BPNN Math Example

↓ For the following NN, $W13 = 0.3$, $W14 = -0.4$, $W15 = 0.6$,
$W23 = -0.9$, $W24 = -0.2$, $W25 = -0.3$, $W36 = -0.4$, $W37 = -0.1$,
$W46 = 0.3$, $W47 = -0.3$, $W56 = 0.4$, $W57 = 0.8$, learning rate $= 0.2$
If $x1 = 1$, $x2 = 0$ and $x = [1, 0]$ and $Y = [0, 1]$, find the
updated weights after one iteration.

Inputs, X   outputs, Y
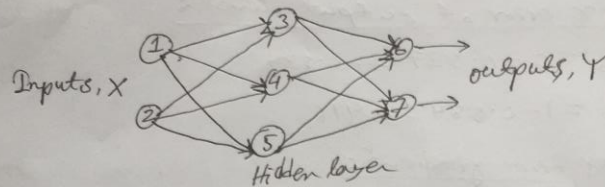
Hidden layer

Sol^n: Given input set $X = [1, 0]$, desired output set $Y = [0, 1]$.

① Let us at first find out the output at hidden layer:

$$y_3^{(1)} = \text{sigmoid}\,(x_1 W_{13} + x_2 W_{23} - \theta_3)$$
$$= \text{sigmoid}\,(1 \times 0.3 + 0 - 0)$$
$$= \text{sigmoid}\,(0.3) = \frac{1}{1 + e^{-0.3}} = 0.574 \;\leftarrow$$

$$y_5^{(1)} = \text{sigmoid}\,(x_1 W_{15} + x_2 W_{25} - \theta_5)$$
$$= \text{sigmoid}\,(1 \times 0.6 + 0 - 0)$$
$$= \text{sigmoid}\,(0.6) = \frac{1}{1 + e^{-0.6}} = 0.646 \;\leftarrow$$

$$y_4^{(1)} = \text{sigmoid}\,(x_1 W_{14} + x_2 W_{24} - \theta_4)$$
$$= \text{sigmoid}\,(1 \times (-0.4) + 0 - 0)$$
$$= \text{sigmoid}\,(-0.4) = \frac{1}{1 + e^{0.4}} = 0.40 \;\leftarrow$$

② Actual outputs at output level:

$$y_6 = \text{sigmoid}\,(y_3 W_{36} + y_4 W_{46} + y_5 W_{56} - \theta_6)$$
$$= \text{sigmoid}\,(0.574(-0.4) + 0.3 \times 0.4 + 0.646 \times 0.4 - 0)$$
$$= \text{sigmoid}\,(-0.23 + 0.12 + 0.2584 - 0)$$
$$= \text{sigmoid}\,(0.1484) = \frac{1}{1 + e^{-0.1484}} = 0.537 \;\leftarrow$$

$$y_7 = \text{sigmoid} (y_3 W_{37} + y_4 W_{47} + y_5 W_{57} - \theta_7)$$
$$= \text{sigmoid} (0.574(-0.1) + 0.4(0.3) + 0.646 \times 0.8 - 0)$$
$$= \text{sigmoid} (-0.0574 - 0.12 + 0.5168)$$
$$= \text{sigmoid} (0.3394)$$
$$= \frac{1}{1 + e^{-0.3394}} = 0.584 \leftarrow$$

③ Now calculating error at output level:

$$e_6 = y_{d6} - y_6 = 0 - 0.537 = -0.537$$
$$e_7 = y_{d7} - y_7 = 1 - 0.584 = 0.416$$

④ Now, calculating error gradients,

$$\delta_6 = y_6 [1 - y_6] \times e_6$$
$$= 0.537 [1 - 0.537] (-0.537) = -0.1335 \leftarrow$$

$$\delta_7 = y_7 [1 - y_7] \times e_7$$
$$= 0.584 [1 - 0.584] \times 0.416 = 0.101 \leftarrow$$

Now updating weights betⁿ o/p and hidden layer:

⑤ $\Delta W_{36} = \alpha \times y_3 \times \delta_6 = 0.2 \times 0.574 (-0.1335)$
$$= -0.0153 \leftarrow$$

$\Delta W_{37} = \alpha \times y_3 \times \delta_7 = 0.2 \times 0.574 \times (0.101)$
$$= 0.012 \leftarrow$$

$\Delta W_{46} = \alpha \times y_4 \times \delta_6 = 0.2 \times 0.4 \times (-0.1335)$
$$= -0.01 \leftarrow$$

$\Delta W_{47} = \alpha \times y_4 \times \delta_7 = 0.2 \times 0.4 \times 0.101$
$$= 0.0081 \leftarrow$$

$$\Delta W_{56} = \alpha \times y_5 \times \delta_6 = 0.2 \times 0.646 \times (-0.1337)$$

$$= -0.017 \longleftarrow$$

$$\Delta W_{57} = \alpha \times y_5 \times \delta_7 = 0.2 \times 0.646 \times 0.101$$

$$= 0.013 \longleftarrow$$

Therefore,

(6) $W_{36}(2) = W_{36}(1) + \Delta W_{36} = -0.4 + (-0.0153) = -0.4153 \longleftarrow$

$W_{37}(2) = W_{37}(1) + \Delta W_{37} = -0.1 + 0.012 = -0.088 \longleftarrow$

$W_{46}(2) = 0.3 - 0.01 = 0.29 \longleftarrow$

$W_{47}(2) = W_{47}(1) + \Delta W_{47} = -0.3 + 0.0081 = -0.2919 \longleftarrow$

$W_{56}(2) = W_{56}(1) + \Delta W_{56} = 0.4 - 0.017 = 0.383 \longleftarrow$

$W_{57}(2) = 0.8 + 0.013 = 0.813 \longleftarrow$

Error gradients at hidden layer:

(7) $\delta_3 = y_3 \cdot [1 - y_3] \times [W_{36} \times \delta_6 + W_{37} \times \delta_7]$

$$= 0.574 \times 0.426 \times [-0.4 \times (-0.1335) + (-0.1)(0.101)]$$

$$= 0.244 [0.0534 - 0.0101]$$

$$= 0.244 \times 0.0433 = 0.0105 \longleftarrow$$

$\delta_4 = y_4 [1 - y_4] \times [W_{46} \times \delta_6 + W_{47} \times \delta_7]$

$$= 0.4 [1 - 0.4] \times [0.3(-0.1335) + (-0.3)(0.101)]$$

$$= 0.24 \times [-0.04005 - 0.0303]$$

$$= 0.24 (-0.07035) = -0.017 \longleftarrow$$

$\delta_5 = y_5 [1 - y_5] \times [W_{56} \times \delta_6 + W_{57} \times \delta_7]$

$$= 0.646 \times [1 - 0.646] \times [0.4 \times (-0.1335) + 0.8 \times 0.101]$$

$$= 0.2287 [-0.0534 + 0.0808]$$

$$= 0.2287 \times 0.0274 = 0.0063 \longleftarrow$$

(8) $\Delta W_{13} = \alpha \times x_1 \times \delta_3 = 0.2 \times 1 \times 0.0105 = 0.0021$ ←

$\Delta W_{14} = \alpha \times x_1 \times \delta_4 = 0.2 \times 1 \times 0.0063 = 0.00126$ ←

$\Delta W_{23} = \alpha \times x_2 \times \delta_3 = 0.2 \times 0 \times 0.0105 = 0$ ←

$\Delta W_{24} = \alpha \times x_2 \times \delta_4 = 0.2 \times 0 \times 0.0157 = 0$ ←

$\Delta W_{25} = \alpha \times x_2 \times \delta_5 = 0.2 \times 0 \times 0.0063 = 0$ ←

Therefore,

(9) $W_{13}(2) = W_{13}(1) + \Delta W_{13} = 0.3 + 0.0021 = 0.3021$ ←

$W_{14}(2) = W_{14} + \Delta W_{14} = -0.4 - 0.0034 = -0.4034$ ←

$W_{15}(2) = W_{15} + \Delta W_{15} = 0.6 + 0.00126 = 0.60126$ ←

$W_{23}(2) = W_{23} + \Delta W_{23} = -0.9 + 0 = -0.9$ ←

$W_{24}(2) = W_{24} + \Delta W_{24} = -0.2 + 0 = -0.2$ ←

$W_{25}(2) = W_{25} + \Delta W_{25} = -0.3 + 0 = -0.3$ ←