3.2.2 Fragile Embedding

The following steps show the embedding of the watermark ($W_{\text{fragile+recov}}$) into the image ($\text{Watermarked}_{\text{r}}$).

- **Step-1** Divide the image ($\text{Watermarked}_{\text{r}}$) into 2×4 size of non-overlapping blocks. Then, sequentially select 12 bits of watermark data from $W_{\text{fragile+recov}}$ for each block.
- **Step-2** Choose the first block and consider that each column is representing a pixel unit (with 2 pixels). Thus, each 2×4 size block has four units (e.g., U1, U2, U3, and U4).
- **Step-3** Convert 12-bit watermark into 9-base number Wat_9 in such a way that it has four digits (e.g., $Wat_9 = d1d2d3d4$).
- **Step-4** Modify pixels of U1 using $d1$ as per the given steps.
  - Compute digit F_{embed} as shown in Eq. (5). Here, P_k is the k^{th} pixel of unit U.

$$F_{\text{embed}} = \left(\sum_{k=1}^{n} 3^{k-1} P_k \right) \mod 3^n \quad \text{where} \quad n = 2 \tag{5}$$

  - Calculate x as given in Eq. (6).

$$x = \left(d - F_{\text{embed}} + \left\lfloor \frac{3^n - 1}{2} \right\rfloor \right) \mod 3^n \tag{6}$$

  - Change x into x' by converting into 3-base number as $x' = y_1 y_2 \dots y_n$, where y_i denotes the i^{th} digit of x' for $1 \leq i \leq n$. Next, get $x'' = z_1 z_2 \dots z_n$, where $z_i = y_i - 1$.
  - Add digits of x'' to the pixels of unit U to get the updated pixels as shown in Eq. (7).

$$P_{\text{new}_k} = P_k + z_i \quad \text{where} \quad \begin{cases} 1 \leq k \leq n \\ i = n - k + 1 \end{cases} \tag{7}$$

- Repeat the steps with U2, U3, and U4 to embed $d2$, $d3$, and $d4$, respectively.
- **Step-5** Repeat step 2, 3, and 4 for each block to get the dual watermarked image W_{img}.