Tutorial (Week 9)

23. $$Y^L_{\ } = \begin{cases} 0, & X < d^* \\ (1+r)X - d, & d^* \le X < u^* \\ u - d, & X \ge u^* \end{cases}$$

$r = 0.1$ (next year)

$d = 11$

$u - d = 11 \Rightarrow u = 22$

$r = 0$ (this year)

$$E(Y^L) = (1+r)\left[E(X \wedge u^*) - E(X \wedge d^*)\right]$$

Thm 8.7

$$u^* = \frac{u}{1+r}, \quad d^* = \frac{d}{1+r}$$

For this year, $E(Y^L) = E(X \wedge 22) - E(X \wedge 11) = 7.1$

For next year, $E(Y^L) = 1.1\left[E(X \wedge 20) - E(X \wedge 10)\right] = 7.975$

$\therefore$ the required ratio $= \dfrac{7.975}{7.1} = 1.1254$.

24. We want $Var(Y^L) = E[(Y^L)^2] - [E(Y^L)]^2$.

Here $r = 0$, $\alpha = 1$, $u = \infty$, $d = 100$

Thm 8.7 gives $E(Y^L) = E(X) - E(X \wedge d)$

$$\underset{\substack{\text{exponential} \\ \theta = 1000}}{=} \theta - \theta\left(1 - e^{-d/\theta}\right) = \theta e^{-d/\theta}$$

$$= 1000 e^{-0.1}$$

Thm 8.8 gives $E[(Y^L)^2] = E(X^2) - E[(X \wedge d)^2] - 2d\,E(Y^L)$

$$= 2\theta^2 - 2\theta^2\Gamma(3; d/\theta) + d^2 e^{-d/\theta} - 2d\,E(Y^L)$$

Thm A.1

$$\Gamma(3; x) = 1 - e^{-x}\left(1 + x + \frac{x^2}{2}\right)$$

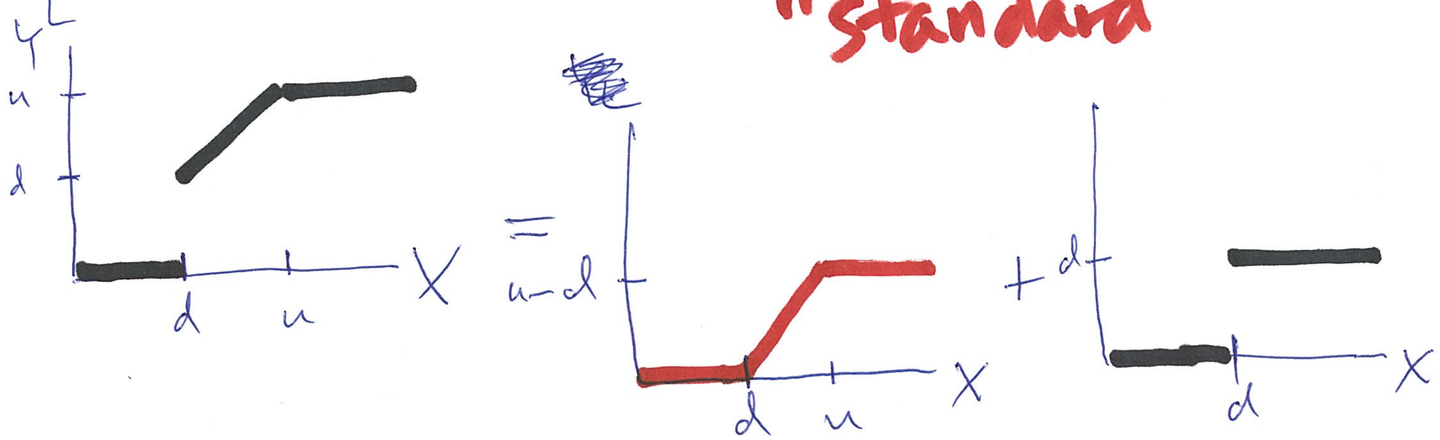
$$= 2\theta^2 e^{-d/\theta}\left(1 + \frac{d}{\theta} + \frac{(d/\theta)^2}{2}\right) + d^2 e^{-d/\theta} - 2d\,E(Y^L)$$

$\therefore Var(Y^L) = 990,944$

25. $Y^L = \begin{cases} 0, & X < 50{,}000 = d \\ X, & d \le X < 100{,}000 = u \\ u, & X \ge u \end{cases}$

$$= \begin{cases} 0, & X<d \\ (X-d)+d, & d\le X<u \\ (u-d)+d, & X\ge u \end{cases} = \boxed{\begin{cases} 0, & X<d \\ X-d, & d\le X<u \\ u-d, & X\ge u \end{cases}} + \begin{cases} 0, & X<d \\ d, & X\ge d \end{cases}$$

"standard"



$$E(Y^L) = \underbrace{E(X\wedge u) - E(X\wedge d)}_{\text{Thm 8.7}} + d\,\underbrace{\Pr(X\ge d)}_{1-F(d)} = 16{,}229.65$$

26.

$$f(x) = \begin{cases} 0.3 \times \frac{1}{50}, & 0 \le x < 50 \\ 0.36 \times \frac{1}{50}, & 50 < x < 100 \\ 0.18 \times \frac{1}{100}, & 100 < x < 200 \\ 0.16 \times \frac{1}{200}, & 200 < x < 400 \end{cases}$$

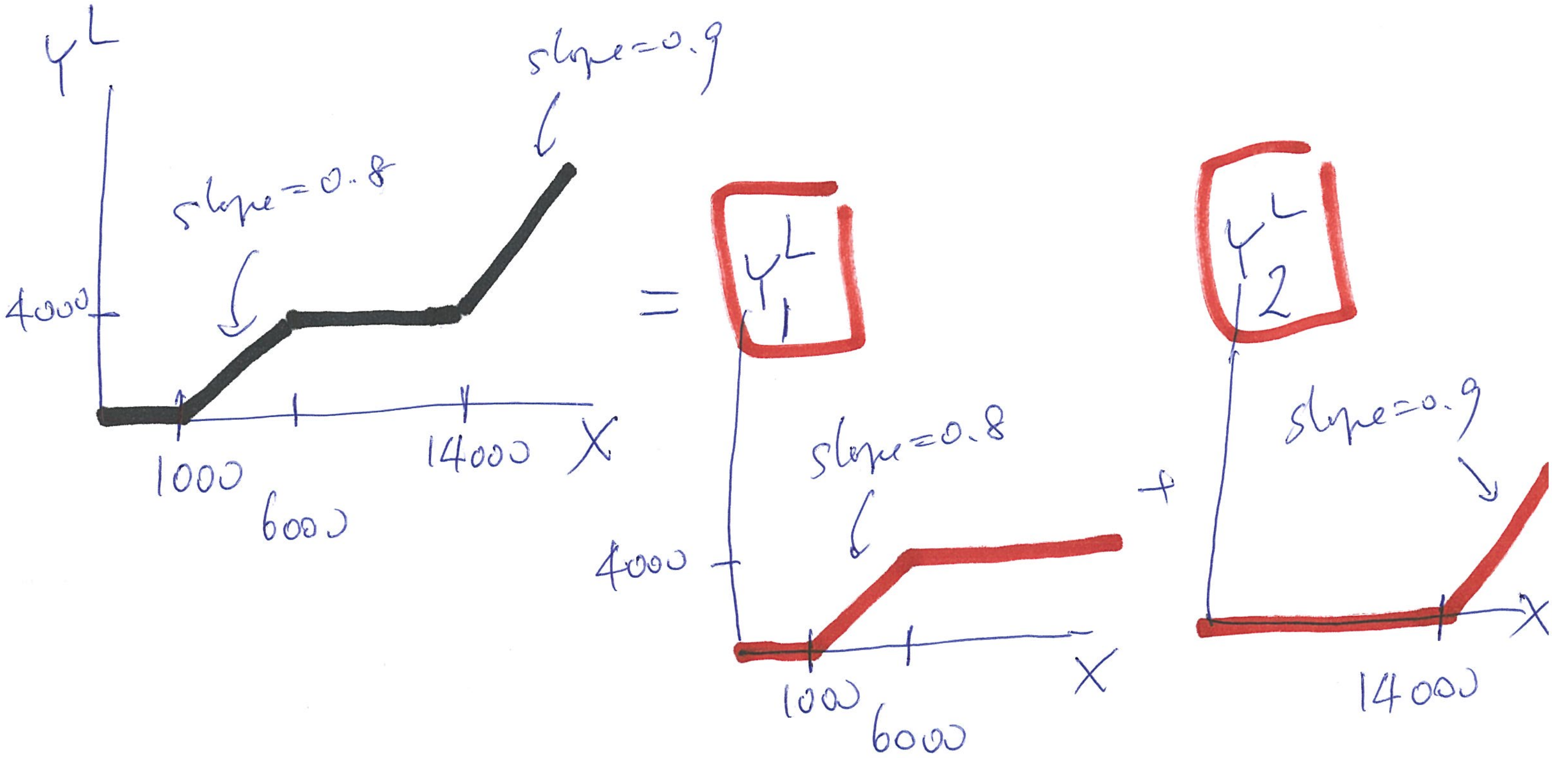
$$E\left[(X\wedge 350)^2\right] = \int_0^{400} (x\wedge 350)^2 f(x)\,dx$$

$$= \int_0^{50} \frac{0.3}{50} x^2\,dx + \int_{50}^{100} \frac{0.36}{50} x^2\,dx + \int_{100}^{200} \frac{0.18}{100} x^2\,dx$$

$$+ \int_{200}^{350} \frac{0.16}{200} x^2\,dx + \int_{350}^{400} \frac{0.16}{200} (350^2)\,dx = 20{,}750$$

27.

$$Y^L = \begin{cases} 0, & X<1000 \\ 0.8(X-1000), & 1000\le X<6000 \\ 4{,}000, & 6000\le X< 6000+8{,}000=14{,}000 \\ 4{,}000+0.9(X-14{,}000), & X\ge 14{,}000 \end{cases}$$



Now, $E(Y_1^L) \underset{\text{Thm 8.7}}{=} 0.8\left[E(X\wedge 6000) - E(X\wedge 1000)\right] =$

and $E(Y_2^L) \underset{\text{Thm 8.7}}{=} 0.9\left[E(X) - E(X\wedge 14000)\right] =$

$\therefore\ E(Y^L) = E(Y_1^L) + E(Y_2^L) = 2700.$