HW2 R10945004 孫欽鈜

Problem 1. No collaborator

	Worse-case time-complexity	Is stable?	if seq. is sorted	if seq. is reversed
Bubble Sort	n^2	stable	$\Theta(n^2)$	$\Theta(n^2)$
Merge Sort	$n\log n$	stable	$\Theta(n\log n)$	$\Theta(n\log n)$
Insertion Sort	n^2	stable	$\Theta(n)$	$\Theta(n^2)$
Quick Sort	n^2	not stable	$\Theta(n\log n)$	$\Theta(n\log n)$
Heap Sort	$n\log n$	not stable	$\Theta(n\log n)$	$\Theta(n\log n)$

Problem 2. Collaborator : R10945020 黃靖婷

assume $a_1 = 0$, $a_2 = n$, ... a_n and n is a power of 2

Set a n-size table → table(| -1 | -1 | -1 | ..... | -1 |) index=0 1 2 n-1

for (a_i in a_n) : —— $\Theta(n)$
    if $a_i \le n$: —— $\Theta(1)$
        table[rounddown(a_i)] = a_i — $\Theta(1)$

Find the first element before -1 in table and —— $O(n)$
the first element after -1 in table.

Correctness : because the table size is n and there will be $n+1$ integer between $[0, n]$, and there will have max n points ($\frac{n+1}{n} > 1$). So it will find out at least one pair → $|a_i - a_j| > 1$

Run Time : $O(n)$

Problem 3. Collaborator : R10945020 黄靖婷

1.

$\because$ Sort every $2k+1$ elements $\Rightarrow$ Sort(Array[0, 2k]) $\rightarrow$ Sort(Array[k, k+2k]) $\rightarrow \cdots$

$\rightarrow$ Sort(Array[ik, ik+2k])

$\therefore$ $ik+2k = n \quad i = \frac{n-2k}{k} = \frac{n}{k} - 2$

for $i=0$ to $\frac{n}{k}-2$ —— total $\frac{n}{k}-1$ rounds

MergeSort(Array[ik, ik+2k]) —— $O((2k+1)\log(2k+1))$

Correctness : When we sort $2k+1$ elements, there will be k element is right, because every element is at most k spots away from its actual locations.

Time : $O\left(\left(\frac{n}{k}-1\right)(2k+1)\log(2k+1)\right)$

$\Rightarrow O(n\log k)$ #

2. We have all possible case is $((2k)!)^{\frac{n}{k}-1}$

According to the decision tree, time complexity

$$
\begin{aligned}
O(n) &\geq \log_2((2k)!)^{\frac{n}{k}-1} \\
&\geq \left(\frac{n}{k}-1\right)\log_2((2k)!) \\
&\geq \frac{n}{k}\log_2 k! \\
&= \frac{n}{k}\Big(\underbrace{\log_2 k}_{\geq \log_2 \frac{k}{2}} + \underbrace{\log_2(k-1)}_{\geq \log_2 \frac{k}{2}} + \cdots \underbrace{\log_2 \frac{k}{2}}_{\geq \log_2 \frac{k}{2}} + \underbrace{\cdots \log_2 1}_{\geq 0}\Big) \quad \geq \frac{k}{2}\log_2\frac{k}{2} \\
&\geq \frac{n}{2}\log_2\frac{k}{2} \\
&\in \Omega(n\log k)
\end{aligned}
$$

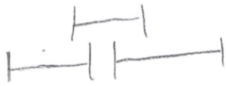
Problem 4.

Collaborator : R10942104 林[illegible]

Least conflict :

opt :

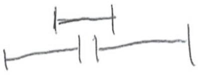
$\Rightarrow$ difference :

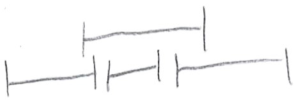


$\therefore$ the least conflict solution $= \frac{1}{2}$ opt. solution

proof : the least conflict has at least $\frac{K}{2}$ solution

by prove that the least conflict hasn't $\frac{K}{3}$ solution

① 

② 

$\therefore$ in case ①, greedy choose $\frac{K}{2}$ solutions

in case ②, greedy choose K solutions

$\therefore$ greedy can't have $\frac{K}{3}$ solution.

$\therefore$ the least confict will always select $\lceil \frac{K}{2} \rceil$ meetings.

Problem 5.　　　　Collaborator : R10942104 林冠廷

① $M_1, M_2, \cdots\cdots, M_n$

$s_1, s_2, \cdots\cdots, s_n$

$t_1, t_2, \cdots\cdots, t_n$

Want $\underset{S}{\text{Max}} \sum_S (t_i - s_i)$ subject to $\sum_S M_i \geq k$

Let $S[i, j]$ = max time (i times interval, $M_1, M_2, \cdots M_j$)

$T[i, j]$ = accepted meeting in $S[i, j]$

$S[0, j] = 0$, $S[i, 0] = t_i - s_i$, $S[0, 0] = 0$

for $i = 1$ to n — $\Theta(n)$

for $j = 1$ to n — $\Theta(n)$

$$S[i, j] = \max \begin{cases} S[i-1, j] & \text{not accept } M_j \\ S[i-1, j-\text{size}(C)] + (t_i - s_i) & \text{accept } M_j \end{cases}$$

Find Set C for $C = [M_j \cap \{M_{j-1} \sim M_1\}]$ — $\Theta(n)$

if $\text{Sum}(C) < M_j$ and $\text{size}(C) > j$

accept M_j

else

not accept M_j

$$T[i, j] = \begin{cases} 0 \\ 1 \end{cases}$$

<u>Correctness</u>

Accepted Groups

check $T[i, j]$ for meeting M_j

if $T[i, j] = 0$, reject M_j

check $T[i, j-1]$ for meeting M_{j-1}

if $T[i, j] = 1$, accept M_j

check $T[i-1, j-M_j]$ for meeting $M_j - 1$

<u>Run-time</u> $\Theta(n^3)$

② Can't both chose M_i and M_{i+2}

Let $d_{odd}[i]$: $\overset{i=1\sim\lceil\frac{n}{2}\rceil}{M_1, M_3, \ldots, M_{2i-1}}$ max total meeting time

$d_{even}[i]$: $\overset{i=1\sim\lfloor\frac{n}{2}\rfloor}{M_2, M_4, \ldots M_{2i}}$ 中 max total meeting time

$d'_{odd}[i]$: 不含 $M_{i\,odd}$, $d'_{even}[i]$ 不含 $M_{i\,even}$

$d_{odd}[1] = t_1 - s_1$, $d'_{odd}[1] = 0$

$d_{even}[1] = t_2 - s_2$, $d'_{even}[1] = 0$

For $i=1$ to len(d_{odd}): — $\frac{n}{2}$ rounds → $\Theta(n^2)$

if $i = 1$

$d_{odd}[i] = t_1 - s_1$, $d'_{odd}[i] = 0$

else if M_i doesn't conflict — $\Theta(\frac{n}{2})$

$d'_{odd}[i] = d_{odd}[i-1]$

$$d_{odd}[i] = \max\begin{cases} d'_{odd}[i] \\ (t_i - s_i) + d'_{odd}[i] \end{cases}$$

else if M_i conflict — $\Theta(\frac{n}{2})$

$d'_{odd}[i] = d_{odd}[i-1]$

$$d_{odd}[i] = \max\begin{cases} d'_{odd}[i] \\ d'_{odd}[i] - \underbrace{\sum(t_i^* - s_i^*)}_{\text{conflict time}} + (t_i - s_i) \end{cases}$$

Do same for $d_{even}[i]$,

then Set $R = \{[M_1, M_2], [M_2, M_3], [M_3, M_4], \ldots [M_{n-1}, M_n]\}$ — $\Theta(n)$

for $i=1$ to size(R)

12 23 34 45 56

if $R[i]$ conflict

$$ans[i] = \max\begin{cases} d_{odd}[\text{select odd in } R[i]] \\ d_{even}[\text{select even in } R[i]] \end{cases}$$

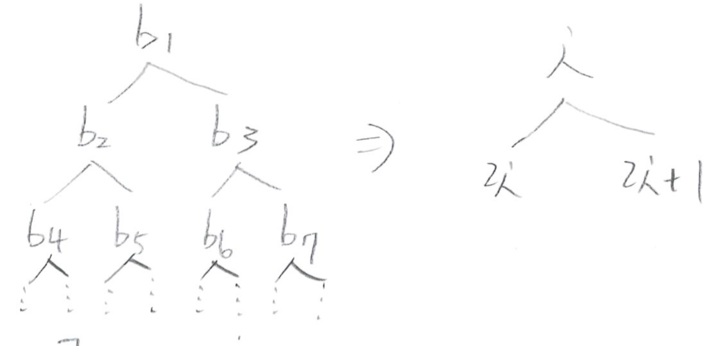
else $ans[i] = d_{odd}[\text{select odd in } R[i]] + d_{even}[\text{select even in } R[i]]$

∴ take $ans[n]$ is the solution.

Run-time: $\Theta(n^2)$

Problem 6. No collaborator

Want to Max $\sum p_i$ $\Rightarrow$



Let $T'[i] = T[2i] \cup T[2i+1]$

$T[i]$ = Node i

$A'[i]$ = max score from $T'[i]$

$A[i]$ = max score from $T[i]$

For $i = m$ to 1 —— m rounds

if i is a leaf —— $\Theta(1)$

$A[i] = p_i$, $A'[i] = 0$ —— $\Theta(1)$

else

$A'[i] = A[2i] + A[2i+1]$ — $\Theta(1)$

$A[i] = \max \begin{cases} A'[i] \\ p_i + A'[2i] + A'[2i+1] \end{cases}$ — $\Theta(1)$

run-time : $\Theta(m)$