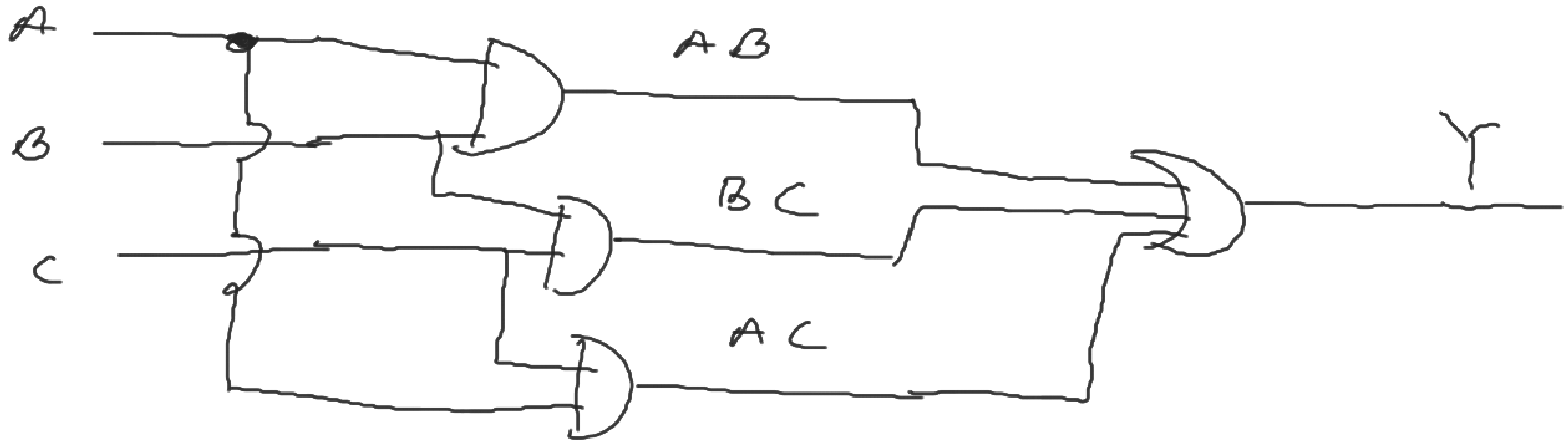
$$Y = AB + BC + AC$$



EX-OR gate :

A	B	Y
0	0	0
0	1	1 ✓
1	0	1 ✓
1	1	0

$$Y = \bar{A}B + A\bar{B}$$

$$Y = A \oplus B$$

$\boxed{A \oplus B = \bar{A}B + A\bar{B}}$

→ Save 4 logic gates

→ 5 logic gates

EX-NOR gate :

A	B	Y
0	0	1 ✓
0	1	0
1	0	0
1	1	1 ✓

$$Y = \bar{A}\bar{B} + AB$$

$$Y = \overline{A \oplus B}$$

$\boxed{\overline{A \oplus B} = AB + \bar{A}\bar{B}}$

→ Save 4 logic gates

Design the logic circuit from the given statements:

1) Design a logic circuit where, there are 3 inputs A, B, C and output Y; output will be HIGH only when the majority of inputs are HIGH.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 ✓
1	0	0	0
1	0	1	1 ✓
1	1	0	1 ✓
1	1	1	1 ✓

$$Y = \bar{A}BC + A\bar{B}C + \underline{AB\bar{C} + ABC}$$

$$= \bar{A}BC + A\bar{B}C + AB$$

$$= \bar{A}BC + A(B + \bar{B}C)$$

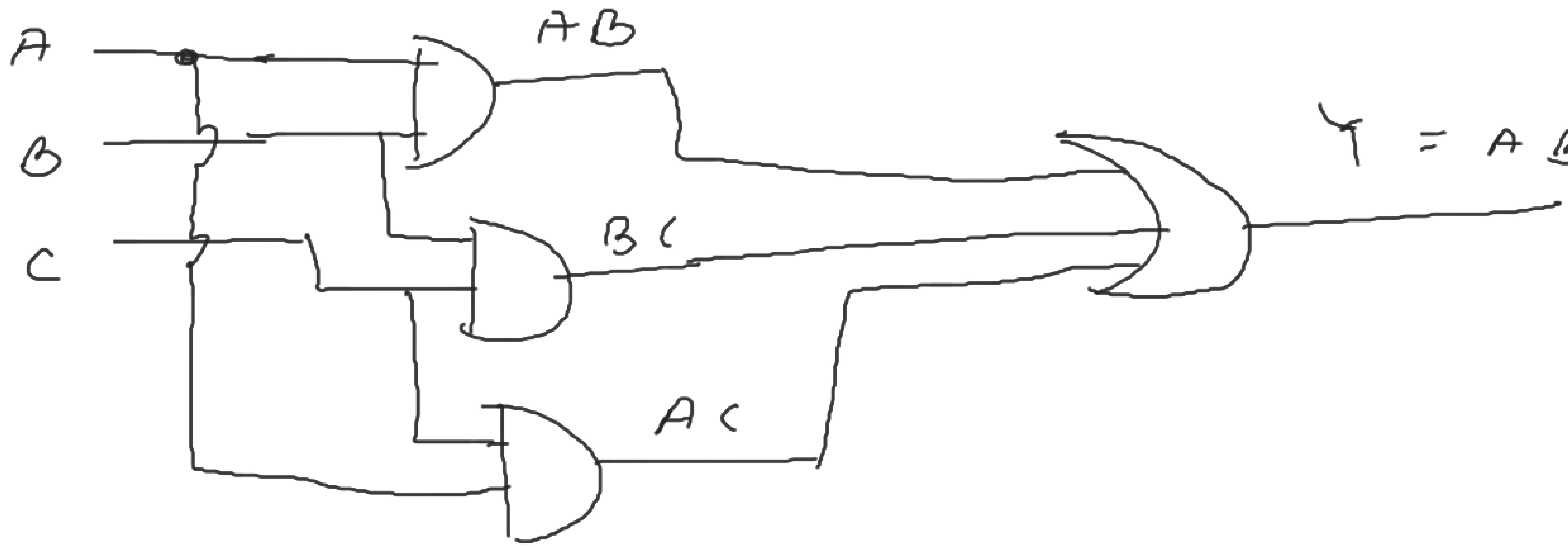
$$= \bar{A}BC + A(B + C)$$

$$= \bar{A}BC + AB + AC$$

$$= B(A + \bar{A}C) + AC$$

$$= B(A + C) + AC$$

$$= AB + BC + AC$$



$Y = AB + BC + AC$

2) A, B, C are inputs and Y is output. Output Y will be HIGH only when majority of inputs are Low. Design The circuit.

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{B} + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{A}(\bar{B} + B\bar{C}) + A\bar{B}\bar{C}$$

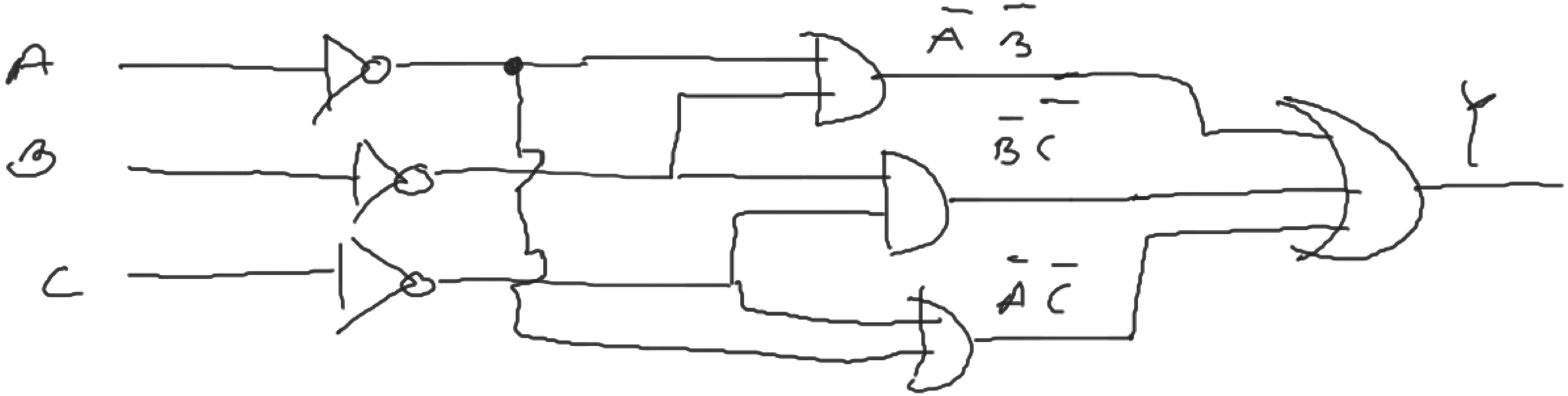
$$= \bar{A}(\bar{B} + \bar{C}) + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{B} + \bar{A}\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{B} + \bar{C}(\bar{A} + A\bar{B})$$

$$= \bar{A}\bar{B} + \bar{C}(\bar{A} + \bar{B})$$

$$= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$$



3) A, B, C ⇒ Inputs X ⇒ Output

$$\begin{cases} X = A \rightarrow \text{B and C are same} \\ X = 1 \rightarrow \text{B and C are different} \end{cases}$$

A	B	C	X
0	0	0	0
0	0	1	1 ✓
0	1	0	1 ✓
0	1	1	0
1	0	0	1 ✓
1	0	1	1 ✓
1	1	0	1 ✓
1	1	1	1 ✓

$$X = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B} + AB$$

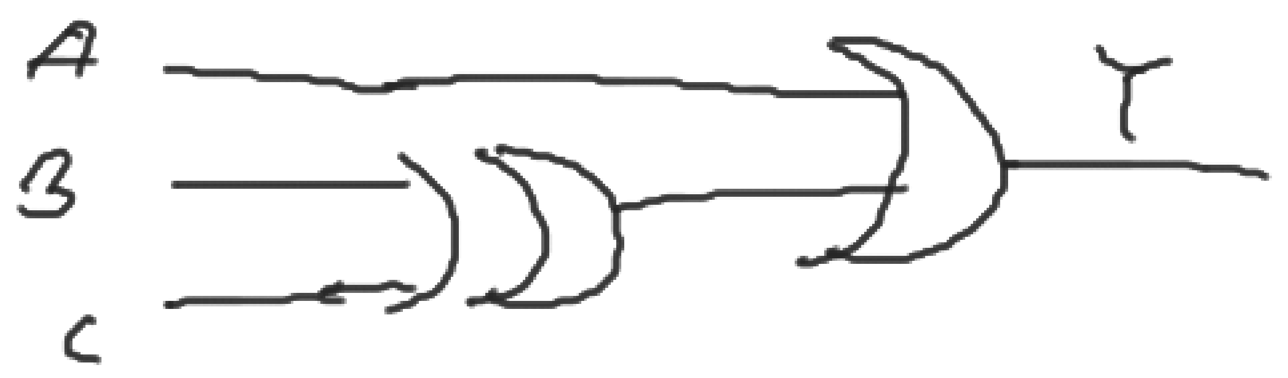
$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A$$

$$= \bar{A}(\bar{B}C + B\bar{C}) + A$$

$$= A + \bar{A}(B \oplus C)$$

$$x + \bar{x}y = x + y$$

$$\boxed{X = A + (B \oplus C)}$$



4)

D	C	B	A	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$> 0010_2 \qquad < 1010_2$

$$\boxed{3_{10} \le Y \le 9_{10}}$$

$$x + \bar{x}y = x + y$$

$$Y = AB\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D$$

$$= AB\bar{C}\bar{D} + \bar{B}C\bar{D}(\bar{A} + A) + BC\bar{D}(\bar{A} + A) + \bar{B}\bar{C}D(\bar{A} + A)$$

$$= AB\bar{C}\bar{D} + \bar{B}C\bar{D} + BC\bar{D} + \bar{B}\bar{C}D$$

$$= AB\bar{C}\bar{D} + C\bar{D}(\bar{B} + B) + \bar{B}\bar{C}D$$

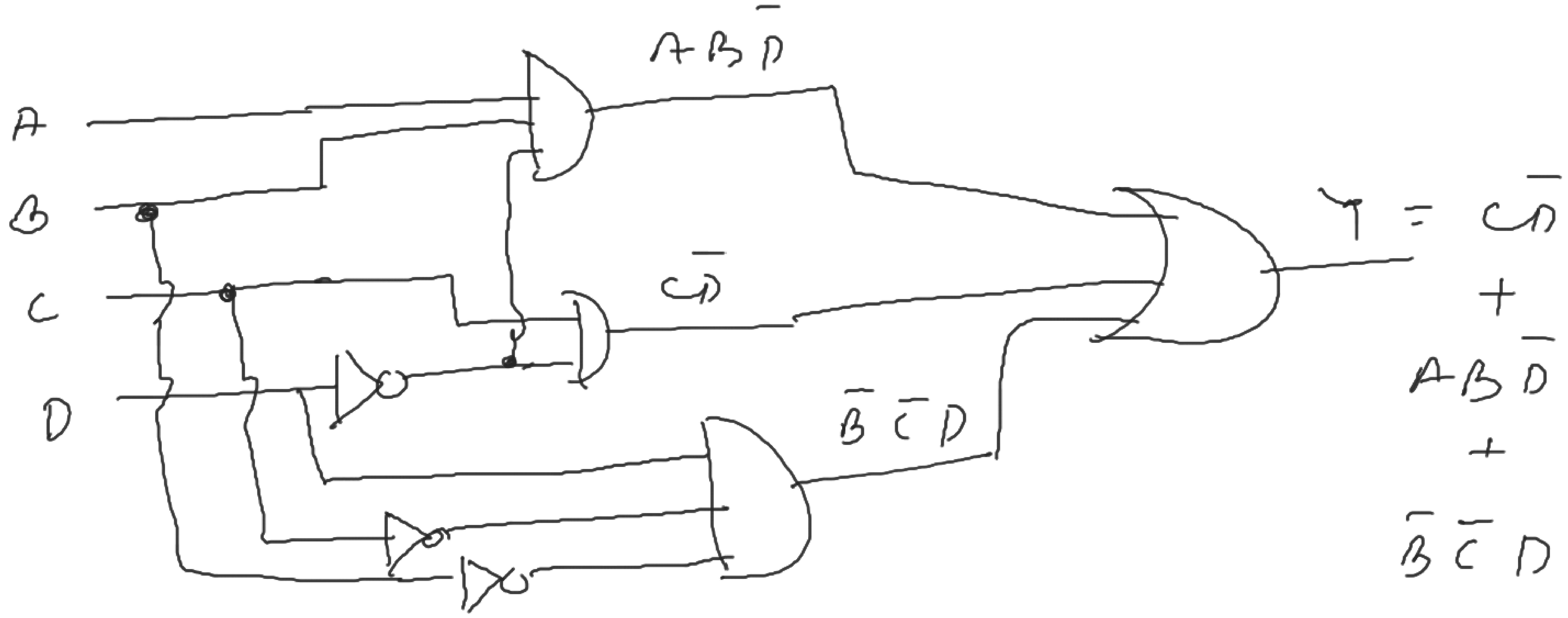
$$= AB\bar{C}\bar{D} + C\bar{D} + \bar{B}\bar{C}D$$

$$= \bar{D}(C + \bar{C}AB) + \bar{B}\bar{C}D$$

$$= \bar{D}(C + AB) + \bar{B}\bar{C}D$$

$$= C\bar{D} + AB\bar{D} + \bar{B}\bar{C}D$$

$$Y = C\bar{D} + AB\bar{D} + \bar{B}\bar{C}D$$



6)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(A is HIGH) AND (At least 2 other inputs are HIGH)

$ACD + AB(CD + \overline{C}D)$

$ACD + AB(C \oplus D)$

$$Y = A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + ABCD$$

$$= A\bar{B}CD + AB\bar{C}D + ABC$$

$$= A\bar{B}CD + AB(C + \bar{C}D)$$

$$= A\bar{B}CD + AB(C + D)$$

$$= A\bar{B}CD + ABC + ABD$$

$$= AC(B + \bar{B}D) + ABD$$

$$= AC(B + D) + ABD$$

$$= ABC + ACD + ABD$$

$$= A(BC + CD + BD)$$

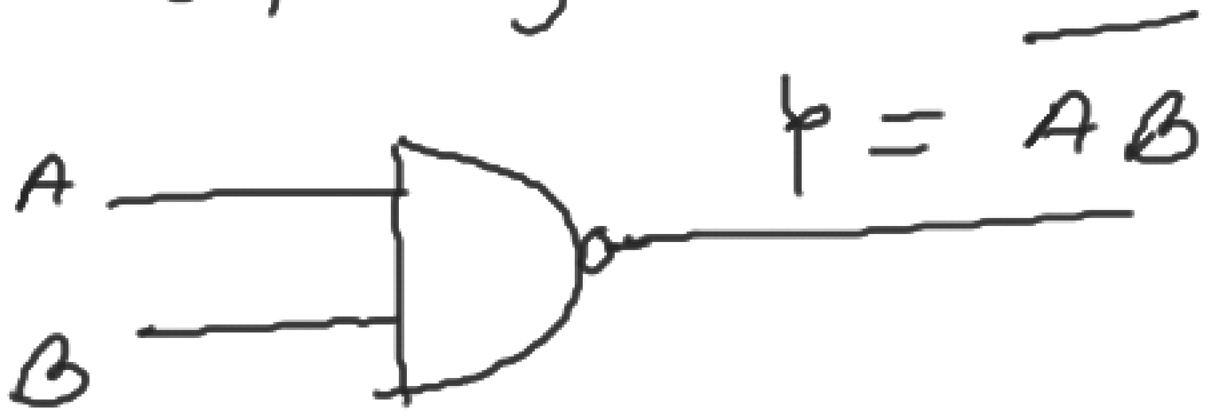
Draw the circuit.

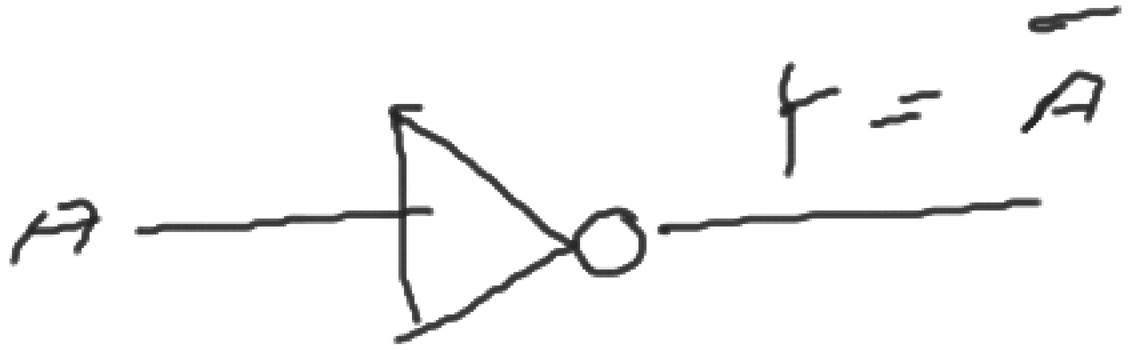
Universality of NAND gate :

NAND $\longrightarrow$ Basic gate

NAND $\rightarrow$ NOT, AND, OR

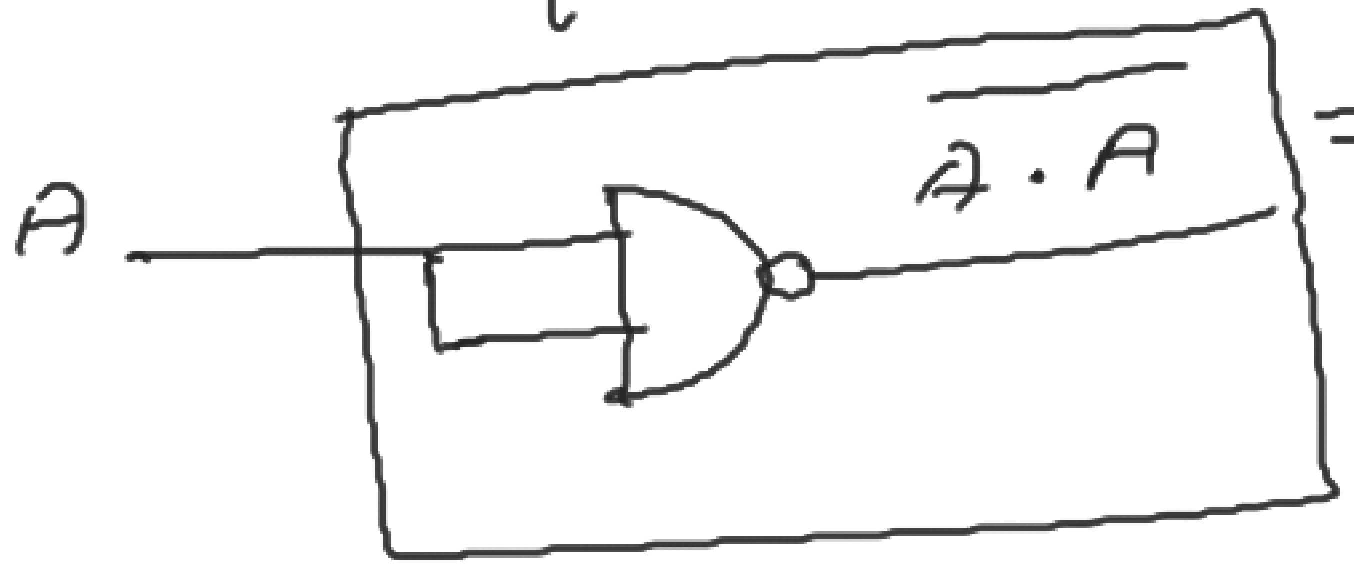
1) NAND to NOT :





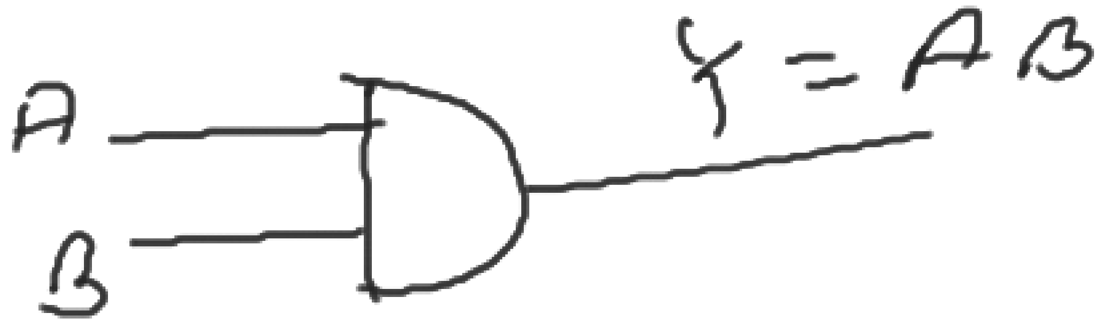
$$Y = \overline{A}$$

$$Y = \overline{A \cdot A}$$

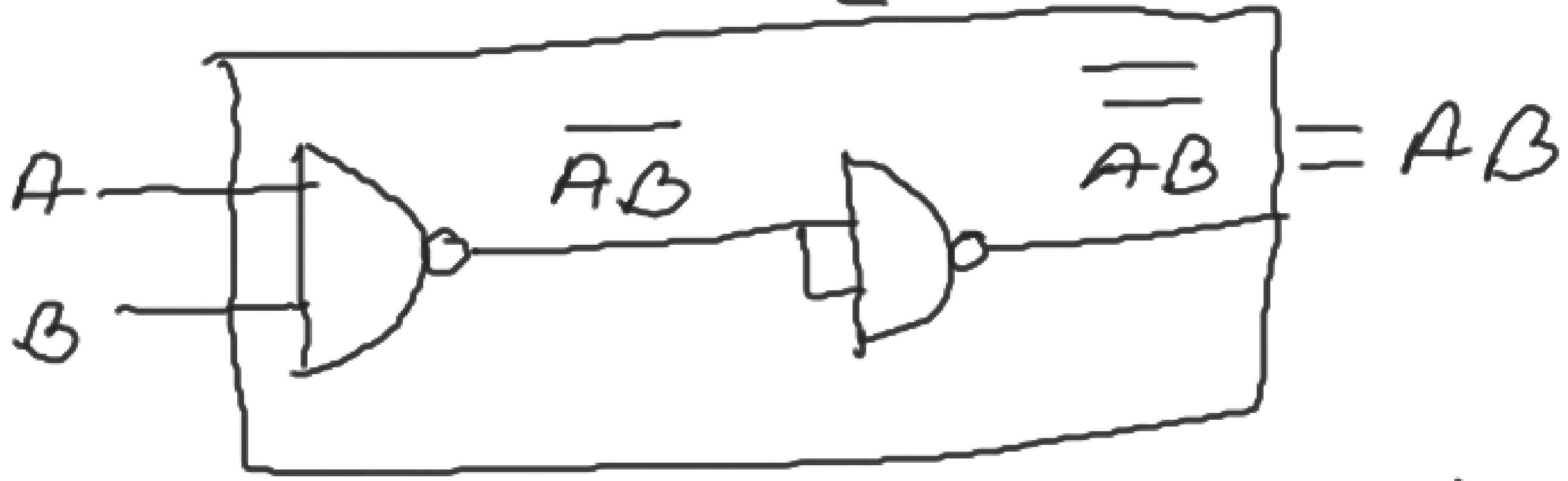


$= \overline{A}$

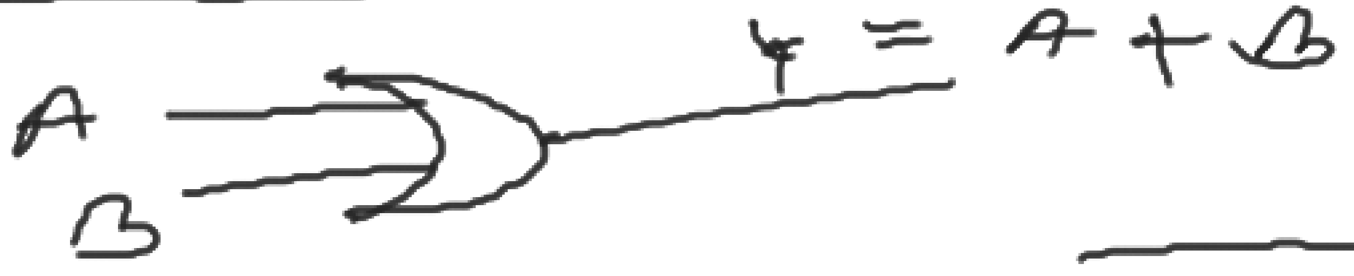
2) NAND to AND:



$$Y = AB = \left(\overline{\overline{AB}}\right)$$

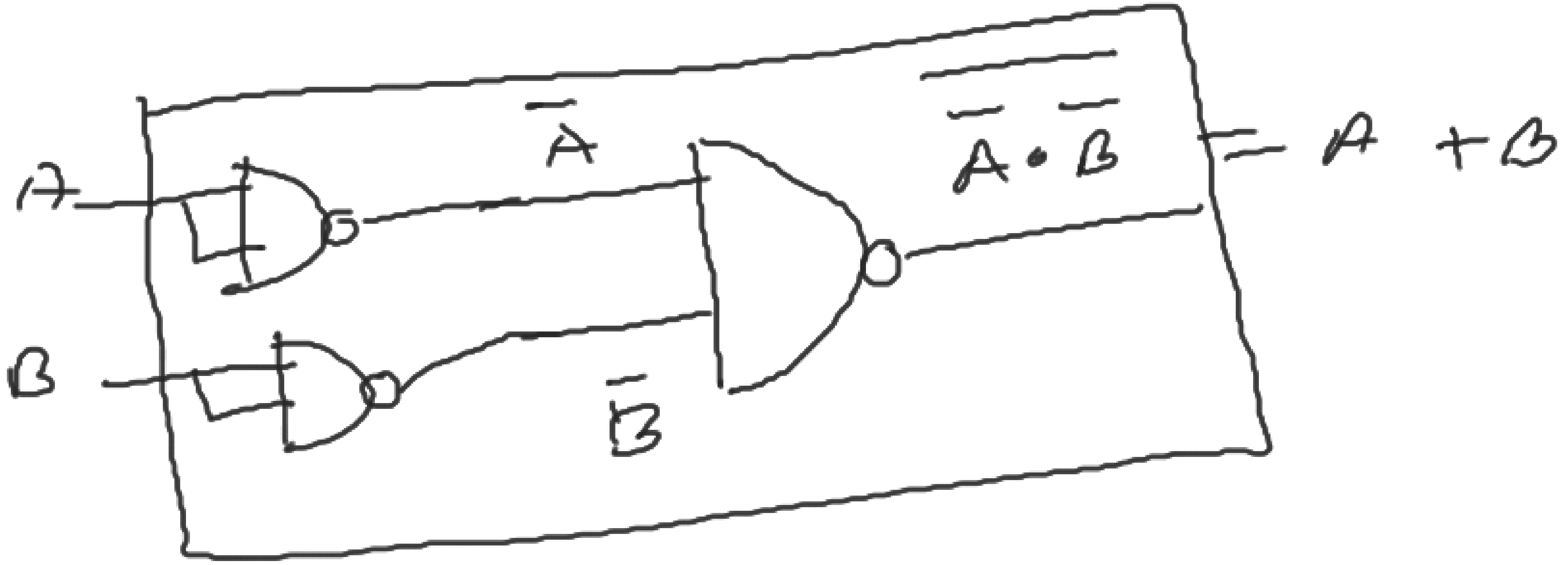


3) NAND to OR:

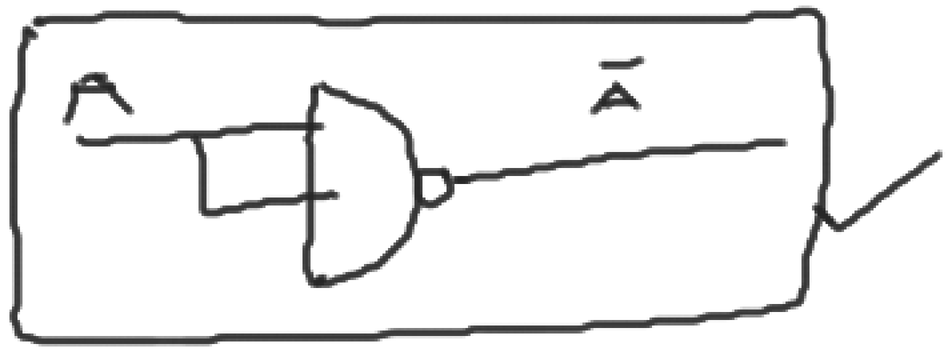


$$Y = A + B = \overline{\overline{(A + B)}}$$

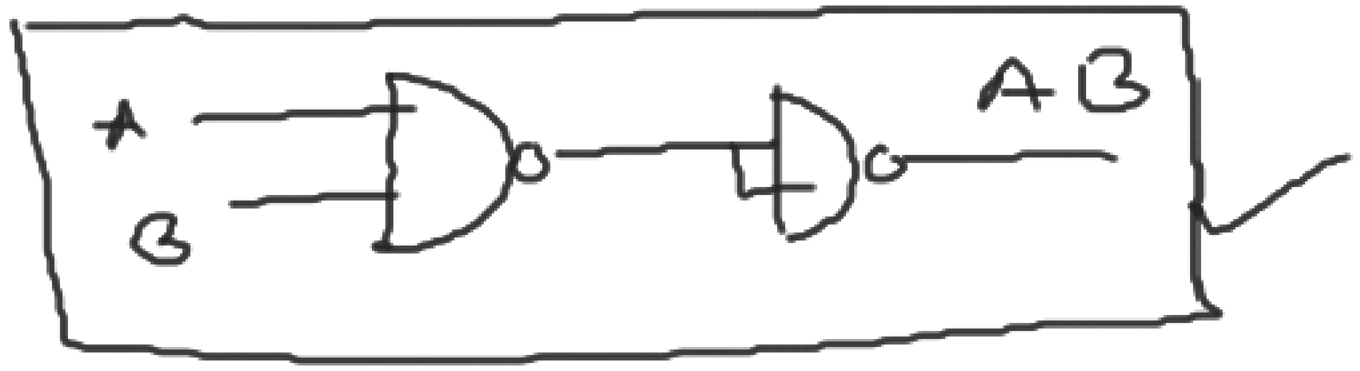
$$= \overline{\bar{A} \cdot \bar{B}}$$



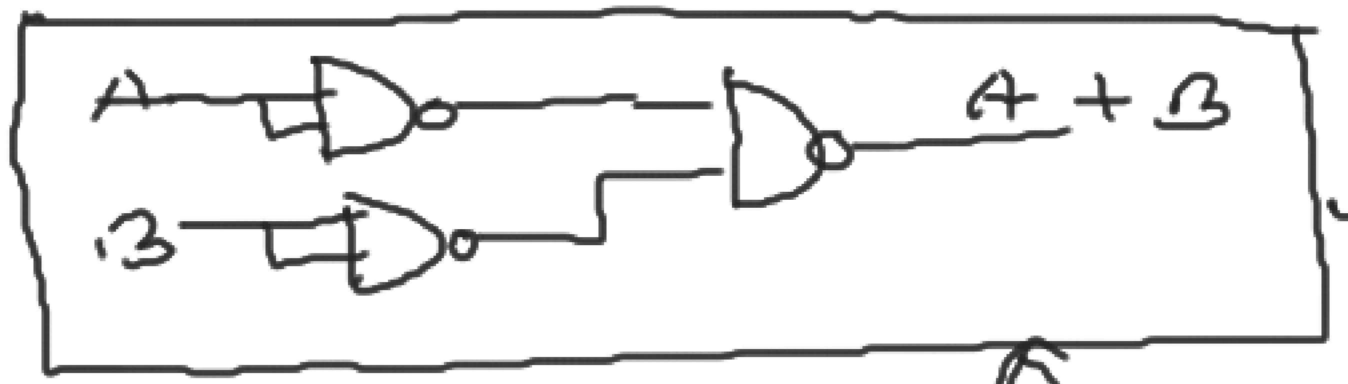
So, we can design all the basic gates using NAND gate only. Therefore NAND gate is called universal gate.



NOT



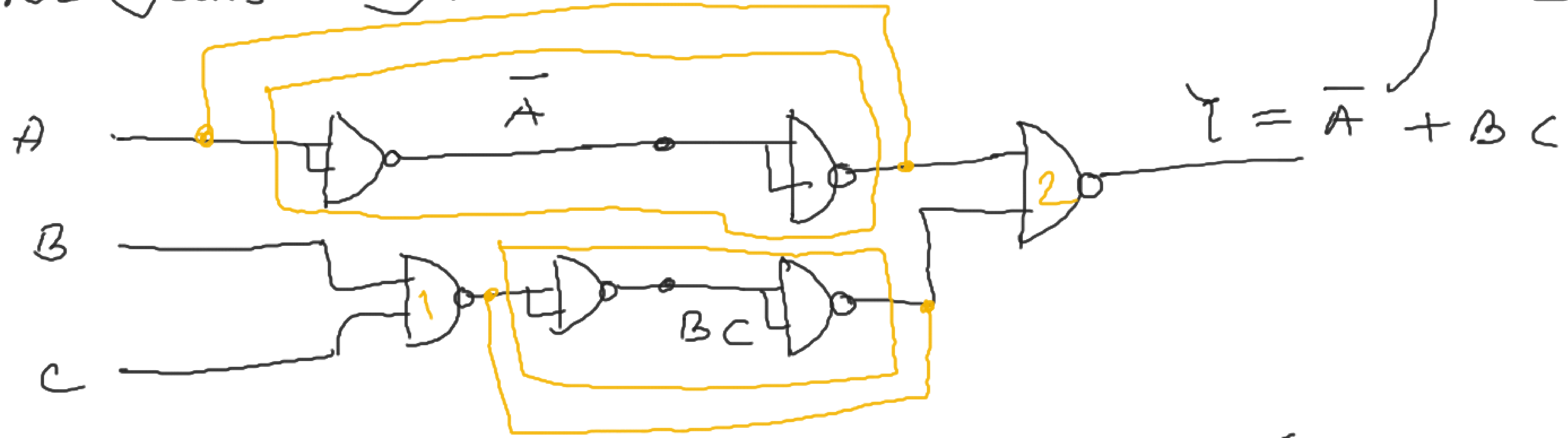
AND

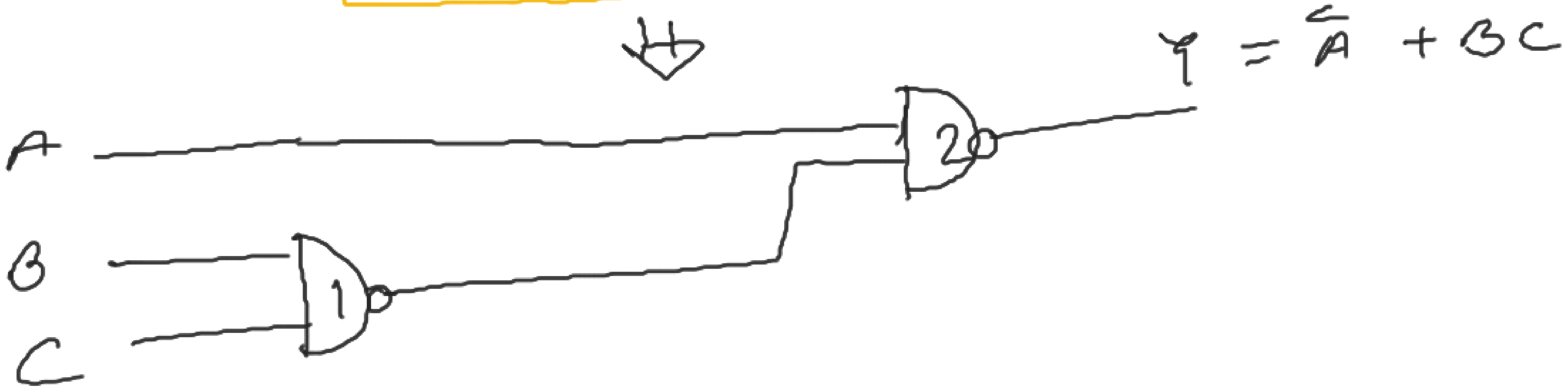


OR

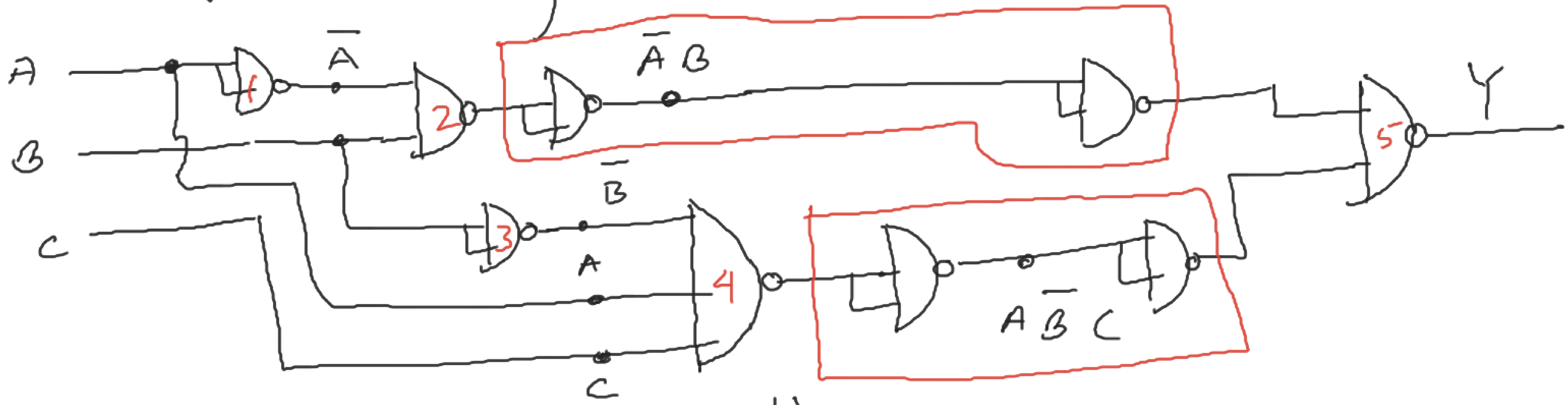
$\boxed{\bar{\bar{x}} = x}$

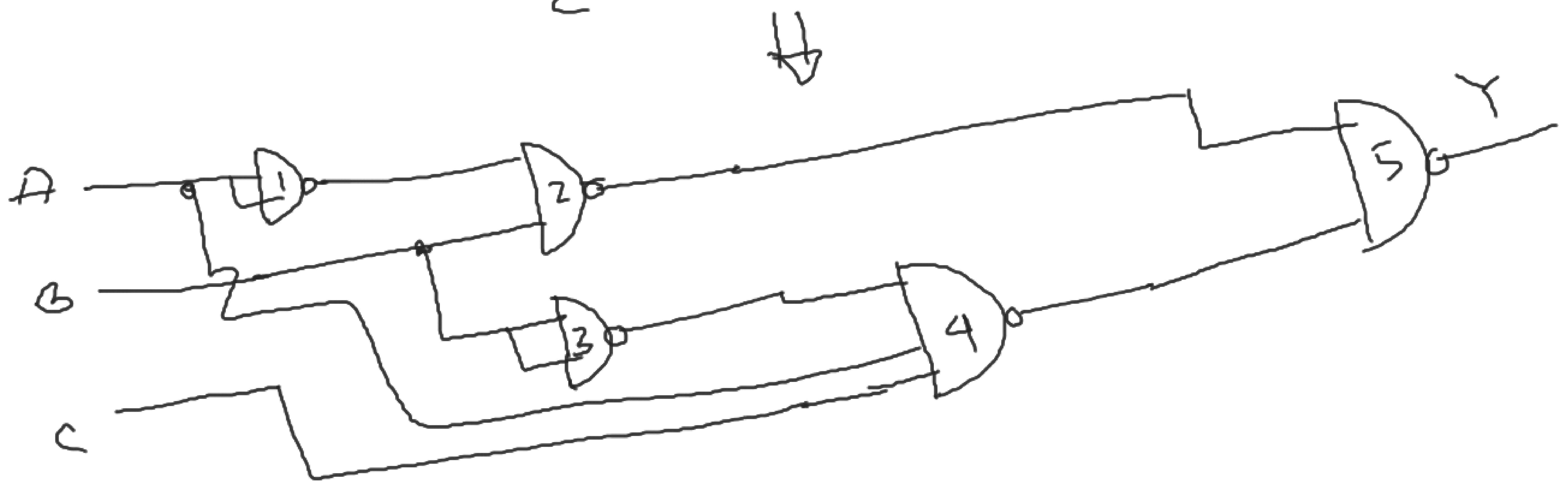
Draw the circuit diagram of $y = \bar{A} + BC$ using NAND gates only.



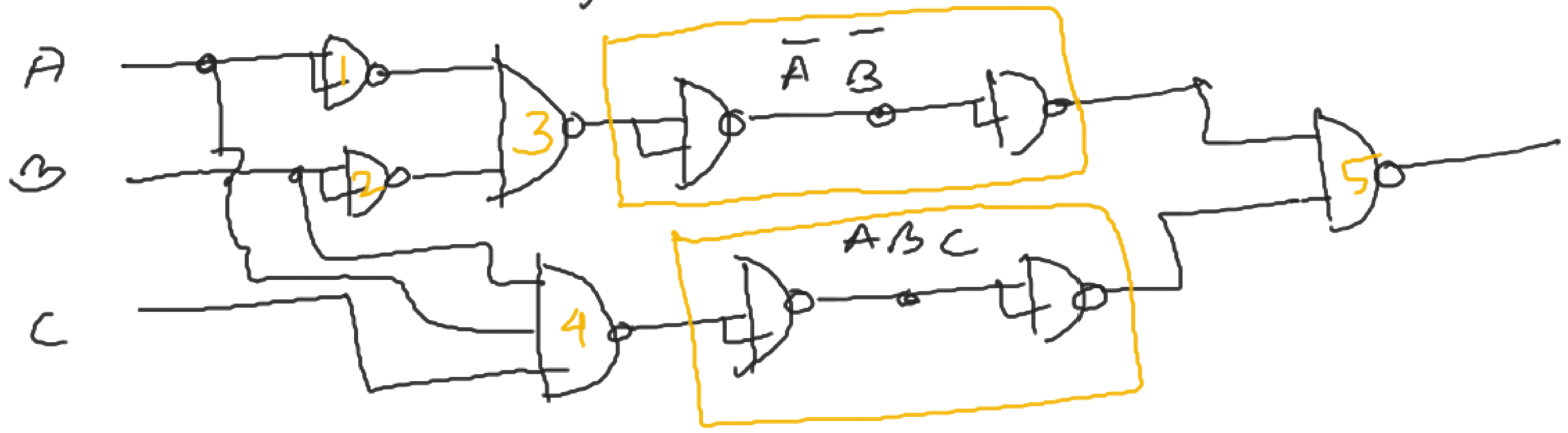


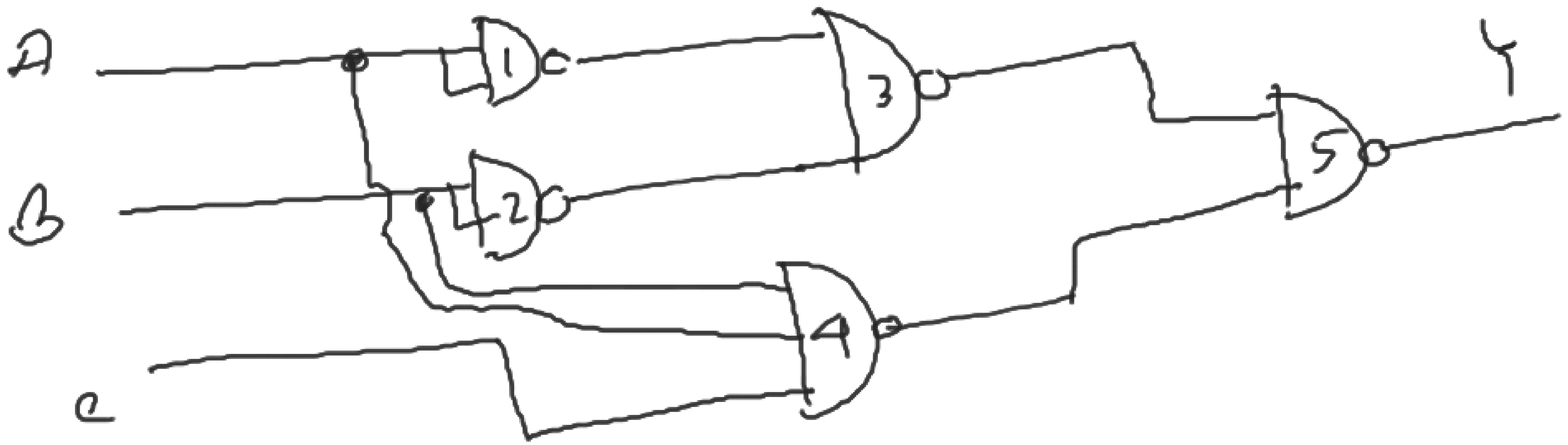
Draw $y = \bar{A}B + A\bar{B}C$ using NAND gates only.





Draw $y = \bar{A}\bar{B} + ABC$ using NAND gates only.



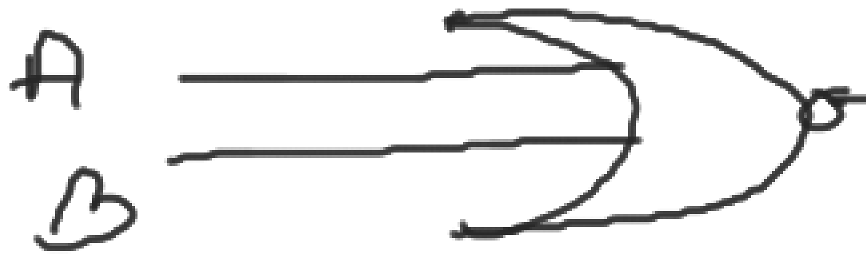


Universality of NOR gate:

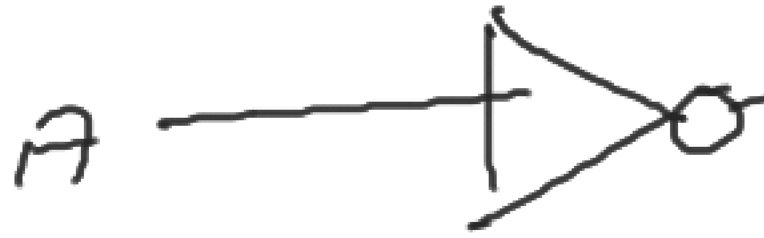
$x = x + x$

NOR → NOT
NOR → OR
NOR → AND

1) NOR to NOT:

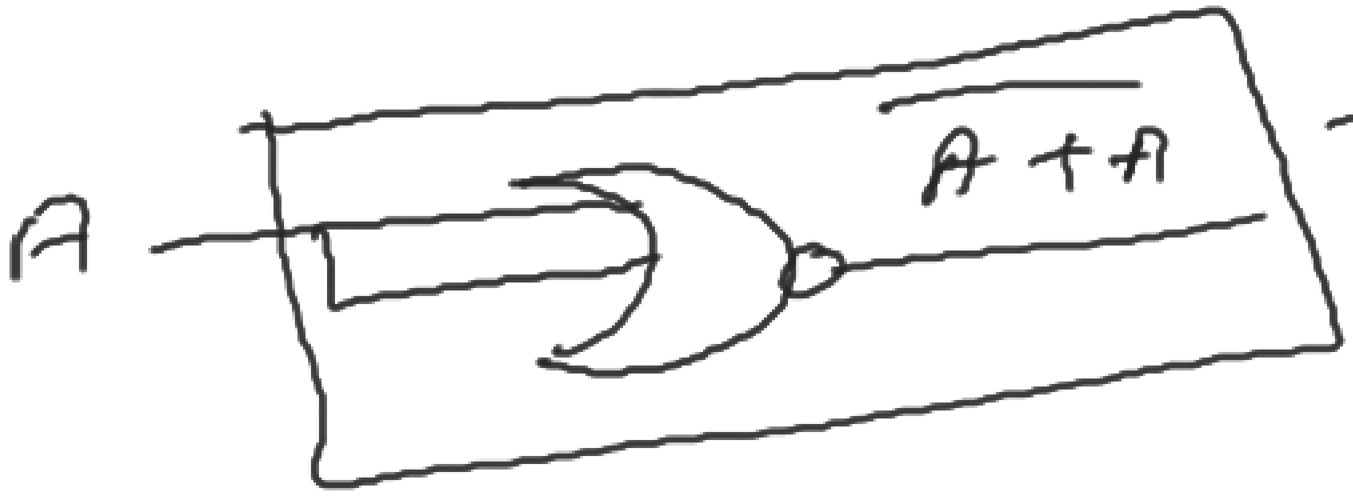


$Y = \overline{A+B}$



$Y = \overline{A}$

$$Y = \overline{A}$$
$$= \overline{A+A}$$



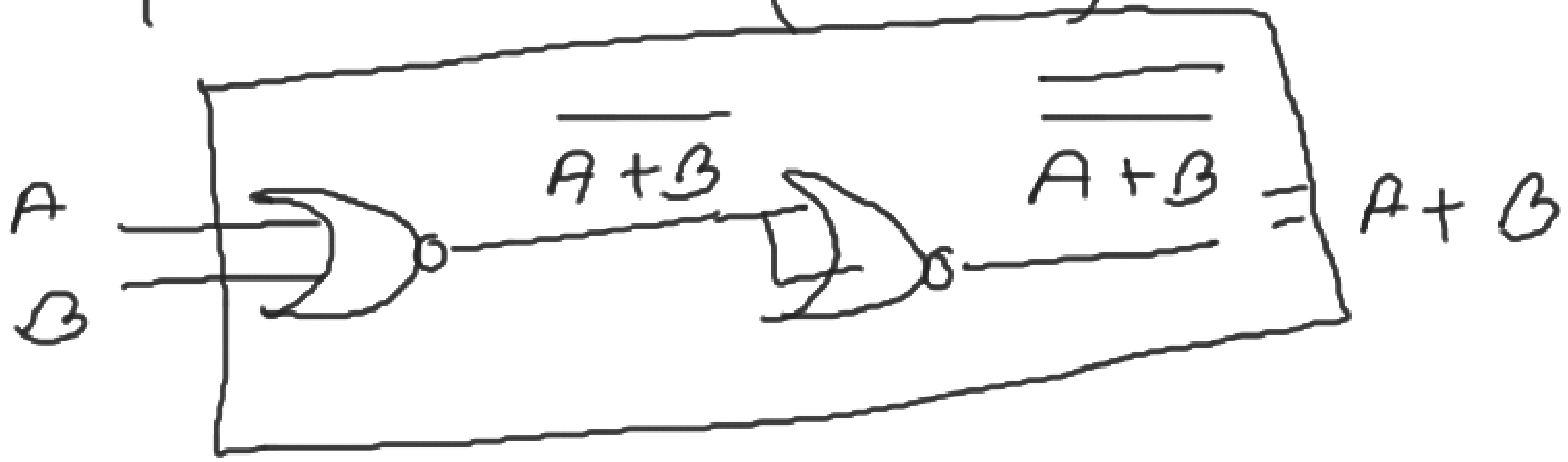
$\overline{A+A} = \overline{A}$

2) NOR to OR:

$Y = \overline{A+B}$

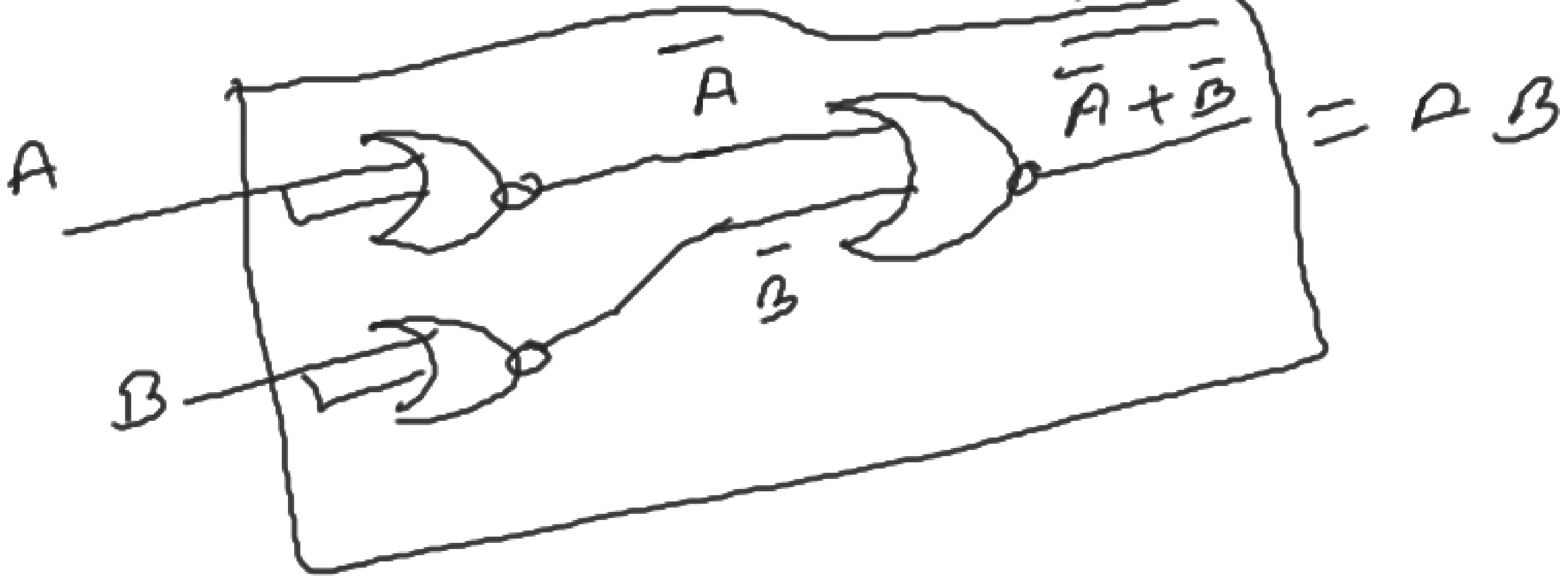
$Y = A + B$

$$Y = A + B = \left(\overline{\overline{A+B}}\right)$$



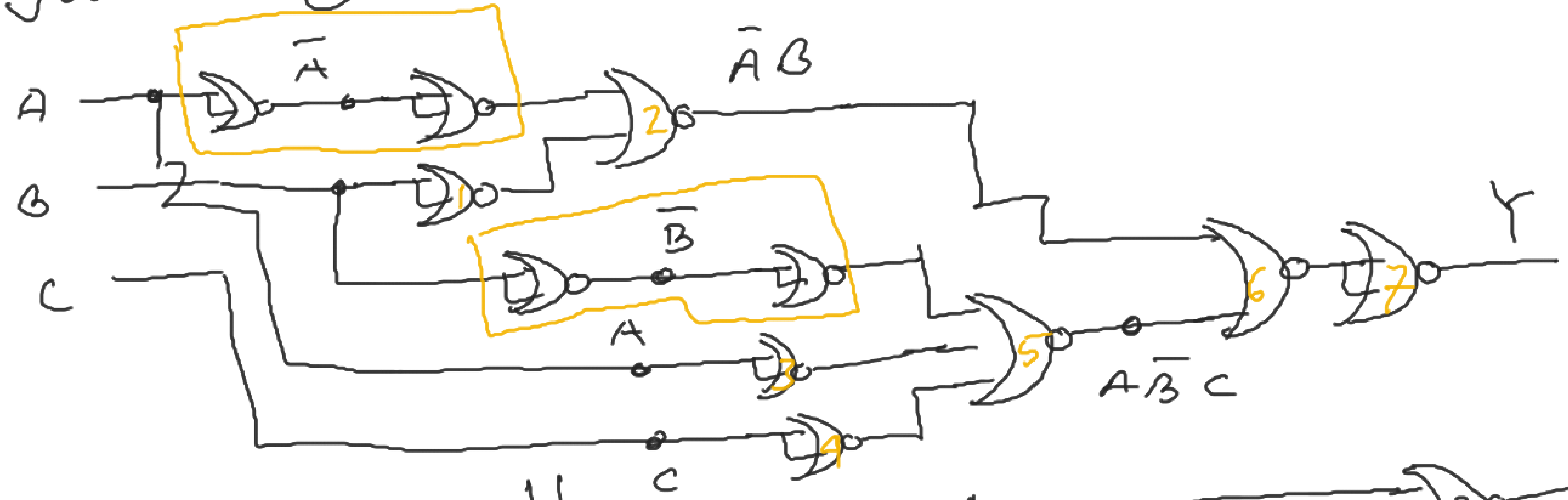
3) NOR to AND:

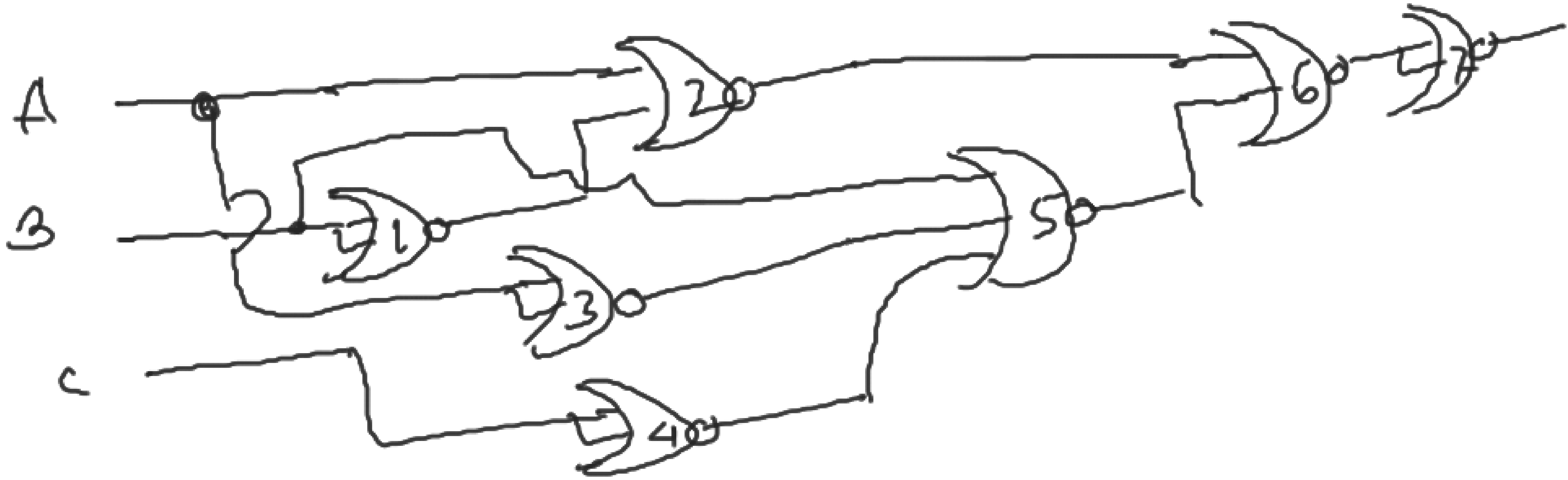
$$Y = AB = \overline{\overline{AB}} = \overline{\bar{A} + \bar{B}}$$



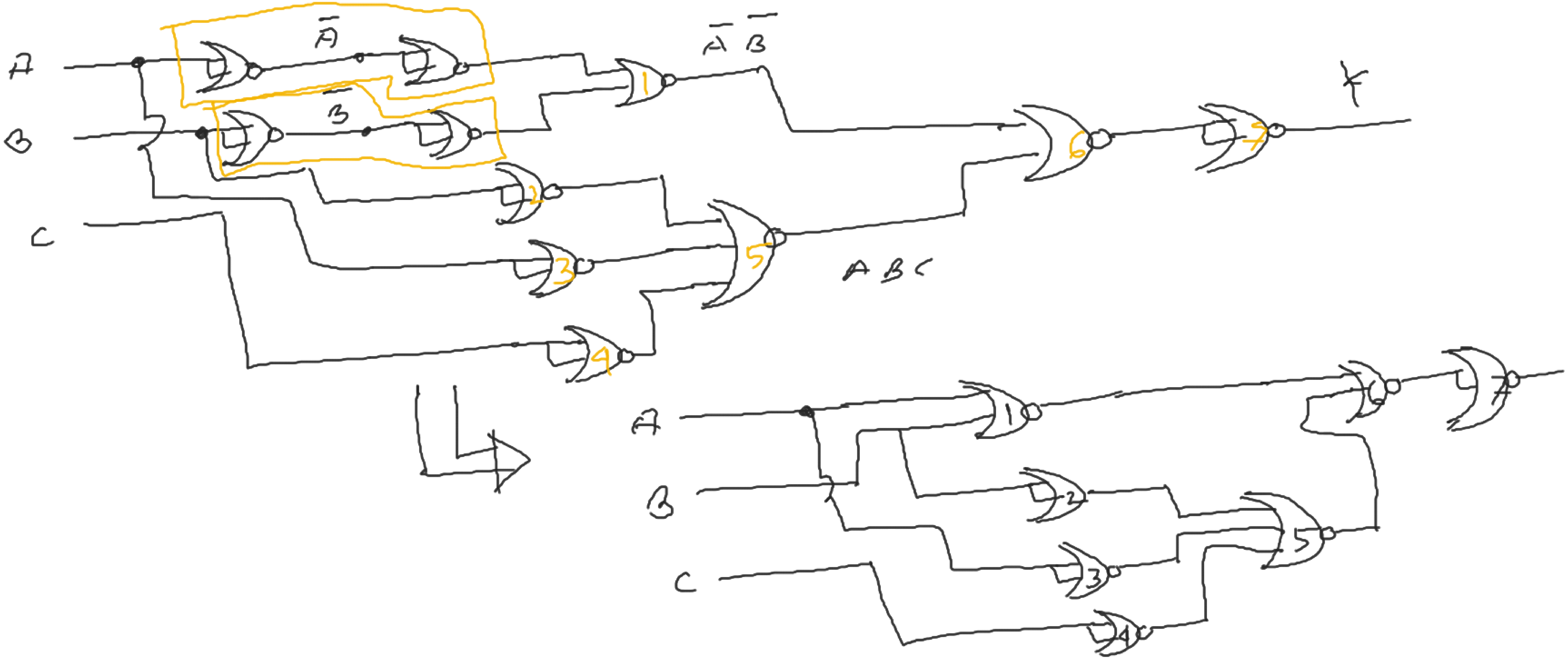
We can design all the basic gates using NOR gate only. So NOR gate is the universal gate.

Draw the circuit of $Y = \bar{A}B + A\bar{B}C$ using NOR gates only.





Draw the circuit of $y = \bar{A}\,\bar{B} + ABC$ using NOR gates only.



Simplification of logic expression using Karnough Map (K-map):

Boolean Algebra:

$$A B C + A B \bar{C}$$
$$= A B (C + \bar{C})$$
$$= A B$$

What is K-map?

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}$	1	1	1	1
C	1		1	1

AB

2 X 4 K-map

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	1	1	x
$C\bar{D}$	1	1	1	1
CD	1		1	
$\bar{C}D$	1	1		

4 X 4 K-map

"What is adjacent 1":

$$A B C \bar{D} + A B C D$$
$$= A B C (\bar{D} + D)$$
$$= A B C$$

ABC

$$\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$+ ABCD + ABC\bar{D}$$

Looping:

1. pair → 2 adjacent '1'
2. Quad
3. octec

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1			1
$C\bar{D}$	1	1	1	1
CD	1	1	1	
$\bar{C}D$	1			

$\rightarrow \bar{A}B\bar{D}$

$\rightarrow AC$ ✓

$\rightarrow \bar{A}\bar{B}$

1. Pair $\rightarrow$ 2 adjacent '1's
2. Quad $\rightarrow$ 4 adjacent '1's

$$A\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}CD + ABCD$$
$$= AC\bar{D}(\bar{B}+B) + ACD(\bar{B}+B)$$
$$= AC\bar{D} + ACD$$
$$= AC(\bar{D}+D)$$
$$= AC$$

$\rightarrow A\bar{C}$

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
$\bar{C}\bar{D}$	1	1	1	1
$C\bar{D}$	1	1	1	1
CD				
$\bar{C}D$	1		1	1

$\rightarrow \bar{A}\bar{C}$

$\rightarrow C\bar{D}$

$$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}D$$
$$= \bar{A}\bar{C}\bar{D}(\bar{B}+B) + \bar{A}\bar{C}D(B+\bar{B})$$
$$= \bar{A}\bar{C}\bar{D} + \bar{A}\bar{C}D$$
$$= \bar{A}\bar{C}(\bar{D}+D)$$
$$= \bar{A}\bar{C}$$