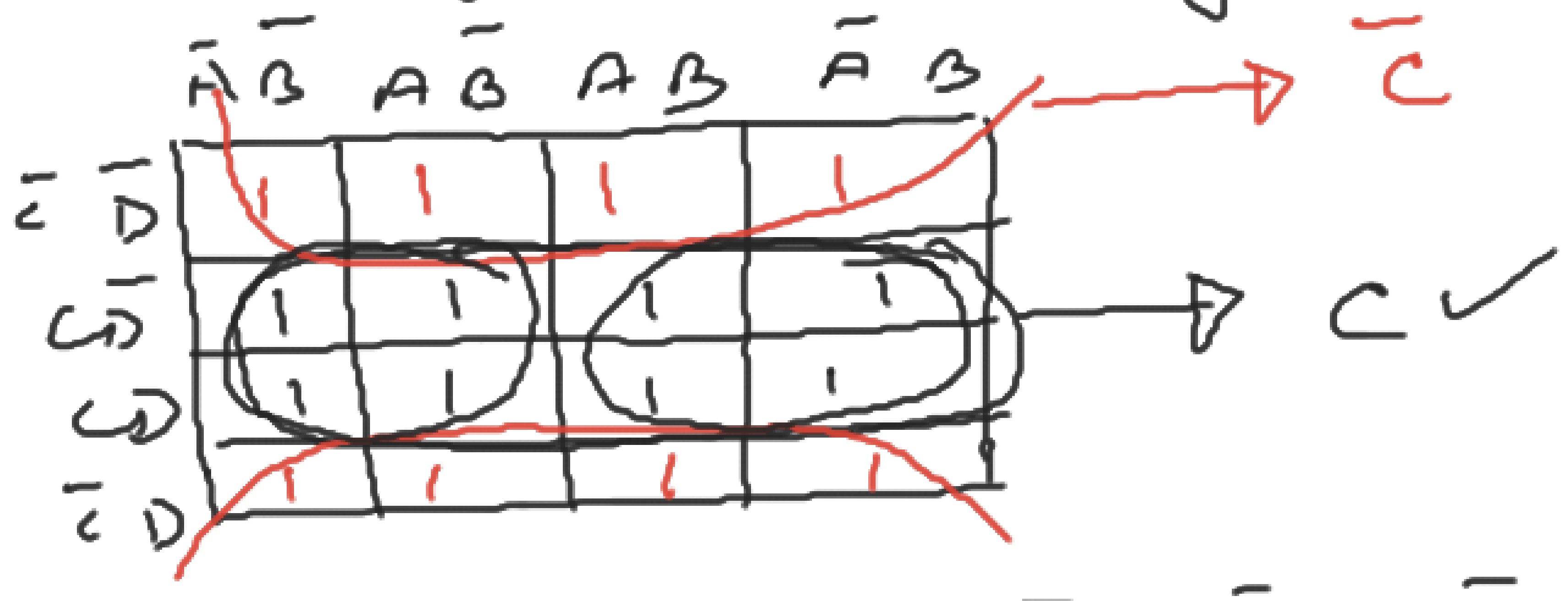
3) Octec $\rightarrow$ 8 adjacent '1's



$\bar A\bar B C\bar D + A\bar B C\bar D + ABC\bar D + \bar A BC\bar D + \bar A\bar B CD + A\bar B CD + ABCD + \bar A BCD$

$= \bar B C\bar D(\bar A + A) + BC\bar D(A+\bar A) + \bar B CD(\bar A + A) + BCD(A+\bar A)$

$= \bar B C\bar D + BC\bar D + \bar B CD + BCD$

$= C\bar D(\bar B + B) + CD(\bar B + B)$

$= C\bar D + CD$

$= C(\bar D + D)$

$= C$

Pair $\rightarrow$ Reduce 1 variable

Quad $\rightarrow$ Reduce 2 variables

Octec $\rightarrow$ Reduce 3 variables

Octec > Quad > Pair > nothing

Algorithm of K-map:

1. Draw the K-map according to number of inputs and insert '1's in appropriate position.
2. Find '1's (if any) that has no adjacent '1'. That 1 is called isolated 1. If so then loop itself.
3. Find '1's that has only one adjacent '1'. If so then pair them.
4. Find octet (if any).
5. Find quad (if any).
6. Find pair (if any).
7. Write down the result of all the loops as a sum form.

$$y = \bar{A} B C + A B \bar{C} + A \bar{B} C + A B C$$

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}$			1	
C		1	1	1

$L1$

L_3

L_2

$$y = AB + AC + BC$$

1. ✓

2. ✓

3. ✓

7. ✓

5)

D	C	B	A	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

L5, L4

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	0	0	0
$C\bar{D}$	1	1	1	1
CD	0	0	1	0
$\bar{C}D$	1	1	0	1

L4, L2, L3

1. ✓
2. ✓
3.

$$y = ABC + \bar{A}\bar{C}D + \bar{B}\bar{C}D + C\bar{D} + \bar{A}\bar{B}\bar{D}$$

4 to 10, 0, 15

$F(A,B,C,D) = \sum(0,4,5,6,7,8,9,10,15)$

6)

A	B	C	D		
1	0	1	1	1	✓
1	1	0	1	1	✓
1	1	1	0	1	✓
1	1	1	1	1	✓

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$				
$C\bar{D}$			1	
CD		1	1	
$\bar{C}D$			1	

L_1

L_2

L_3

$$y = ABC + ACD + ABD$$

L2 2 3

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	1	1	1
$C\bar{D}$	1	1	0	0
CD	0	0	0	1
$\bar{C}D$	0	0	1	1

L1

$$y = \bar{A}BD + \bar{B}\bar{D} + B\bar{C}$$

1. ✓
2. ✓
3. ✓
4. ✓
5. ✓ ? ? ?

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	0	1	1
$C\bar{D}$	1	0	0	1
CD	0	0	0	0
$\bar{C}D$	1	0	1	1

L2 L1 L3

$$y = \bar{A}\bar{C} + \bar{A}\bar{D} + B\bar{C}$$

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	0	0	1	1
$C\bar{D}$	1	0	1	1
CD	1	0	1	1
CD	0	0	1	1

$$y = B + \bar{A}C$$

$$y = ABC + \bar{A}BC + A\bar{B}\bar{C} + AB + C$$

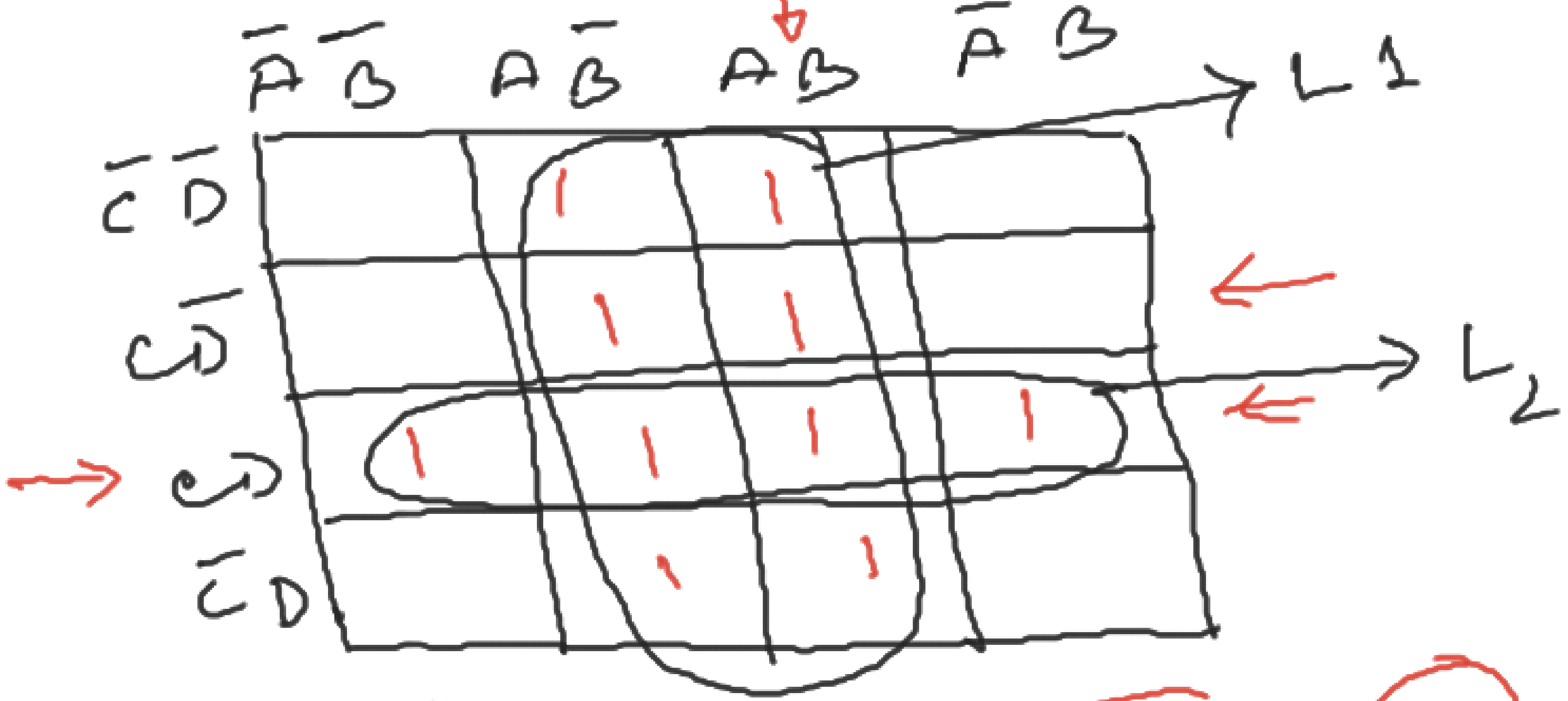
$$AB(C+\bar{C})$$
$$= ABC + AB\bar{C}$$

$$= C(A+\bar{A})$$
$$= AC + \bar{A}C$$
$$= AC(B+\bar{B})$$
$$\bar{A}C(A+\bar{A})$$

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}$		1	1	
C	1	1	1	1

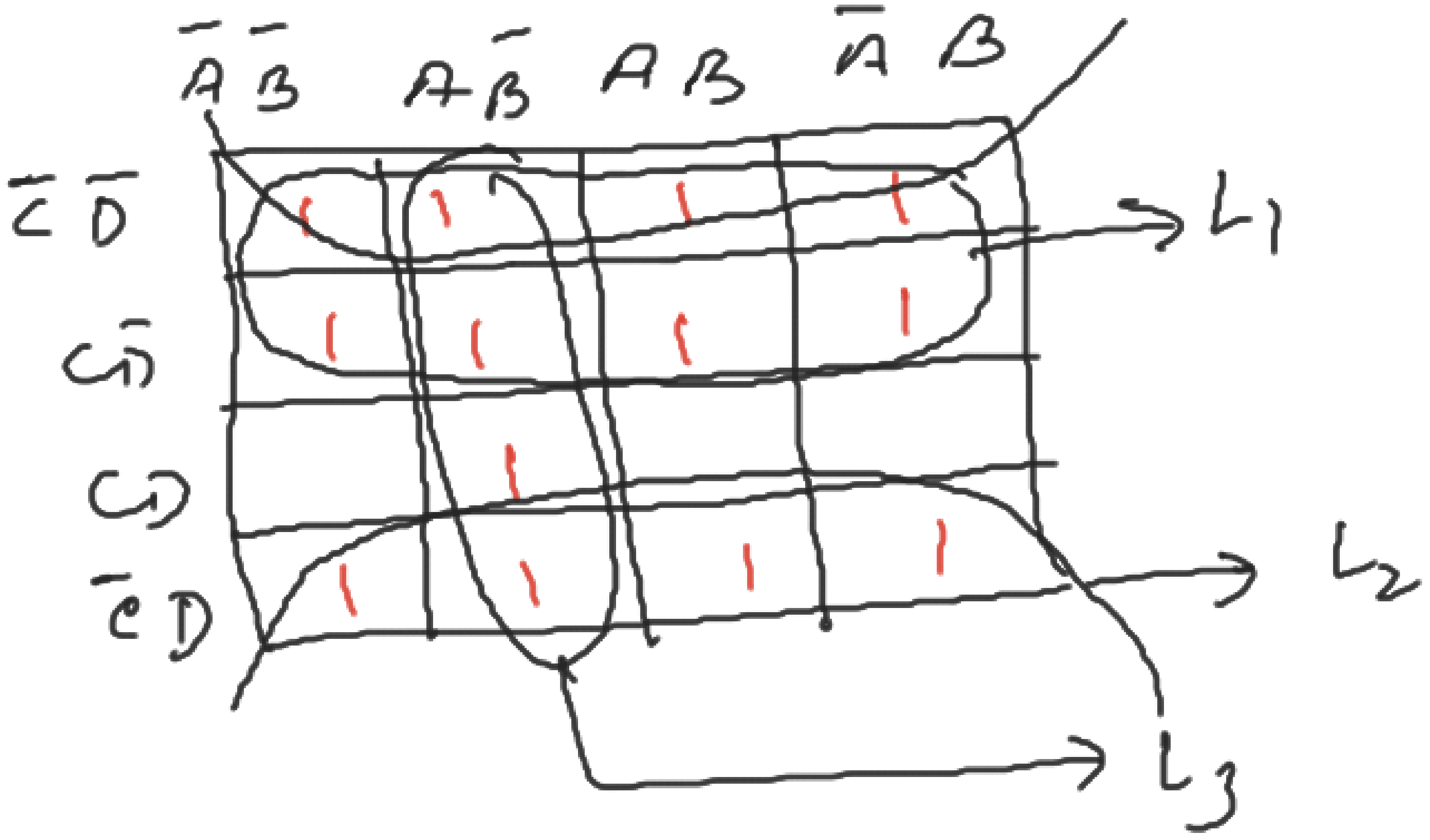
$$\boxed{y = C + A}$$

9) $y = ABC\bar{D} + \bar{A}\bar{B}CD + AB\bar{C}\bar{D} + ABC + AB + CD + A$



$= A + CD$

10) $y = A\bar{B}\bar{C}D + \bar{C}D + A\bar{B}C + \bar{D}$



$y = \bar{D} + \bar{C} + A\bar{B}$

Implement / simplify the following function:

$\rightarrow F(\overset{LSB}{A}, B, \overset{MSB}{C}) = \sum(0, 1, 4, 6, 7)$

	C	B	A	F
0	0	0	0	1 ✓
1	0	0	1	1 ✓
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1 ✓
5	1	0	1	0
6	1	1	0	1 ✓
7	1	1	1	1 ✓

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}$	1	1	0	0
C	1	0	1	1

L1

L2

L3

$$F(A,B,C) = \underbrace{\bar{B}\bar{C} + BC}_{} + \bar{A}\bar{B}$$

$$= \overline{B \oplus C} + \bar{A}\bar{B}$$

Don't draw the circuit

$$F(A,B,C,D) = \Sigma(0,1,2,3,5,7,9,11,12,13,14,15)$$

D	C	B	A	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	
0	1	0	1	1
0	1	1	0	
0	1	1	1	1
1	0	0	0	
1	0	0	1	1
1	0	1	0	
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	1	1	1
$C\bar{D}$		1	1	
CD	1	1	1	1
$\bar{C}D$		1	1	

L_1

L_2

L_3

$$F(A,B,C,D) = A + \underbrace{\bar{C}\bar{D} + CD}$$

$$= A + \overline{C \oplus D}$$

Indexing : D C B A

MSB (D)   LSB (A)

	$\bar A \bar B$	$A \bar B$	AB	$\bar A B$
$\bar C$	0	1	3	2
C	4	5	7	6

2 X 4 K-Map

	$\bar A \bar B$	$A \bar B$	AB	$\bar A B$
$\bar C \bar D$	0	1	3	2
$C \bar D$	4	5	7	6
CD	12	13	15	14
$\bar C D$	8	9	11	10

4 X 4 K-map

1011

$$F(A,B,C,D) = \sum(0,1,4,6,10,12,13,14,15)$$

	$\bar A \bar B$	$A \bar B$	AB	$\bar A B$
$\bar C \bar D$	1	1	0	0
$C \bar D$	1	0	0	1
CD	1	1	1	1
$\bar C D$	0	0	0	1

L2   L4   L1   L3

$$F(A,B,C,D) = \bar A B D + \bar B \bar C \bar D + CD + \bar A C$$

7) $F(A,B,C,D) = \sum(0,1,2,3,6,7,9,11,14,15)$

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$	
$\bar{C}\bar{D}$	1	1	1	1	$\rightarrow$ L1 $= \bar{C}\bar{D} + BC + A\bar{C}$
$C\bar{D}$	0	0	1	1	$\rightarrow$ L2
CD	0	0	1	1	
$\bar{C}D$	0	1	1	0	$\rightarrow$ L3

8) $F(A,B,C,D) = \sum(0,2,3,4,8,9,12,13,14,15)$

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$	
$\bar{C}\bar{D}$	1	.	1	1	$\rightarrow$ L1
$C\bar{D}$	1	.	.	.	$\rightarrow$ L2
CD	1	1	1	1	$\rightarrow$ L3
$\bar{C}D$	1	1	.	.	$\rightarrow$ L4

$$= B\bar{C}\bar{D} + \bar{A}\bar{B} + CD + \bar{B}D$$

Chapter 6: Digital Arithmetics

Binary Addition:

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 00 \end{array} \qquad \begin{array}{r} 0 \\ +\ 1 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \end{array}$$

Sum
Carry

Half Adder:

Truth Table:

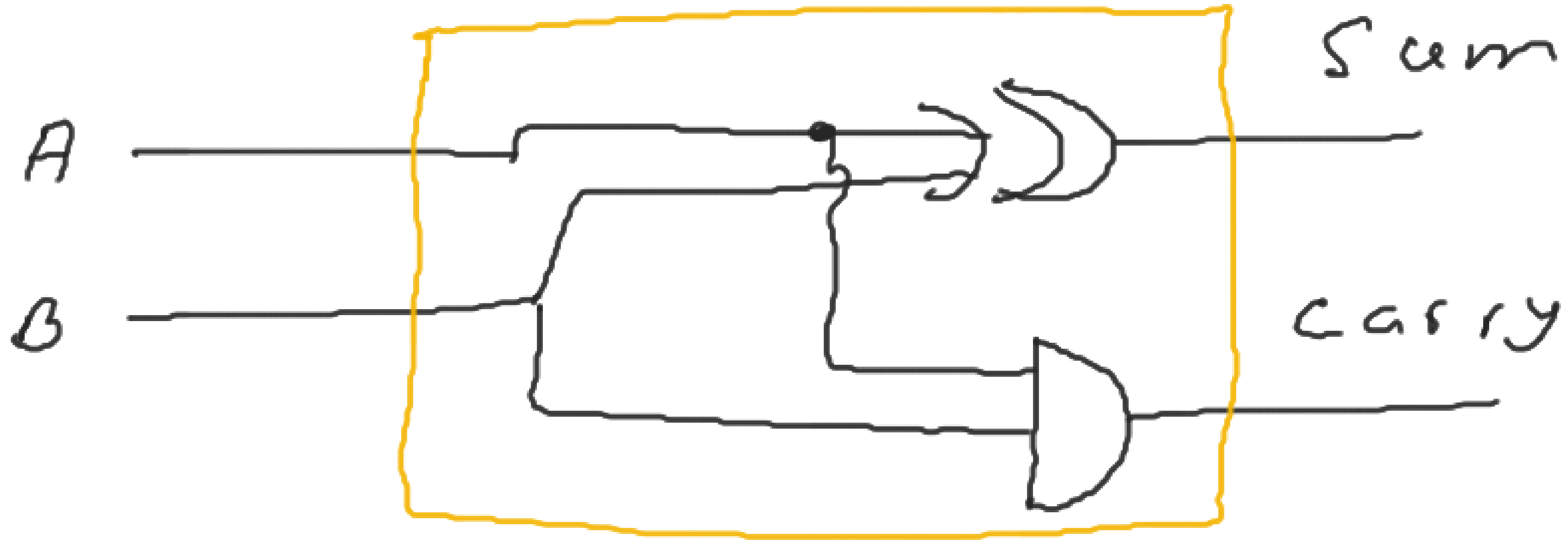
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Simplification:

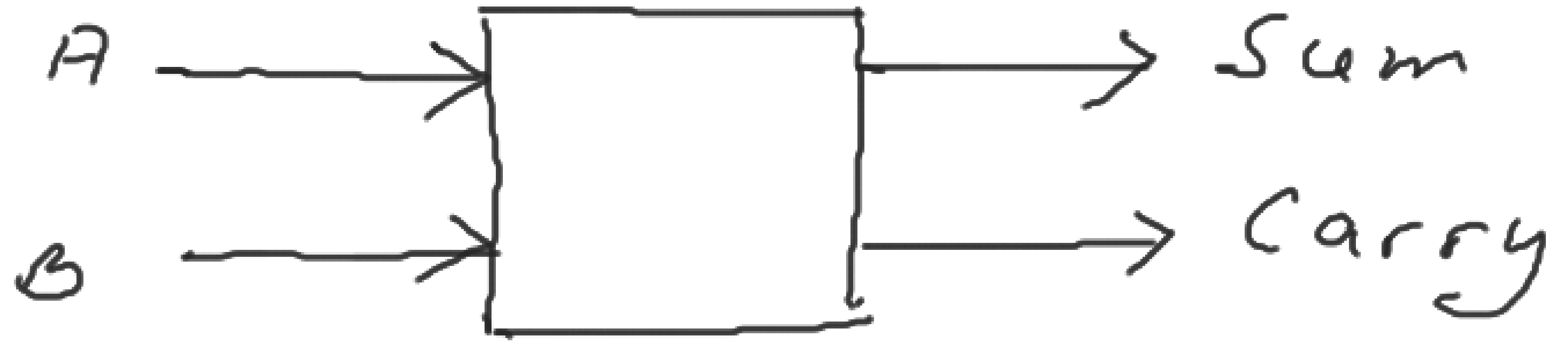
$$\text{Sum} = \bar{A}B + A\bar{B} = A \oplus B$$

$$\text{Carry} = AB$$

Circuit:



Block diagram:



Full Adder : Truth Table

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1 ✓	0
0	1	0	1 ✓	0
0	1	1	0	1 ✓
1	0	0	1 ✓	0
1	0	1	0	1 ✓
1	1	0	0	1 ✓
1	1	1	1 ✓	1 ✓

C_{in} → Carry Input

C_{out} → Carry Output

$$
\begin{array}{rcrrrrr}
 & & 1 & 1 & 0 & 0 & \\
12 & \rightarrow & & 1 & 1 & 0 & 0 \\
+\,13 & \rightarrow & & 1 & 1 & 0 & 1 \\
\hline
25 & \leftarrow & 1 & 1 & 0 & 0 & 1
\end{array}
$$

simplification:

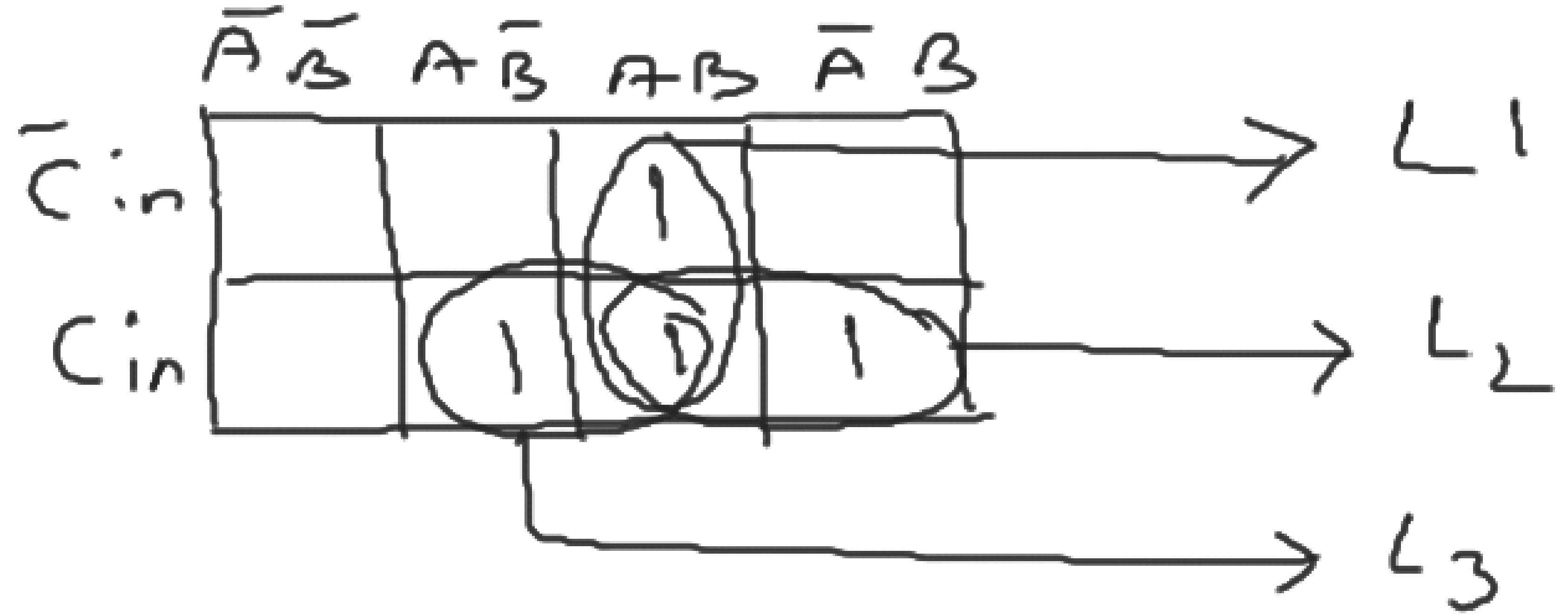
Sum ⇒

	$\bar A \bar B$	$A \bar B$	AB	$\bar A B$
$\bar C_{in}$		1		1
C_{in}	1		1	

$$
\begin{aligned}
Sum &= A\bar B\bar C + \bar A B\bar C + \bar A\bar B C + ABC \\
&= \bar C(A\bar B + \bar A B) + C(\bar A\bar B + AB) \\
&= \bar C(A \oplus B) + C(\overline{A \oplus B}) \\
&= \bar C X + C\bar X \qquad \boxed{\text{Let } A \oplus B = X} \\
&= C \oplus X \\
&= C \oplus A \oplus B
\end{aligned}
$$

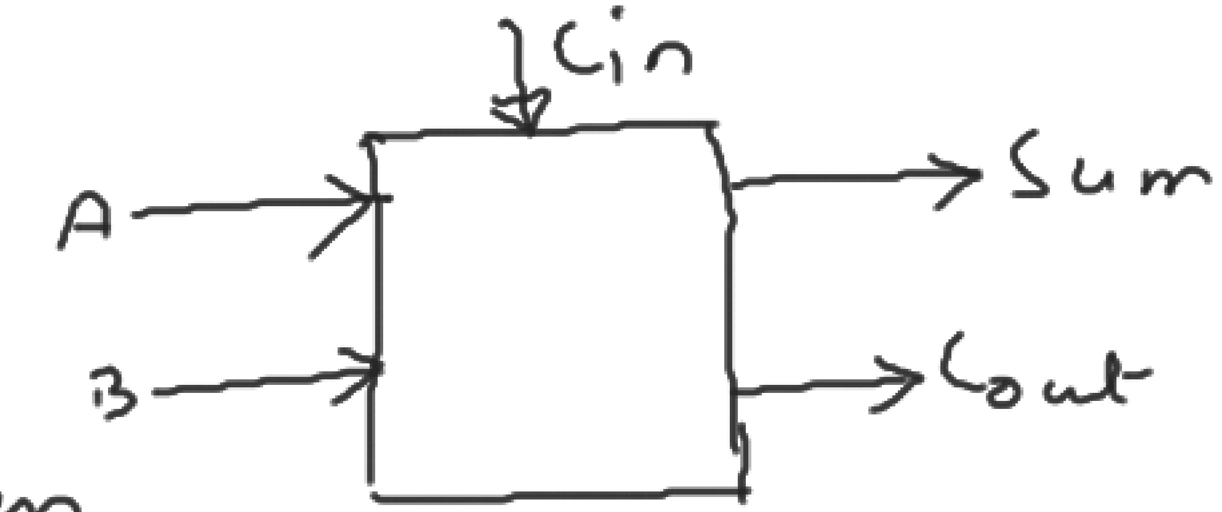
$$\boxed{Sum = A \oplus B \oplus C_{in}}$$

$C_{out} \Rightarrow$

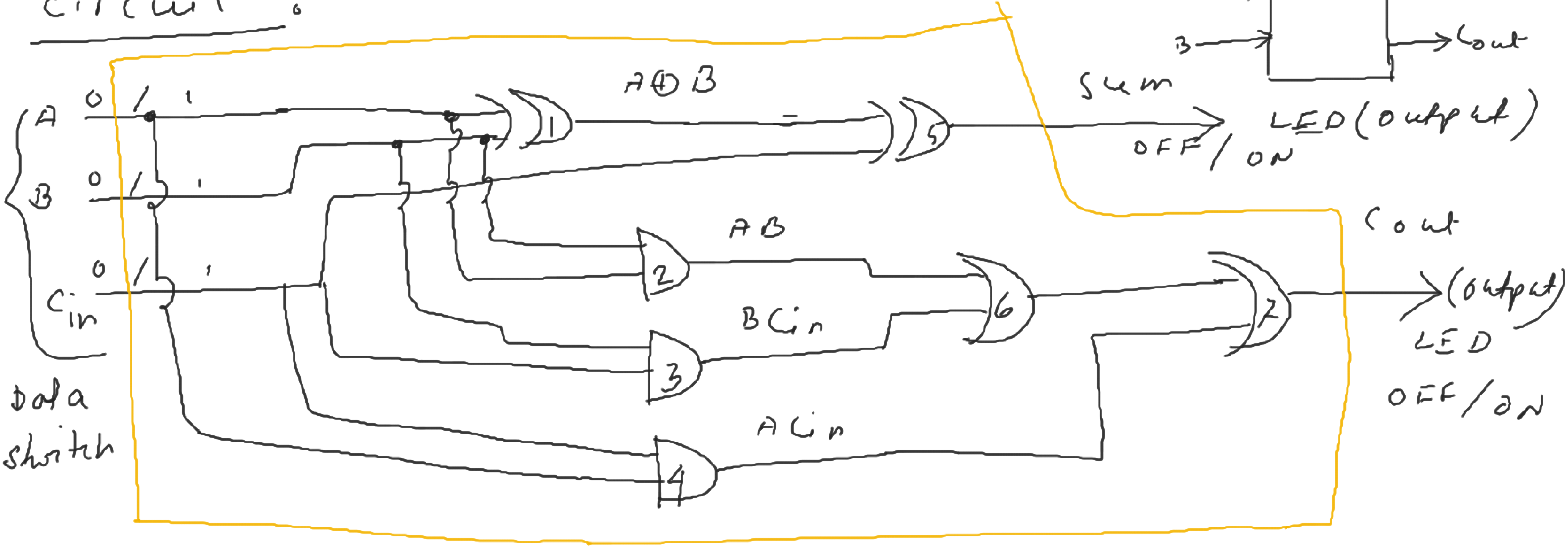


$$\boxed{C_{out} = AB + BC_{in} + AC_{in}}$$

Block diagram



circuit :



4-bit parallel adder:

IC # 7483

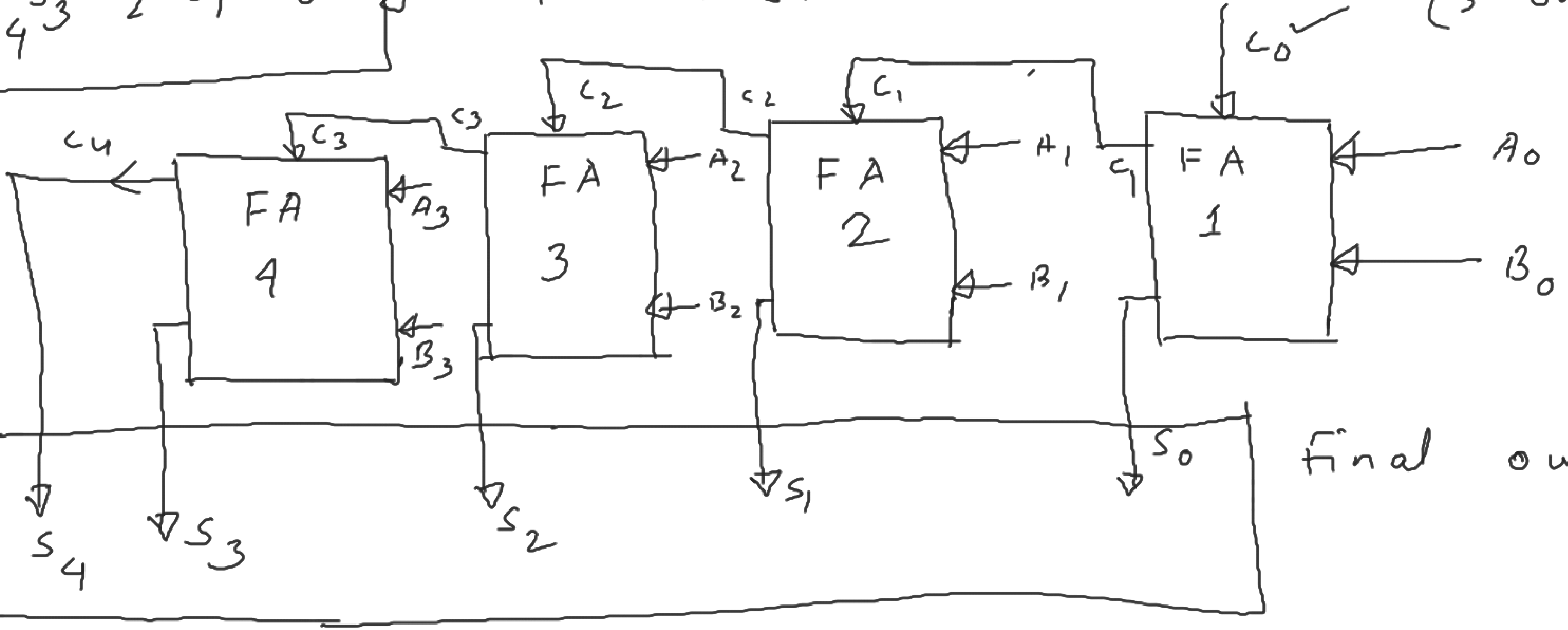
Internal circuit

	c_4	c_3	c_2	c_1	c_0
		A_3	A_2	A_1	A_0
+		B_3	B_2	B_1	B_0
	S_4	S_3	S_2	S_1	S_0

← Final Result

			1	1	0	0	0
	$12 \rightarrow$		1	1	0	0
+	$13 \rightarrow$		1	1	0	1
	$25 \leftarrow$	1	1	0	0	1

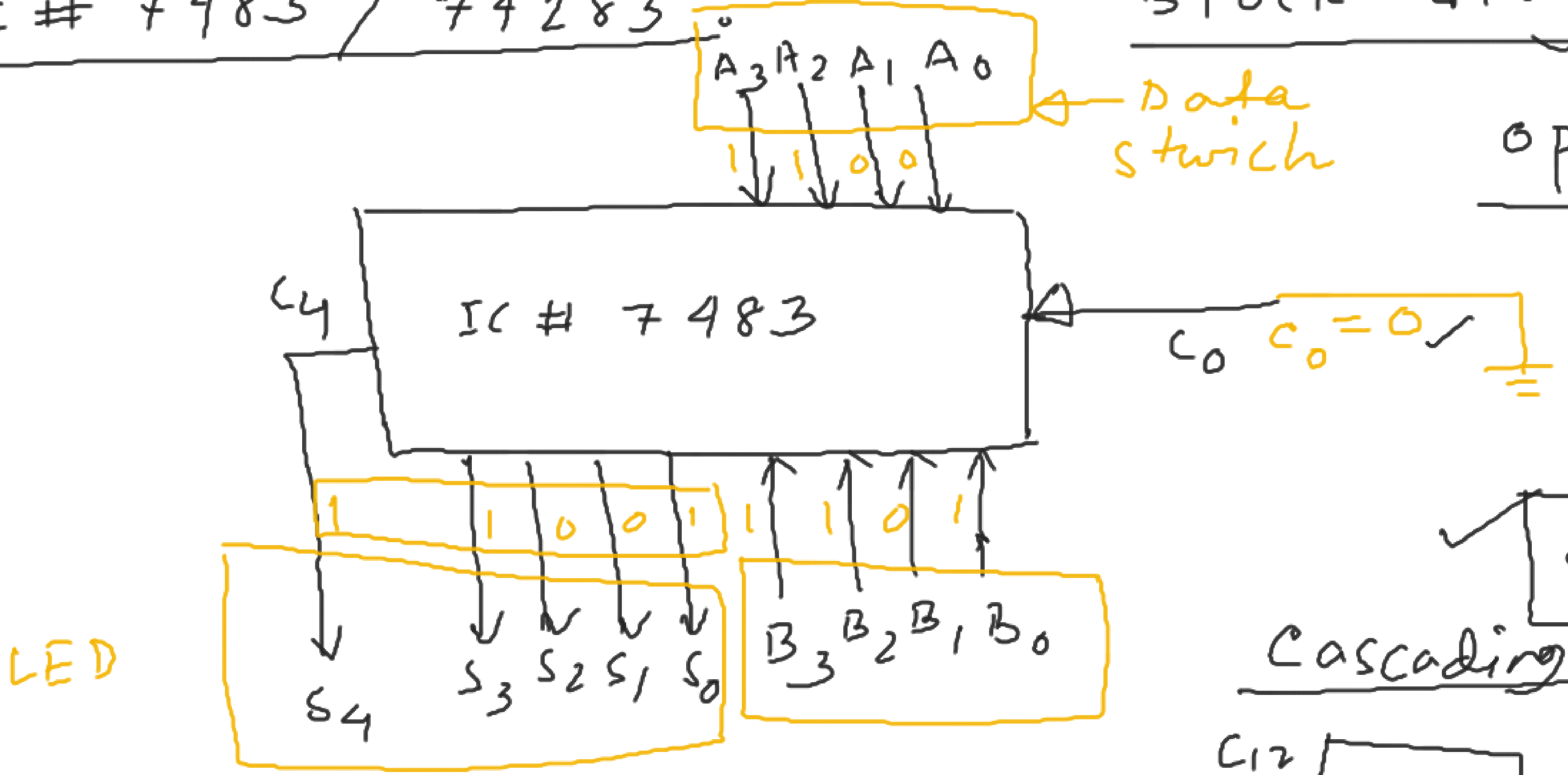
9 inputs
5 outputs



final output

IC # 7483 / 74283 :

Block diagram



Operation of IC # 7483

12 + 13

$C_0 \rightarrow$ open

subtract →

$S_4 \rightarrow$ ON

$S_3 \rightarrow$ ON

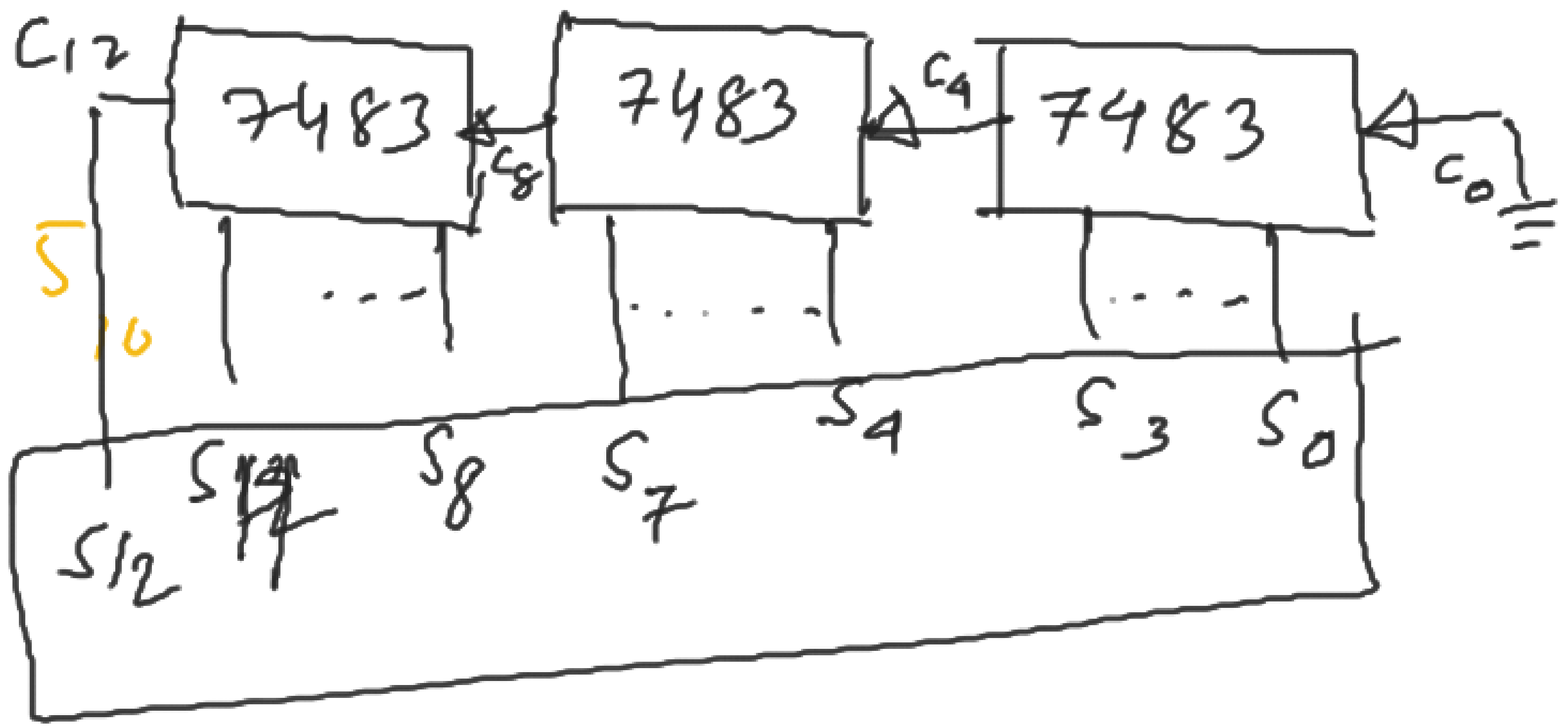
$S_2 \rightarrow$ OFF

$S_1 \rightarrow$ OFF

$S_0 \rightarrow$ ON

$11001_2 = 25_{10}$

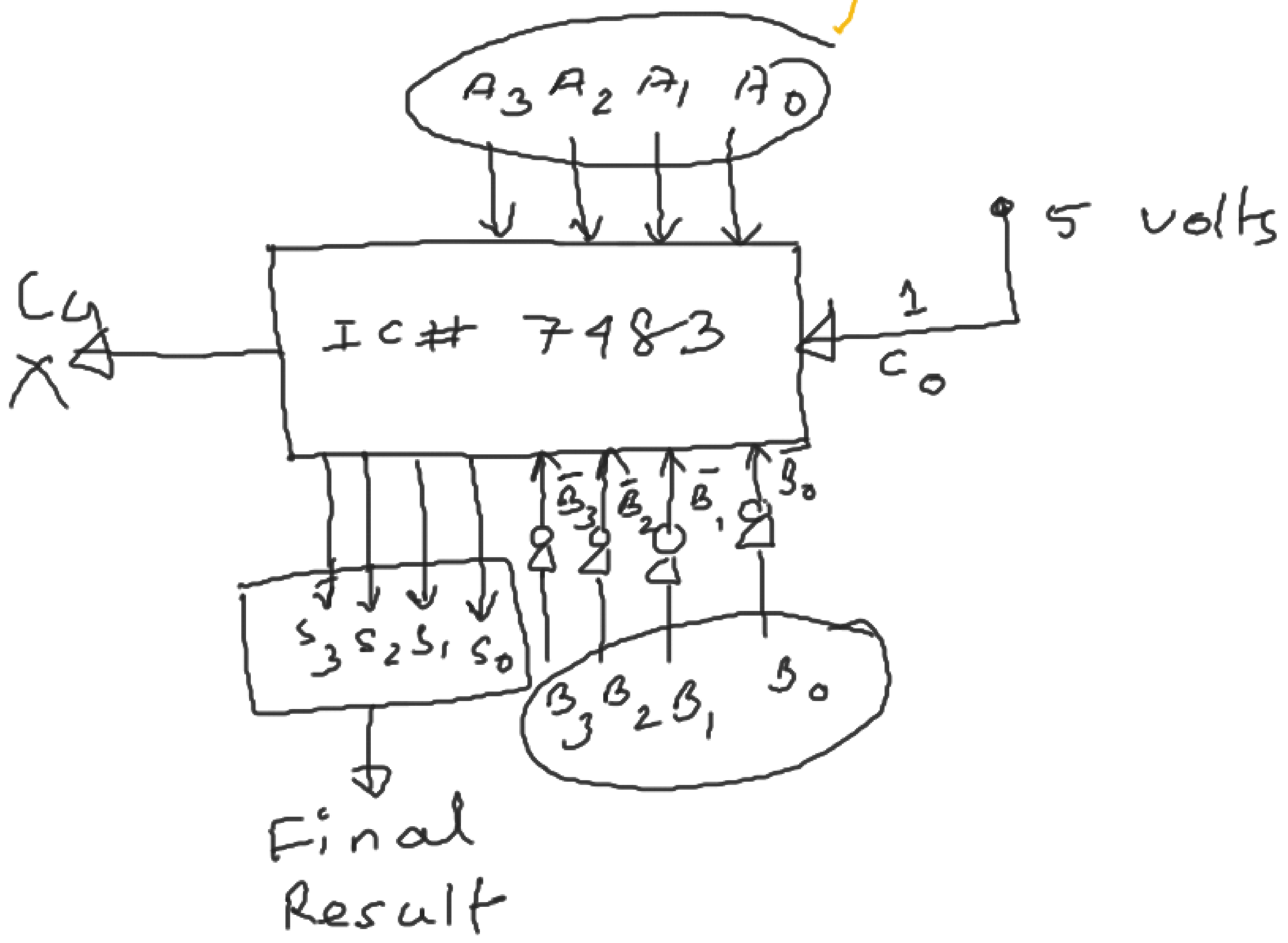
Cascading :



Design 4-bit subtractor using IC# 7483. You can use other logic gates, if necessary.

$\Rightarrow \quad A - B = A + (-B)$ ← 2's complement

$$\begin{array}{l} \quad A_3\ A_2\ A_1\ A_0 \\ (-)\ B_3\ B_2\ B_1\ B_0 \leftarrow \\ \hline \boxed{\bar{B}_3\ \bar{B}_2\ \bar{B}_1\ \bar{B}_0} \leftarrow \text{1's complement} \\ \qquad\qquad +1 \leftarrow \boxed{C_0 = 1} \\ \hline \_\ \_\ \_\ \_ \leftarrow \text{2's} \end{array}$$



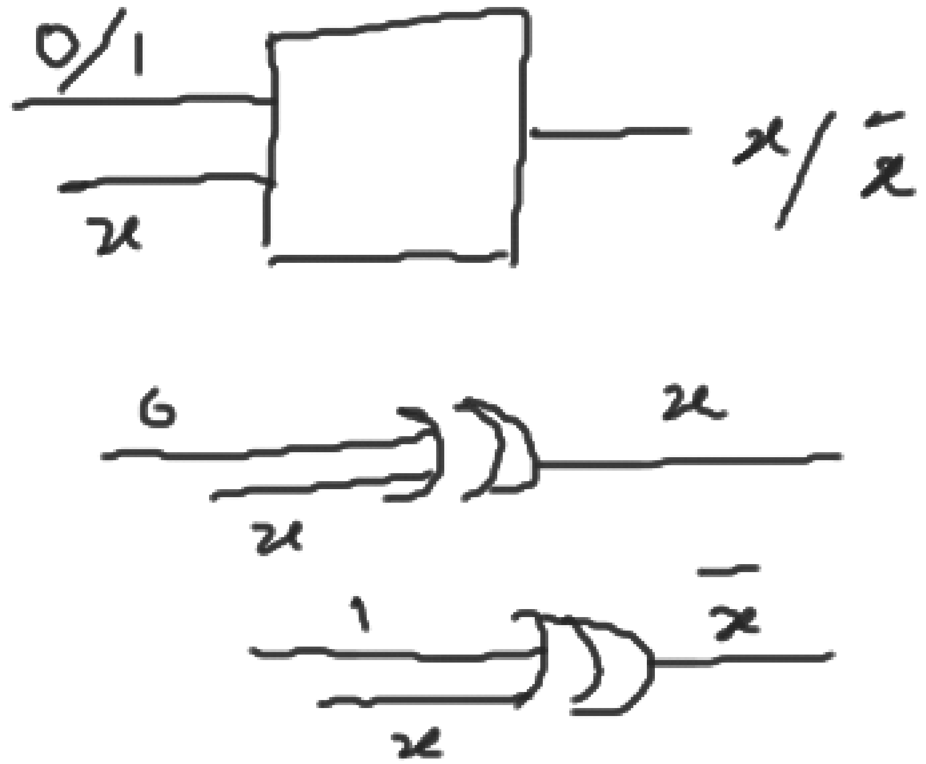
Design 4-bit parallel Adder/subtractor using IC# 7483. You can use other logic gates, if necessary.

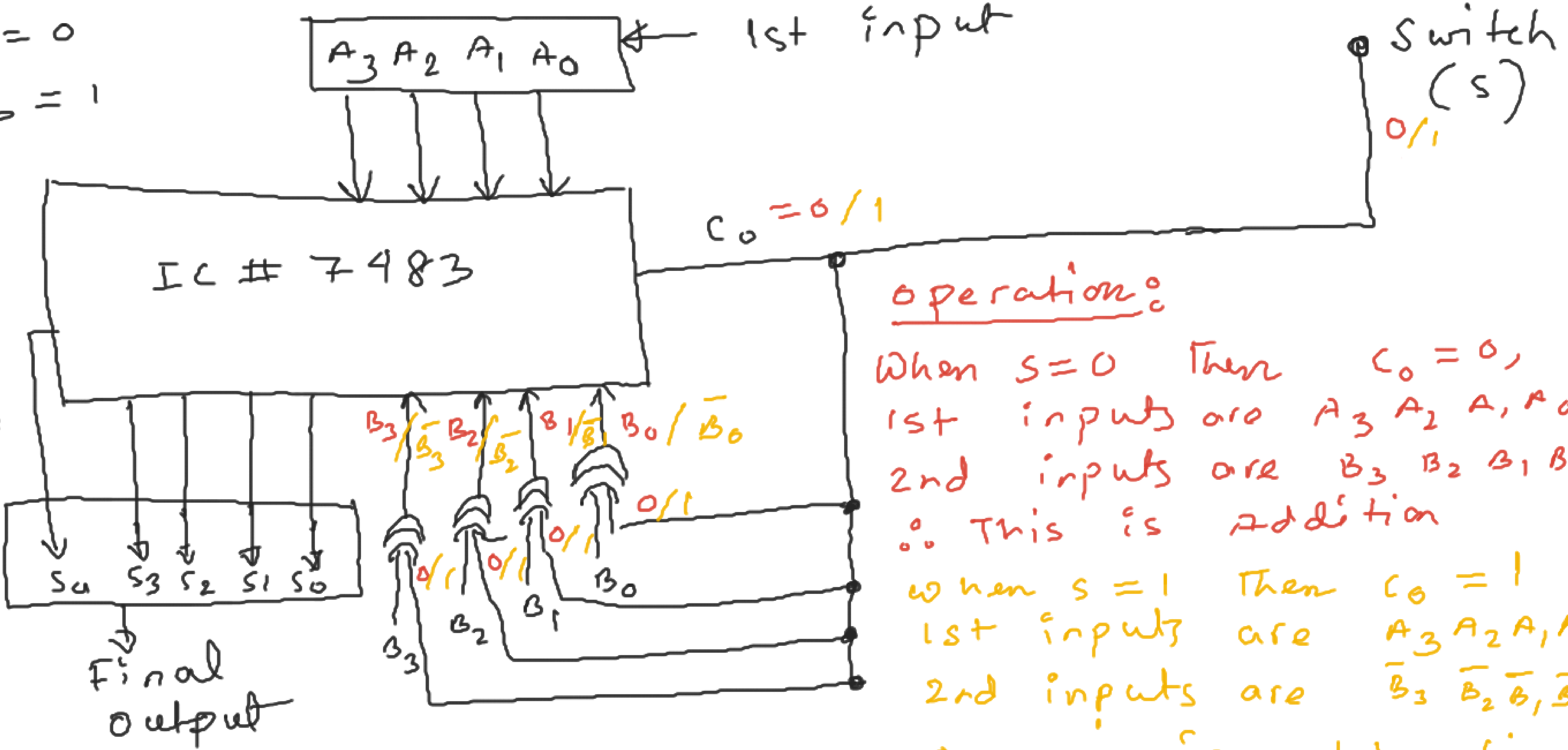
⇒ switch(S) ⇒ If $S = 0$ Then Addition $(A + B)$
If $S = 1$ Then subtraction $(A - B)$

If $S = 0$ Then $C_0 = 0$
If $S = 1$ Then $C_0 = 1$

$\therefore \boxed{S = C_0}$

$S = 0$, B_0, B_1, B_2, B_3
$S = 1$, $\bar{B}_0, \bar{B}_1, \bar{B}_2, \bar{B}_3$





operation:
When $S = 0$ Then $C_0 = 0$,
1st inputs are $A_3\ A_2\ A_1\ A_0$
2nd inputs are $B_3\ B_2\ B_1\ B_0$
∴ This is addition

when $S = 1$ Then $C_0 = 1$
1st inputs are $A_3 A_2 A_1 A_0$
2nd inputs are $\bar{B}_3\ \bar{B}_2\ \bar{B}_1\ \bar{B}_0$
∴ This is subtraction