COMP3121 Homework Q5

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1 Answer a

Here $f(n) = (log(n))^2$ and $g(n) = log((n^{log(n)})^2)$. Using $log(a)^b = blog(a)$,

$$g(n) = 2 * log(n) * log(n)$$
$$g(n) = 2 * (log(n))^2$$

Hence here we see $g(n) = c * f(n)$ Therefore we can say that $f(n) \leq c * g(n)$.
Hence $f(n) = O(g(n))$
We also see that $g(n) \leq c * f(n)$ for any $c > 2$ and so we can also say that $g(n) = O(f(n))$ Since both properties are holding

$$f(n) = \Theta(g(n))$$

2 Answer b

Here $f(n) = n^{10}$ and $g(n) = 2^{\sqrt[10]{n}}$.
We need to show that $f(n) \leq c * g(n)$ to prove that $f(n) = O(g(n))$
If we can prove that $\lim_{n\to+\infty} f(n)/g(n) = 0$ than we can say that $f(n) \leq c*g(n)$
Since $\lim_{n\to+\infty} f(n) = \infty$ and $\lim_{n\to+\infty} g(n) = \infty$
We can use L'hopital rule to derive $\lim_{n\to+\infty} f(n)/g(n)$

$$\lim_{n\to+\infty} f(n)/g(n) = \lim_{n\to+\infty} f'(n)/g'(n)$$

Here $f'(n) = 10n^9$ and $g'(n) = (1/10)n^{-9/10} * log(2) * 2^{\sqrt[10]{n}}$. As both these functions will also approach infinity when $n \to +\infty$, We can again differentiate them and keep doing it until it goes to infinity. After 10 derivations we observe that there is no n left in $f(n)$. Hence the limit will become zero and we conclude that $f(n) \leq c * g(n)$ for sufficiently large n. Theerfore,

$$f(n) = O(g(n))$$

3 Answer c

Here $f(n) = n^{1+(-1)^n}$ and $g(n) = n$. We observe that $f(n) = 1$ for all even values of n and n^2 for all odd values of n. So when n is even $f(n) < g(n)$ and when n is odd, $f(n) > g(n)$. Hence the answer is neither of $f(n) = O(g(n))$ or $g(n) = O(f(n))$.