Assignment 1

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Section: A

** $L\{(te^t \sin t)''\}$

we know that $L\{\sin t\} = \frac{1}{s^2+1}$

$$L\{t \sin t\} = (-1)^1 \frac{d}{ds} \frac{1}{s^2+1}$$

$$= (-1) \frac{d}{ds} (s^2+1)^{-1}$$

$$= (s^2+1)^{-2} \cdot 2s = \frac{2s}{(s^2+1)^2}$$

$$L\{e^t t \sin t\} = \frac{2(s-1)}{\{(s-1)^2+1\}^2}$$

$$= \frac{2s-2}{(s^2-2s+1+1)^2}$$

$$= \frac{2s-2}{(s^2-2s+2)^2} = f(s)$$

Now Let $F(t) = te^t \sin t$

$F(0) = 0$

$F''(t) = t(e^t \sin t + e^t \cos t) + e^t \sin t$

$F'(0) = 0$

$$\because L\{F''(t)\} = s^2 f(s) - sF(0) - F'(0)$$

$$= s^2 \frac{2s-2}{(s^2-2s+2)^2} - 0 - 0$$

$$= \frac{2s^3-2s^2}{(s^2-2s+2)^2}$$

$$= \frac{2(s^3-s^2)}{(s^2-2s+2)^2} \text{ (Ans)}$$

** $L\left\{\left[\frac{1}{t}(1-\cos t)\right]'\right\}$

we can write that

$$L\{1-\cos t\}$$

$$=\frac{1}{s}-\frac{s}{s^2+1}$$

Let, $s^2+1=z$

$2s=dz$

$s=\frac{dz}{2}$

$$\therefore L\left\{\frac{1}{t}(1-\cos t)\right\}=\int_s^\infty\left(\frac{1}{s}-\frac{s}{s^2+1}\right)ds$$

$$=\left[\log s-\frac{1}{2}\log(s^2+1)\right]_s^\infty$$

$$=\log\infty-\frac{1}{2}\log\infty-\log s+\frac{1}{2}\log(s^2+1)$$

$$=\infty-\infty+\frac{1}{2}\log(s^2+1)-\log s$$

$$=0+\frac{1}{2}\log(s^2+1)-\log s$$ [using L-Hospital rule]

$$=f(s)$$

Let $F(t)=\frac{1}{t}(1-\cos t)$

$$F(0)=\frac{1}{0}-\frac{1}{0}=0$$

$$\therefore L\{F'(t)\}=s.f(s)-F(0)$$

$$=s.\left(\frac{1}{2}\log s^2+1-\log s\right)-0$$

$$=s.\log\sqrt{s^2+1}-\log s$$

$$=s.\log\left(\frac{\sqrt{s^2+1}}{s}\right)$$

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** $L\left\{\left(\frac{e^{-t}\sin t}{t}\right)''\right\}$

we know that $L\{\sin t\} = \frac{1}{s^2+1}$

$\because L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1}\,ds$

$= \left[\tan^{-1} s\right]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1} s = \frac{\pi}{2} - \tan^{-1} s$

$= \cot^{-1} s$

$\because L\left\{\frac{e^{-t}\sin t}{t}\right\} = \cot^{-1}(s+1)$

$= f(s)$

Let $F(t) = \frac{e^{-t}\sin t}{t}$

$F(0) = \frac{e^{-0}.\sin 0}{0} = 1$ [Applying L-Hopital rules]

$F'(t) = \frac{1}{t}(-e^{-t}\cos t - e^{-t}\sin t) + e^{-t}\sin t\,(t^{-1-1})$

$= \frac{1}{t}(-e^{-t}\cos t - e^{-t}\sin t) - \frac{e^{-t}\sin t}{t^2}$

$F'(0) = \frac{1}{0}(-e^{-0}\cos 0 - e^{-0}\sin 0) - \frac{e^{-0}\sin 0}{0^2}$

$= 0 - 1 = -1$

$\because L\{F''(t)\} = s^2 f(s) - s.F(0) - F'(0)$

$= s^2\cot^{-1}(s+1) - s + 1$

$= s^2\cot^{-1}(s+1) - s + 1$

Ans.