R10945004
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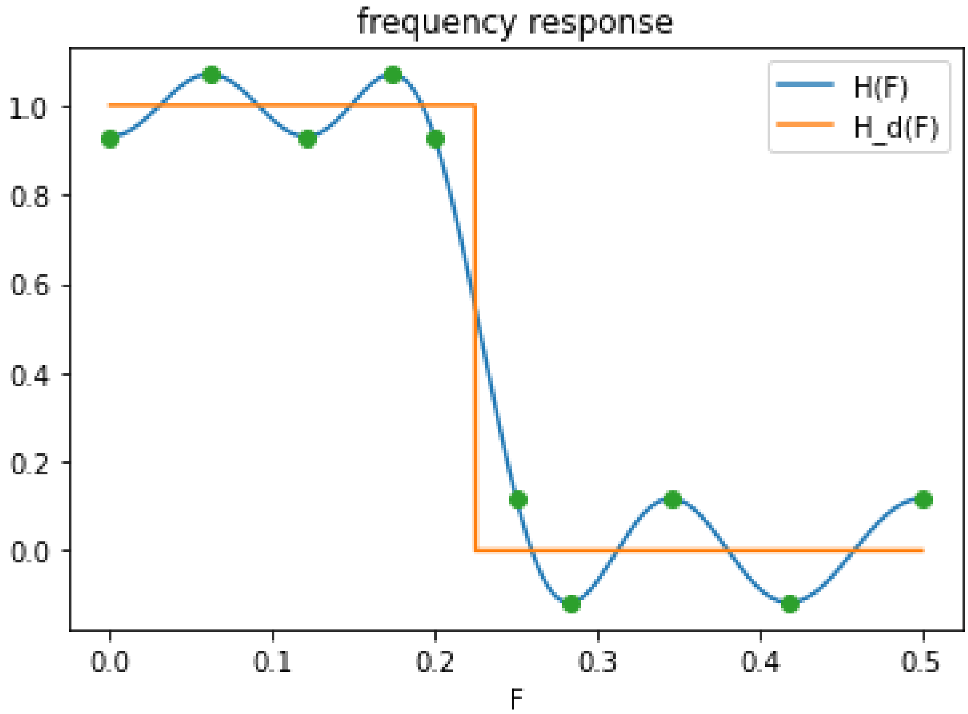
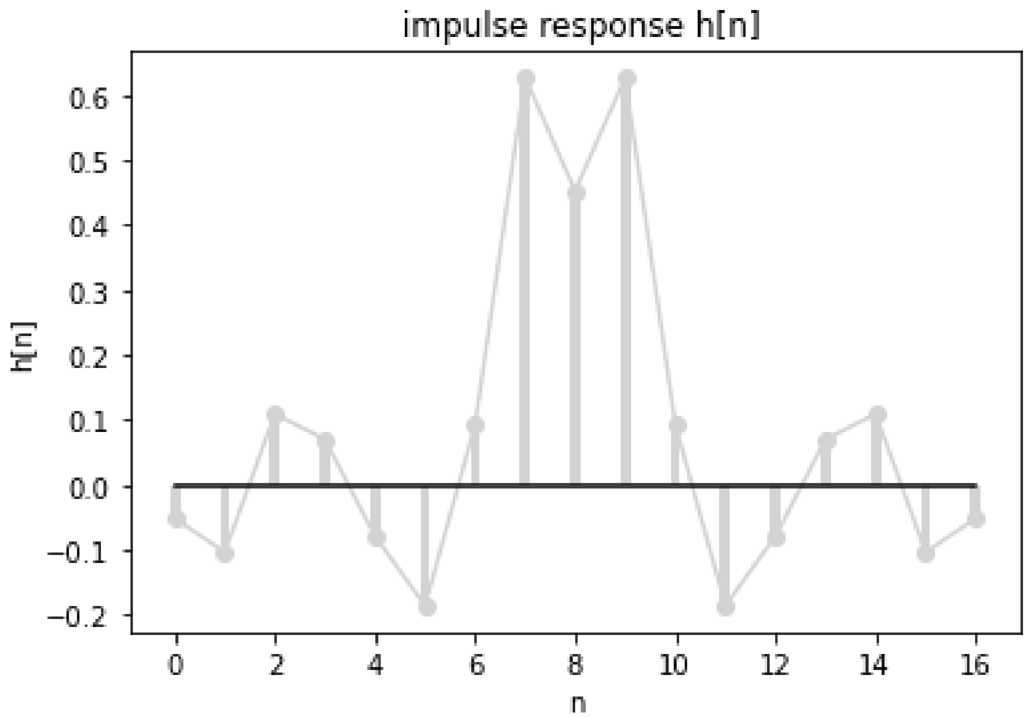
(1) Design a Mini-max lowpass FIR filter such that

(a) the frequency response



(b) the impulse response h[n]



(c) the maximal error for each iteration

iteration	error
1_th	0.3827
2_th	0.2611
3_th	0.3348
4_th	0.4796
5_th	0.0840
6_th	0.0700
7_th	0.0700

2\. (a) ex: 在時域中 convolution，相當於在頻域中相乘，而相乘又可以取log 拆成兩兩相加。
，再將最終結果反傅立葉轉換即可得到結果。

like: $y(t) = x(t) * h(t)$

$$\downarrow \mathcal{F}$$

$$Y(w) = X(w) \times H(w)$$

$$\log(Y(w)) = \log(X(w)) + \log(H(w))$$

get $Y(w) \xrightarrow{\mathcal{F}^{-1}} y(t)$

(b) discrete fourier transform (DFT) 需要比較大量的運算資源。

3\. $y[n] = x[n] * (0.8^n u[n] - 0.6^n u[n])$

$$\overset{z}{\Rightarrow} Y(z) = X(z)\left[\frac{1}{1-0.8z^{-1}} - \frac{1}{1-0.6z^{-1}}\right]$$

$$= X(z)\left(\frac{0.2z^{-1}}{(1-0.8z^{-1})(1-0.6z^{-1})}\right)$$

$$= X(z)\left(\frac{0.2z^{-1}}{1-1.4z^{-1}+0.48z^{-2}}\right)$$

$$Y(z)\left[1-1.4z^{-1}+0.48z^{-2}\right] = X(z)(0.2z^{-1})$$

$$\overset{z^{-1}}{\Rightarrow} y[n] - 1.4y[n-1] + 0.48y[n-2] = 0.2x[n-1]$$

$$y[n] = 0.2x[n-1] + 1.4y[n-1] - 0.48y[n-2]$$

4. (a) Step invariance：在計算上為將原始訊號$h_a(t)$與 step function 作 convolution，$h_{a,u}(t) = h_a(t) * u(t) = \int_{-\infty}^{\infty} h_a(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{t} h_a(\tau)\,d\tau$

$H_{a,u}(f) = \frac{H_a(f)}{j2\pi f}$，當f大時，值變小，f小→值變大，因此在高頻處產生aliasing的機會就降低。

(b) 藉由$\tan^{-1}$ function 將原先$(-\infty, \infty)$的訊號 mapping成$(-\frac{f_s}{2}, \frac{f_s}{2})$，$f_{new} = \frac{f_s}{\pi}\tan^{-1}(\frac{2\pi}{c} f_{old})$ 以此防止產生aliasing。

5. $x[n] = y(0.002n)$，$N = 2000$

(a) $\Delta t = 0.002$，$f_s = \frac{1}{\Delta t} = 500$

$\because X[m] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi mn}{N}}$，$f = \frac{m}{N} f_s$

$\therefore X[300] \Rightarrow f = \frac{300}{2000} \times 500 = 75$

(b) $X[1800] \Rightarrow f = \frac{1800}{2000} \times 500 = 450$

$\because 1800 > \frac{N}{2}(1000)$

$\therefore f = 450 - f_s = -50$ #

6.

(C)，因為 C 的 transition band 0.1 比 (a) 和 (d) 的 transition band 0.4 寬，而 weightfunction pass : stop = 2:1，比 (b) 的 pass : stop = 1:1 還好 所以 C 的 passband error 最低。

7. $H_d(f)=1,\ |F|<0.3,\ H_d(F)=0$ at $0.3<|F|<0.5$, $n=5$

$$s[0]=\int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\,dF \qquad s[n]=2\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos(2\pi nF)H_d(F)\,dF$$

$$=\int_{-\frac{3}{10}}^{\frac{3}{10}} 1\,dF \qquad =2\int_{-\frac{3}{10}}^{\frac{3}{10}}\cos(2\pi nF)\,dF$$

$$=\left[F\Big|_{-\frac{3}{10}}^{\frac{3}{10}}\right] \qquad =2\times\frac{1}{2\pi n}\times\left[\sin(2\pi nF)\Big|_{-\frac{3}{10}}^{\frac{3}{10}}\right]$$

$$=0.6 \qquad =\frac{1}{\pi n}\times\left[\sin(0.6\pi n)+\sin(-0.6\pi n)\right]$$

$$=\frac{2\sin(0.6\pi n)}{\pi n}$$

$$K=\left\lfloor\frac{5}{2}\right\rfloor=2$$

$$h[K]=h[2]=0.6$$

$$h[K+n]=\frac{s[n]}{2}\ ;\ h[K-n]=\frac{s[n]}{2}$$

$$h[2+n]=\frac{s[n]}{2}\ ;\ h[2-n]=\frac{s[n]}{2}$$

$$\therefore h[4]=h[0]=\frac{s[2]}{2}=\frac{\sin(1.2\pi)}{\pi}=-0.187$$

$$h[3]=h[1]=\frac{s[1]}{2}=\frac{2\sin(0.6\pi)}{\pi}=0.605$$

$h[2]=0.6\quad h[5]=0$ #