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Assignment-04

(1) $\mathcal{L}^{-1}\dfrac{s^2+2s+6}{s^3} = \mathcal{L}^{-1}\left[\dfrac{1}{s}+\dfrac{2}{s^2}+\dfrac{6}{s^3}\right]$

$= 1+2t+\dfrac{6t^2}{2}$

$= 1+2t+3t^2$ Ans

(2) $\mathcal{L}^{-1}\dfrac{1}{s^2-7s+12} = \mathcal{L}^{-1}\dfrac{1}{s^2-4s-3s+12}$

$= \mathcal{L}^{-1}\dfrac{1}{s(s-4)-3(s-4)}$

$= \mathcal{L}^{-1}\left[\dfrac{1}{(s-3)(s-4)}\right]$

Let $\dfrac{1}{(s-3)(s-4)} = \dfrac{A}{(s-3)}+\dfrac{B}{(s-4)}$ ------(i)

or, $1 = A(s-4)+B(s-3)$

For, $s=4$ $1 = B(4-3)$ or, $B=1$ | For $s=3$ $1=A(3-4)$ or, $A=-1$

From (i) $\dfrac{1}{(s-3)(s-4)} = \dfrac{-1}{s-3}+\dfrac{1}{s-4}$

$= \dfrac{1}{s-4}-\dfrac{1}{s-3} = e^{4t}-e^{3t}$ Ans

(3) $\mathcal{L}^{-1}\dfrac{s+2}{s^2-4s+13} = \mathcal{L}^{-1}\dfrac{s+2}{s^2-2.2s+2^2+9}$

$= \mathcal{L}^{-1}\dfrac{s+2}{(s-2)^2+3^2}$

$= \mathcal{L}^{-1}\dfrac{s+2-4+4}{(s-2)^2+3^2}$

$= \mathcal{L}^{-1}\left[\dfrac{s-2}{(s-2)^2+3^2}+\dfrac{4}{(s-2)^2+3^2}\right]$

$= e^{2t}\cos 3t+\dfrac{4}{3}e^{2t}\sin 3t$ Ans

(4) $\dfrac{3s+1}{(s-1)(s^2+1)}$

Let $\dfrac{3s+1}{(s-1)(s^2+1)} = \dfrac{A}{(s-1)} + \dfrac{Bs+C}{(s^2+1)}$

or, $(3s+1) = A(s^2+1) + (Bs+C)\cdot(s-1)$ ---- (i)

~~or, (3s+1) = A(s^2+1) +~~

For, $s=1$ $\cdot$ $4 = 2A$

or, $A = 2$

From (i) $\Rightarrow$

$3s+1 = As^2+A + Bs^2 - Bs + Cs - C$ — (ii)

Now equating co-efficients of s^2 from (ii) $\Rightarrow$

$0 = A+B$

or, $B = -A$

$= -2$

No equating co-efficients of s from (ii) $\Rightarrow$

$0 = -B + C$

or, $C = 3+B$

$= 3 + -2$

$= 1$

so, $\dfrac{3s+1}{(s-1)(s^2+1)} = \dfrac{2}{s-1} + \dfrac{-2s+1}{s^2+1}$

$\therefore L^{-1}\dfrac{3s+1}{(s-1)(s^2+1)} = L^{-1}\left[\dfrac{2}{s-1} + \dfrac{-2s+1}{s^2+1}\right]$

$= L^{-1}\left[\dfrac{2}{s-1} - \dfrac{2s}{s^2+1} + \dfrac{1}{s^2+1}\right]$

$= 2e^t - 2\cos t + \sin t$ Ans

⑤ $L^{-1}\dfrac{11s^2-2s+5}{2s^3-3s^2-3s+2}$

$P(s)=11s^2-2s+5$

$Q(s)=2s^3-3s^2-3s+2$

$Q'(s)=6s^2-6s-3$

$Q(s)$ has three distinct zeros. say

$a_1=2$

$a_2=-1$

$a_3=\frac{1}{2}$

Now, $L^{-1}=\left\{\dfrac{11s^2-2s+5}{2s^3-3s^2-3s+2}\right\}$

$=\dfrac{P(2)}{Q'(2)}e^{2t}+\dfrac{P(-1)}{Q'(-1)}e^{-t}+\dfrac{P(\frac{1}{2})}{Q'(\frac{1}{2})}$

$=\dfrac{45}{9}e^{2t}+\dfrac{18}{9}e^{-t}+\dfrac{27/4}{-9/2}e^{\frac{1}{2}t}$

$=5e^{2t}+2e^{-t}-\frac{3}{2}e^{\frac{1}{2}t}$ Ans

⑥ $L^{-1}\dfrac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$

$P(s)=2s^2-6s+5$

$Q(s)=(s-1)(s-2)(s-3)$

$=(s^2-2s-s+2)(s-3)$

$=(s^2-3s+2)(s-3)$

$=s^3-3s^2-3s^2+9s+2s-6$

$=s^3-6s^2+11s-6$

$Q'(s)=3s^2-12s+11$

$Q(s)$ has three distinct zeroes

say, $a_1=1$

$a_2=2$

$a_3=3$

Now, $L^{-1}\left\{\dfrac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right\}$

$=\dfrac{P(1)}{Q'(1)}e^{t}+\dfrac{P(2)}{Q'(2)}e^{2t}+\dfrac{P(3)}{Q'(3)}e^{3t}$

$=\frac{1}{2}e^{t}-e^{2t}+\frac{5}{2}e^{3t}$ Ans

(9) $\mathcal{L}^{-1} \frac{s-4}{(s-1)^2+9} = e^{4t} \cos 3t$ Ans

(10) $\mathcal{L}^{-1} \frac{1}{(s-2)(s^2+1)}$

Let $\frac{1}{(s-2)(s^2+1)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2+1)}$

or, $1 = A(s^2+1)+(Bs+C)(s-2)$ —— (i)

For, $s=2$ $\quad 1 = A(4+1)$

or, $A = \frac{1}{5}$

From (i) $\Rightarrow$ $1 = As^2+A+Bs^2-2Bs+Cs-2C$ --- (ii)

Now equating co-efficients of s^2, s from (ii)

$0 = A+B;$ | $0 = C-2B$
or, $B = -A$ | or, $B = \frac{C}{2} = -\frac{2}{5}$
$= -\frac{1}{5}$

$\therefore \mathcal{L}^{-1}: \frac{1}{(s-2)(s^2+1)} = \frac{1}{5(s-2)} + \frac{-\frac{1}{5}s-2/5}{s^2+1}$

$= \frac{1}{5(s-2)} + \frac{\frac{-s-2}{5}}{s^2+1}$

$= \frac{1}{5(s-2)} - \frac{(s+2)}{5(s^2+1)}$

$= \frac{1}{5}e^{2t} - \frac{1}{5}\cos t - \frac{2}{5}\sin t.$ Ans