STAT743 FOUNDATIONS OF STATISTICS (Part II)

Assignment 3 Due in class at 12:30pm on March 22, 2018

Instructions:

1. Ensure that your full name and student number are clearly marked on your solutions.
2. Solutions need not be typed (although they can be) but must be readable.
3. Start each question on a new page and clearly indicate where each part of the question begins.
4. Submit questions in the same order as given below.
5. You are expected to show all details of your solution and any results taken from my notes or the textbook must be clearly and properly referenced.
6. You must include a commented listing of your R code for any question which requires the use of computing.
7. No extensions to the due date and time will be given except in extreme circumstances and late assignments will not be accepted. Assignments are due at the **start** of class.
8. Students are reminded that submitted assignments must be entirely their own work. Submission of someone else's solution under your name is academic misconduct and will be dealt with as such. Penalties for academic misconduct can include a 0 for the assignment, an F for the course with an annotation on your transcript and/or dismissal from your program of study.

Q. 1 Suppose that $Y_1, \ldots, Y_n$ is a random sample from the Uniform distribution on the interval $[-\theta, \theta]$ and define

$$T = \max\{-Y_{(1)}, Y_{(n)}\}$$

where $Y_{(1)} = \min\{Y_1, \ldots, Y_n\}$ and $Y_{(n)} = \max\{Y_1, \ldots, Y_n\}$.

a) Derive the probability density function of T and show that $W = T/\theta$ is a pivotal quantity.

b) Find an equitailed $100(1-\alpha)\%$ confidence interval for θ based on the pivot W.

c) Derive the likelihood ratio test for testing

$$H_0 : \theta \; = \; \theta_0 \quad \text{V} \quad H_1 : \theta \neq \theta_0$$

and invert this test to construct an alternative confidence interval for θ.

d) For a fixed confidence level (use 95%), verify numerically that the interval based on the likelihood ratio test derived in (c) is shorter than the equitailed interval found in (b) for all sample sizes n.

Q. 2 **a)** Casella and Berger 9.13

b) Casella and Berger 9.36

Q. 3 **a)** Casella and Berger 7.23

b) Casella and Berger 8.11

Q. 4 Suppose that $X_1, \ldots, X_n$ is a random sample from a Poisson distribution with mean λ.

a) Show that the conjugate prior family of distributions for λ is the gamma family of distributions.

b) A scientist believes that the number of times that a certain event occurs in a small interval of time follows a Poisson distribution. Furthermore she believes that the mean of this distribution is approximately 5 and she is 99% sure that the mean is less than 10. Find the conjugate prior distribution which best captures the scientists prior beliefs. *You may restrict your search for the hyper-parameter α to integers*

c) Based on the observed data below and the prior you found in part (c), determine the posterior mean and variance of λ, the posterior probability that $\lambda < 10$.

7	9	7	6	5	10	9	2	7	3
10	4	13	11	9	7	3	13	6	7

d) Find the Jeffrey's Prior for λ and show that it is an improper prior but that it results in a proper posterior distribution. Repeat the analysis of the data in part (c) when using the Jeffrey's prior.