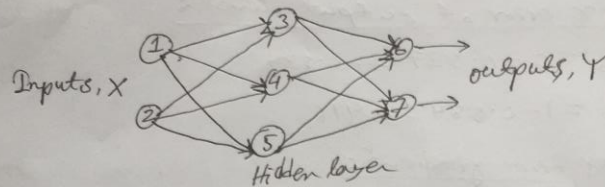


BPNN Math Example

BPNN Math

①

For the following NN, $w_{13}=0.3$, $w_{14}=-0.4$, $w_{15}=0.6$, $w_{23}=-0.9$, $w_{24}=-0.2$, $w_{25}=-0.3$, $w_{36}=-0.4$, $w_{37}=-0.1$, $w_{46}=0.3$, $w_{47}=-0.3$, $w_{56}=0.4$, $w_{57}=0.8$, learning rate = 0.2
If $x_1=1$, $x_2=0$ and $X=[1,0]$ and $Y=[0,1]$, find the updated weights after one iteration.



Solⁿ: Given input set $X=[1,0]$, desired output set $Y=[0,1]$
① Let us at first find out the output at hidden layer:

$$\begin{aligned} y_3^{(1)} &= \text{sigmoid}(x_1 w_{13} + x_2 w_{23} - \theta_3) \\ &= \text{sigmoid}(1 \times 0.3 + 0 - 0) \\ &= \text{sigmoid}(0.3) = \frac{1}{1 + e^{-0.3}} = 0.574 \end{aligned}$$

$$\begin{aligned} y_5^{(1)} &= \text{sigmoid}(x_1 w_{15} + x_2 w_{25} - \theta_5) \\ &= \text{sigmoid}(1 \times 0.6 + 0 - 0) \\ &= \text{sigmoid}(0.6) = \frac{1}{1 + e^{-0.6}} = 0.646 \end{aligned}$$

$$\begin{aligned} y_4^{(1)} &= \text{sigmoid}(x_1 w_{14} + x_2 w_{24} - \theta_4) \\ &= \text{sigmoid}(1 \times (-0.4) + 0 - 0) \\ &= \text{sigmoid}(-0.4) = \frac{1}{1 + e^{0.4}} = 0.40 \end{aligned}$$

② Actual outputs at output level:

$$\begin{aligned} y_6 &= \text{sigmoid}(y_3 w_{36} + y_4 w_{46} + y_5 w_{56} - \theta_6) \\ &= \text{sigmoid}(0.574 \times (-0.4) + 0.3 \times 0.4 + 0.646 \times 0.4 - 0) \\ &= \text{sigmoid}(-0.23 + 0.12 + 0.2584 - 0) \\ &= \text{sigmoid}(0.1484) = \frac{1}{1 + e^{-0.1484}} = 0.537 \end{aligned}$$

$$\begin{aligned}
 y_7 &= \text{sigmoid}(y_3 w_{37} + y_4 w_{47} + y_5 w_{57} - \theta_7) \\
 &= \text{sigmoid}(0.574(-0.1) + 0.4(0.3) + 0.646 \times 0.8 - 0) \\
 &= \text{sigmoid}(-0.0574 - 0.12 + 0.5168) \\
 &= \text{sigmoid}(0.3394) \\
 &= \frac{1}{1 + e^{-0.3394}} = 0.5844
 \end{aligned}$$

③ Now calculating error at output level:

$$e_6 = y_{d6} - y_6 = 0 - 0.537 = -0.537$$

$$e_7 = y_{d7} - y_7 = 1 - 0.584 = 0.416$$

④ Now, calculating error gradients,

$$\begin{aligned}
 \delta_6 &= y_6 [1 - y_6] \times e_6 \\
 &= 0.537 [1 - 0.537] (-0.537) = -0.1335
 \end{aligned}$$

$$\begin{aligned}
 \delta_7 &= y_7 [1 - y_7] \times e_7 \\
 &= 0.584 [1 - 0.584] \times 0.416 = 0.101
 \end{aligned}$$

Now updating weights betⁿ o/p and hidden layer:

$$\begin{aligned}
 \Delta w_{36} &= \alpha \times y_3 \times \delta_6 = 0.2 \times 0.574 \times (-0.1335) \\
 &= -0.0153
 \end{aligned}$$

$$\begin{aligned}
 \Delta w_{37} &= \alpha \times y_3 \times \delta_7 = 0.2 \times 0.574 \times (0.101) \\
 &= 0.012
 \end{aligned}$$

$$\begin{aligned}
 \Delta w_{46} &= \alpha \times y_4 \times \delta_6 = 0.2 \times 0.4 \times (-0.1335) \\
 &= -0.011
 \end{aligned}$$

$$\begin{aligned}
 \Delta w_{47} &= \alpha \times y_4 \times \delta_7 = 0.2 \times 0.4 \times 0.101 \\
 &= 0.0081
 \end{aligned}$$

$$s_6 = \alpha \times y_5 \times \delta_6 = 0.2 \times 0.646 \times (-0.1337) \\ = -0.017 \leftarrow$$

$$\Delta w_{57} = \alpha \times y_5 \times \delta_7 = 0.2 \times 0.646 \times 0.101 \\ = 0.013 \leftarrow$$

Therefore,

$$\textcircled{6} w_{36}(2) = w_{36}(1) + \Delta w_{36} = -0.4 + (-0.0153) = -0.4153 \leftarrow$$

$$w_{37}(2) = w_{37}(1) + \Delta w_{37} = -0.1 + 0.012 = -0.088 \leftarrow$$

$$w_{46}(2) = 0.3 - 0.01 = 0.29 \leftarrow$$

$$w_{47}(2) = w_{47}(1) + \Delta w_{47} = -0.3 + 0.0081 = -0.2919 \leftarrow$$

$$w_{56}(2) = w_{56}(1) + \Delta w_{56} = 0.4 - 0.017 = 0.383 \leftarrow$$

$$w_{57}(2) = 0.8 + 0.013 = 0.813 \leftarrow$$

Error gradients at hidden layer:

$$\textcircled{7} \delta_3 = y_3 \cdot [1 - y_3] \times [w_{36} \times \delta_6 + w_{37} \times \delta_7] \\ = 0.574 \times 0.426 \times [-0.4 \times (-0.1335) + (-0.1) \times (0.101)] \\ = 0.244 [0.0534 - 0.0101] \\ = 0.244 \times 0.0433 = 0.0105 \leftarrow$$

$$\delta_4 = y_4 [1 - y_4] \times [w_{46} \times \delta_6 + w_{47} \times \delta_7] \\ = 0.4 [1 - 0.4] \times [0.3 (-0.1335) + (-0.3) (0.101)] \\ = 0.24 \times [-0.04005 - 0.0303] \\ = 0.24 (-0.07035) = -0.017 \leftarrow$$

$$\delta_5 = y_5 [1 - y_5] \times [w_{56} \times \delta_6 + w_{57} \times \delta_7] \\ = 0.646 \times [1 - 0.646] \times [0.4 \times (-0.1335) + 0.8 \times 0.101] \\ = 0.2287 [-0.0534 + 0.0808] \\ = 0.2287 \times 0.0274 = 0.0063 \leftarrow$$

$$\textcircled{8} \Delta w_{13} = \alpha \times x_1 \times s_3 = 0.2 \times 1 \times 0.0105 = 0.0021 \leftarrow$$

$$\Delta w_{14} = \alpha \times x_1 \times s_4 = 0.2 \times 1 \times 0.0063 = 0.00126 \leftarrow$$

$$\Delta w_{23} = \alpha \times x_2 \times s_3 = 0.2 \times 0 \times 0.0105 = 0 \leftarrow$$

$$\Delta w_{24} = \alpha \times x_2 \times s_4 = 0.2 \times 0 \times 0.0063 = 0 \leftarrow$$

$$\Delta w_{25} = \alpha \times x_2 \times s_5 = 0.2 \times 0 \times 0.0063 = 0 \leftarrow$$

Therefore,

$$\textcircled{9} w_{13}(2) = w_{13}(1) + \Delta w_{13} = 0.3 + 0.0021 = 0.3021 \leftarrow$$

$$w_{14}(2) = w_{14} + \Delta w_{14} = -0.4 - 0.0034 = -0.4034 \leftarrow$$

$$w_{15}(2) = w_{15} + \Delta w_{15} = 0.6 + 0.00126 = 0.60126 \leftarrow$$

$$w_{23}(2) = w_{23} + \Delta w_{23} = -0.9 + 0 = -0.9 \leftarrow$$

$$w_{24}(2) = w_{24} + \Delta w_{24} = -0.2 + 0 = -0.2 \leftarrow$$

$$w_{25}(2) = w_{25} + \Delta w_{25} = -0.3 + 0 = -0.3 \leftarrow$$