CSE 209 : Digital Logic & System Design

(DLSD)

Why DLSD ?

What DLD ?

0 → False → LOW (0 volts)

1 → True → HIGH (5 volts)

Logic →

Binary number

Logic gates → ALU → (DLSD)

chapter → 1 and 2

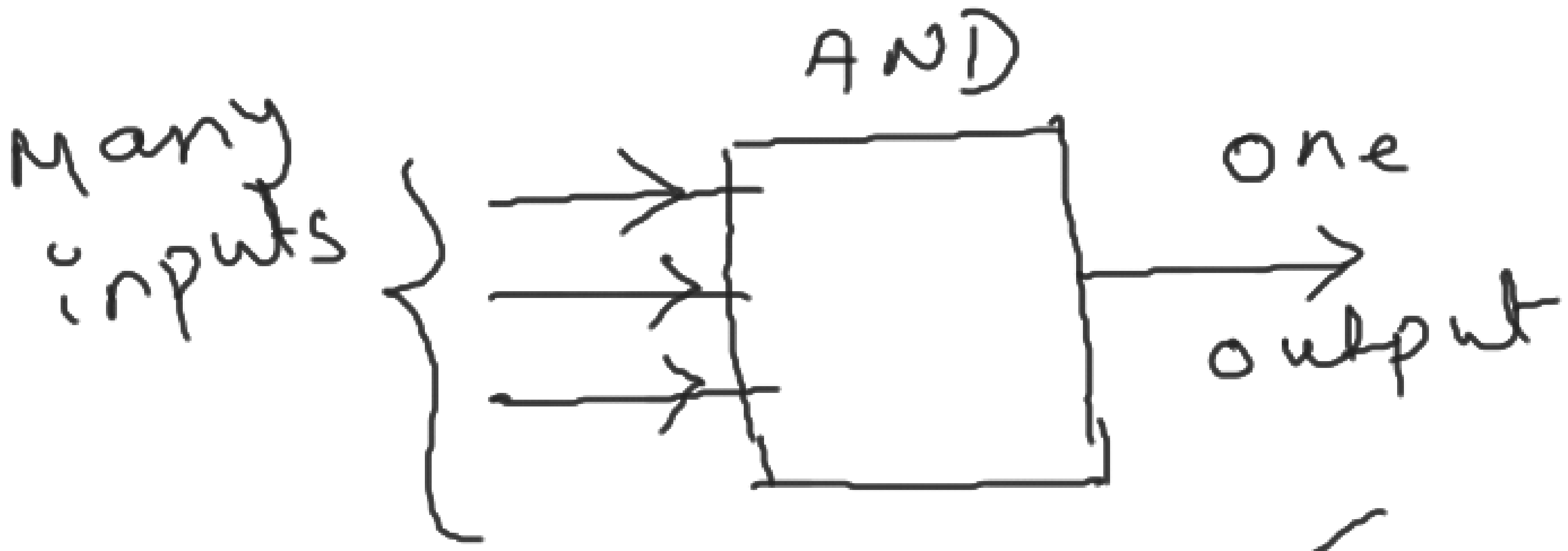
→ Ronal J. Tocci

Chapter 3 : Logic gates & Boolean Algebra

Logic gates :

1. AND gate (conjunction) 2. OR gate (Disjunction) 3. NOT gate (Negation)

1. AND gate :



Output will be HIGH when all The inputs are HIGH; OThrwise output will be LOW.

Truth Table :

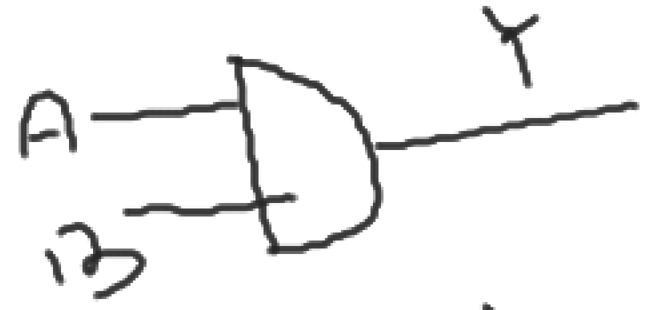
2 inputs

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

3 inputs

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

• symbol

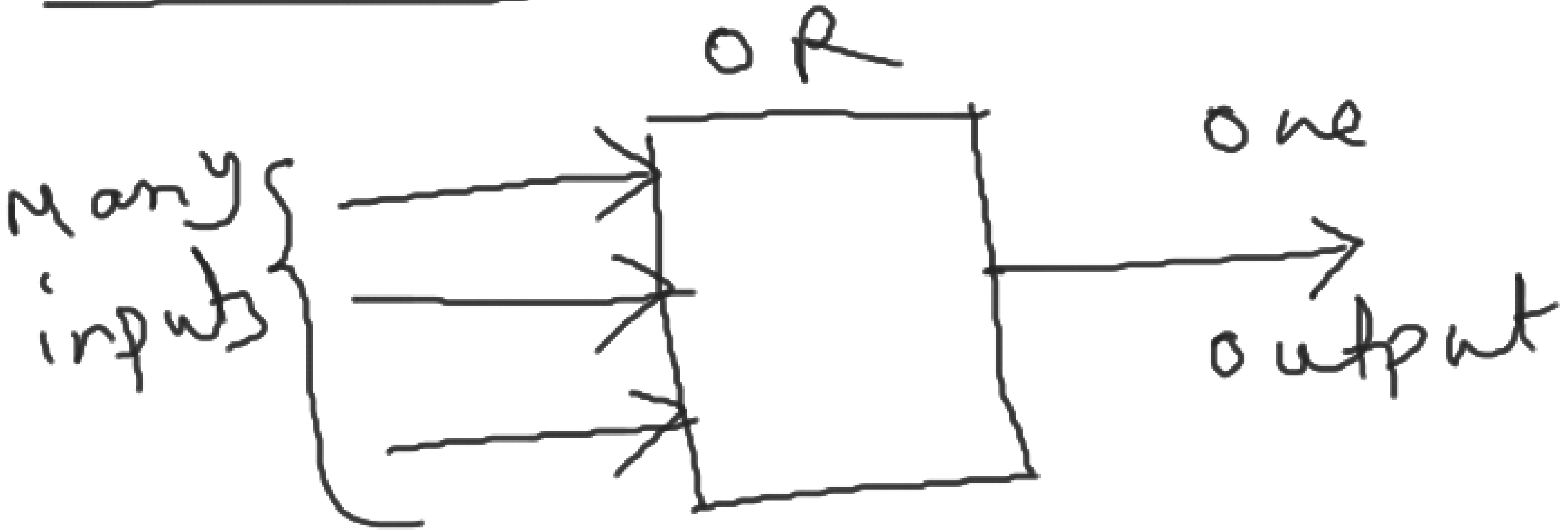


$Y = ABC$

Mathematical Notation

$$Y = A \cdot B = AB$$

2) OR gate



Output will be LOW only when all the inputs are LOW; otherwise output will be HIGH.

Truth Table

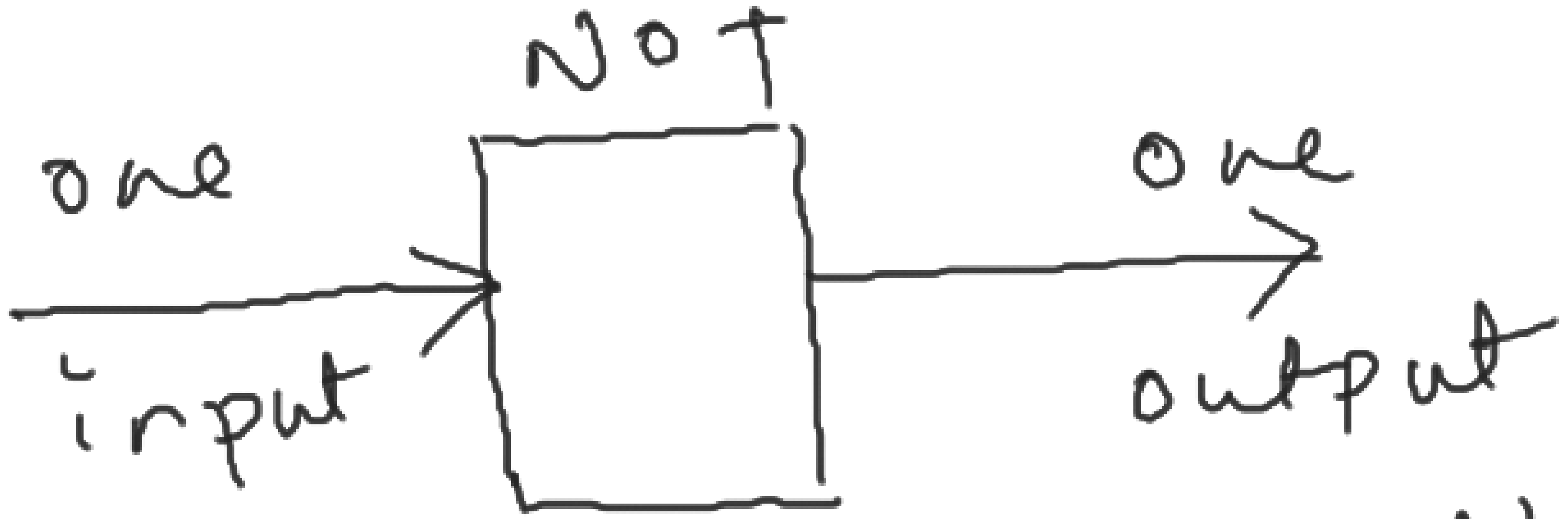
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

symbol :

$Y = A + B$ $\qquad$ $Y = A + B + C$

3) NOT gate:

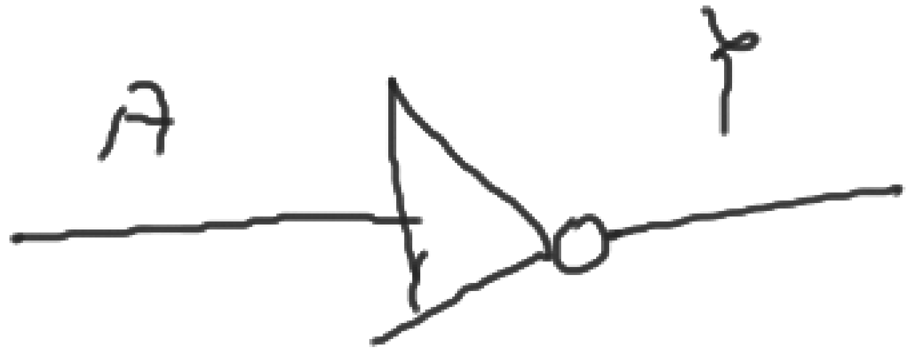


Output will be opposite of input.

Truth Table

A	Y
0	1
1	0

Symbol



$Y = \bar{A}$

AND 

$Y = AB$

OR 

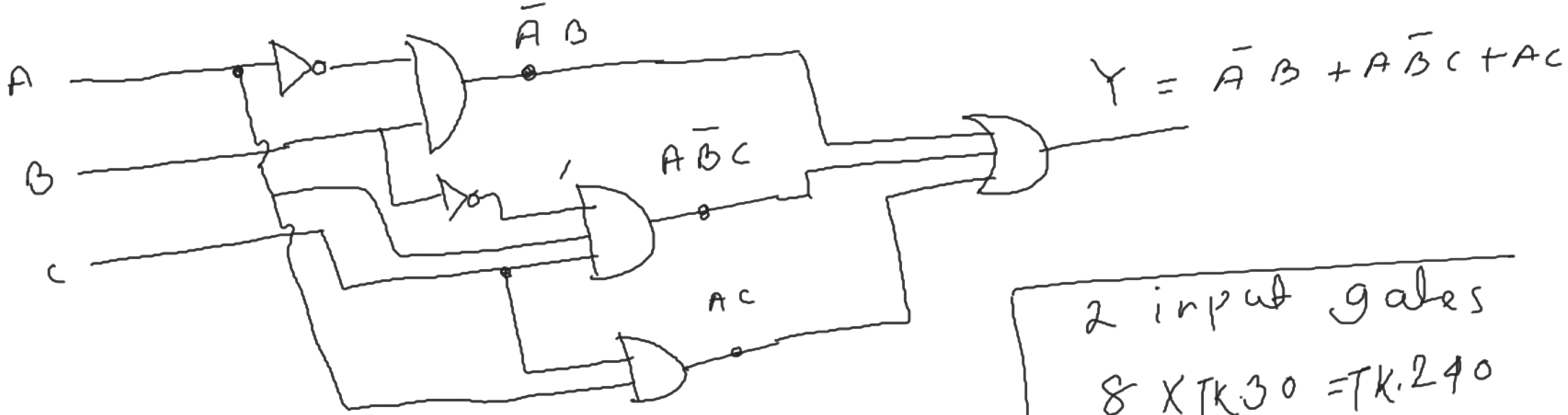
$Y = A + B$

NOT 

$Y = \bar{A}$

$\boxed{\text{Basic logic gate}}$

Draw the logic circuit of $y = \bar{A}B + A\bar{B}C + AC$



2 input gates

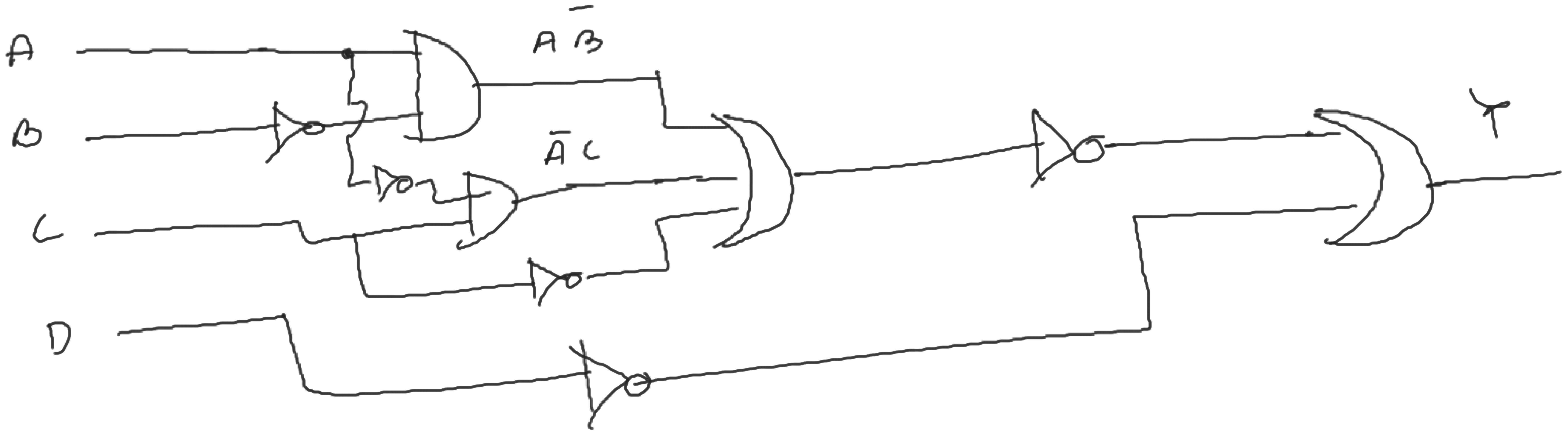
$8 \times Tk.30 = Tk.240$

Draw the truth Table of $Y = \bar{A}B + A\bar{B}C + AC$

A	B	C	$\bar{A}B$	$A\bar{B}C$	AC	Y
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	1	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	0	0
1	0	1	0	1	1	1
1	1	0	0	0	0	0
1	1	1	0	0	1	1

$$Y = \overline{(A\bar{B} + \bar{A}C + \bar{C})} + \bar{D}$$

Draw the logic circuit of above expression.

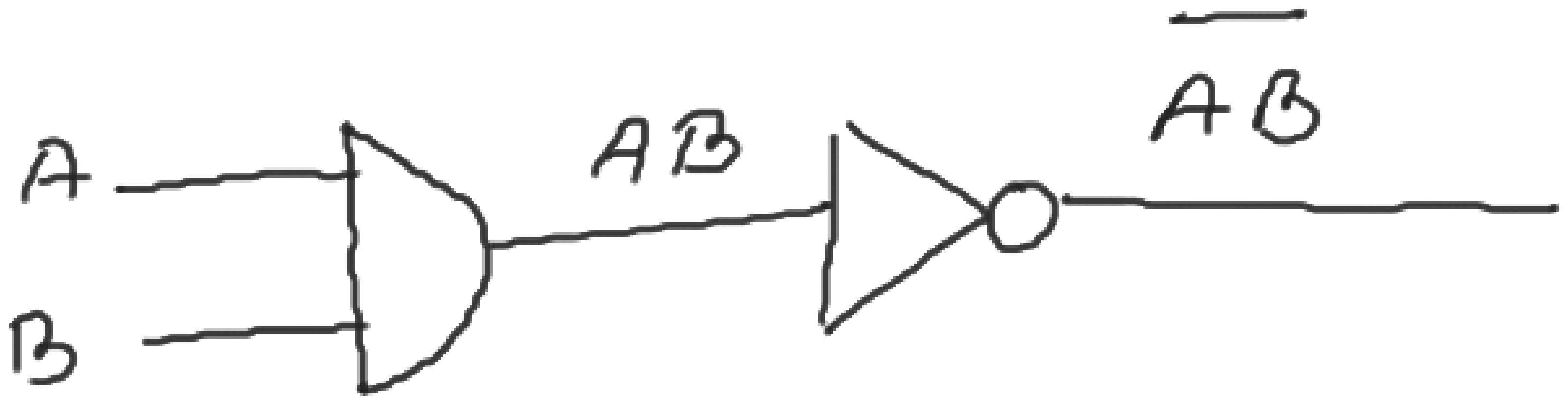


Compound Logic gates:

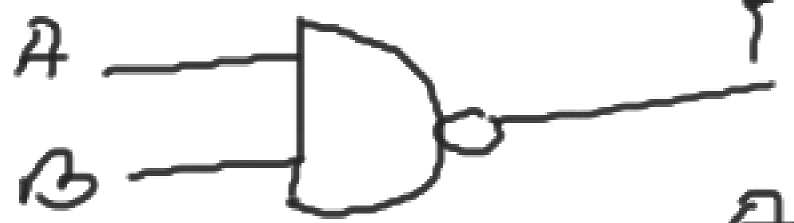
4. NAND gate 5. NOR gate
6. Exclusive OR gate 7. Exclusive NOR gate.

4. NAND gate:

N$\boxed{\text{OT}}$ AND = NAND



$\Downarrow$



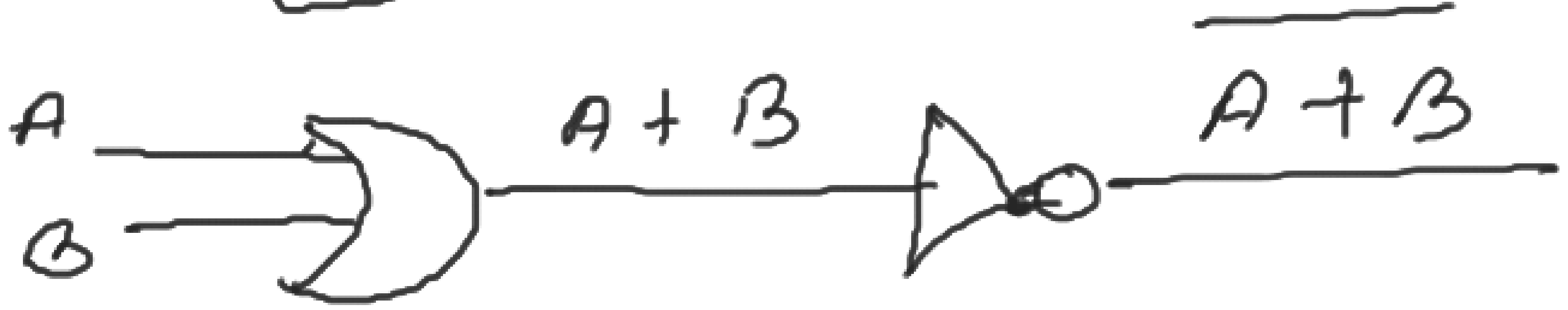
$Y = \overline{AB}$

Truth Table:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

5. NOR gate:

N$\boxed{\text{OT}}$ OR $\Rightarrow$ NOR



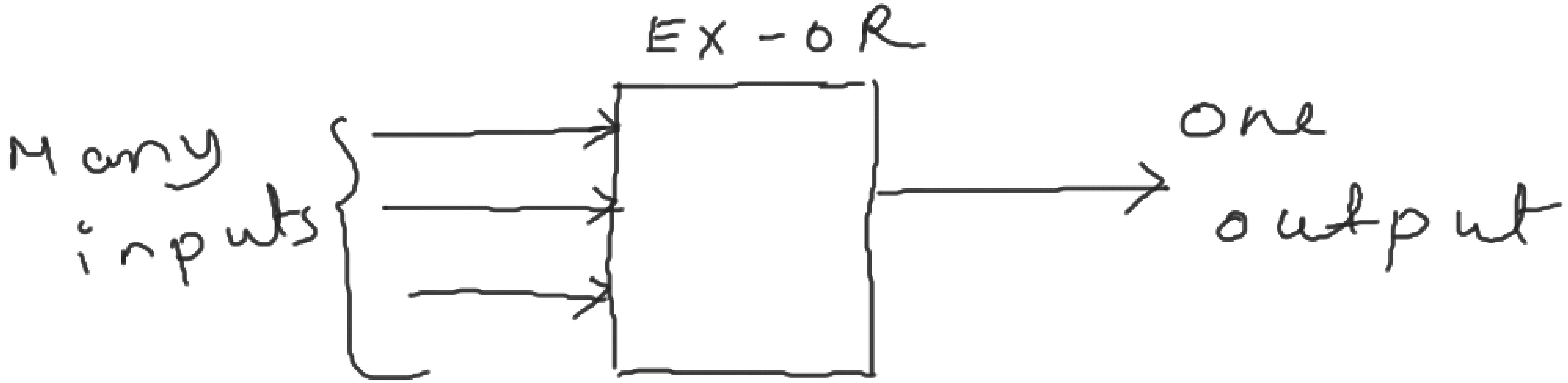
$\Downarrow$

$Y = \overline{A+B}$

Truth Table:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

6. Exclusive OR (EX-OR / X-OR) gate:



Output will be HIGH when inputs are different, otherwise output will be LOW.

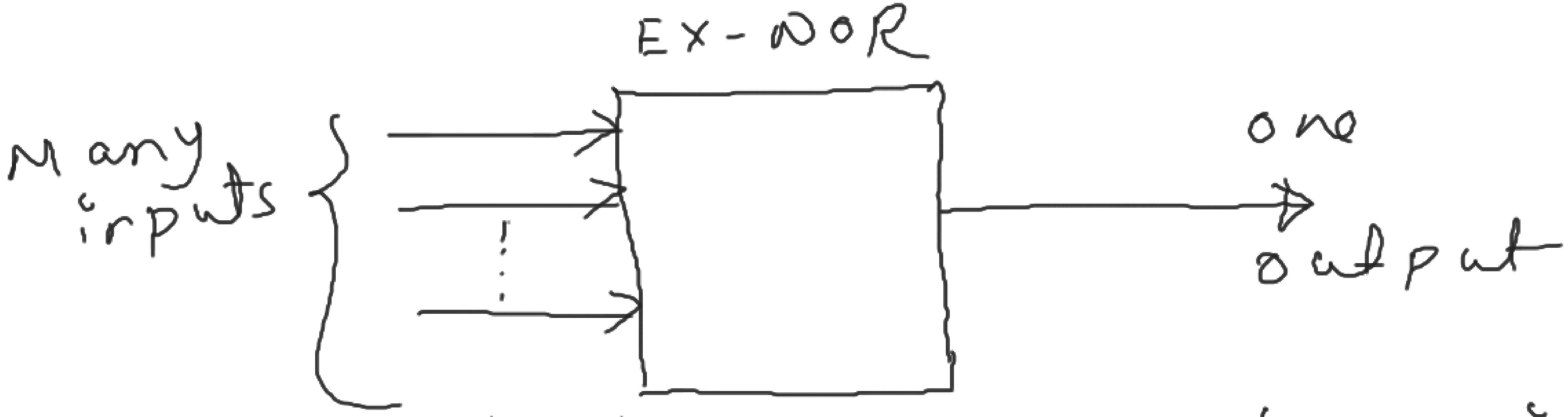
Truth Table:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Symbol:

$$Y = A \oplus B$$

7. Exclusive NOR (EX-NOR / X-NOR) gate :



Output will be HIGH when inputs are same, otherwise output will be LOW.

Truth Table

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

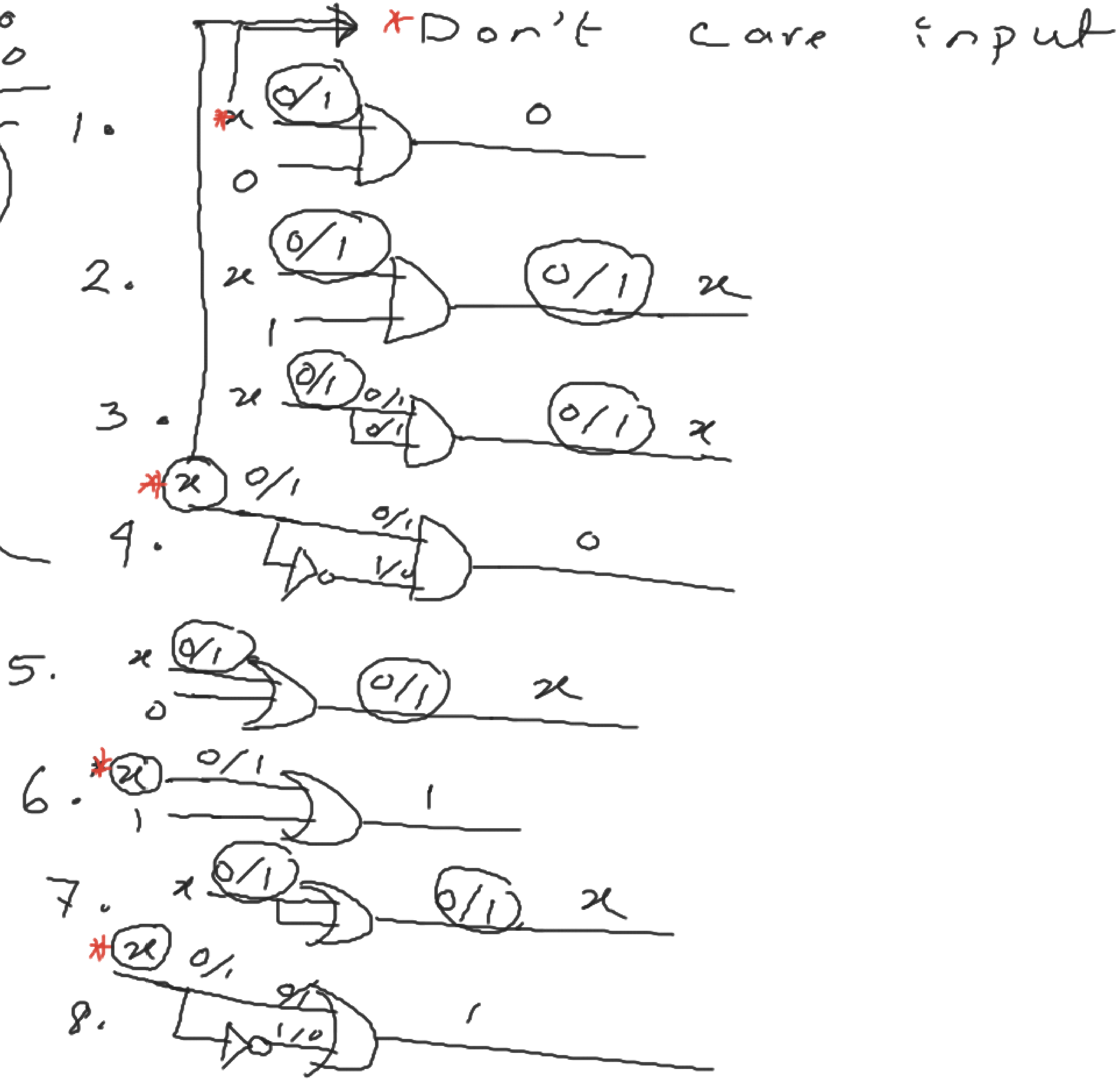
symbol :

$$Y = \overline{A \oplus B}$$

Boolean Algebra:

1. $x \cdot 0 = 0$
2. $x \cdot 1 = x$
3. $x \cdot x = x$
4. $x \cdot \bar{x} = 0$

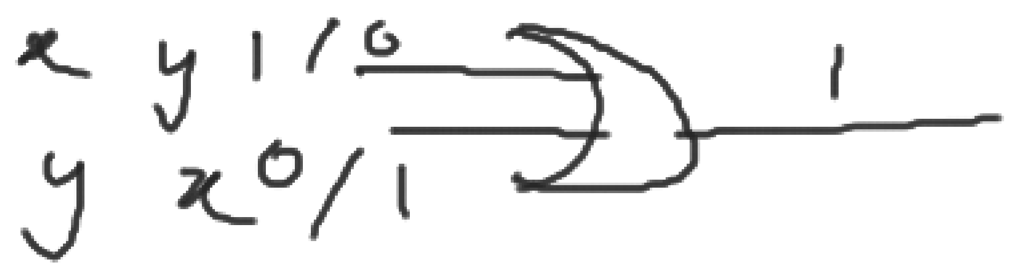
5. $x + 0 = x$
6. $x + 1 = 1$
7. $x + x = x$
8. $x + \bar{x} = 1$



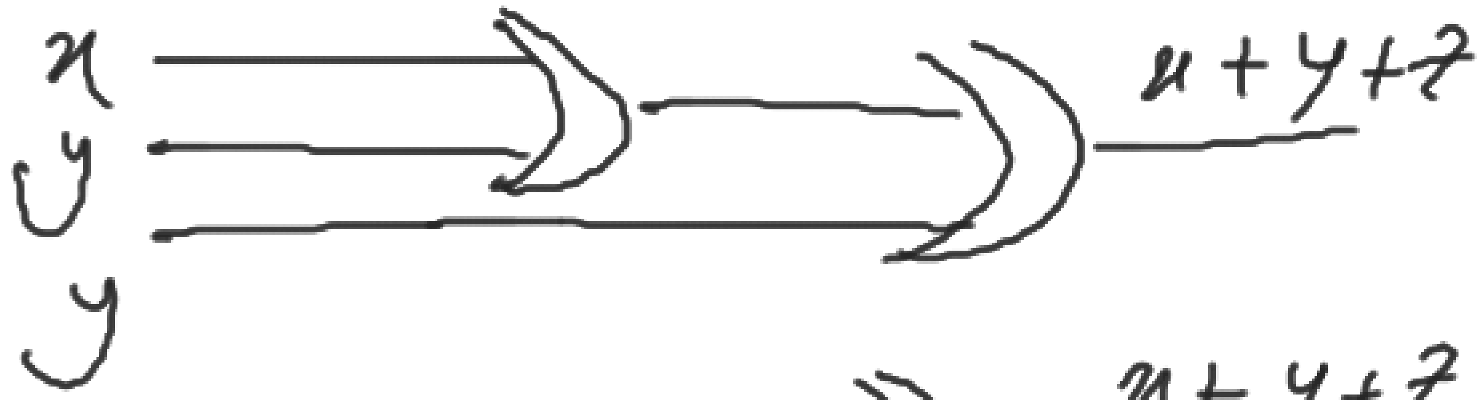
9) $x + y = y + x$

10) $x \cdot y = y \cdot x$

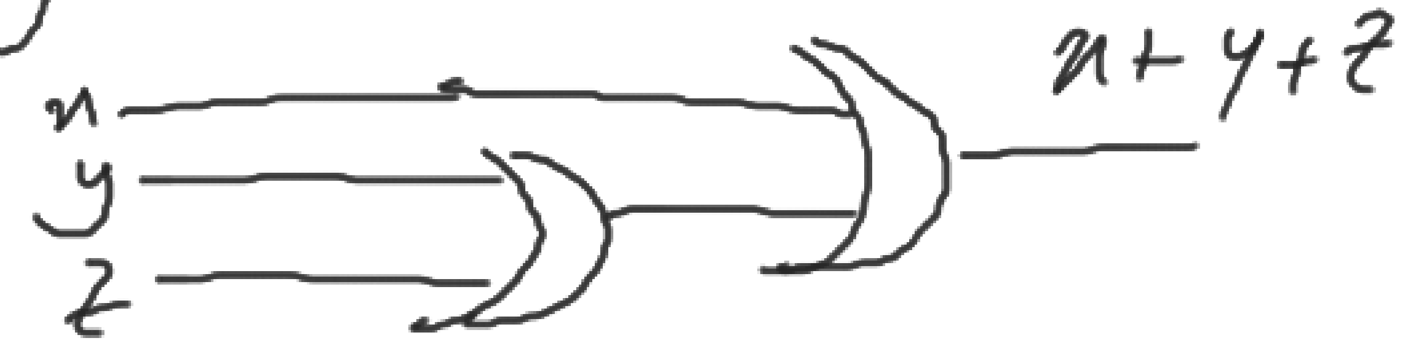
Commutative Law



11) $x + (y + z) = (x + y) + z = x + y + z$



12) $x(yz) = (xy)z = xyz$



Associative Law

13) a) $x(y + z) = xy + xz$

b) $(x + y)(z + w) = xz + xw + yz + yw$

Distributive Law

14. $\boxed{\underline{x + xy} = \underline{x}}$

Save 2 logic gates

$x + xy$

$= x(1 + y)$

$= x \cdot 1$

$= \underline{\underline{x}}$

15) a) $\boxed{x + \bar{x} y = x + y}$ → Save 2 logic gates
b) $\boxed{\bar{x} + x y = \bar{x} + y}$

$x + \bar{x} y$

$= x + xy + \bar{x} y$

$= x + y(x + \bar{x})$

$= x + y \cdot 1$

$= x + y$

$x \cdot 1$

$= x \cdot (1 + y)$

$= x + xy$

16) $\overline{x + y} = \bar{x} \cdot \bar{y}$

17) $\overline{xy} = \bar{x} + \bar{y}$

De Morgan's Theorem

H/W

Simplification of logic expression using Boolean Algebra:

$$Y = A\bar{B}D + A\bar{B}\bar{D}$$
$$= A\bar{B}(D + \bar{D})$$
$$= A\bar{B}\cdot 1$$
$$= A\bar{B} \longrightarrow$$

8 logic gates
$8 \times$ TK30 = TK240

2 logic gates
$2 \times$ TK.30
= TK. 60

$$\overline{x+y} = \bar{x}\cdot\bar{y}$$

x	y	$x+y$	$\overline{x+y}$	$\bar{x}$	$\bar{y}$	$\bar{x}\cdot\bar{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$$\overline{x+y} = \bar{x}\cdot\bar{y}$$

$$\overline{xy} = \bar{x} + \bar{y}$$

x	y	xy	$\overline{xy}$	$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$$\overline{xy} = \bar{x} + \bar{y}$$

$$Z = (\bar{A} + B)(A + B) \longrightarrow 4 \text{ logic gates}$$

$$= A\bar{A} + \bar{A}B + AB + BB \longrightarrow \text{Distributive Law}$$

$$= 0 + \underline{\bar{A}B + AB} + B \qquad x \cdot x = x$$

$$= B(A + \bar{A}) + B$$

$$= B \cdot 1 + B$$

$$= B + B$$

$$= \underline{\underline{B}}$$

$$x = ACD + \bar{A}BCD \longrightarrow 7 \text{ logic gates}$$

$$= CD(A + \bar{A}B)$$

$$= CD(A + B) \longrightarrow 3 \text{ logic gates}$$

$$= ACD + BCD \longleftarrow 5 \text{ logic gates}$$

$$y = \overline{(\bar{A} + C)\cdot(B + \bar{D})}$$
$$= \overline{\bar{A} + C} + \overline{B + \bar{D}}$$
$$= \bar{\bar{A}}\cdot\bar{C} + \bar{B}\cdot\bar{\bar{D}}$$
$$= A\bar{C} + \bar{B}D$$

$$A + \bar{A}BCD$$
$$= A + BCD$$
$$x + \bar{x}y = x + y$$
$$\bar{A} + ACDEF$$
$$= \bar{A} + CDEF$$
$$\bar{x} + xy = \bar{x} + y$$
$$\boxed{1 + \bar{x}y = 1}$$

$$y = ABC + A\bar{B}\cdot\overline{(\bar{A}\cdot\bar{C})}$$
$$= ABC + A\bar{B}(\bar{\bar{A}} + \bar{\bar{C}})$$
$$= ABC + A\bar{B}(A + C)$$
$$= ABC + AA\bar{B} + A\bar{B}C$$
$$= ABC + A\bar{B} + A\bar{B}C$$
$$= ABC + A\bar{B}(1 + C)$$
$$= ABC + A\bar{B}$$

$$= ABC + A\bar{B}$$
$$= A(\bar{B} + BC)$$
$$= A(\bar{B} + C)$$
$$= A\bar{B} + AC$$

$$Z = A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$A(\bar{B} + C)$$

$$= A\bar{B}(\bar{C} + C) + ABC$$

$$= A\bar{B} + ABC$$

$$= A(\bar{B} + BC)$$

$$= A(\bar{B} + C)$$

$$y = \bar{A}C\,\overline{(\bar{A}BD)} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$= \bar{A}C(A + \bar{B} + \bar{D}) + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$= A\bar{A}C + \bar{A}\bar{B}C + \bar{A}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$= 0 + \bar{A}\bar{B}C + \bar{A}\bar{D}(C + \bar{C}B) + A\bar{B}C$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{D}(C + B) + A\bar{B}C$$

$$= \bar{B}C(\bar{A} + A) + \bar{A}\bar{D}C + \bar{A}\bar{D}B$$

$$= \bar{B}C + \bar{A}C\bar{D} + \bar{A}B\bar{D} = \bar{B}C + \bar{A}\bar{D}(B + C)$$

$$y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$= \bar{A}BC + A\bar{B}C + AB(\bar{C} + C)$$

$$= \bar{A}BC + A\bar{B}C + AB$$

$$= \bar{A}BC + A(B + \bar{B}C)$$

$$= \bar{A}BC + A(B + C)$$

$$= \bar{A}BC + AB + AC$$

$$= B(A + \bar{A}C) + AC$$

$$= B(A + C) + AC$$

$$= AB + BC + AC$$

Design the circuit from the given Truth Table:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$\bar{A}\cdot BC$

$A\bar{B}C$

$AB\bar{C}$

ABC

Write down Y as a sum of product form of A, B, C

$$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$
$$= \bar{A}BC + A\bar{B}C + AB(\bar{C}+C)$$
$$= \bar{A}BC + A\bar{B}C + AB$$
$$= \bar{A}BC + A(B+\bar{B}C)$$
$$= \bar{A}BC + A(B+C)$$
$$= \bar{A}BC + AB + AC$$
$$= B(A+\bar{A}C) + AC$$
$$= B(A+C) + AC$$
$$= AB + BC + AC$$