Mathematical Modeling in Systems Biology problem sets: 7.8.26 Noisy toggle switch

7.8.26 *Noisy toggle switch. Stochastic systems can exhibit a range of bistable-like behaviours, ranging from ‘true’ bistability to frequent noise-induced transitions between two nominally stable states. To explore this behaviour, consider a stochastic system that recapitulates the bistable toggle switch discussed in Section 7.2.3:

R_1 : (synthesis)	$\dashrightarrow P_1$	propensity: $\frac{\alpha}{1+N_2^{\beta}}$
R_2 : (synthesis)	$\dashrightarrow P_2$	propensity: $\frac{\alpha}{1+N_1^{\beta}}$
R_3 : (decay)	$P_1 \dashrightarrow$	propensity: δN_1
R_4 : (decay)	$P_2 \dashrightarrow$	propensity: δN_2

Here N_1 and N_2 are the molecular counts for the two repressors. The Hill-type propensities for the synthesis reactions are not well-justified at the molecular level, but these expressions nevertheless provide a simple formulation of a bistable stochastic system. Take parameter delta = 1 and beta_ = 4. The corresponding deterministic system (i.e. dp_i/dt = alpha_/(1 + p_{4j}) − p_i) is bistable for any alpha_ > 1. Run simulations of the stochastic system for alpha = 5, 50, 500, and 5000. Be sure to run the simulations sufficiently long so that the steady trend is clear (i.e. at least 10,000 reaction steps). Verify that for alpha = 5000 the system exhibits bistability (with about 5000 molecules of the dominant species, in the long term). In contrast, verify that with alpha = 5, noise dominates and the system shows no signs of bistability. What about at alpha = 50 and 500? Comment on how the steadystate molecule abundance affects system behaviour. (Note: it may be necessary to run multiple simulations to confirm your findings.)

Agent-based modeling

Simulates viral spread in a population using a SIR model.

Please simulate a combination of three beta_max parameters = 0.05, 0.03, 0.01 (representing personal hygiene: e.g., Washing hands, wearing a mask) and three isolated parameters = 0.0, 0.5, 0.9. (Representing social distancing and/or lockdown measures) Therefore, there would be nine simulations. Plot the number of infected individuals over 5000 steps with three beta_max parameters, each isolated parameter on a different plot. (That is, plot #1 contains three curves: isolated = 0.0 for beta_max = 0.05, 0.03, 0.01, and plot #2 contains three curves: isolated = 0.5 for beta_max = 0.05, 0.03, 0.01, and plot #3 contains three curves: isolated = 0.9 for beta_max = 0.05, 0.03, 0.01)

List the maxima (peaks) of infected individuals in the nine simulations. Which parameter set is the most effective in "flattening the curve" (having the lowest peak infected individuals)?

Compared to decreasing the transmission rate (the beta_max parameter), is increasing the "isolated" parameter more effective in flattening the curve?