COMP3121 Homework Q1

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1 Answer

In this problem let n be the number of symbols and $n-1$ be the number of operators between them. We have to place parenthesis around them to make them into small expressions with operators between them which eventually will evaluate to TRUE. Here we need to solve the sub problems which is evaluating the sub expressions which might evaluate to TRUE or FALSE. Let $T(i,j)$ be the sub problem which is solving the expression from symbol ith to symbol jth to evaluate it to TRUE (T), and the second sub problem be solving the expression from symbol ith to symbol jth to evaluate to FALSE (F).

Just like problem 5 from Tut 4, the base case will $T(i,i)$ is 1 if i is TRUE or 0 if i is FALSE and vice versa for $F(i,i)$.

For each sub problem, we split the expressions around the operator so that everything to right of that is in its own bracket and everything to the left is in its own bracket. Hence we form two sub smaller expressions. We then recursively solve each the sub expressions and combine the results together depending on the type of operator we are splitting by and the final result we want to evaluate to whether TRUE or FALSE. We solve the sub problems parellely:

$$T(i,j) = \sum_{m=i}^{j-1} TSplit(i,m,j)$$

$$F(i,j) = \sum_{m=i}^{j-1} FSplit(i,m,j)$$

$$TSplit(i,m,j) = \begin{cases} T(i,m) * T(m+1,j) & \text{If operator is 'AND'} \\ T(i,m) * F(m+1,j) + T(i,m) * T(m+1,j) + F(i,m) * T(m+1,j) & \text{If operator is 'OR'} \\ T(i,m) * F(m+1,j) + F(i,m) * F(m+1,j) + F(i,m) * T(m+1,j) & \text{If operator is 'NAND'} \\ F(i,m) * F(m+1,j) & \text{If operator is 'NOR'} \end{cases}$$

$$FSplit(i,m,j) = \begin{cases} T(i,m) * F(m+1,j) + F(i,m) * F(m+1,j) + F(i,m) * T(m+1,j) & \text{If operator is 'AND'} \\ F(i,m) * F(m+1,j) & \text{If operator is 'OR'} \\ T(i,m) * T(m+1,j) & \text{If operator is 'NAND'} \\ T(i,m) * F(m+1,j) + T(i,m) * T(m+1,j) + F(i,m) * T(m+1,j) & \text{If operator is 'NOR'} \end{cases}$$

The equations inside the TSplit and FSplit are corresponding with the truth tables of those operators. The complexity for this is $O(n^3)$ because of n^2 different ranges of i and j and each split of T and F covers n different splitting points. Reference to this question is taken from Tutorial 4 Problem 5 and https://www.youtube.com/watch?v=oyj9tRZhmis.