Final Assignment

(Dated: Please hand in before Jan. 8th, 2025)

1. Given a state $\rho = \begin{bmatrix} \frac{1}{2} & \frac{1}{4}e^{-i\varphi} \\ \frac{1}{4}e^{i\varphi} & \frac{1}{2} \end{bmatrix}$, what is the corresponding Bloch vector? Apply the unitary rotation about x-axis and y-axis at $\pi/2$ to ρ sequentially, what is the final state? What is the equivalent (single) unitary rotation? What is the generator of the equivalent unitary rotation?

2. Perform the measurement $\widehat{\mathrm{H}} = \vec{r}\cdot\hat{\sigma}$ on the above state ρ. What are the measurement operators corresponding to the measurement outcomes? What are the probabilities for obtaining the outcomes?

3. Given the Werner state

$$\rho^{\mathrm{AB}} = (1-z)\frac{\widehat{I}}{4} + z|\psi_+\rangle\langle\psi_+|, \tag{1}$$

where $|\psi_+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ is one of the Bell states. Find the conditions on z for ρ^{AB} violating CHSH inequality and being entangled, respectively.

4. (a) Show that the mutual information $\mathrm{I(A:B)}$ of a bipartite system ρ^{AB} can be expressed in terms of relative entropy

$$\mathrm{I(A:B)} = \mathrm{S}(\rho^{\mathrm{AB}}||\rho^{\mathrm{A}} \otimes \rho^{\mathrm{B}}). \tag{2}$$

(b) And show the Subadditivity inequality

$$\mathrm{S}(\rho^{\mathrm{AB}}) \leq \mathrm{S}(\rho^{\mathrm{A}}) + \mathrm{S}(\rho^{\mathrm{B}}), \tag{3}$$

with equality if and only if subsystem A and B are non-correlated.

5. Given the rotation operation $-\boxed{R_y(\theta)}- = e^{-i\frac{\theta}{2}\hat{\sigma}_y}$. Please quantify the entanglement of state $|\Psi_1\rangle$ by using negativity as a function of θ in the following quantum circuit. Performing the x-, y-, and z-measurement on the first qubit, what is the assemblage of the second qubit after these measurements on the first qubit?