utorid: zhaizixu student email: shawn.zhai@mail.utoronto.ca

ECE368: Probabilistic Reasoning

Lab 2: Bayesian Linear Regression

Name: Shawn Zhai	Student Number: 1006979389

You should hand in: 1) A scanned .pdf version of this sheet with your answers (file size should be under 2 MB); 2) four figures for Question 2 and three figures for Question 4 in the .pdf format; and 3) one Python file regression.py that contains your code. All these files should be uploaded to Quercus.

1. Express the posterior distribution $p(\mathbf{a}|x_1, z_1, \ldots, x_N, z_N)$ using $\sigma^2, \beta, x_1, z_1, x_2, z_2, \ldots, x_N, z_N$. **(1 pt)**

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} \quad Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix} \quad \Sigma_a^{-1} \begin{bmatrix} 1/\beta & 0 \\ 0 & 1/\beta \end{bmatrix} \quad \Sigma_w^{-1} = 1/\sigma^2$$

$$\underline{\mu}_{a|x_1, z_1, \cdots, x_N, z_N} = (\Sigma_a^{-1} + X^T \Sigma_w^{-1} X)^{-1} \cdot (A^T \Sigma_w^{-1} Z)$$

$$\Sigma_{a|x_1, z_1, \cdots, x_N, z_N} = (\Sigma_a^{-1} + X^T \Sigma_w^{-1} X)^{-1}$$

$$P(a|x_1, z_1, \cdots, x_N, z_N) \sim N(\underline{\mu}_{a|x_1, \cdots, z_N}, \Sigma_{a|x_1, \cdots z_N})$$

2. Let $\sigma^2 = 0.1$ and $\beta = 1$. Draw four contour plots corresponding to the distributions $p(\mathbf{a})$, $p(\mathbf{a}|x_1, z_1)$, $p(\mathbf{a}|x_1, z_1, \ldots, x_5, z_5)$, and $p(\mathbf{a}|x_1, z_1, \ldots, x_{100}, z_{100})$. In all contour plots, the x-axis represents a_0, and the y-axis represents a_1. Please save the figures with names **prior.pdf, posterior1.pdf, posterior5.pdf, posterior100.pdf**, respectively. **(1.5 pt)**

3. Suppose that there is a new input x, for which we want to predict the corresponding target value z. Write down the distribution of the prediction z, i.e, $p(z|x, x_1, z_1, \ldots, x_N, z_N)$. **(1 pt)**

$$X' = [1 \quad x] \quad \underline{\mu}_{z|x, x_1, \cdots, z_n} = X' \cdot \underline{\mu}_{a|x_1, \cdots, z_N}$$

from part 1

$$\Sigma_{z|x, x_1, \cdots, z_n} = \sigma^2 + X' \cdot \Sigma_{a|x_1, \cdots, x_N} \cdot X'^T$$

$$P(z|x, x_1, z_1, \cdots x_N, z_N) \sim N(\underline{\mu}_{z|x, x_1, \cdots z_N}, \Sigma_{z|x, x_1, \cdots z_n})$$

4. Let $\sigma^2 = 0.1$ and $\beta = 1$. Given a set of new inputs $\{-4, -3.8, \ldots, 3.8, 4\}$, plot three figures, whose x-axis is the input and y-axis is the prediction, corresponding to three cases:

   (a) The predictions are based on one training sample, i.e., based on $p(z|x, x_1, z_1)$.
   (b) The predictions are based on 5 training samples, i.e., based on $p(z|x, x_1, z_1, \ldots, x_5, z_5)$.
   (c) The predictions are based on 100 training samples, i.e., based on $p(z|x, x_1, z_1, \ldots, x_{100}, z_{100})$.

The range of each figure is set as $[-4, 4] \times [-4, 4]$. Each figure should contain the following three components: 1) the new inputs and the corresponding predicted targets; 2) a vertical interval at each predicted target, indicating the range within one standard deviation; 3) the training sample(s) that are used for the prediction. Use plt.errorbar for 1) and 2); use plt.scatter for 3). Please save the figures with names **predict1.pdf, predict5.pdf, predict100.pdf**, respectively. **(1.5 pt)**