

Q.1. Write short notes on:

a) Write notes on CSG and B-REP technique.

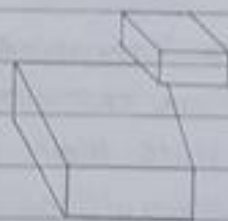
Ans. CSG Method:

i) Another technique for solid modelling is to combine the volumes occupied by overlapping three-dimensional objects using Boolean set operations.

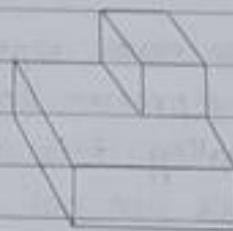
ii) It is also called Constructive Solid Geometry (CSG).

It creates a new volume by applying Boolean operators such as union, intersection, or difference to two specified objects.

iii) Fig. (a) and (b) and (c) show the example for forming new shapes using Boolean set operations. The Fig. a(i) shows that two rectangular blocks are placed adjacent to each other. We can obtain the combined object with the union operation as shown in Fig. a(ii).



(i) Objects



(ii) Combined objects

Fig. (a) Combined object by using union operator.

iv) The Fig. (b) shows the result of intersection operation obtained by overlapping cylinder & cube.

v) With the difference operation, we can obtain the resulting solid as shown in Fig. (c).

vi) The CSG method uses three dimensional objects such as blocks, pyramids, cylinders, cones, spheres, &

- P.L.
- a) closed spline surfaces to generate other solid objects in the method, an object is stored as a tree with operators at the internal nodes & simple primitives at the leaves.

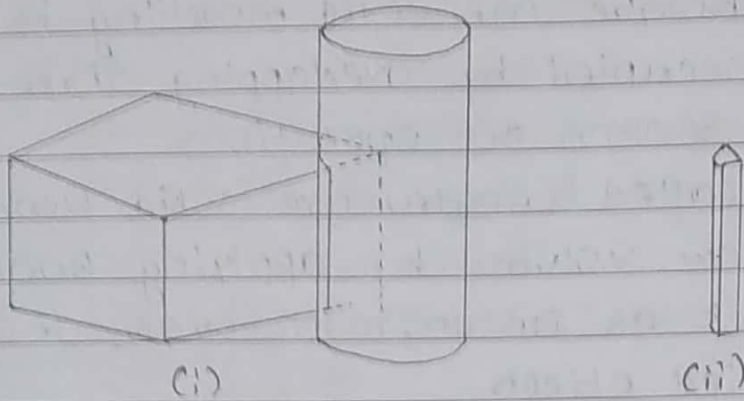


Fig. (b)



Fig. (c)

vii) Some nodes represent Boolean Operators, whereas other represent operations such as translation, rotation, and scaling. It is important to note that Boolean operations are not, in general, commutative. Therefore the edges of the trees must be in proper order.

Boundary Representation (B-reps):

- i) Boundary representation, B-rep in short, can be considered as an extension to the wireframe model.
- ii) The merit of a B-rep is that a solid is bounded by its surface & has its interior & exterior. The surface of a solid consists of a set of well-organized faces, each of which is a piece of some surface.

Q.1.

- a) iii) Faces may share vertices & edges that are curve segments. Therefore, B-rep is an extension to the wireframe model by adding face information to the latter.
- iv) There are 2-types of information in a B-rep topological and geometric.
- v) Topological information provide the relationships among vertices, edges & faces similar to that used in a wireframe model. In addition to connectivity, topological information also include orientation of edges & faces.
- vi) Geometric information are usually equations of the edges & faces. The orientation of each face is important.
- vii) Normally a face is surrounded by a set of vertices using the right-handed rule, the ordering of these vertices for describing a particular face must guarantee that the normal vector of the face is pointing to the exterior of the solid.
- viii) Normally, the order is counter clockwise. Therefore, by inspecting normal vectors one can immediately tell the inside/outside of a solid under B-rep.

Q.1.

b) Back surface detection method.

Ans. i) Object surfaces that are oriented away from the viewer are called back faces.

ii) The back faces of a cube are completely blocked by the cube itself & hidden from view. Therefore, we can identify & remove these back-faces.

iii) We know that equation of plane is given by,

$$Ax + By + Cz + D = 0 \quad \text{--- (1)}$$

In object space method, the identification of back-faces is based on above equation. From the above equation, we can say, if a point (x, y, z) satisfies the equation then the point (x, y, z) is lying on the plane. But,

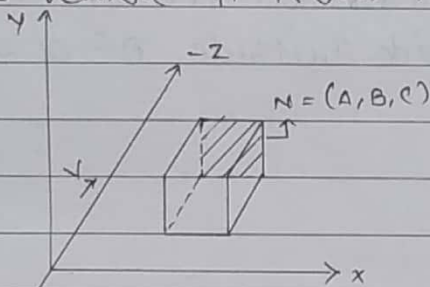
$$\text{if } Ax + By + Cz + D < 0 \quad \text{--- (2)}$$

It means (x, y, z) lies on negative side.

$$\text{And if } Ax + By + Cz + D > 0 \quad \text{--- (3)}$$

It means (x, y, z) lies on positive side.

iv) If we consider any point (x, y, z) as viewing point, then any plane which satisfies equation (2) must be a back face. After finalising the back face we have to remove it from the further visibility.



v) Let $N = (A, B, C)$ be the normal vector, in the right handed system with viewing direction along the negative z axis, the polygon is a back face if $C < 0$.

vi) Also, we cannot see any face whose normal has z component $C = 0$. Thus we can say any polygon is classified as a back face when its normal vector is negative or equal to zero i.e. $C \leq 0$.

Q.1.

c) Z-Buffer depth buffer method.

Ans. i) Another way to handle hidden lines & surface is z buffers. It is also called as depth buffer algorithm. Here we are sorting the polygons according to their position in space. And then in frame buffer itself. We are sorting polygons which are closer to viewer. We know that frame buffer is used to store images which we want to display on monitor. Here for visibility test we are making use of z buffer along with frame buffer.

ii) The z buffer is a large array to hold all the pixels of display. z buffer is somewhat similar to frame buffer. In frame buffer we are having arrays to store x and y coordinates of an image. Similarly z buffer contains z-coordinates of pixels which we want to display.

iii) When there is nothing to display on monitor i.e. frame buffer is empty, at that time we have to initialize z buffer elements to a very large negative values. A large negative values on z axis represents a point beyond which there is nothing i.e. setting background color. $z_{\text{buffer}}(x, y) = z_{\text{initial value}}$.

iv) If the new surface has z value greater than z_{buffer} then it lies in front. So we have to modify the contents of $z_{\text{buffer}}(x, y)$ by new z values & set the pixel value at (x, y) to the color of the polygon at (x, y) .

v) If the new value of new surface is smaller than it lies in behind some polygon which was previously entered.

Q.1.

d) Area subdivision method.

Ans. i) The area-subdivision method takes advantage by locating those view areas that represent part of a single surface.

ii) Divide the total viewing area into smaller & smaller rectangle until each small area is the projection of part of a single visible surface or no surface at all.

iii) Continue this process until the subdivisions are easily analysed as belonging to a single surface or until they are reduced to the size of a single pixel.

iv) The algorithm is a recursive procedure based on a 2-step strategy:

1) Decide which polygons overlap the given area on the screen.

2) Which polygons are visible in that area.

v) Categories of polygons are:

1) Surrounding polygon.

2) Intersection polygon.

3) Contained polygon.

4) Disjoint polygon.

Q.2. Derive all the necessary matrices required to perform 3D rotation about arbitrary axis.

Ans. Our Translation matrix is,

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_1 & -y_1 & -z_1 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x_1 & y_1 & z_1 & 1 \end{bmatrix}$$

The matrix for rotation about x-axis, in anti-clockwise direction is,

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By replacing θ with angle M & then with values of $\cos M$ and $\sin M$,

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos M & \sin M & 0 \\ 0 & -\sin M & \cos M & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & R/A & Q/A & 0 \\ 0 & -Q/A & R/A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_x^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & R/A & -Q/A & 0 \\ 0 & Q/A & R/A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix for rotation about y-axis in clockwise direction

$$R_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y^{-1} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q.2. NOW we have to perform rotation about y-axis in clockwise direction by angle N .

$$\therefore \sin N = \frac{\text{Opposite side}}{\text{hypotenuse}} = \frac{P}{B}$$

$$\& \cos N = \frac{\text{Adjacent side}}{\text{hypotenuse}} = \frac{A}{B}$$

Replacing θ by angle N & then replacing $\cos N$ & $\sin N$,

$$R_y = \begin{bmatrix} \cos N & 0 & \sin N & 0 \\ 0 & 1 & 0 & 0 \\ -\sin N & 0 & \cos N & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} A/B & 0 & P/B & 0 \\ 0 & 1 & 0 & 0 \\ -P/B & 0 & A/B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y^{-1} = \begin{bmatrix} A/B & 0 & -P/B & 0 \\ 0 & 1 & 0 & 0 \\ P/B & 0 & A/B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here at this stage we have matched an arbitrary axis to z-axis. Now we can use our normal rotation about z-axis.

$$\therefore R_z = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, the final transformation matrix will be,

$$\therefore R_A = T \cdot R_x \cdot R_y \cdot R_z \cdot R_y^{-1} \cdot R_x^{-1} \cdot T^{-1}$$

Q. 3. Explain Warnock's algorithm used to remove hidden surface with example.

Ans. i) It is also called as subdivision algorithm.

ii) In painter's algorithm, we have seen procedure to remove hidden surfaces. It means painter's algorithm deals with the procedure.

iii) But Warnock's algorithm does not deal with the procedure to remove hidden surfaces. It just tries to display the final picture on screen.

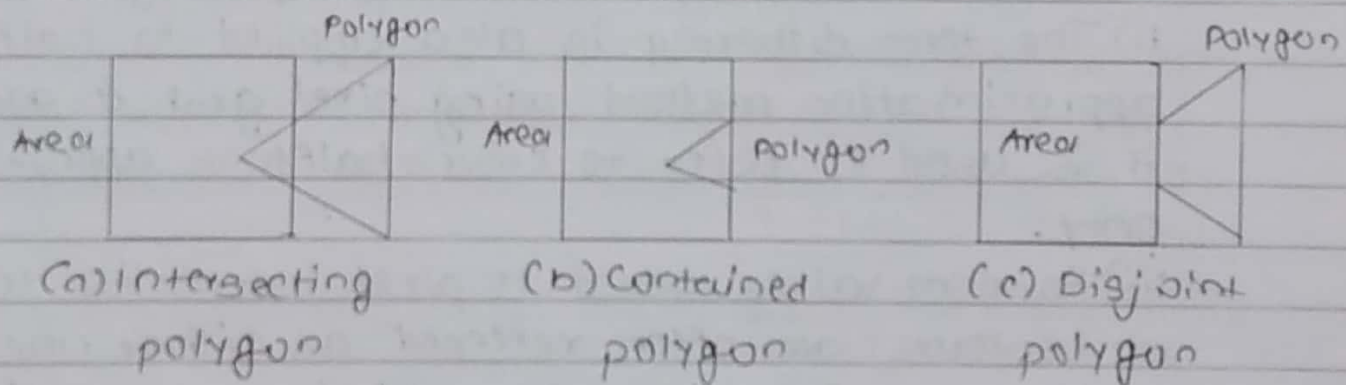
iv) The basic idea is as the resolution of the display increases the size of the picture also increases & hence its correctness also increases.

The algorithm is a recursive procedure based on a 2-step strategy:

1) Decide which polygon overlap the given area on the screen.

2) Which polygon are visible in that area.

v) So, the Warnock's algorithm checks whether the polygon is partially or fully visible in the given area.



Q.4. Explain Halftone & Dithering technique.

Ans. Half toning:

- i) Many displays & hardcopy devices are bi-level.
- ii) They can only produce two intensity levels.
- iii) In such displays or hardcopy devices we can create an apparent increase in the number of available intensity value.
- iv) When we view a very small area from a sufficient large viewing distance, our eyes average fine details within the small area & record only the overall intensity of the area.
- v) The phenomenon of apparent increase in the number of available intensities by considering combine intensity of multiple pixels is known as half toning.
- vi) Commonly used in black & white photographs in newspapers, magazine & books.

Dithering Techniques:

- i) It refers to techniques for approximating halftone without reducing the resolution, as pixel grid patterns.
- ii) The term dithering is also applied to halftone approximation method using pixel grid, & something it is used to refer to color halftone approximations only.
- iii) Random values added to pixel intensities to break up contours are often referred as dither noise.
- iv) Number of methods is used to generate intensity variations.
- v) Different methods generate intensity variations with a one-to-one mapping of points in a scene to the display pixel.

Q.5. Explain Gouraud and Phong shading.

Ans. Gouraud Shading:

1) It was developed in the 1970s by Henri Gouraud.

2) It is the interpolation technique.

3) Intensity levels are calculated at each vertex & interpolated across the surface.

4) Intensity values for each polygon are matched with the values of adjacent polygons along the common edges.

5) This eliminates the intensity discontinuities that can occur in flat shading.

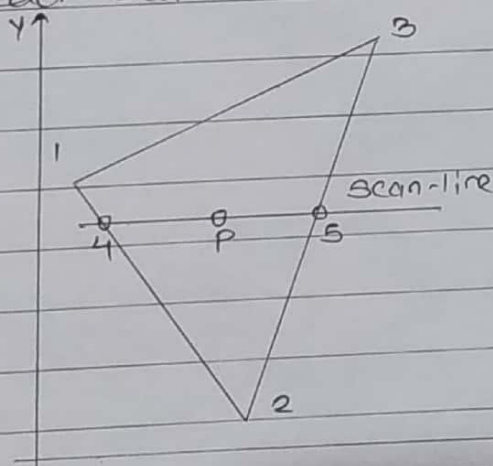
6) To render a polygon, Gouraud surface rendering proceeds as follows:

a) Determine the average unit normal vector at each vertex of the polygon.

b) Apply an illumination model at each polygon vertex to obtain the light intensity at that position.

c) Linearly interpolate the vertex intensities over the projected area of the polygon.

Illumination values are linearly interpolated across each scan-line as shown below:



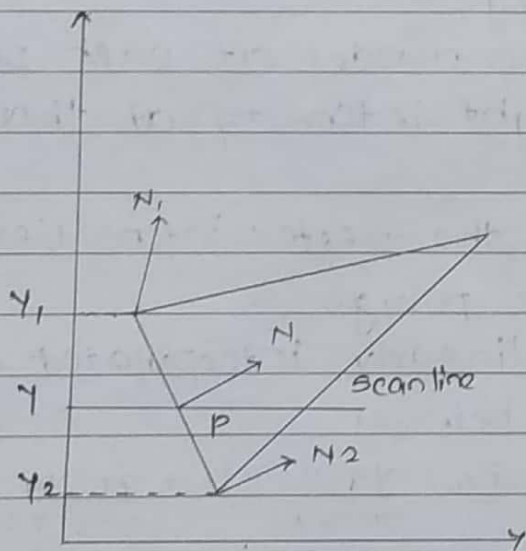
$$I_4 = \frac{y_4 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_4}{y_1 - y_2} I_2$$

$$I_5 = \frac{y_5 - y_2}{y_3 - y_2} I_3 + \frac{y_3 - y_5}{y_3 - y_2} I_2$$

$$I_P = \frac{x_5 - x_P}{x_5 - x_4} I_4 + \frac{x_P - x_4}{x_5 - x_4} I_5$$

Q.5. Phong Shading:

- 1) A more accurate interpolation based approach for rendering a polygon was developed by Phong Bui Tuong.
- 2) Basically the Phong surface rendering model is called as normal-vector interpolation rendering.
- 3) It interpolates normal vectors instead of intensity values.
- 4) To render a polygon, Phong surface rendering follows:
 - a) Determine the average unit normal vector at each vertex of the polygon.
 - b) Linearly interpolate the vertex normal over the projected area of the polygon.
 - c) Apply an illumination model at positions along scan lines to calculate pixel intensities using the interpolated normal vectors.



$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$