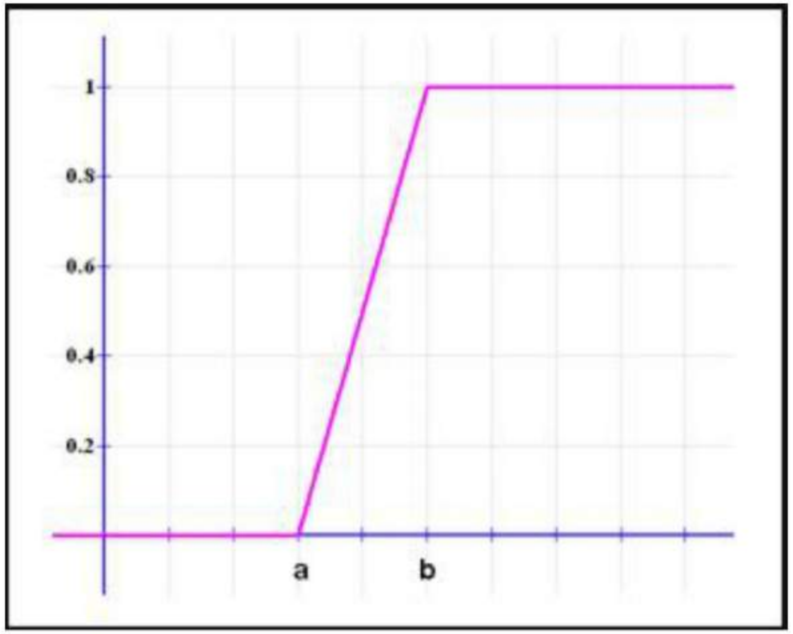
$$\mu_A(x) = \begin{cases} 0, & x > d \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 1, & x < c \end{cases}$$



Standard Membership Functions

L-Functions: with parameters c = d = + ∞

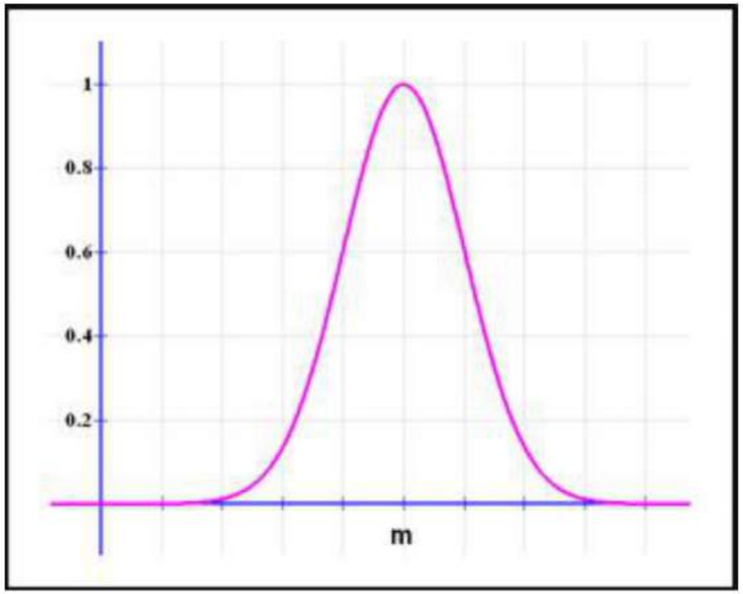
$$\mu_A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$$



Standard Membership Functions

Gaussian function: defined by a central value **m** and a standard deviation **k > 0**. The smaller k is, the narrower the "bell" is.

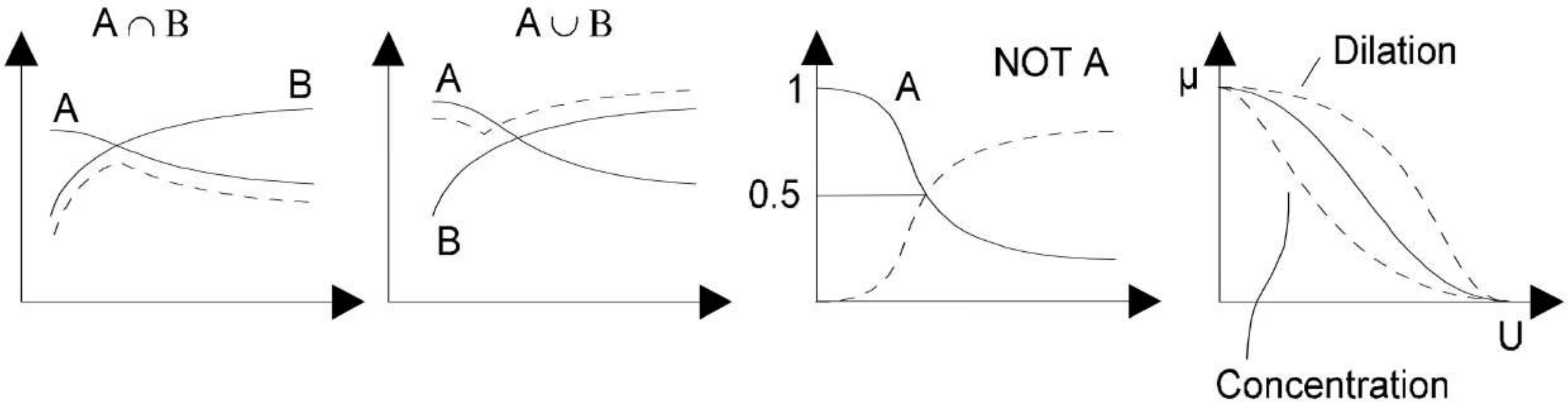
$$\mu_A(x) = e^{-\frac{(x-m)^2}{2k^2}}$$



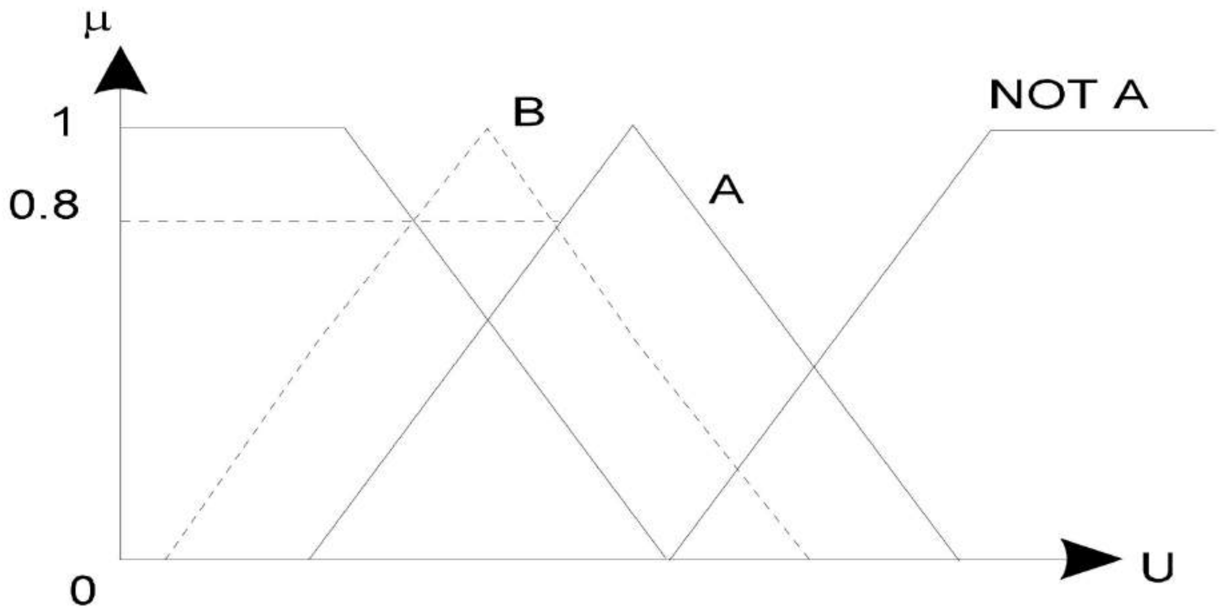
Operations on Fuzzy Sets

- Operations on fuzzy sets is done by means of their membership functions. Let *A* and *B* be fuzzy sets on a mutual universe.

1. Intersection of *A* and *B* is: $A \cap B =$	= *A* **min** *B*, corresponding items in *a* and *b*.
2. Union of *A* and *B* is: $A \cup B =$	= *A* **max** *B*, corresponding items in *a* and *b*.
3. Complement of A is: A =	= 1 - A
4. Dilation (increases the degree of membership of all members by spreading out the curve) of A is:	=DIL (A) = $(u_A(x)^{1/n})$ for all x in U
5. Concentration (decreases the degree of membership of all members) of A is:	=CON (A) = $(u_A(x)^{n})$ for all x in U



Operations with Fuzzy Sets..



- A graphical representation of **similarity S**, between **two fuzzy sets B and A** based on possibility P and necessity N measures (see the formulas in the text).

Linguistic Variables and Hedges

- *A linguistic variable is a fuzzy variable.* A linguistic variable takes **words or sentences** as values.
- For example, the fuzzy variable ‘temperature’, might have values as ‘hot’, ‘medium’, and ‘cold’.
- **Example1:** Let **x** be a linguistic variable with the label **age**. Terms of this linguistic variables, which are fuzzy sets, could be: “old”, “young”, “very old” from the term set
- *T={Old, VeryOld, NotSoOld, MoreOrLessOld, QuiteOld, Young, VeryYoung, NotSoYoung, etc}*
- If *wind* is strong Then sailing is **good**
- If *project duration* is long Then completion risk is **low**
- If *speed* is slow Then stopping distance is **short**

Linguistic Variables and Hedges

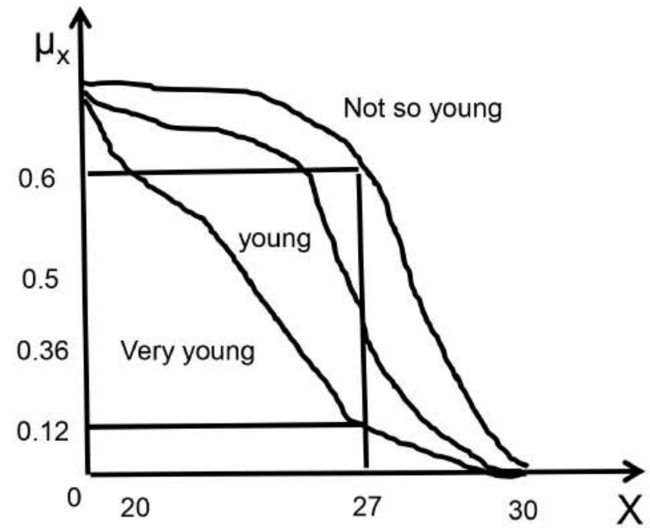
- *A linguistic **hedge** or modifier is an operation that modifies the meaning of a term or fuzzy set.*
- *Hedges are some word that modify the linguistic variable of a fuzzy set.*
- *For example, if **weak pressure** is a **fuzzy set**, then:*
- *{very weak pressure, more-or-less weak pressure, extremely weak pressure, and not so-weak pressure}*

  are examples of hedges which are applied to this fuzzy set.

Fuzzy Modifier..

- A linguistic modifier is an operation that modifies the meaning of a term. For example, in the sentence, **very close to 0**, the word *very* modifies ***Close to 0*** which is a fuzzy set. Examples of other modifiers are a little, more or less, possibly, definitely.

Very a = a^2	Somewhat a = More or Less a = Not so a = $a^{1/2}$
Extremely a = a^3	Little bit a = $a^{1.3}$ Slightly a = $a^{1/3}$
Very Very a= a^4	



- Young = [1 0.36 0.01 0   0]
- Very very young = young^4 = [1   0.13   0   0   0]

Normalization:

- A fuzzy set is normalized if its largest membership value equals 1.
- A fuzzy set can be normalize by dividing each membership value by the largest membership value in the set, a/max(a).

Psychological Continuum

- *Very young, young, not so young, old, very old, not so old -------> derived from young and old.*
- *Pressure ----->Strong, low, Okay, high (100-2000 Pascal)*
- *Bitter, sweet, sour, salt, hot, ...*

Fuzzy systems...

- *A simple fuzzy rule for the smoker and the risk of cancer case example.*

Rule: IF a person is a "heavy_smoker"
THEN the risk of cancer is "high",

where the two fuzzy concepts "heavy-smoker" and "high" can be represented by their membership functions, for example:

A fuzzy concept "heavy-smoker":

No.of cigarettes per day	0	2	4	6	8	10
Grade (membership)	0	0.1	0.6	0.8	0.9	1.0

A fuzzy concept: "High risk of cancer"

Level of risk:	1	2	3	4	5
Grade (membership)	0.0	0.2	0.7	0.9	1.0

Fuzzy Inference and Rules

Fuzzy Inference: The process of reasoning based on Fuzzy logic.
Example:

If the power transformer is slightly overloaded,
Then keep this load for a while.

Fuzzy rule: A conditional statement in the form:

IF x is A THEN y is B, where x and y are linguistic variables, and A and B are linguistic values determined by fuzzy sets.

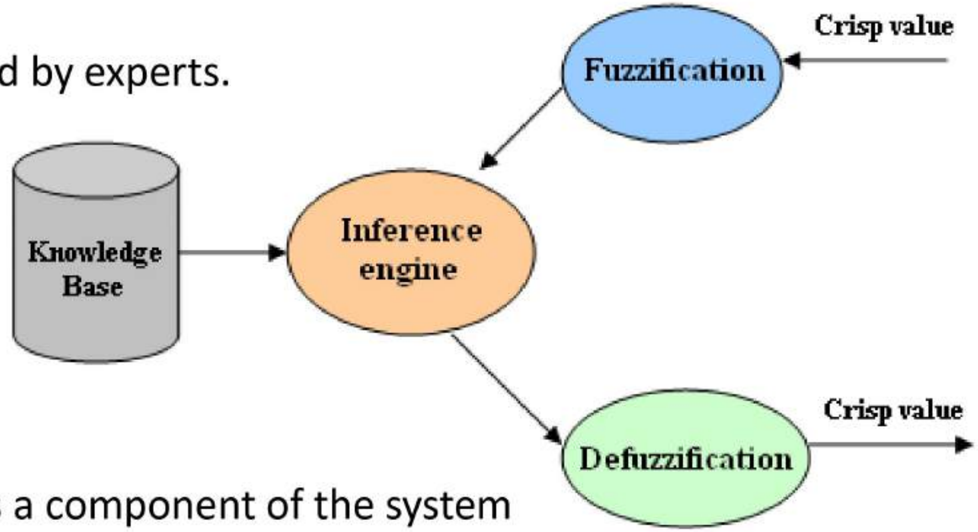
- ***Rule 1***: *IF (GPA is high) and (Exam is satisfactory) and (Approach is Smart) then (Decision is Select)*
- ***Rule 2***: *IF (GPA is low) and (Exam is bad) or (Approach is Stupid) then (Decision is Reject)*

Structure of Fuzzy Inference System

In general, a fuzzy inference system consists of **four modules**:

1. Fuzzification Module: Transforms the system inputs, which are crisp numbers, into fuzzy sets. This is done by applying a fuzzification function.

2. Knowledge Base: (KB) Stores IF-THEN rules provided by experts.



3. Inference Engine: Simulates the human reasoning process by making fuzzy inference on the inputs and IF-THEN rules.

In the field of artificial intelligence, inference engine is a component of the system that applies logical rules to the knowledge base to deduce new information.

4. Defuzzification Module: Transforms the fuzzy set obtained by the inference engine into a crisp value.

Fuzzy Rules and Fuzzy Inference

- *Inputs to a fuzzy system can be:*
  - fuzzy, e.g. (Score = Moderate), defined by membership functions
  - exact, e.g.: (Score = 190); (Theta = 35), defined by crisp values.

- *Outputs from a fuzzy system can be:*
  - fuzzy, i.e. a whole membership function, or
  - exact, i.e. a single value is produced on the output.

Mamdani-Style Inference

Four steps: fuzzification, rule evaluation, aggregation of the rules, defuzzification

Rule 1: IF x is A3 OR y is B1 THEN z is C1	**Rule 1:** IF project_funding is adequate OR project_staffing is small THEN risk is low
Rule 2: IF x is A2 AND y is B2 THEN z is C2	**Rule 2:** IF project_funding is marginal AND project_staffing is large THEN risk is normal
Rule 3: IF x is A1 THEN z is C3	**Rule 3:** IF project_funding is inadequate THEN risk is high

Fuzzy Expert System

- An expert system that uses fuzzy logic instead of Boolean logic.
- A fuzzy expert system is a collection of fuzzy rules and membership functions that are used to reason about data.

Problem

- **A 4-person family wants to buy a house. An indication of how comfortable they want to be is the no. of bedrooms in the house. But they also want a large house.**

Let u={1,2,3,4,5,6,7,8,9,10} be the set of available houses described by their number of bedrooms. Then the fuzzy set C (for comfortable) may be described as

C=[0.2 0.5 0.8 1 0.7 0.3 0 0 0 0]

Let, L be the fuzzy set large defined as:

L= [0 0 0.2 0.4 0.6 0.3 1 1 1 1]

The intersection of Comfortable and Large is then

=[0 0 0.2 0.4 0.6 0.3 0 0 0 0]

To interpret this, five bedrooms is optimal, but only satisfactory to the grade of 0.6. The second best solution is four bedrooms.

The union of Comfortable and Large is:

[0.2 0.5 0.8 1 0.7 0.8 1 1 1 1]

Here, four bedrooms is fully satisfactory because it is comfortable, and 7-10 bedrooms also, because that would mean a large house.

These properties are important, because **they help to predict the outcome** of long sentences.

Applications of Fuzzy Logic

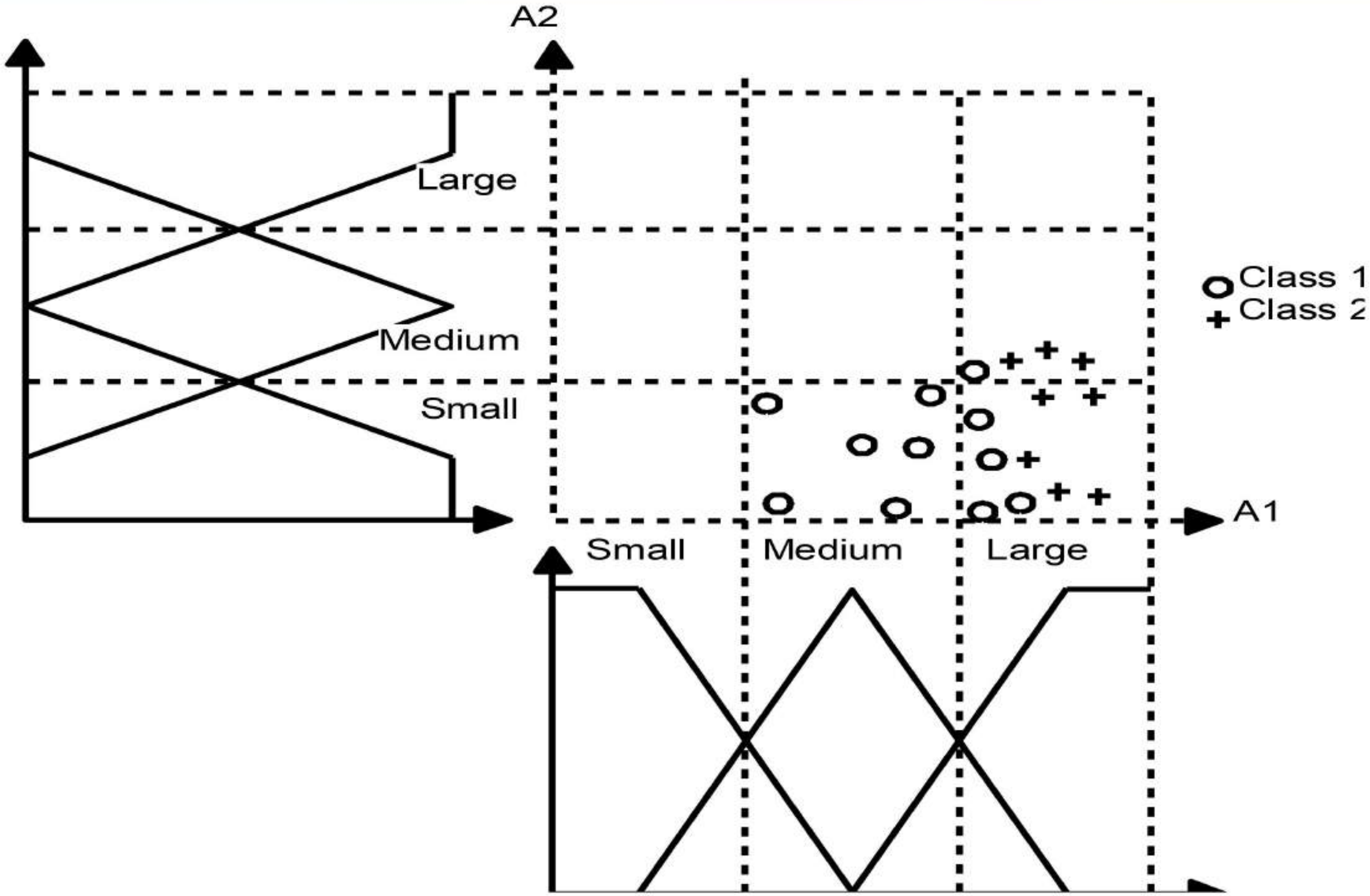
Control Applications: Aircraft control, Sendai subway operation (Hitachi), Cruise control (Nissan), Automatic Transmission (Nissan, Sabaru), Self parking model car (Tokyo University), Space Shuttle docking (NASA)

Scheduling and Optimization:
Elevator Scheduling (Hitachi, Fujitsu, Mitsubishi)

Stock Market Analysis: Yamichi Securities

Signal Analysis for Tuning and Interpretation:
TV picture adjustment (Sony Corporation)
Handwriting Recognition: Sony Palmtop computer
Video Camera Autofocus: Sony and Canon
Video Image Stabilizer: Matsushita, Panasonic

Pattern Recognition and Classification



Fuzzy System Applications

- Pattern recognition and classification
- Fuzzy clustering
- Image and speech processing
- Fuzzy systems for prediction
- Fuzzy control
- Monitoring
- Diagnosis
- Optimization and decision making
- Group decision making

Acknowledgement

- AIMA = Artificial Intelligence: A Modern Approach by Stuart Russell and Peter Norving (3rd edition)
- U of toronto
- Other online resources

Thank You