Emergence of Numerals in Multi-Agent Autonomous Communication System

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Abstract

This project aims to propose a new computational simulation method for the emergence of numerals based on multi-agent autonomous communication system following deep reinforcement learning methodology.

Acknowledgements

Any acknowledgements go here.

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Chapter 1

Introduction

Natural language processing (NLP) is an important and long-standing topic in artificial intelligence (AI), in which a core question is natural language understanding (NLU). With the rapid development of deep learning (DL), most current statae-of-the-art methods in NLP, e.g. [Socher et al., 2013, Mikolov et al., 2013, Kim, 2014], are based on DL models trained on massive static textual corpora. From an information processing perspective, the information flow of NLP-based human-computer interaction systems is illustrated in Figure 1.1 given as follow. As the diagram shows, the input of NLP systems are various kinds of textual materials generated by human beings to descibe their experiences/perceptions (E/P). Under such a perspective, symbols in natural languages are actually feature representations of the original E/P, whereas most current NLP systems directly take these symbols as original features.

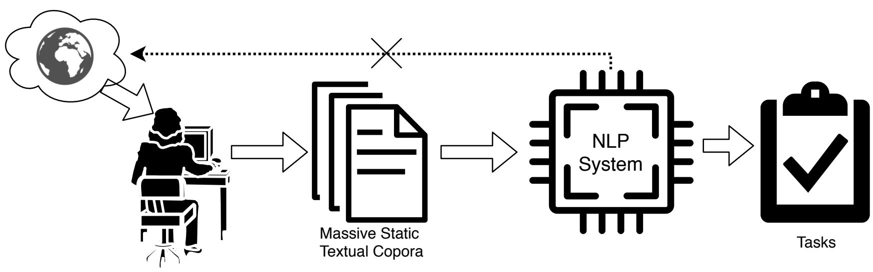

Figure 1.1: An overview of information flow in current NLP systems.

Considering the missing original E/P, grounded language learning (GLL) argues that models need a grounded environment to learn and understand language[Matuszek, 2018]. However, natural languages of the time have been developed for at least tens of thousands of years[Berwick and Chomsky, 2016] and already became very sophisticated. Thus, to verify that computational agents can truly ground symbols in natural languages to correspongding E/P and can complete the specified tasks, it is necessary to facilitate them to discover and develop various kinds of characteristics of natural language during autonomous communication of agents. There are already lots of works, e.g. [Hill et al., 2017, Havrylov and Titov, 2017, Yu et al., 2018, Kottur et al., 2017], aiming to facilitate the emergence of "natural language" in multi-agent autonomous communication systems. However, one significant limitation of previous works is that, only referential objects/attributes in environments, e.g. shapes and colors, were considered and to which discrete symbols were grounded to.

This project, on the other hand, aims to explore and analyse the grounding of abstractions which are non-referential (?) in the original experiences/perceptions of human beings. However, as it is too huge a topic to tackle, our project is limited to cardinal numerals for the following reasons: i) numeral systems are relatively simple and self-contained[Hurford, 1999]; ii) concepts related to cardinal numerals are more straightforward to model with numeric representations; iii) functions of emergent cardinal numerals can be formalised and verified more reliably in simulation.

In this work, our main contributions are given as follows:

1. We propose a language game in which we can define numerals as symbols indicating numbers of replicating tokens when generating outputs.

2. Based on the language game, we successfully train agents to transmit numerical concepts which corresponds to function words in natural languages.

3. We transformed iterated learning proposed by [Smith et al., 2003] to train DL models and find that it does help to improve the compoitionality of the emergent language.

4. We further analyse and discuss the compositionality of the emergent communication protocal and HERE!.

Chapter 2

Background

As we demonstrate in Chapter 1, there are 2 almost disjointly developed research topics that motivates this project, i.e. computer simulation methods in evolutionary linguistics and multi-agent games in GLL. Thus, in the following 2 sections, we will introduce works which are highly related to our project from these 2 different areas.

2.1 Computer Simulation Methods in Evolutionary Linguistics

The emergence and evolution of natural language have always been critical questions to the field of evolutionary linguistics [MacWhinney, 2013] and one important issue is to use quantitative methods to overcome the time limit on unpreserved pre-historic linguistic behaviors[Lieberman, 2006, Evans and Levinson, 2009]. Since it was first introduced by [Hurford, 1989], computer simulation methods have attracted a rapidly growing attention, e.g. [Hurford et al., 1998, Knight et al., 2000, Briscoe, 2002, Christiansen and Kirby, 2003, Bickerton and Szathmáry, 2009, Cangelosi and Parisi, 2012]. As we introduced in Chapter 1, one of our objectives is to facilitate computational agents to discover and develop various kinds of natural langauge phenonmana, which shares a same objective and motivation of computer simulation methods in evolutionary linguistics.

To imply and verify linguistics theories, there are 2 necessary component: i) environments, in which agents can execute actions and communicate with each other; ii) pre-defined elementary linguistic knowledge that can be manipulated and altered by agents. Further, we could categorise the environments into the following 3 different types according to their simulation objectives:

- *Iterated learning* introduced by [Kirby, 1999] which aims at simulating cultural transitions from generation to generation.

- *Language games* introduced by [Wittgenstein, 1953] which takes the emergent communication protocal in cooperation between individuals as a prototype of language.

- *Genetic evolution* introduced by [Briscoe, 1998] which aims at simluating evolution of languages as a kind of natural selection procedure[Darwin, 1859].

With environments and pre-defined elementary linguistic knowledge, computational agents can then learn bi-directional meaningutterance mapping functions[Gong and Shuai, 2013]. With diffrent kinds of resulting linguistic phenonmana, this simulation procedure can be broadly categorised intto 2 classes:

- lexical models, e.g. [Steels, 2005, Baronchelli et al., 2006, Puglisi et al., 2008], whose main concern is whether a common lexicon can form during the communication in agent community;

- syntactic and grammatical models, e.g. [Kirby, 1999, Vogt, 2005], in which agents mainly aim to map meanings (represented in various ways) to utterances (either structured or unstructured).

However, no matter how these mapping functions are learnt, e.g. by neural network models [Munroe and Cangelosi, 2002] or by mathematical equations [Minett and Wang, 2008, Ke et al., 2008], the most basic elements of linguistics, e.g. meanings to communicate about and a signalling channel to employ, are all pre-defined.

In contract, although we also follow the framework of language games and train agents in an iterated learning fashion, the basic linguistics elements in our project are not provided from the outset any more and computational agents can specify meanings of symbols/utterances by themselves.

2.2 Multi-agent Games in Grounded Language Learning

Unlike how we human beings learn and understand language, the current DL-based NLP techiniques learn semantics from only large-scaled static textual materials. Thus, GLL argues that computational also need to learn and understand languages by interacting with environments and grounding language into their E/P. With the recent rapid development of deep reinforcement learning (DRL), it is proven that computational agents can master a variaty of complex cognitive activities, e.g. [Mnih et al., 2015, Silver et al., 2017]]. Thus, a bunch of works in GLL apply DRL techiniques to facilitate agents to learn or invent natural languages[1], such as [Hermann et al., 2017, Mordatch and Abbeel, 2018, Havrylov and Titov, 2017, Hill et al., 2017].

To verify language abilities of computational agents, previous works in GLL usually follow the framework of language games, of which are mainly variants of referential games introduced by [Lewis, 1969], e.g. [Hermann et al., 2017, Havrylov and Titov, 2017]. Also, some works are more motivated by the competence instead of cooperation such as [Cao et al., 2018].

From another perspetive, based on the number of participated agents, we can broadly categorise language games in GLL into the following 2 types:

- *Single-agent games* usually need to be done by one agent and a human participator, in which the main concern is to explore how could computational agents learn the compositionality of semantics.

- *Multi-agent games* are usally completed by an agent population, in which the main concern is to explore how various kinds of natural language phenonmana emerge and envolve during communicating among agents.

However, like we mentioned in Chapter 1, whichever kind of langauge game they follow in previous works of GLL, the objects/attributes the symbols grounded to are all referential. We, on the other hand, aim to explore the feasibility of grounding symbols to non-referential objects (specifically, numerals) during the game.

[1] Strictly speaking, "invent natural language" should be called as "invent communication protocals sharing compoitionality like natural languages". However, as our project is to facilitate compositionality in multi-agent communication protocals, we thus call these emergent communication protocals a kind of "langauge" invented by agents

Chapter 3

Game, Models and Evaluation Methods

In this chapter, we first describe the proposed language game and the definition of numerals in our game. We then introduce the architecture of models we used and also the iterated learning for training models.

3.1 Game Description

Unlike traditional simulation methods in evolutionary linguistics introduced in Section 2.1, there are 3 necessary components in our architecture and they are given as follows:

- *Environment*: To imply our linguistic assumption as well as make the size of environment limited and thus analysable, all perceptions in the established environment are sequences of objects represented by one-hot vectors. For ease of demonstration, we denote these objects as $o \in \{A, B, C, D\}$ in the following sections.
- *Agents*: There are 2 kinds of agents in our project: i) *speakers* S that can observe objects in the environment and emit messages m_i; ii) *listeners* L that can receive the messages and generate a sequence of obejcts.
- *Dynamics*: In this project, the dynamics mean not only the manually designed reward function for agents but also the training mechanism, e.g. iterated learning and blank vocabulary. The details will be introduced in Subsection 3.2.4 and Subsection 3.2.5. It worth mentioning that one premise of our project is that all the linguistic hypotheses are purely implied by dynamics.

3.1.1 Game Procedure

The overall view of the proposed Set-Forward game is illustrated in Figure 3.1 given as follow.

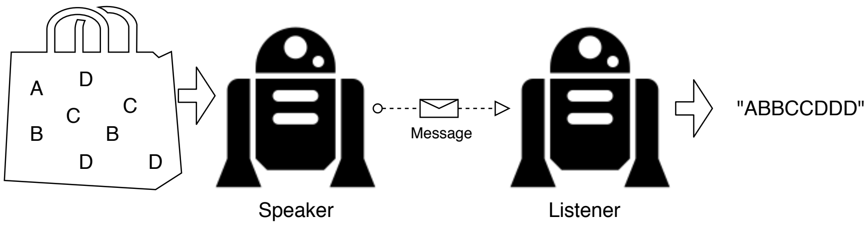

Figure 3.1: Diagram of Game Playing Procedure.

According to the steps of playing games at iteration i, the components of our games are illustrated as follows:

1. Perceptions: the perception from environments is a **set** of objects, i.e. $s_i = \{o_{i_1}, o_{i_2}, \ldots\}$.

2. Speaker observation and message generation: after observing and encoding the perception, speaker S would generate a message $m_i = \{s_{i_1}, s_{i_2}, \ldots, s_M\}$ where M is the maximum length of messages and s_k are selected from a randomly initialised vocabulary such that the symbols in the initial vocabulary are meaningless;

3. Listener receiving message and perception reproduction: after receiving and encoding the message m_i, the listener would generate a **sequence** $\hat{s}_i = \{\hat{o}_{i_1}, \hat{o}_{i_2}, \ldots\}$ whose symbols are identical to those in the original perception s_i;

4. Reward and parameter update: by comparing s_i and $\hat{s}_i$, we take the cross-entropy between them as the reward for both listener and speaker and update parameters of both speaker and listener with respect to it.[1]

One thing that needs to be highlighted is that the perceptions s_i are sets and thus order of objects would not make any diffrence. Further, we argue that the only important feature that need to be transmitted is actually the numbers of different objects which correponds to the function of numerals in natural language.

[1]Different ways of updating parameters are instroduced in Section 3.2.

3.1.2 Functions of Numerals in the Game

Broadly speaking, numerals are words that can describe the numerical quantities and usually act as determiners to specify the quantities of nouns, e.g. ”two dogs” and ”three people”. Also, under most scenarios, numerals correpond to non-referential concepts[Da Costa and Bond, 2016]. Considering the objective of listeners L in our language game, we define a numeral as a symbol s^n at **position** i indicating a function that reproduce some object o_i exactly n times:

$$s^n : o_i \rightarrow \{ \overbrace{o_i, \ldots, o_i}^{n \text{ elements}} \} \tag{3.1}$$

Note that, the meaning of a symbol is not only decided by itself but also its position in message, as L would encode meanings of symbols according to their appearance in messages. Also, in our models, there is no specific mechanism to separate the meanings of symbols from their positions.

From the side of speakers S, a numeral preferred to be defined as a symbol s^n at **position** i that represents the numbers of specific object o_i. Thus, we expect S would first learn to count the number of different objects and then encode them into a sequence of discrete symbols. As [Siegelmann and Sontag, 1992] shows that Recurrent Neural Networks (RNNs) are Turing-complete and Long-short Term Memory (LSTM) model proposed by [Hochreiter and Schmidhuber, 1997] is a super set of RNN, it is safe to claim that LSTM is also Turing-complete and thus capable of counting numbers of objects.

3.1.3 A Variant: Set-Select Game

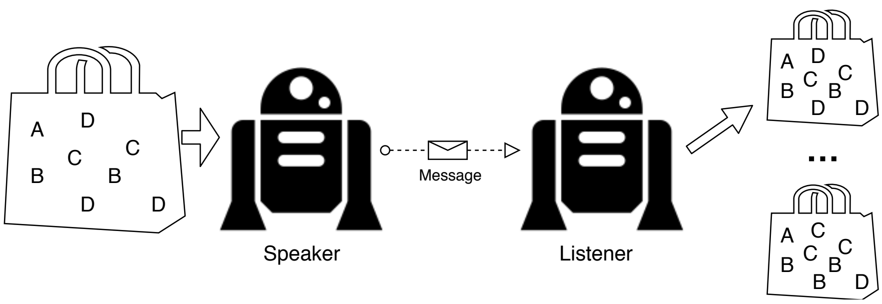

Figure 3.2: Diagram of Referential Game Playing Procedure.

We illustrate the Set-Select game, a variant of Set-Forward game, in Figure 3.2 given above. The only difference is that listeners need to select the correct set of objects among a bunch of distractors[2] instead of generating it.

3.2 Proposed Models

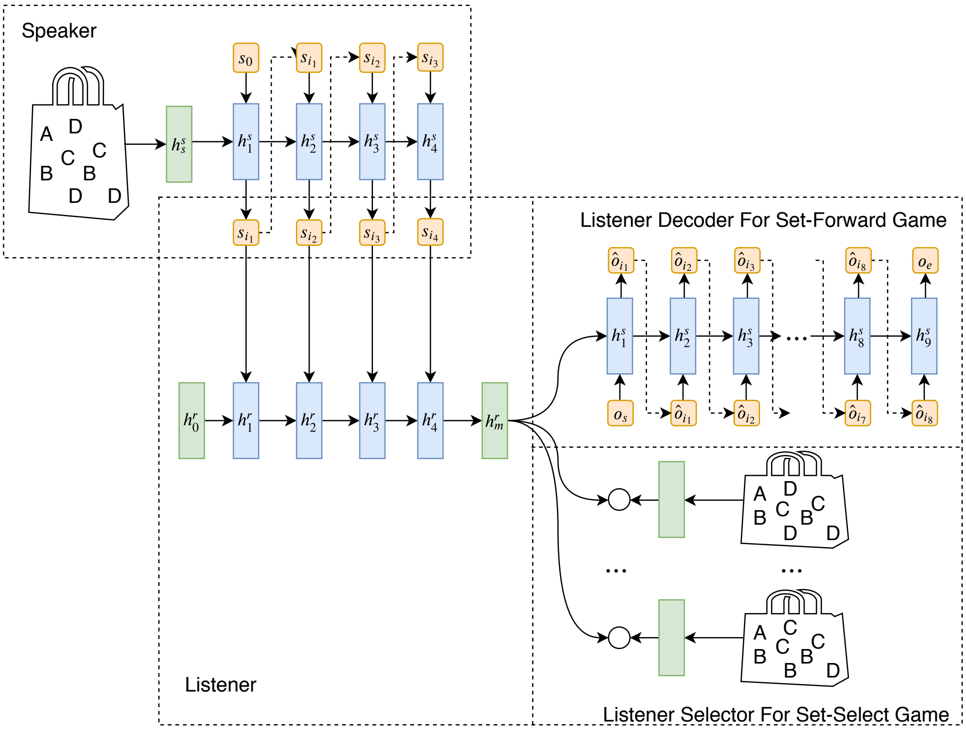

Figure 3.3: Overall Diagram of Model Architectures for Playing Games.

We illustrate the overall architecture of our models in Figure 3.3 given above, in which it is straightforward to see that a speaker S consists of a set encoder and a standard LSTM sequence decoder that can generate messages. As for a listeners L, it would first encode messages with a LSTM sequence encoder and get the feature vector h^l_m. Then, in the Set-Forward game, L would take h^l_m as the initial hidden state and predict a sequence of objects with a LSTM sequence decoder. As for in Set-Select game, L would compare h^l_m with a bunch of sets which are encoded by set encoders of L and select the one shown to S based on the dot product between h^l_m and feature vectors of each set.

Further details are shown in the following subsections.

[2]A distractor is a set that contains different numbers of objects as the correct one.

3.2.1 Speaker

The architecture of our speaking agents is very similar to the Seq-to-Seq model proposed by [Sutskever et al., 2014] except that replace the encoder for input sequences with a set encoder whose details would be introduced in the following subsubsection. As Seq-to-Seq model is quite popular nowadays, we skip details about how to generate sequences which correpond to the messages in our games, and focus on how to encode sets of objects.

3.2.1.1 Set Encoder

Our set encoder shares an almost same architecture of inputting sets proposed by [Vinyals et al., 2015]. However, as there is an addition in softmax function and it would introduce counting bias into the feature representation of sets, we replace equation (5) in [Vinyals et al., 2015] with the following operation in order to avoid exposing counting system to models:

$$a_{i,t} = \sigma(e_{i,t}) \tag{3.2}$$

where σ is sigmoid function.

In our implementation, the number of attention operations is set to be the same as the number of all types of objects, as we want to help models to represent number of each kind of objects as features in the vector representation of input set.

3.2.1.2 Message Generator

To generate the message m_i, we follow [Havrylov and Titov, 2017] and adopt a LSTM-based sequence decoder with 2 different kinds of sampling mechanisms: i)direct sampling that directly sample from the correponding categorical distribution specified by $softmax(Wh_k + b)$; ii) GUMBEL-softmax estimator proposed by [Jang et al., 2016] with straight-through trick introduced in [Bengio et al., 2013]. Beside, the learning mechanisms also vary for these 2 different sampling methods, which is further discussed in Subsection 3.2.4.

Note that the length of each message m_i is fixed to $|M|$ and symbols $s_{i_1}, \dots, s_{i_{|M|}}$ are all from an initially meaningless vocabulary whose size is $|L|$. The effect of $|L|$ and $|M|$ on the emergent language is further discussed in Chapter 4.

3.2.2 Listener in Set2Seq2Seq Model

3.2.3 Listener in Set2Seq2Choice Model

3.2.4 Loss/Reward and Learning

3.2.5 Numeral Iterated Learning

3.2.6 Baseline Models

3.3 Evaluation Methods

Chapter 4

Experiment Results and Discussion

4.1 Future Works

Chapter 5

Conclusions

5.1 Final Reminder

The body of your dissertation, before the references and any appendices, *must* finish by page 40. The introduction, after preliminary material, should have started on page 1.

You may not change the dissertation format (e.g., reduce the font size, change the margins, or reduce the line spacing from the default 1.5 spacing). Over length or incorrectly-formatted dissertations will not be accepted and you would have to modify your dissertation and resubmit. You cannot assume we will check your submission before the final deadline and if it requires resubmission after the deadline to conform to the page and style requirements you will be subject to the usual late penalties based on your final submission time.

5.2 Further Discussion

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