CHAPTER 5

Project Scheduling Models

5.1 Introduction

- A project is a collection of tasks that must be completed in minimum time or at minimal cost.
- Objectives of Project Scheduling
  - Completing the project as early as possible by determining the earliest start and finish of each activity.
  - Calculating the likelihood a project will be completed within a certain time period.
  - Finding the minimum cost schedule needed to complete the project by a certain date.

5.1 Introduction

- A project is a collection of tasks that must be completed in minimum time or at minimal cost.
- Objectives of Project Scheduling
  - Investigating the results of possible delays in activity's completion time.
  - Progress control.
  - Smoothing out resource allocation over the duration of the project.

Task Designate

- **Tasks are called "activities."**
  - Estimated completion time (and sometimes costs) are associated with each activity.
  - Activity completion time is related to the amount of resources committed to it.
  - The degree of activity details depends on the application and the level of specificity of data.

5.2 Identifying the Activities of a Project

- To determine optimal schedules we need to
  - Identify all the project's activities.
  - Determine the precedence relations among activities.
- Based on this information we can develop managerial tools for project control.

Identifying Activities, Example

KLONE COMPUTERS, INC.

- KLONE Computers manufactures personal computers.
- It is about to design, manufacture, and market the Klonepalm 2021 palmbook computer.

KLONE COMPUTERS, INC

- There are three major tasks to perform:
  - Manufacture the new computer.
  - Train staff and vendor representatives.
  - Advertise the new computer.
- KLONE needs to develop a precedence relations chart.
- The chart gives a concise set of tasks and their immediate predecessors.

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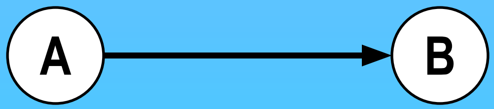
	Activity	Description
Manufacturing activities	A	Prototype model design
	B	Purchase of materials
	C	Manufacture of prototype model
	D	Revision of design
	E	Initial production run
Training activities	F	Staff training
	G	Staff input on prototype models
	H	Sales training
Advertising activities campaign	I	Pre-production advertising
	J	Post-redesign advertising campaign

KLONE COMPUTERS, INC

From the activity description chart, we can determine immediate predecessors for each activity.



Activity A is an immediate predecessor of activity B, because it must be competed just prior to the commencement of B.

KLONE COMPUTERS, INC

Precedence Relationships Chart

Activity	Immediate Predecessor	Estimated Completion Time
A	None	90
B	A	15
C	B	5
D	G	20
E	D	21
F	A	25
G	C,F	14
H	D	28
I	A	30
J	D,I	45

5.3 The PERT/CPM Approach for Project Scheduling

- The PERT/CPM approach to project scheduling uses network presentation of the project to
  - Reflect activity precedence relations
  - Activity completion time
- PERT/CPM is used for scheduling activities such that the project's completion time is minimized.

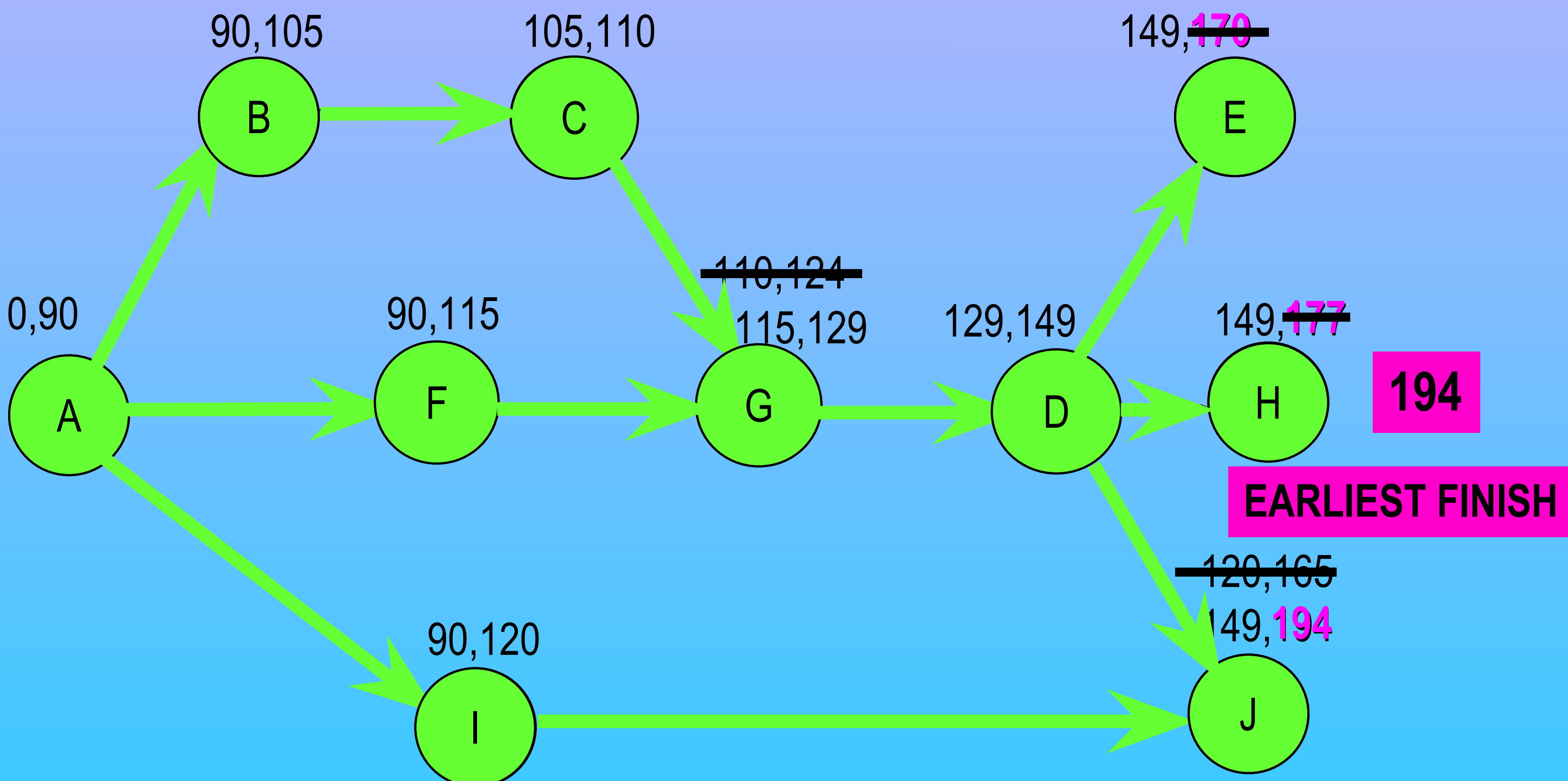
KLONE COMPUTERS, INC. - Continued

- Management at KLONE would like to schedule the activities so that the project is completed in minimal time.
- Management wishes to know:
  - The earliest and latest start times for each activity which will not alter the earliest completion time of the project.
  - The earliest finish times for each activity which will not alter this date.
  - Activities with rigid schedule and activities that have slack in their schedules.

Earliest Start Time / Earliest Finish Time

- Make a forward pass through the network as follows:
  - Evaluate all the activities which have no immediate predecessors.
    - The earliest start for such an activity is zero ES = 0.
    - The earliest finish is the activity duration EF = Activity duration.
  - Evaluate the ES of all the nodes for which EF of all the immediate predecessor has been determined.
    - ES = Max EF of all its immediate predecessors.
    - EF = ES + Activity duration.
  - Repeat this process until all nodes have been evaluated
    - EF of the finish node is the earliest finish time of the project.

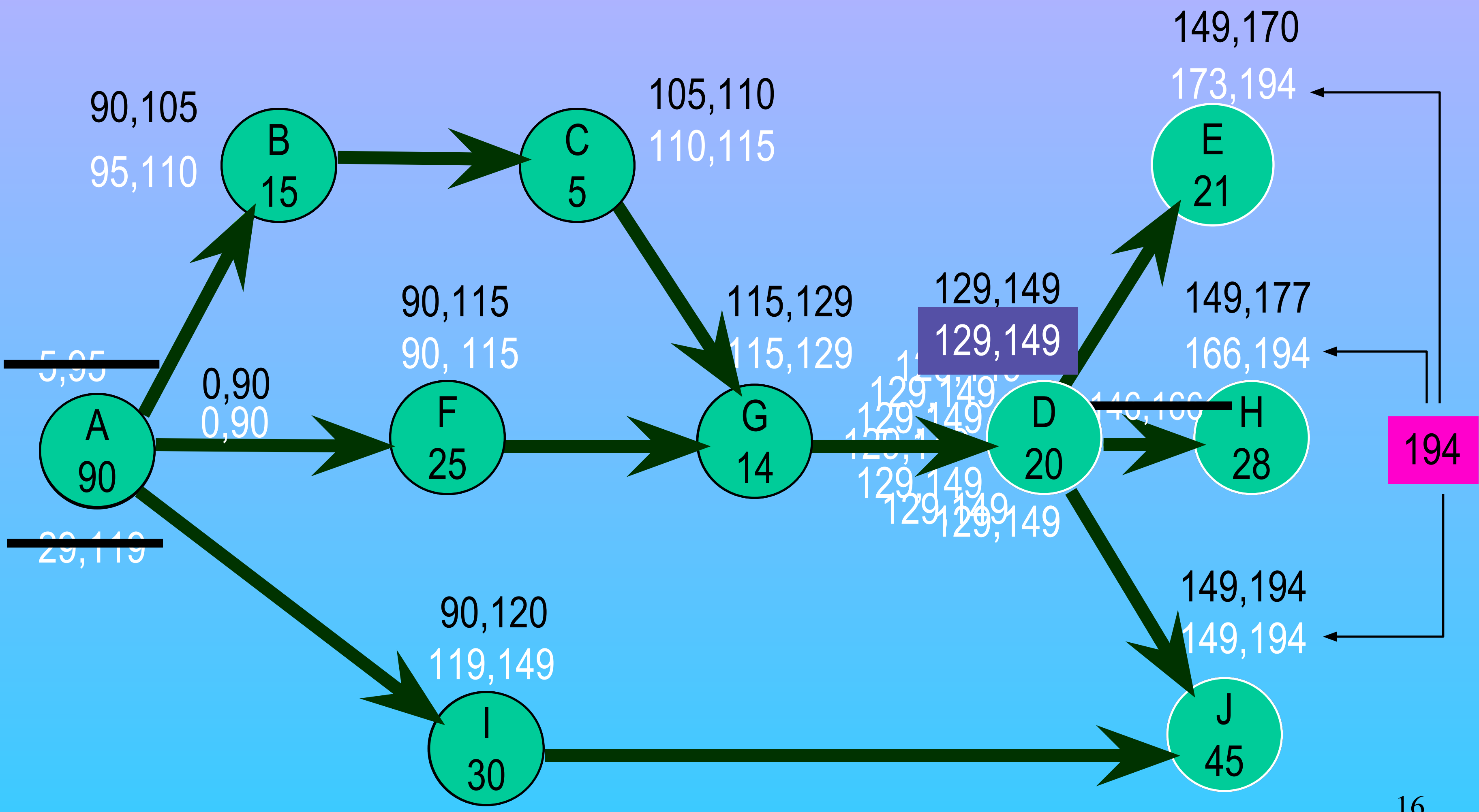
Earliest Start / Earliest Finish – Forward Pass



Latest start time / Latest finish time

- Make a backward pass through the network as follows:
  - Evaluate all the activities that immediately precede the finish node.
    - The latest finish for such an activity is LF = minimal project completion time.
    - The latest start for such an activity is LS = LF - activity duration.
  - Evaluate the LF of all the nodes for which LS of all the immediate successors has been determined.
    - LF = Min LS of all its immediate successors.
    - LS = LF - Activity duration.
  - Repeat this process backward until all nodes have been evaluated.

Latest Start / Latest Finish – Backward Pass



Slack Times

- Activity start time and completion time may be delayed by planned reasons as well as by unforeseen reasons.
- Some of these delays may affect the overall completion date.
- To learn about the effects of these delays, we calculate the **slack time**, and form the **critical path**.

Slack Times

– Slack time is the amount of time an activity can be delayed without delaying the project completion date, assuming no other delays are taking place in the project.

Slack Time = LS - ES = LF - EF

Slack time in the Klonepalm 2000 Project

Activity	LS - ES	Slack
A	0 -0	0
B	95 - 90	5
C	110 - 105	5
D	119 - 119	0
E	173 - 149	24
F	90 - 90	0
G	115 - 115	0
H	166 - 149	17
I	119 - 90	29
J	149 - 149	0

Critical activities must be rigidly scheduled

The Critical Path

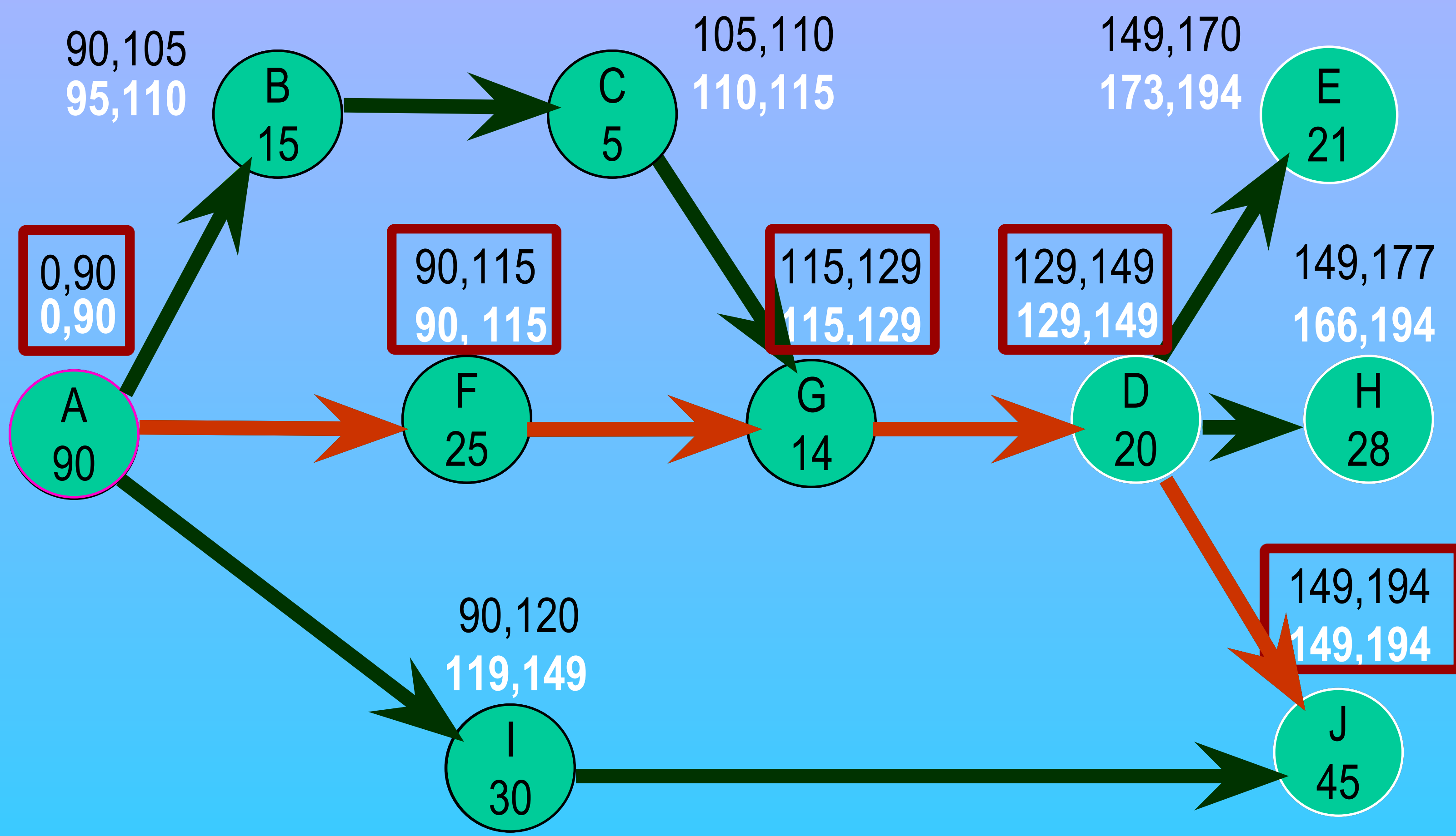
- The critical path is a set of activities that have no slack, connecting the START node with the FINISH node.

- The critical activities (activities with 0 slack) form at least one critical path in the network.

- A critical path is the longest path in the network.

- The sum of the completion times for the activities on the critical path is the minimal completion time of the project.

The Critical Path



Possible Delays

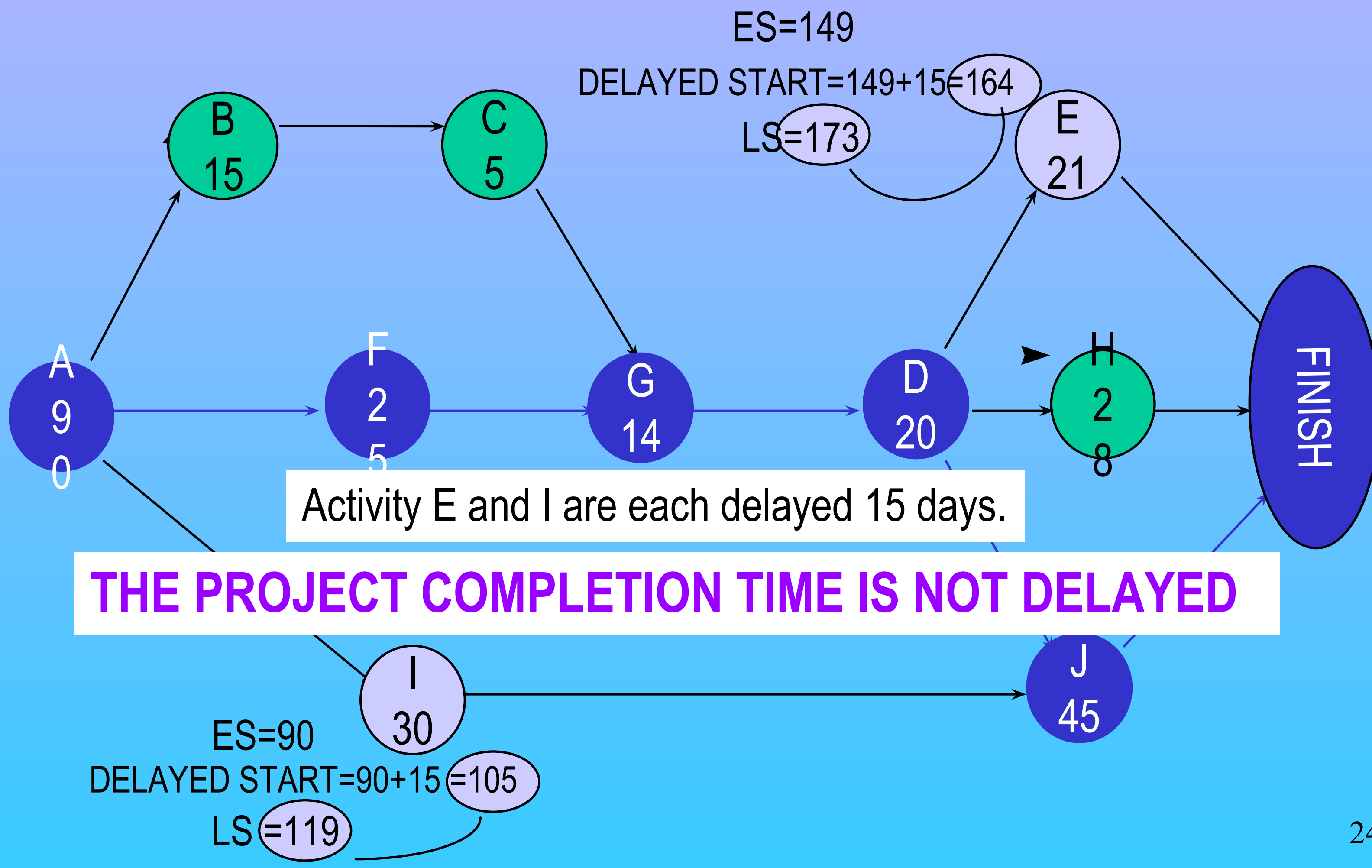
- We observe two different types of delays:
  - Single delays.
  - Multiple delays.
- Under certain conditions the overall project completion time will be delayed.
- The conditions that specify each case are presented next.

Single delays

- A delay of a certain amount in a ***critical activity***, causes the entire project to be delayed by the same amount.

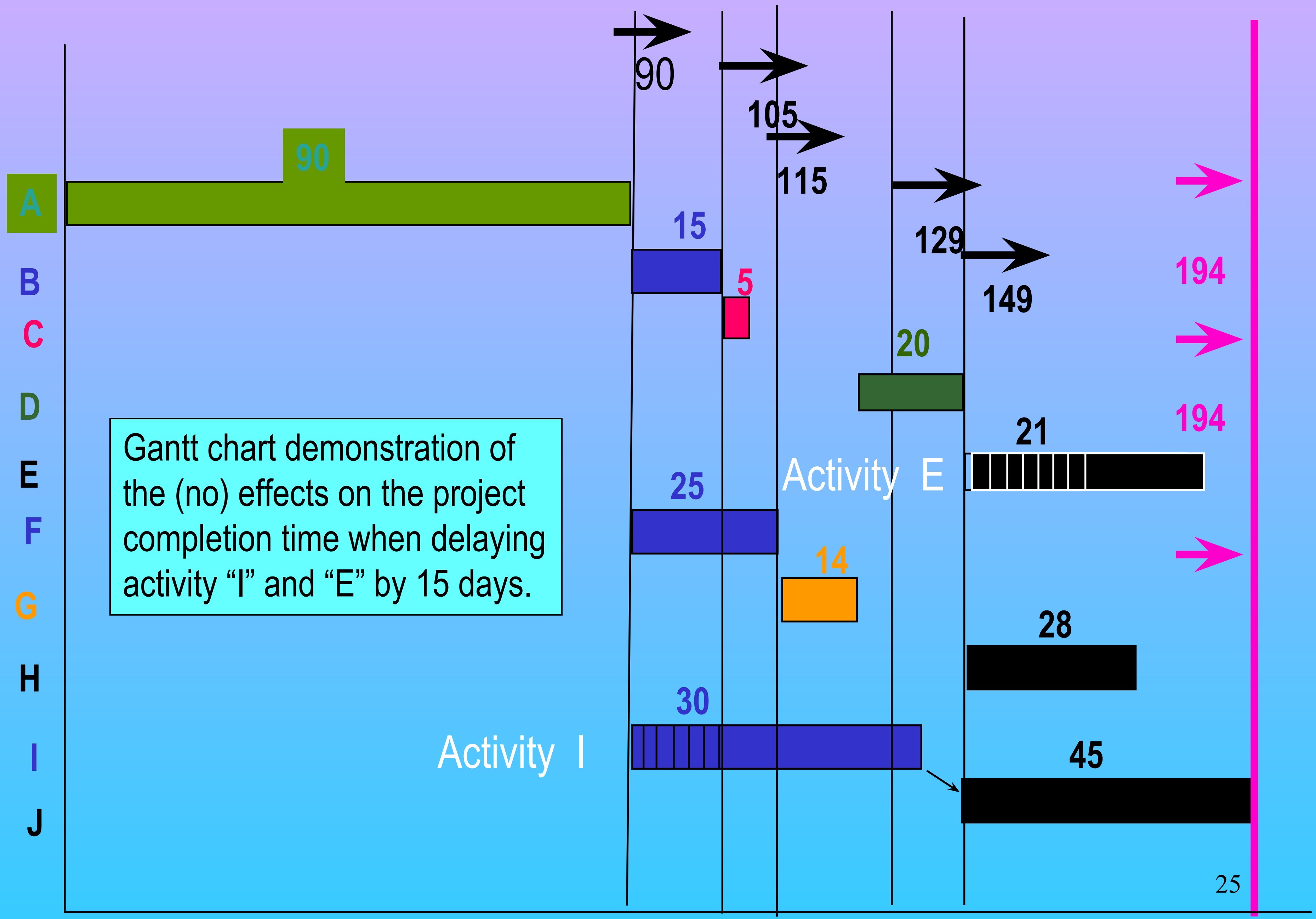
- A delay of a certain amount in a ***non-critical*** *activity* will delay the project by the amount the delay exceeds the slack time. When the delay is less than the slack, the entire project is not delayed.

Multiple delays of non critical activities: Case 1: Activities on different paths

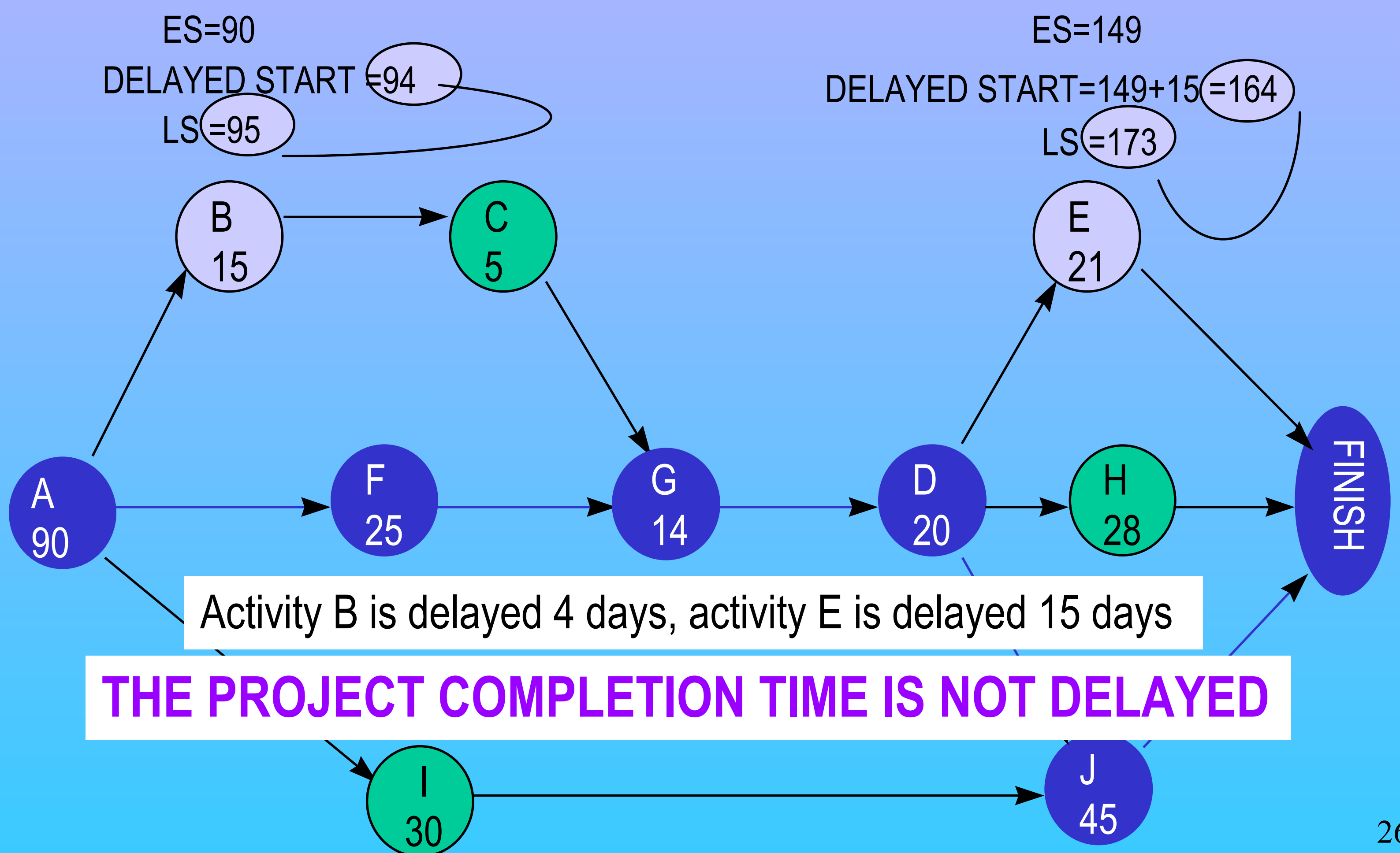


Activity E and I are each delayed 15 days.

THE PROJECT COMPLETION TIME IS NOT DELAYED


Gantt chart demonstration of the (no) effects on the project completion time when delaying activity “I” and “E” by 15 days.
A
B
C
D
E
F
G
H
I
J
90
90
15
5
105
115
129
149
20
21
25
14
28
30
45
194
194
Activity E
Activity I

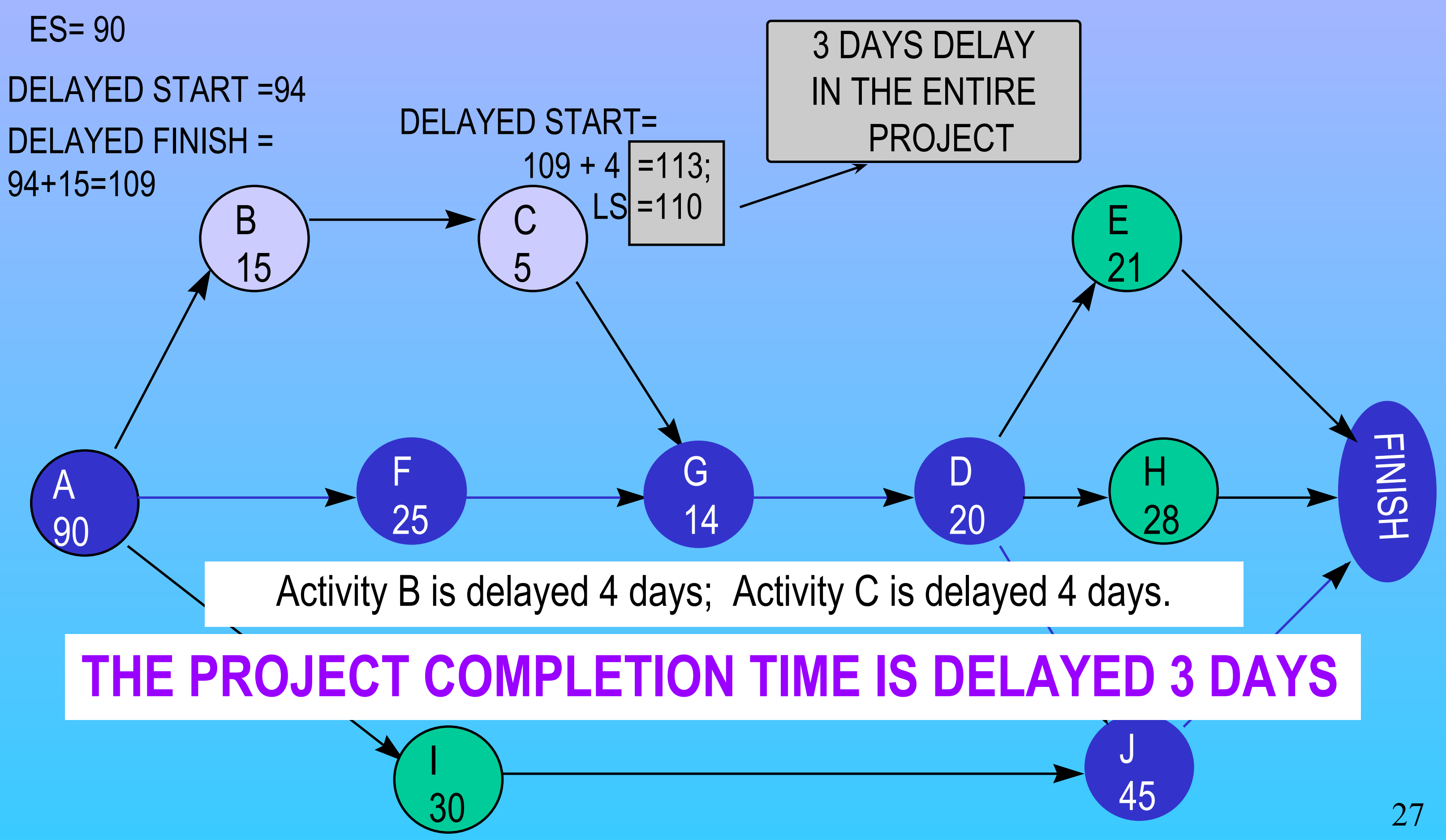
Multiple delays of non critical activities: Case 2: Activities are on the same path, separated by critical activities.



Activity B is delayed 4 days, activity E is delayed 15 days

THE PROJECT COMPLETION TIME IS NOT DELAYED

Multiple delays of non critical activities: Case 2: Activities are on the same path, no critical activities separating them.



5.4 A Linear Programming Approach to PERT/CPM

- Variables
  - X_i = The start time of the activities for i=A, B, C, …,J
  - X(FIN) = Finish time of the project
- Objective function
  - Complete the project in minimum time.
- Constraints
  - For each arc $L \rightarrow M$ a constraint states that the start time of M must not occur before the finish time of its immediate predecessor, L.

A Linear Programming Approach

Define X(FIN) to be the finish time of the project.

The objective then is

Minimize X(FIN)

While this objective function is intuitive other objective functions provide more information, and are presented later.

A Linear Programming Approach

Minimize X(FIN)

ST

$X(FIN) \geq X_E + 21$

$X(FIN) \geq X_H + 28$

$X(FIN) \geq X_J + 45$

$X_D \geq X_G + 14$

$X_E \geq X_D + 20 \quad X_G \geq X_C + 5$

$X_H \geq X_D + 20 \quad X_G \geq X_F + 25$

$X_J \geq X_D + 20 \quad X_I \geq X_D + 90$

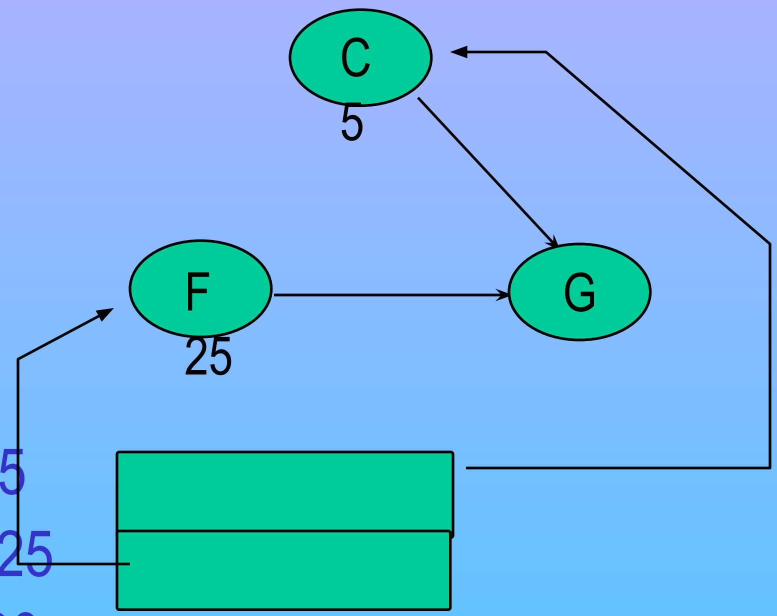
$X_J \geq X_I + 30 \quad X_F \geq X_A + 90$

$X_C \geq X_B + 15$

All X s are nonnegative

$X_D \geq X_G + 14$

$X_B \geq X_A + 90$



A Linear Programming Approach

Minimize $X_A+X_B+\ldots+X_J$

This objective function ensures that the optimal X values are the **earliest start** times of all the activities. The project completion time is minimized.

Maximize $X_A+X_B+\ldots+X_J$

S.T. $X(FIN) = 194$

and all the other constraints as before.

This objective function and the additional constraint ensure that the optimal X values are the **latest start** times of all the activities.

5.5 Obtaining Results Using Excel

CRITICAL PATH ANALYSIS

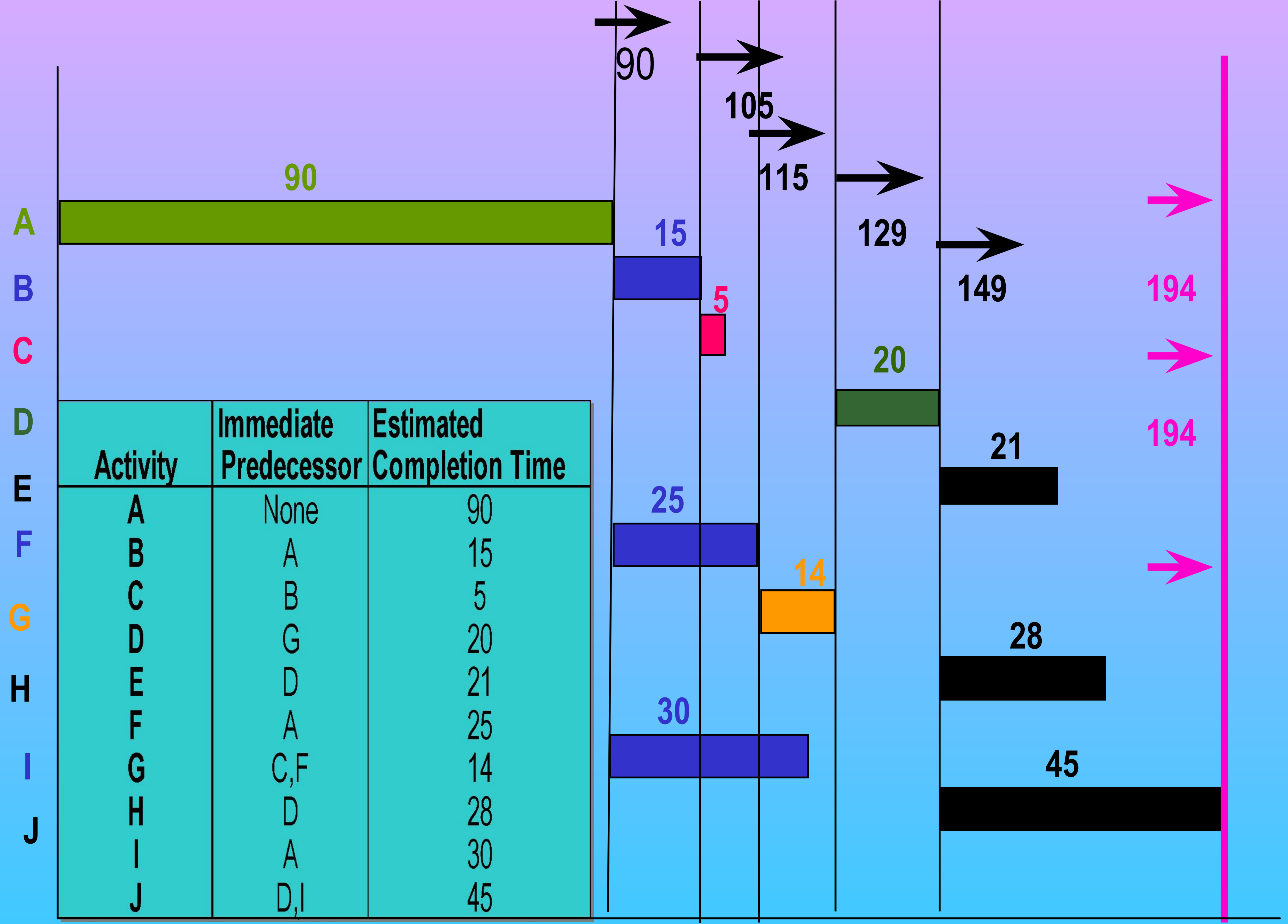
MEAN	194		
STANDARD DEVIATION*	0	* Assumes all critical activities are on one critical path	
VARIANCE*	0	If not, enter in gold box, the **variance** on one critical path of interest.	
PROBABILITY COMPLETE BEFORE		=	

Acitivty	Node	Critical	μ	σ	σ^2	ES	EF	LS	LF	Slack
Design	A	*	90			0	90	0	90	0
Materials	B		15			90	105	95	110	5
Manufacture	C		5			105	110	110	115	5
Design Revision	D	*	20			129	149	129	149	0
Production Run	E		21			149	170	173	194	24
Staff Training	F	*	25			90	115	90	115	0
Staff Input	G	*	14			115	129	115	129	0
Sales Training	H		28			149	177	166	194	17
Preprod. Advertise	I		30			90	120	119	149	29
Post. Advertise	J	*	45			149	194	149	194	0

5.6 Gantt Charts

- Gantt charts are used as a tool to monitor and control the project progress.
- A Gantt Chart is a graphical presentation that displays activities as follows:
  - Time is measured on the horizontal axis. A horizontal bar is drawn proportionately to an activity' s expected completion time.
  - Each activity is listed on the vertical axis.
- In an *earliest time Gantt chart* each bar begins and ends at the earliest start/finish the activity can take place.

Here‘s how we build an Earliest Time Gantt Chart for KLONEPALM 2000



Activity	Immediate Predecessor	Estimated Completion Time
A	None	90
B	A	15
C	B	5
D	G	20
E	D	21
F	A	25
G	C,F	14
H	D	28
I	A	30
J	D,I	45

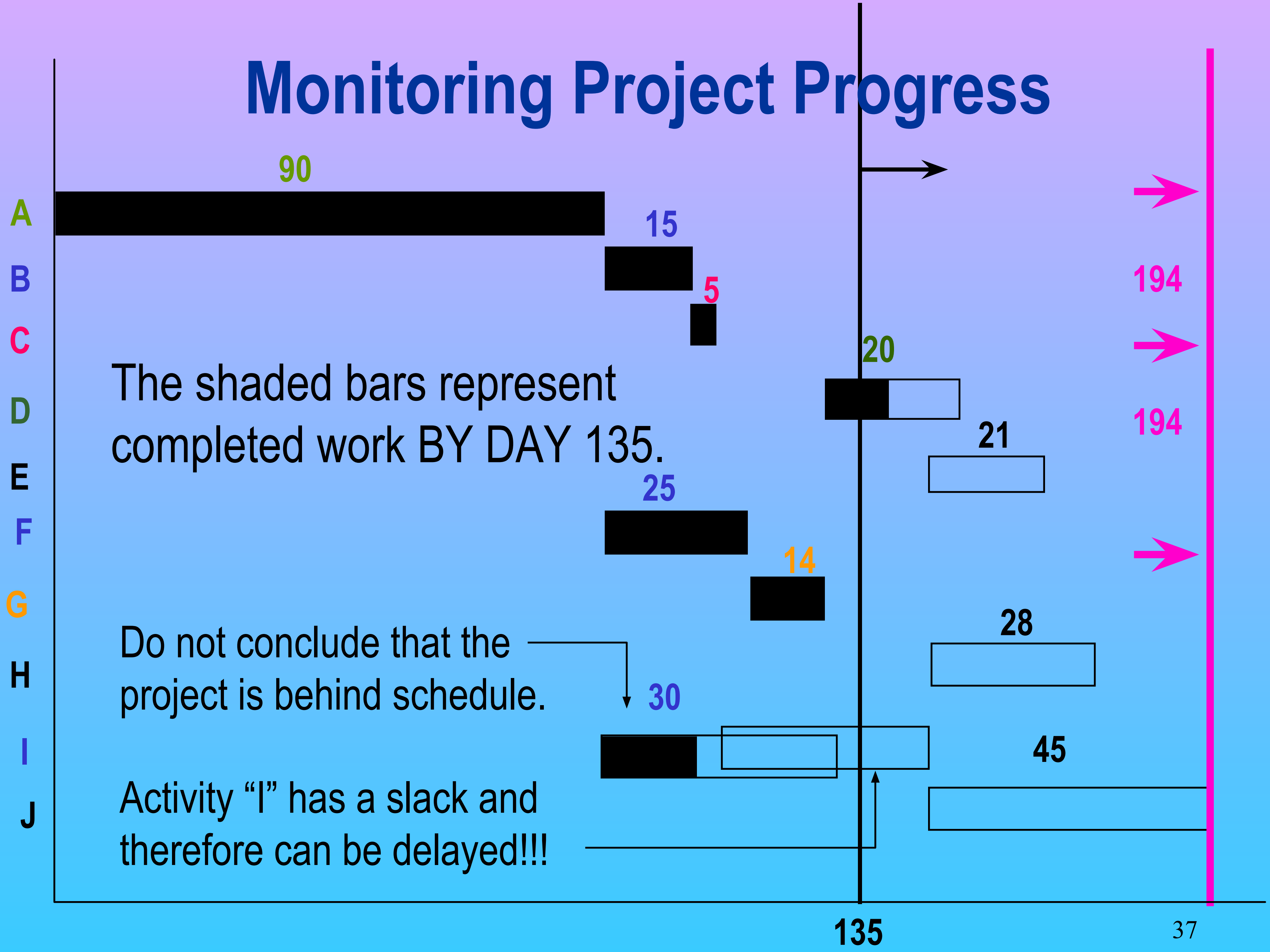
Gantt Charts-
Monitoring Project Progress

- Gantt chart can be used as a visual aid for tracking the progress of project activities.

- Appropriate percentage of a bar is shaded to document the completed work.

- The manager can easily see if the project is progressing on schedule (with respect to the earliest possible completion times).

Monitoring Project Progress



Gantt Charts – Advantages and Disadvantages

- Advantages.
  - Easy to construct
  - Gives earliest completion date.
  - Provides a schedule of earliest possible start and finish times of activities.

- Disadvantages
  - Gives only one possible schedule (earliest).
  - Does not show whether the project is behind schedule.
  - Does not demonstrate the effects of delays in any one activity on the start of another activity, thus on the project completion time.

5.7 Resource Leveling and Resource Allocation

- It is desired that resources are evenly spread out throughout the life of the project.
- Resource leveling methods (usually heuristics) are designed to:
  - Control resource requirements
  - Generate relatively similar usage of resources over time.

Resource Leveling – A Heuristic

- A heuristic approach to “level” expenditures
  - Assumptions
    - Once an activity has started it is worked on continuously until it is completed.
    - Costs can be allocated equally throughout an activity duration.

Step 1: Consider the schedule that begins each activity at its ES.

Step 2: Determine which activity has slack at periods of peak spending.

Step 3: Attempt to reschedule the non-critical activities performed during these peak periods to periods of less spending, but within the time period between their ES and LF.

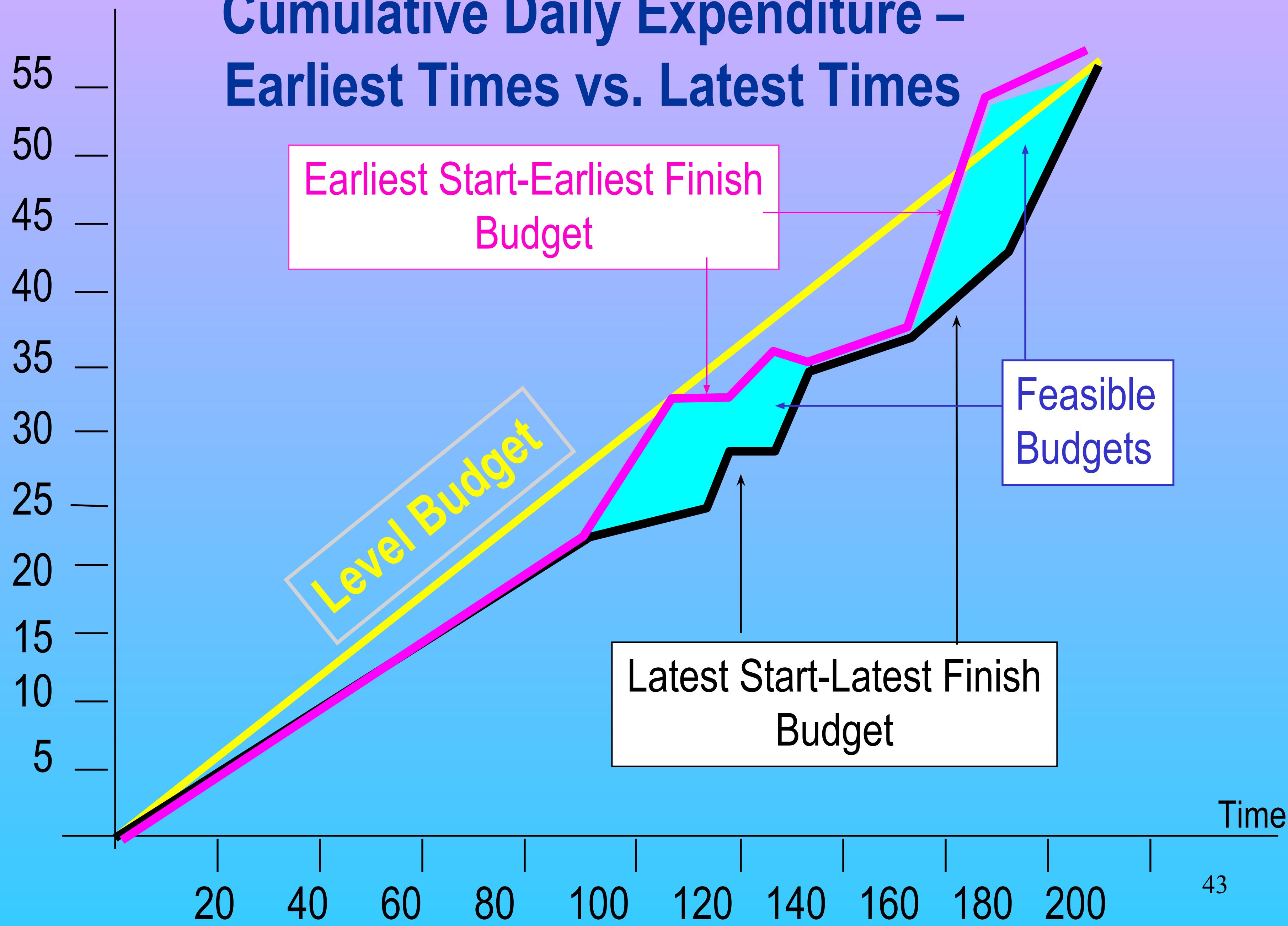
Resource Leveling –
KLONE COMPUTERS, Inc. - continued

- Management wishes to schedule the project such that
  - Completion time is 194 days.
  - Daily expenditures are kept as constant as possible.
- To perform this analysis cost estimates for each activity will be needed.

Resource Leveling –
KLONE COMPUTERS, Inc. – cost estimates

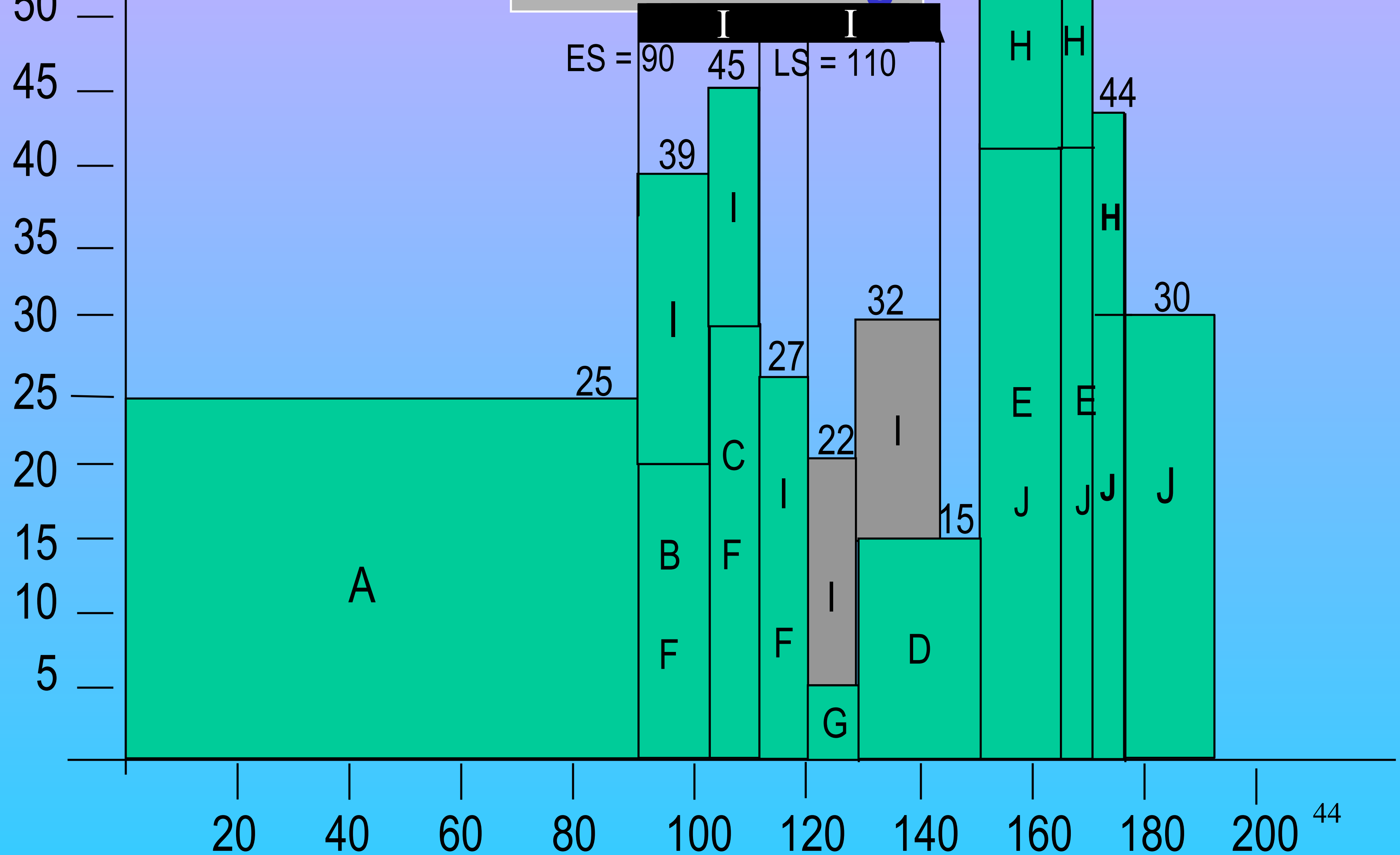
Activity	Description	Total Cost (x10000)	Total Time (days)	Cost per Day
A	Prototype model design	2250	90	25
B	Purchase of materials	180	15	12
C	Manufacture of prototype	90	5	18
D	Revision of design	300	20	15
E	Initial production run	231	21	11
F	Staff training	250	25	10
G	Staff input on prototype	70	14	5
H	Sales traini ng	392	28	14
I	Pre-production advertisement	510	30	17
J	Post-production advertisement	1350	45	30
	Total cost =	5,623		

Cumulative Daily Expenditure – Earliest Times vs. Latest Times

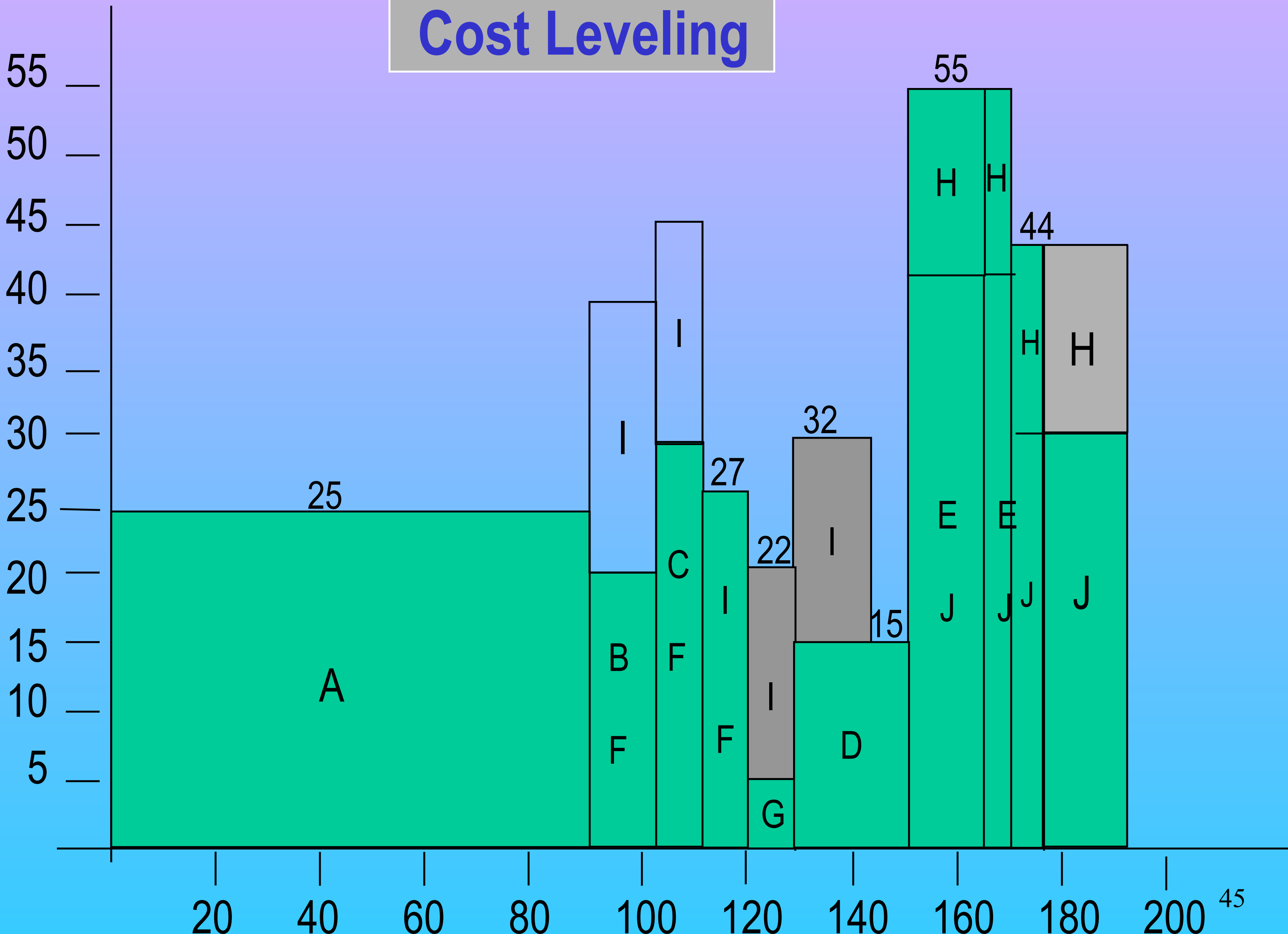


Daily Expenditure of the ES Schedule

Cost Leveling



Cost Leveling



5.8 The Probability Approach to Project Scheduling

- Activity completion times are seldom known with 100% accuracy.
- PERT is a technique that treats activity completion times as random variables.
- Completion time estimates are obtained by the *Three Time Estimate approach*

The Probability Approach – Three Time Estimates

- **The T*hree Time Estimate approach* provides completion time estimate for each activity.**
- **We use the notation:**

a = an optimistic time to perform the activity.

m = the most likely time to perform the activity.

b = a pessimistic time to perform the activity.

The Distribution, Mean, and Standard Deviation of an Activity

Approximations for the mean and the standard deviation of activity completion time are based on the <u>Beta</u> distribution.

$$\mu = \text{the mean completion time} = \frac{a + 4m + b}{6}$$

$$\sigma = \text{the standard deviation} = \frac{b - a}{6}$$

The Project Completion Time Distribution - Assumptions

To calculate the mean and standard deviation of the project completion time we make some simplifying assumptions.

The Project Completion Time Distribution - Assumptions

- Assumption 1
  - A critical path can be determined by using the mean completion times for the activities.
  - The project mean completion time is determined solely by the completion time of the activities on the critical path.
- Assumption 2
  - The time to complete one activity is independent of the time to complete any other activity.
- Assumption 3
  - There are enough activities on the critical path so that the distribution of the overall project completion time can be approximated by the normal distribution.

The Project Completion Time Distribution

The three assumptions imply that the overall project completion time is normally distributed, the following parameters:

Mean = Sum of mean completion times along the critical path.

Variance = Sum of completion time variances along the critical path.

Standard deviation = $\sqrt{\text{Variance}}$

The Probability Approach –

KLONE COMPUTERS

Activity	Optimistic	Most Likely	Pessimistic
A	76	86	120
B	12	15	18
C	4	5	6
D	15	18	33
E	18	21	24
F	16	26	30
G	10	13	22
H	24	18	32
I	22	27	50
J	38	43	60

The Probability Approach –
KLONE COMPUTERS

- Management at KLONE is interested in information regarding the completion time of the project.
- The probabilistic nature of the completion time must be considered.

KLONE COMPUTERS –
Finding activities' mean and variance

$\mu_A = [76+4(86)+120]/6 = 90$

$\sigma_A = (120 - 76)/6 = 7.33$ $\sigma_A^2 = (7.33)^2 = 53.78$

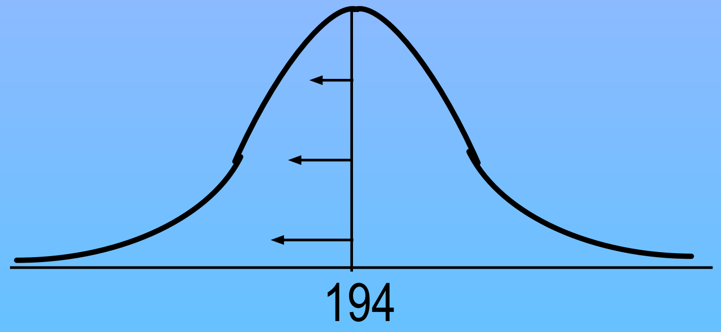
Activity	μ	σ	σ^2
A	90	7.33	53.78
B	15	1.00	1.00
C	5	0.33	0.11
D	20	3.00	9.00
E	21	1.00	1.00
F	25	2.33	5.44
G	14	2.00	4.00
H	28	1.33	1.78
I	30	4.67	21.78
J	45	3.67	13.44

KLONE COMPUTERS –
Finding mean and variance for the critical path

- The mean times are the same as in the CPM problem, previously solved for KLONE.
- Thus, the critical path is A - F- G - D – J.
  - Expected completion time = $\mu_A + \mu_F + \mu_G + \mu_D + \mu_J = 194$.
  - The project variance $= \sigma_A^2 + \sigma_F^2 + \sigma_G^2 + \sigma_D^2 + \sigma_J^2 = 85.66$
  - The standard deviation = $\sqrt{\sigma^2}$ = 9.255

The Probability Approach – Probabilistic analysis

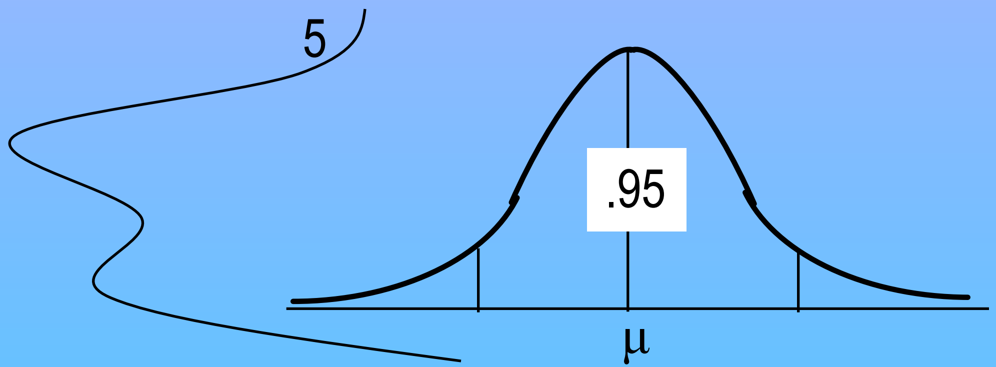
- The probability of completion in 194 days =



$$P(X \le 194) = P(Z \le \frac{194 - 194}{9.255}) = P(Z \le 0) = 0.5$$

The Probability Approach – Probabilistic analysis

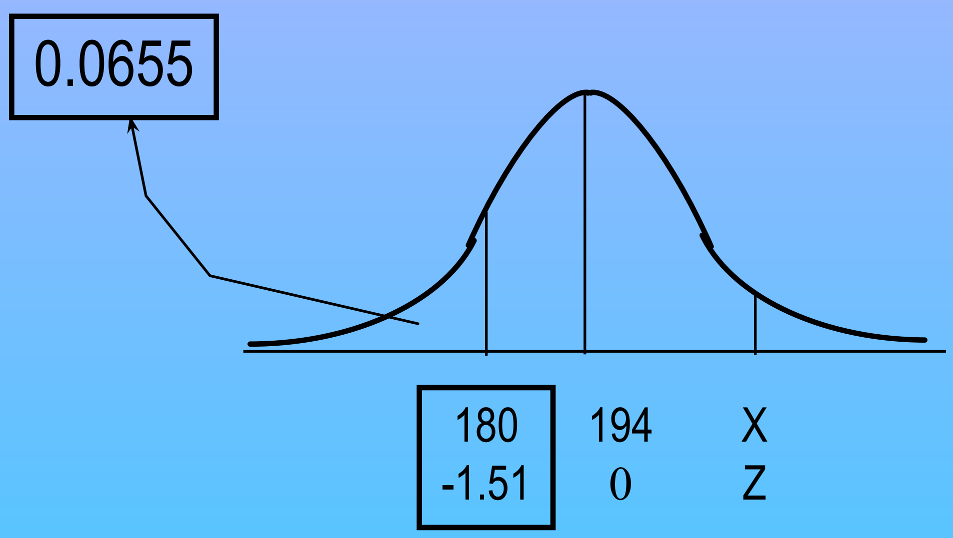
- An interval in which we are reasonably sure the completion date lies is $\mu \pm z_{0.025}\ \sigma$



- The interval is = $194 \pm 1.96(9.255) \cong [175, 213]$ days.
- The probability that the completion time lies in the interval [175,213] is 0.95.

The Probability Approach – Probabilistic analysis

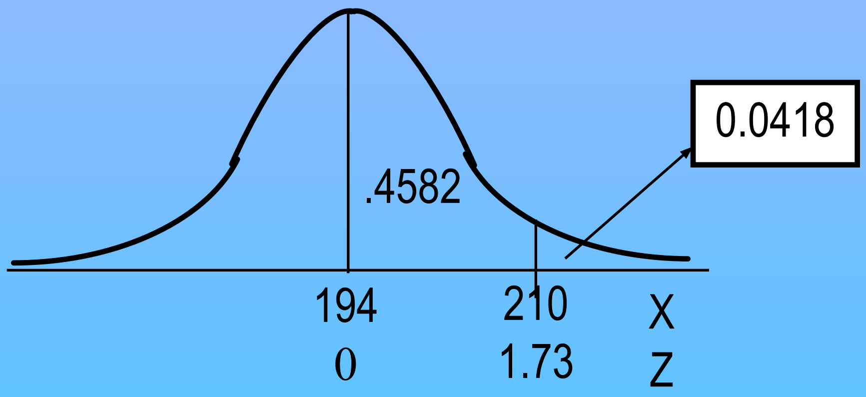
- The probability of completion in 180 days =



$$P(X \le 180) = P(Z \le -1.51) = 0.5 - 0.4345 = 0.0655$$

The Probability Approach – Probabilistic analysis

- The probability that the completion time is longer than 210 days =

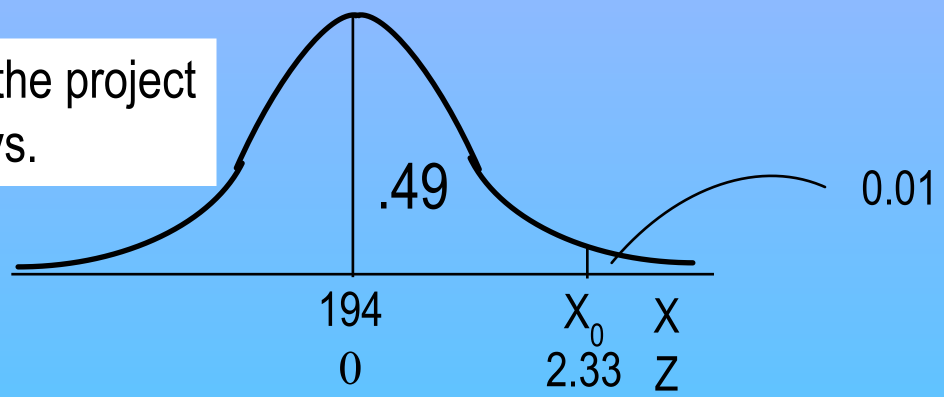


$$P(X \geq 210) = P(Z \geq 1.73) = 0.5 - 0.458 = 0.0418$$

The Probability Approach – Probabilistic analysis

- Provide a completion time that has only 1% chance to be exceeded.

There is 99% chance that the project is completed in 215.56 days.



$P(X \geq X_0) = 0.01$, or $P(Z \geq [(X_0 - \mu)/\sigma] = P(Z \geq Z_0) = .01$

$P(Z \geq 2.33) = 0.01$; $X_0 = \mu + Z_0\sigma = 194 + 2.33(9.255) = 215.56$ days.

The Probability Approach –
Probabilistic analysis with a spreadsheet

	A	B	C	D
1	mu=	194	sigma=	9.255
2				
3	1	Probability(X < 194)		0.5
4				
5	2	LCL		175.8606
6		UCL		212.1394
7				
8	3	Probability(X < 180)		0.065178
9				
10	4	Probability(X > 210)		0.041923
11				
12	5	99% chance of completing by		215.5303

NORMDIST(194, 194, 9.255, TRUE)

NORMINV(.025, 194, 9.255)

NORMINV(.975, 194,

NORMDIST(180, 194, 9.255, TRUE)

1 - NORMDIST(210, 194, 9.255, TRUE)

NORMINV(.99, 194, 9.255)

The Probability Approach –
Critical path spreadsheet

CRITICAL PATH ANALYSIS

MEAN	194			
STANDARD DEVIATION*	9.255629	* Assumes all critical activities are on one critical path		
VARIANCE*	85.66667	If not, enter in gold box, the **variance** on one critical path of interest.		
PROBABILITY COMPLETE BEFORE		180	=	0.065192

Acitivty	Node	Critical	μ	σ	σ^2	ES	EF	LS	LF	Slack
Design	A	*	90	7.333333	53.77778	0	90	0	90	0
Materials	B		15	1	1	90	105	95	110	5
Manufacture	C		5	0.333333	0.111111	105	110	110	115	5
Design Revision	D	*	20	3	9	129	149	129	149	0
Production Run	E		21	1	1	149	170	173	194	24
Staff Training	F	*	25	2.333333	5.444444	90	115	90	115	0
Staff Input	G	*	14	2	4	115	129	115	129	0
Sales Training	H		28	1.333333	1.777778	149	177	166	194	17
Preprod. Advertise	I		30	4.666667	21.77778	90	120	119	149	29
Post. Advertise	J	*	45	3.666667	13.44444	149	194	149	194	0

The Probability Approach – critical path spreadsheet

CRITICAL PATH ANALYSIS

MEAN
STANDAR
VARIANCE
PROBABIL

Acitivty	lack
Design	0
Materials	0
Manufacture	0
Design Revis	0
Production R	24
Staff Trainin	6
Staff Input	0
Sales Trainir	17
Preprod. Adv	24
Post. Adverti	0

<u>A comment – multiple critical paths</u>

In the case of multiple critical paths (a not unusual situation), determine the probabilities for each critical path separately using its standard deviation.

However, the probabilities of interest (for example, $P(X \geq x)$) cannot be determined by each path alone. To find these probabilities, check whether the paths are independent.

If the paths are independent (no common activities among the paths), multiply the probabilities of all the paths:
[Pr(Completion time≥x) = Pr(Path 1≥x)P(Path 2≥x)…Path k≥x)]

If the paths are dependent, the calculations might become very cumbersome, in which case running a computer simulation seems to be more practical.

5.9 Cost Analysis Using the Expected Value Approach

- Spending extra money, in general should decrease project duration.
- Is this operation cost effective?
- The expected value criterion is used to answer this question.

KLONE COMPUTERS -
Cost analysis using probabilities

- Analysis indicated:
  - Completion time within 180 days yields an additional profit of $1 million.
  - Completion time between 180 days and 200 days, yields an additional profit of $400,000.
  - Completion time reduction can be achieved by additional training.

KLONE COMPUTERS -
Cost analysis using probabilities

- Two possible activities are considered for training.
  - *Sales personnel training:*
    - C
    - N ... s.
  - *Technical staff training:*
    - Cost $250,000;
    - New time estimates are $a = 12$, $m = 14$, and $b = 16$.

Which option should be pursued?

KLONE COMPUTERS -
Cost analysis using probabilities

- Evaluation of spending on *sales personnel training.*
  - This activity (H) is not critical.
  - Under the assumption that the project completion time is determined solely by critical activities, this option should not be considered further.
- Evaluation of spending on *technical staff training.*
  - This activity (F) is critical.
  - This option should be further studied as follows:
    - Calculate expected profit when not spending $250,000.
    - Calculate expected profit when spending $250,000.
    - Select the decision with a higher expected profit.

KLONE COMPUTERS - Cost analysis using probabilities

– *Case 1: Do not spend $250,000 on training.*

- Let X represent the project's completion time.
- Expected gross additional profit = E(GP) = $P(X<180)(\$1 \text{ million}) + P(180<X<200)(\$400{,}000) + P(X>200)(0)$.
- Use Excel to find the required probabilities: $P(X<180) = .065192$; $P(180<X<200) = .676398$; $P(X>200) =.25841$
- Expected gross additional profit = . $.065192(1M)+.676398(400K)+ .25841(0) = \$335{,}751.20$

KLONE COMPUTERS -
Cost analysis using probabilities

– *Case 2: Spend $250,000 on training.*
  - The revised mean time and standard deviation estimates for activity F are:

    $\mu_F = (12 + 4\ (14) + 16)/6 = 14$
    $\sigma_F = (16 - 12)/6 = 0.67$
    $\sigma_F^2 = 0(.67)^2 = 0.44$
  - Using the Excel PERT-CPM template we find a new critical path (A-B-C-G-D-J), with a mean time = 189 days, and a standard deviation of = 9.0185 days.

KLONE COMPUTERS -
Cost analysis using probabilities

– The probabilities of interest need to be recalculated. From Excel we find:
  - $P(X < 180) = .159152$;
  - $P(180 < X < 200) = .729561$
  - $P(X > 200) = .111287$

– Expected Gross Additional Revenue =
P(X<180)(1M)+P(180<X<200)(400K)+P(X>200)(0)
= .159152(!M)+ .729561(400K)+ .111287(0)
= $450,976.40

KLONE COMPUTERS -
Cost analysis using probabilities

The expected net additional profit =

450,976-250,000 = \$200,976 < \$335,751

Expected additional net profit when spending \$250,000 on training

Expected profit without spending \$250,000 on training

Conclusion: Management should not spend money on additional training of technical personnel.

5.10 Cost Analyses Using
The Critical Path Method (CPM)

- The critical path method (CPM) is a deterministic approach to project planning.
- Completion time depends only on the amount of money allocated to activities.
- Reducing an activity's completion time is called "crashing."

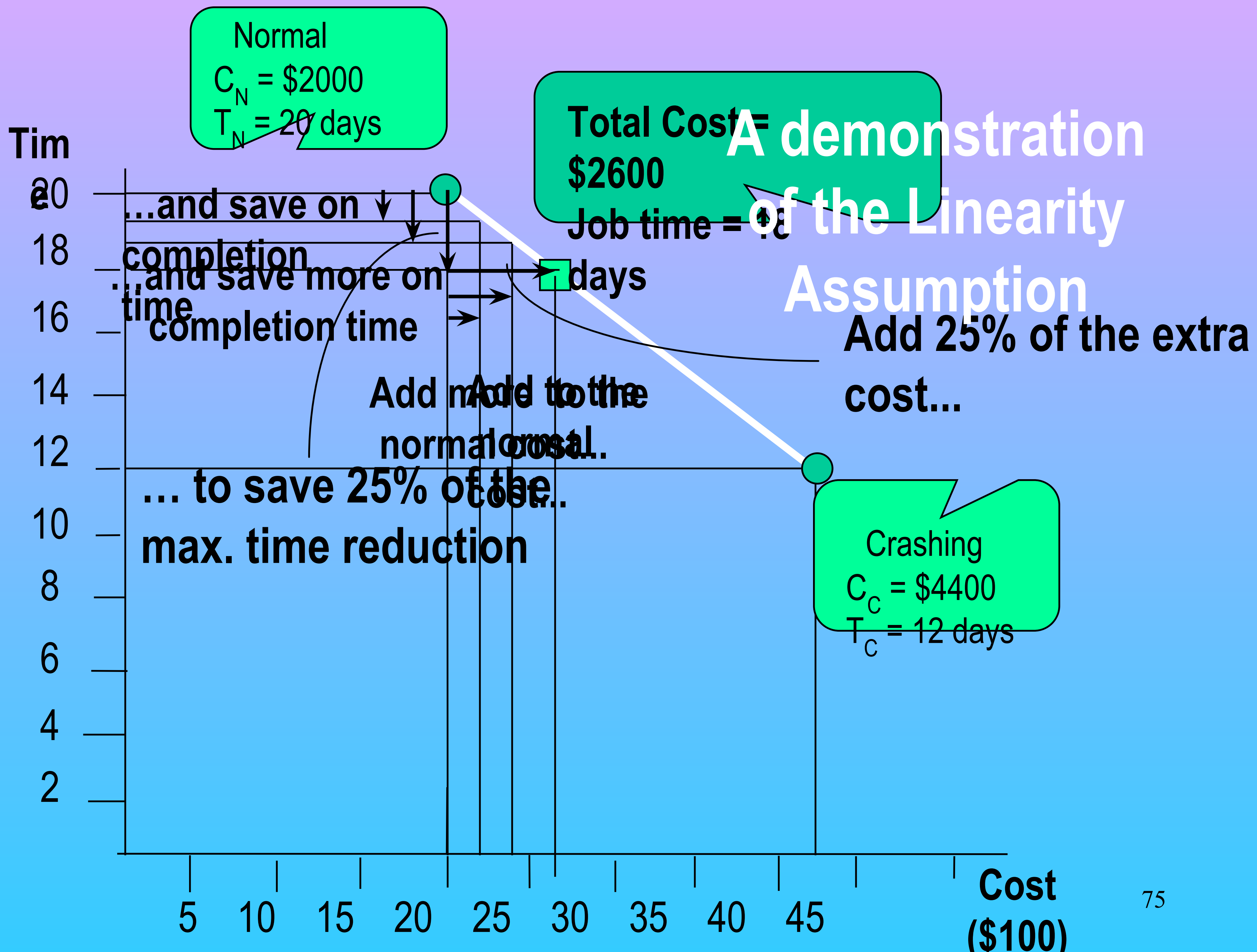
Crash time/Crash cost

- There are two crucial completion times to consider for each activity.
  - Normal completion time (T_N).
  - Crash completion time (T_C), the minimum possible completion time.
- The cost spent on an activity varies between
  - Normal cost (C_N). The activity is completed in T_N.
  - Crash cost (C_C). The activity is completed in T_C.

Crash time/Crash cost – The Linearity Assumption

- The maximum crashing of activity completion time is $T_C - T_N$.
- This can be achieved when spending $C_N - C_C$.
- Any percentage of the maximum extra cost ($C_N - C_C$) spent to crash an activity, yields the same percentage reduction of the maximum time savings ($T_C - T_N$).

A demonstration of the Linearity Assumption



Crash time/ Crash cost - The Linearity Assumption

$$\text{Marginal Cost} = \frac{\text{Additional Cost to get Max. Time Reduction}}{\text{Maximum Time reduction}}$$

$$= (4400 - 2000)/(20 - 12) = \$300 \text{ per day}$$

$$M = \frac{E}{R}$$

Crashing activities –
Meeting a Deadline at Minimum Cost

- If the deadline to complete a project cannot be met using normal times, additional resources must be spent on crashing activities.
- The objective is to meet the deadline at minimal additional cost.

Baja Burrito Restaurants – Meeting a Deadline at Minimum Cost

- Baja Burrito (BB) is a chain of Mexican-style fast food restaurants.
- It is planning to open a new restaurant in 19 weeks.
- Management wants to
  - Study the feasibility of this plan,
  - Study suggestions in case the plan cannot be finished by the deadline.

Baja Burrito Restaurants –

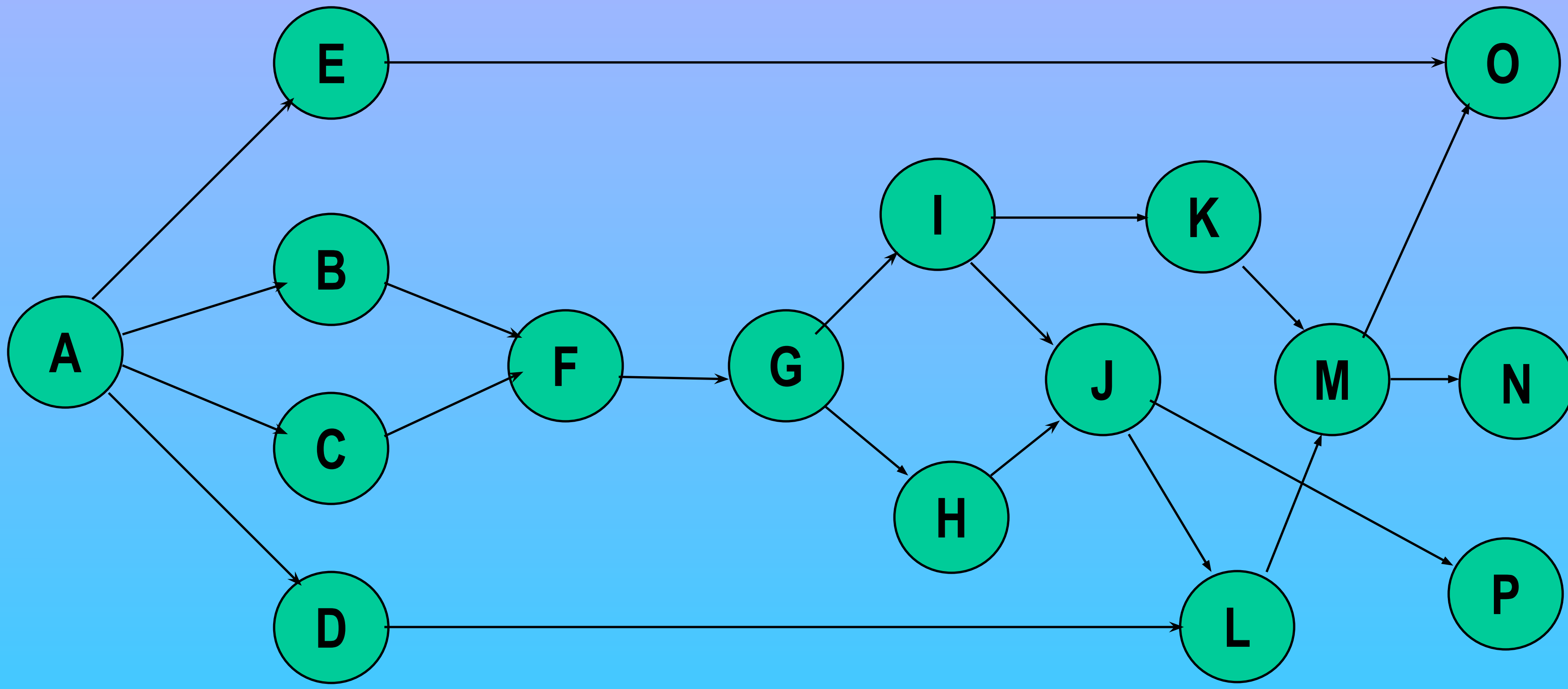
Activity	Immediate Predecessors	Normal Time	Normal Cost	Crash Time	Crash Cost
A-Plan Revisions/Approvals	---	5	25	3	36
B-Grade Land	A	1	10	0.5	15
C-Purchase Materials	A	3	18	1.5	22
D-Order/Receive Equipment	A	2	8	1	12
E-Order/Receive Furniture	A	4	8	1.5	15
L-Install Equipment	D, J	3	14	1.5	22
M-Finish/Paint Inside	K, L	3	10	1.5	18
N-Tile Floors	M	3	6	1	9
O-Install Furniture	E, M	4	8	2.5	14
P-Finish/Paint Outside	J	4	18	2.5	26
TOTAL		**29***	**$200**	**17***	**$300**

Without spending any extra money, the restaurant will open in 29 weeks at a normal cost of $200,000.

When all the activities are crashed to the maximum, the restaurant will open in 17 weeks at crash cost of $300,000.

Determined by the PERT.xls template

Baja Burrito Restaurants – Network presentation



Baja Burrito Restaurants – Marginal costs

Activity	Maximum Reduction R	Extra Cost E	Cost Per Week Reduction M = E/R
A	2.0	11	5.50
B	0.5	5	10.00
C	1.5	4	2.67
D	1.0	4	4.00
E	2.5	7	2.80
F	0.5	[illegible]	6.00
G	1.5	[illegible]	6.67
H	0.5	[illegible]	10.00
I	1.5	[illegible]	5.33
J	0.5	6	12.00
K	1.0	4	4.00
L	1.5	8	5.33
M	1.5	8	5.33
N	2.0	3	1.50
O	1.5	6	4.00
P	1.5	8	5.33

$R = T_N - T_C = 5 - 3 = 2$

$E = C_C - C_N = 36 - 25 = 11$

$M = E/R = 11/2 = 5.5$

Baja Burrito Restaurants – Heuristic Solution

- Small crashing problems can be solved heuristically.
- Three observations lead to the heuristic.
  - The project completion time is reduced only when critical activity times are reduced.
  - The maximum time reduction for each activity is limited.
  - The amount of time a critical activity can be reduced before another path becomes critical is limited.

Baja Burrito Restaurants – Linear Programming

- Linear Programming Approach
  - Variables

    X_j = start time for activity j.

    Y_j = the amount of crash in activity j.
  - Objective Function

    Minimize the total additional funds spent on crashing activities.
  - Constraints
    - No activity can be reduced more than its Max. time reduction.
    - Start time of an activity takes place not before the finish time of all its immediate predecessors.
    - The project must be completed by the deadline date D.

Baja Burrito Restaurants – Linear Programming

$$\text{Min } 5.5Y_A+10Y_B+2.67Y_C+4Y_D+2.8Y_E+6Y_F+6.67Y_G+10Y_H+5.33Y_I+12Y_J+4Y_K+5.33Y_L+1.5Y_N+4Y_O+5.33Y_P$$

Minimize total crashing costs

Activity	Maximum	Extra Cost	Cost Per Week ...uction = E/R
A	2.0	11	5.50
B	0.5	5	10.00
C	1.5	4	2.67
D	1.0	4	4.00
E	2.5	7	2.80
F	0.5	3	6.00
G	1.5	10	6.67
H	0.5	5	10.00
I	1.5	8	5.33
J	0.5	6	12.00
K	1.0	4	4.00
L	1.5	8	5.33
M	1.5	8	5.33
N	2.0	3	1.50
O	1.5	6	4.00
P	1.5	8	5.33

Linear Programming

$$\text{Min } 5.5Y_A+10Y_B+2.67Y_C+4Y_D+2.8Y_E+6Y_F+6.67Y_G+10Y_H+5.33Y_I+12Y_J+4Y_K+5.33Y_L+1.5Y_N+4Y_O+5.33Y_P$$

ST

$X(FIN) \le 19$

Meet the deadline

Activity	Maximum Reduction R	Extra Cost E	Cost Per Week Reduction M = E/R
	2.0		5.50
	0.5		10.00
	1.5		2.67
	1.0		4.00
	2.5		2.80
F	0.5		6.00
G	1.5		
H	0.5		
I	1.5		
J	0.5		
K	1.0		
L	1.5		5.33
M	1.5		5.33
N	2.0		1.50
O	1.5		4.00
P	1.5		5.33

$Y_A \le 2.0$

$Y_B \le 0.5$

$Y_C \le 1.5$

$Y_D \le 1.0$

$Y_E \le 2.5$

$Y_F \le 0.5$

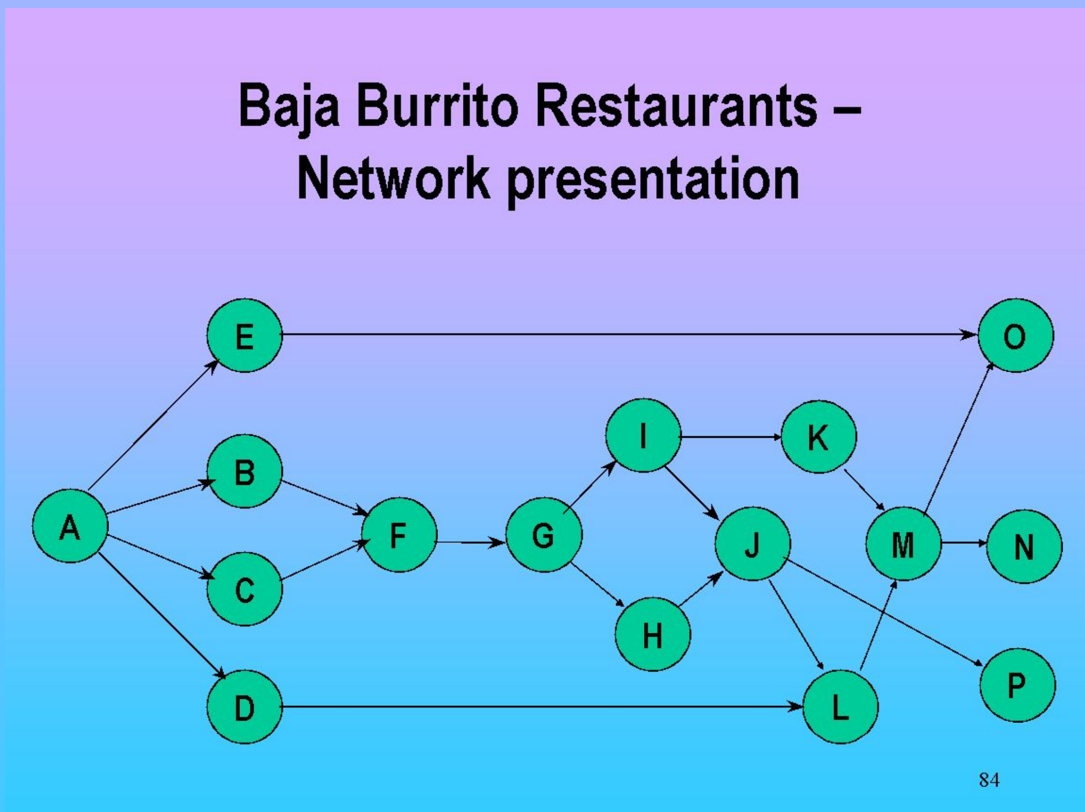
$Y_G \le 1.5$

$Y_H \le$ ……0.5

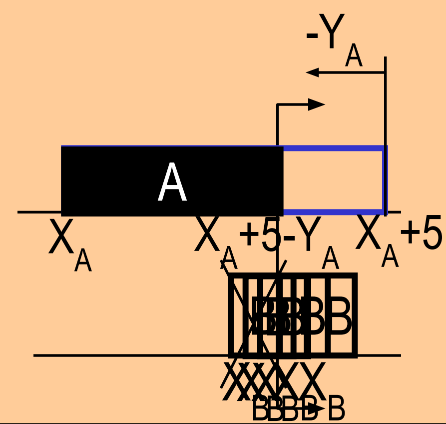
Maximum time-reduction constraints

Linear Programming

$$\text{Min } 5.5Y_A+10Y_B+2.67Y_C+4Y_D+2.8Y_E+6Y_F+6.67Y_G+10Y_H+5.33Y_I+12Y_J+4Y_K+5.33Y_L+1.5Y_N+4Y_O+5.33Y_P$$



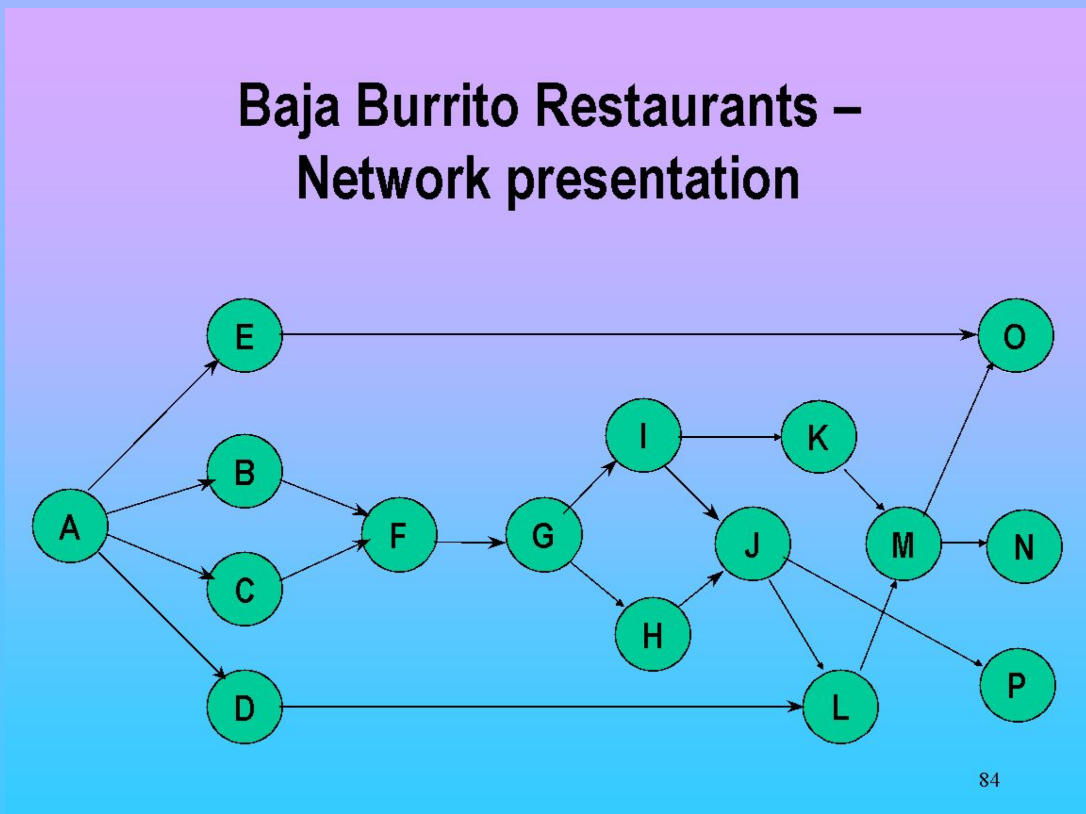
$$X_B \geq X_A + (5 - Y_A)$$



Activity can start only after all the Predecessors are completed.

Linear Programming

$$\text{Min } 5.5Y_A+10Y_B+2.67Y_C+4Y_D+2.8Y_E+6Y_F+6.67Y_G+10Y_H+$$
$$5.33Y_I+12Y_J+4Y_K+5.33Y_L+1.5Y_N+4Y_O+5.33Y_P$$



$$X_B \geq X_A + (5 - Y_A)$$
$$X_C \geq X_A + (5 - Y_A)$$
$$X_D \geq X_A + (5 - Y_A)$$
$$X_e \geq X_A + (5 - Y_A)$$
$$X_F \geq X_A + (5 - Y_A)$$
$$X_B \geq X_B + (1 - Y_B)$$
$$X_F \geq X_C + (3 - Y_C)$$
$$X_G \geq X_F + (1 - Y_F)$$

....

$$X(FIN) \geq X_N + (3 - Y_N)$$
$$X(FIN) \geq X_O + (4 - Y_O)$$
$$X(FIN) \geq X_P + (4 - Y_P)$$

Activity can start only after all the predecessors are completed.

Baja Burrito Restaurants – Deadline Spreadsheet

CRASHING ANALYSIS

TOTAL PROJECT COST	248.75	PROJECT NORMAL COST	200
COMPLETION TIME	19	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	3	0	3	2	11	36
Grade Land	B	1	3	4	0	0	10
Purchase Materials	C	1.5	3	4.5	1.5	4	22
Order Equipment	D	2	3	5	0	0	8
Order Furniture	E	4	12.5	16.5	0	0	8
Concrete Floor	F	0.5	4.5	5	0.5	3	15
Erect Frame	G	4	5	9	0	7.87637E-11	20
Install Electrical	H	2	9	11	0	0	12
Install Plumbing	I	2.5	9	11.5	1.5	8	21
Install Drywall/Roof	J	2	11.5	13.5	0	0	10
Bathrooms	K	2	13	15	0	0	8
Install Equipment	L	1.5	13.5	15	1.5	8	22
Finish/Paint Inside	M	1.5	15	16.5	1.5	8	18
Tile Floors	N	2.5	16.5	19	0.5	0.75	6.75
Install Furniture	O	2.5	16.5	19	1.5	6	14
Finish/Paint Outside	P	4	13.5	17.5	0	0	18

Baja Burrito Restaurants – Operating within a fixed budget

- Baja Burrito has the policy of not funding more than 12.5% above the “normal cost” projection.

Crash budget = (12.5%)(200,000) = 25,000

- Management wants to minimize the project completion time under the budget constraint.

Baja Burrito Restaurants –
Operating within a fixed budget

The crash funds become a constraint

Minimize:

Minimize X(FIN)

$$5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H + 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 1.5Y_N + 4Y_O + 5.33Y_P$$

The completion time becomes the objective function

$X(FIN) \leq 19$

$$5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H + 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 1.5Y_N + 4Y_O + 5.33Y_P \leq 25$$

The other constraints of the crashing model remain the same.

Baja Burrito Restaurants – Budget Spreadsheet

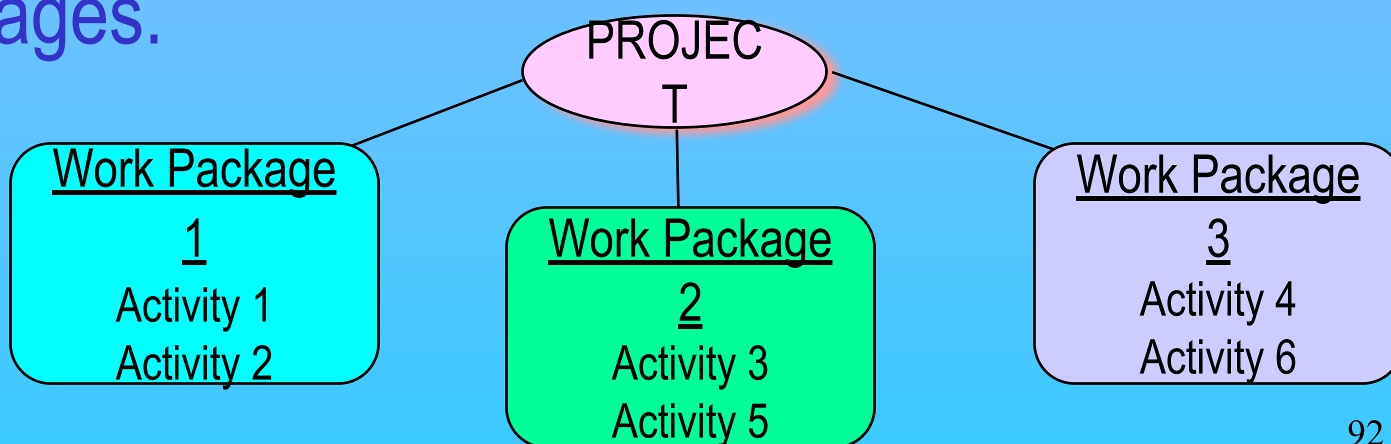
CRASHING ANALYSIS

TOTAL PROJECT COST	225	PROJECT NORMAL COST	200
COMPLETION TIME	23.3125	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	5	0	5	0	0	25
Grade Land	B	1	5	6	0	0	10
Purchase Materials	C	1.5	5	6.5	1.5	4	22
Order Equipment	D	2	5	7	0	0	8
Order Furniture	E	4	16.3125	20.3125	0	0	8
Concrete Floor	F	1	6.5	7.5	0	0	12
Erect Frame	G	4	7.5	11.5	0	0	20
Install Electrical	H	2	12	14	0	0	12
Install Plumbing	I	2.5	11.5	14	1.5	8	21
Install Drywall/Roof	J	2	14	16	0	0	10
Bathrooms	K	2	14	16	0	0	8
Install Equipment	L	1.5	16	17.5	1.5	8	22
Finish/Paint Inside	M	2.8125	17.5	20.3125	0.1875	1	11
Tile Floors	N	3	20.3125	23.3125	0	0	6
Install Furniture	O	3	20.3125	23.3125	1	4	12
Finish/Paint Outside	P	4	19.3125	23.3125	0	0	18

7.11 PERT/COST

- PERT/Cost helps management gauge progress against scheduled time and cost estimates.
- PERT/Cost is based on analyzing a segmented project. Each segment is a collection of work packages.



Work Package - Assumptions

– Once started, a work package is performed continuously until it is finished.

– The costs associated with a work package are spread evenly throughout its duration.

Monitoring Project progress

- For each work package determine:
  - **Work Package Forecasted Weekly cost** =

$$\frac{\text{Budgeted Total Cost for Work Package}}{\text{Expected Completion Time for Work Package (weeks)}}$$

  - **Value of Work to date** = p(Budget for the work package)
    where p is the estimated percentage of the work package completed.
  - **Expected remaining completion time** =
    (1 – p)(Original Expected Completion Time)

Monitoring Project progress

- **Completion Time Analysis**

  Use the expected remaining completion time estimates, to revise the project completion time.

- **Cost Overrun/Underrun Analysis**

  For each work package (completed or in progress) calculate

Cost overrun =
[Actual Expenditures to Date] - [Value of Work to Date].

Monitoring Project Progress – Corrective Actions

- A project may be found to be behind schedule, and or experiencing cost overruns.
- Management seeks out causes such as:
  - Mistaken project completion time and cost estimates.
  - Mistaken work package completion times estimates and cost estimates.
  - Problematic departments or contractors that cause delays.

Monitoring Project Progress – Corrective Actions

- Possible Corrective actions, to be taken whenever needed.
  - Focus on uncompleted activities.
  - Determine whether crashing activities is desirable.
  - In the case of cost underrun, channel more resources to problem activities.
  - Reduce resource allocation to non-critical activities.

TOM LARKIN's MAYORAL CAMPAIGN

- Tom Larkin is running for Mayor.
- Twenty weeks before the election the campaign remaining activities need to be assessed.
- If the campaign is not on target or not within budget, recommendations for corrective actions are required.

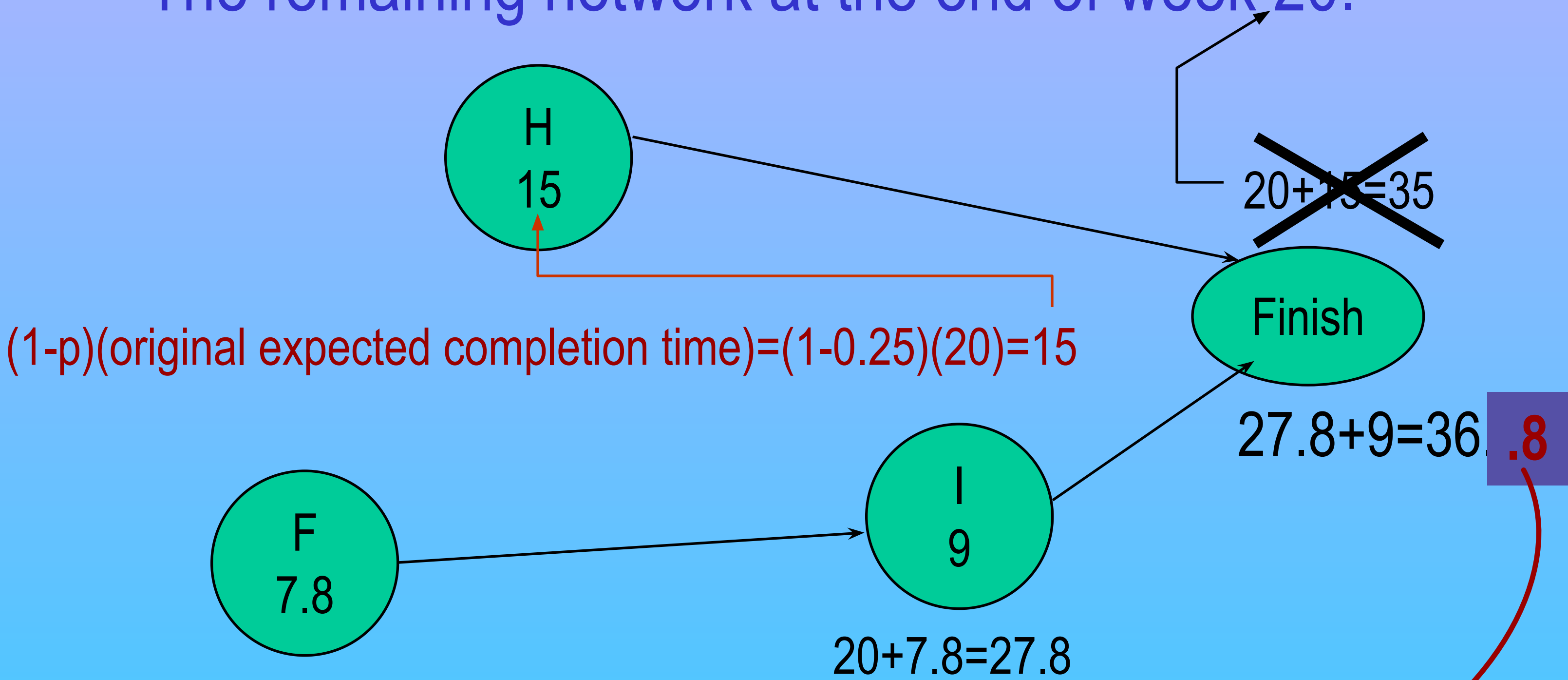
MAYORAL CAMPAIGN –
Status Report

	Work Package	Expenditures ($)	Status
A	Hire campaign staff	2,600	Finish
B	Prepare position paper	5,000	Finish
C	Recruit volunteers	3,000	Finish
D	Raise funds	5,000	Finish
E	File candidacy papers	700	Finish
F	Prepare campaign material	5,600	40% complete
G	Locate/staff headquarter	700	Finish
H	Run personal campaign	2,000	25% complete
I	Run media campaign	0	0% complete

Work packages to focus on

MAYORAL CAMPAIGN –
Completion Time Analysis

– The remaining network at the end of week 20.



The remaining activities are expected to take 0.8 weeks longer than the deadline of 36 weeks.

MAYORAL CAMPAIGN –
Project Cost Control

Estimated work value to date=(13,000)(0.40)=$5,200

Cost overrun = 5600 - 5200 = 400

Work Package	[illegible]ted Values Time	Total Cost	Percent Completed	Estimated Value	Actual Value	Cost Ocerrun
A	4	[illegible]	100%	[illegible]	2,600	600
B	6	[illegible]	100%	[illegible]	5,000	2000
C	4	4,500	[illegible]	4,500	[illegible]	-1500
D	6	2,500	[illegible]	2,500	[illegible]	2,500
E	2	500	[illegible]	[illegible]	700	200
F	13	13,000	40%	[illegible]	5,600	400
G	1	1,500	100%	1,500	700	-800
H	20	6,000	25%	1,500	2,000	-500
I	9	7,000	0%	0	0	0
Total		40,000		20,700	24,600	3,900

MAYORAL CAMPAIGN –
Results Summary

- The project is currently .8 weeks behind schedule
- There is a cost over-run of $3900.
- The remaining completion time for uncompleted work packages is:
  - Work package F: 7.8 weeks,
  - Work package H: 15 weeks,
  - Work package I: 9 weeks.
- Cost over-run is observed in
  - Work package F: $400,
  - Work package H: $500.