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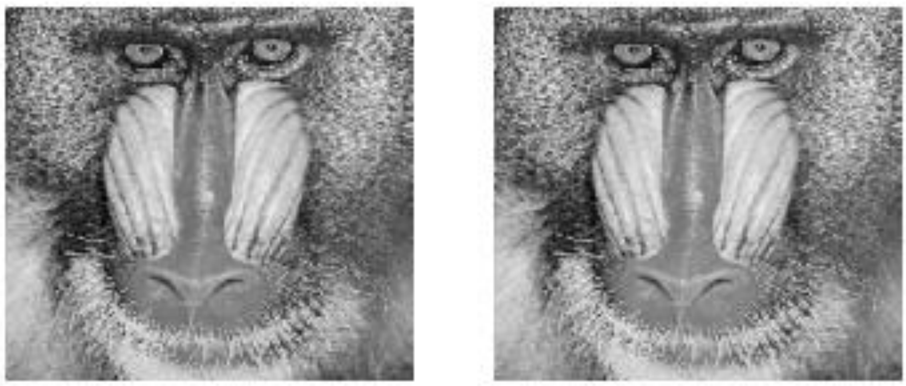
1. Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

　　使用老師上課講義舉例的這三張圖片，透過截圖的方式取得，由於截圖無法精確擷取到三張圖片的 size 一致，故程式內也有使用 resize 的 function 來讓三張圖片的 size 一致，可以看出結果趨勢是跟講義上是一致的。

SSIM : 0.1103
c1=0.0626 c2=0.0626

SSIM : 0.7920
c1=0.0626 c2=0.0626



R10945007 [illegible]

2.

$$X[m] = \sum_{n=0}^{4} \cos\left(\frac{\pi}{5} m\left(n+\frac{1}{2}\right)\right) x[n] \qquad \begin{matrix} n = 0, 1, 2, 3, 4 \\ m = 0, 1, 2, 3, 4 \end{matrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \cos(\frac{\pi}{5}\times\frac{1}{2}) & \cos(\frac{\pi}{5}\times\frac{3}{2}) & \cos(\frac{\pi}{5}\times\frac{5}{2}) & \cos(\frac{\pi}{5}\times\frac{7}{2}) & \cos(\frac{\pi}{5}\times\frac{9}{2}) \\ \cos(\frac{2\pi}{5}\times\frac{1}{2}) & \cos(\frac{2\pi}{5}\times\frac{3}{2}) & \cos(\frac{2\pi}{5}\times\frac{5}{2}) & \cos(\frac{2\pi}{5}\times\frac{7}{2}) & \cos(\frac{2\pi}{5}\times\frac{9}{2}) \\ \cos(\frac{3\pi}{5}\times\frac{1}{2}) & \cos(\frac{3\pi}{5}\times\frac{3}{2}) & \cos(\frac{3\pi}{5}\times\frac{5}{2}) & \cos(\frac{3\pi}{5}\times\frac{7}{2}) & \cos(\frac{3\pi}{5}\times\frac{9}{2}) \\ \cos(\frac{4\pi}{5}\times\frac{1}{2}) & \cos(\frac{4\pi}{5}\times\frac{3}{2}) & \cos(\frac{4\pi}{5}\times\frac{5}{2}) & \cos(\frac{4\pi}{5}\times\frac{7}{2}) & \cos(\frac{4\pi}{5}\times\frac{9}{2}) \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0.9511 & 0.5878 & 0 & -0.5878 & -0.9511 \\ 0.8090 & -0.3090 & -1 & -0.3090 & 0.8090 \\ 0.5878 & -0.9511 & 0 & 0.9511 & -0.5878 \\ 0.3090 & -0.8090 & 1 & -0.8090 & 0.3090 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix}$$

$$X_0 = 1 \times (x_0 + x_1 + x_2 + x_3 + x_4)$$

$$\begin{bmatrix} X_1 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0.9511 & 0.5878 \\ 0.5878 & -0.9511 \end{bmatrix} \begin{bmatrix} x_0 - x_4 \\ x_1 - x_3 \end{bmatrix} = \begin{bmatrix} 0.5878 & 0.5878 \\ 0.5878 & -0.5878 \end{bmatrix} \begin{bmatrix} x_0 - x_4 \\ x_1 - x_3 \end{bmatrix} + \begin{bmatrix} 0.3633 & 0 \\ 0 & -1.5389 \end{bmatrix} \begin{bmatrix} x_0 - x_4 \\ x_1 - x_3 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0.8090 & -0.3090 \\ 0.3090 & -0.8090 \end{bmatrix} \begin{bmatrix} x_0 + x_4 \\ x_1 + x_3 \end{bmatrix} + \begin{bmatrix} -x_2 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8090 & -0.8090 + \frac{1}{2} \\ 0.8090 - \frac{1}{2} & -0.8090 \end{bmatrix} \begin{bmatrix} x_0 + x_4 \\ x_1 + x_3 \end{bmatrix} + \begin{bmatrix} -x_2 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8090 & -0.8090 \\ 0.8090 & -0.8090 \end{bmatrix} \begin{bmatrix} x_0 + x_4 \\ x_1 + x_3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} x_0 + x_4 \\ x_1 + x_3 \end{bmatrix} + \begin{bmatrix} -x_2 \\ x_4 \end{bmatrix}$$

(a)

x_5 $0_{mul} - X_0 = x_0 + x_1 + x_2 + x_3 + x_4$

$3_{mul} - Z_1 = 0.5878 \times (x_0 - x_4 + x_1 - x_3)$, $Z_2 = 0.3633 \times (x_0 - x_4)$, $Z_3 = -1.5389 \times (x_1 - x_3))$

$0_{mul} - X_1 = Z_1 + Z_2$

$0_{mul} - X_2 = Z_1 + Z_3$

$1_{mul} - Z_4 = 0.8090 \times (x_0 + x_4 - x_1 - x_3)$

$0_{mul} - Z_5 = \frac{1}{2}(x_1 + x_3) - x_2 \Rightarrow \because$ trivial multiplications

$0_{mul} - Z_6 = -\frac{1}{2}(x_0 + x_4) + x_4$

$0_{mul} - X_3 = Z_4 + Z_5$

$0_{mul} - X_4 = Z_4 + Z_6$

$\therefore$ Total nontrivial multiplications is 4 !! #

3.

$e^{j\theta} = \cos\theta + j\sin\theta$

設 $x = a + jb$

$\therefore\ x \times e^{j\theta} = (a+jb)(\cos\theta + j\sin\theta)$

$$= \underbrace{(a\cos\theta - b\sin\theta)}_{c} + j\,\underbrace{(b\cos\theta + a\sin\theta)}_{d}$$

$$\Rightarrow \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

① if $\sin\theta = \pm\cos\theta$, $\theta = \frac{n\pi}{4}\ \forall n \in 1, 3, 5, \cdots\cdots$

s.t. $\sin\theta = \cos\theta$

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos\theta \\ \cos\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -2\cos\theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

s.t. $\sin\theta = -\cos\theta$

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos\theta \\ \cos\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -2\cos\theta & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$\therefore$ when $\theta = \frac{n\pi}{4}$, $\forall n \in 1, 3, 5, \cdots\cdots$, only need 2 muls

② if $\cos\theta = \pm 2^{-k}$, $\theta = \cos^{-1}(2^{-k})$, $\forall k \in \mathbb{N}$

$c = a \times (\pm 2^{-k}) - b \times \sin\theta$

$d = a \times \sin\theta + b \times (\pm 2^{-k})$

$\therefore$ when $\theta = \cos^{-1}(2^{-k})$, $\forall k \in \mathbb{N}$, need 2 muls

③ if $\sin\theta = \pm 2^{-k}$, $\begin{cases} c = a \times \cos\theta - b \times (\pm 2^{-k}) \\ d = a \times (\pm 2^{-k}) + b \times \cos\theta \end{cases}$

$\therefore$ when $\theta = \sin^{-1}(2^{-k})$, $\forall k \in \mathbb{N}$, need 2 muls

4. (a) 220-point DFT

$220 = 2^2 \times 5 \times 11$

$$
\begin{aligned}
MUL_{220} &= 55\,MUL_4 + 4\,MUL_{55} \\
&= 55\,MUL_4 + 4(5\,MUL_{11} + 11\,MUL_5) \\
&= 55 \times 0 + 4(5 \times 40 + 11 \times 10) \\
&= 1240
\end{aligned}
$$

(b) 231-point DFT

$231 = 3 \times 7 \times 11$

$$
\begin{aligned}
MUL_{231} &= 3\,MUL_{77} + 77\,MUL_3 \\
&= 3 \times (7\,MUL_{11} + 11\,MUL_7) + 77 \times 2 \\
&= 3 \times (7 \times 40 + 11 \times 16) + 154 \\
&= 1522
\end{aligned}
$$

(c) 245-point DFT

$245 = 5 \times 7^2$

$$
\begin{aligned}
MUL_{245} &= 49\,MUL_5 + 5\,MUL_{49} \\
&= 49 \times 10 + 5 \times (7\,MUL_7 + 7\,MUL_7 + 3 \times 6 \times 6) \\
&= 490 + 5 \times (7 \times 16 + 7 \times 16 + 36 \times 3) \\
&= 2150
\end{aligned}
$$

5.

① 所需要的計算時間為線性，$O(n)$

② 因為 $P = L+M-1$，所以使用的硬體架構可以固定（不隨著 input n 而改變）。

6.

$$x_s[n] = x[n+3]h[-3] + x[n+2]h[-2] + x[n+1]h[-1] + x[n]h[0]$$
$$+ x[n-3]h[3] + x[n-2]h[2] + x[n-1]h[1]$$

$$= 0.03\left(x[n+3] + x[n-3]\right) + 0.06\left(x[n+2] + x[n-2]\right)$$
$$+ 0.24\left(x[n+1] + x[n-1]\right) + 0.34\,x[n]$$

$$= 0.03\times\left(x[n+3] + x[n-3] + 2x[n+2] + 2x[n-2] + 2^3x[n+1] + 2^3x[n-1]\right)$$
$$+ (1 - 0.66)\,x[n]$$

$$= x[n] + 0.03\times\left(x[n+3] + x[n-3] + 2x[n+2] + 2x[n-2] + 2^3x[n+1] + 2^3x[n-1] - 2^4x[n] - 2^2x[n] - 2x[n]\right)$$

∴ Need 1 non-trivial real multiplication.

7. $len(x[n]) = 1100$

(a) $len(y[n]) = 200$

① Directly computing : $3NM = 3 \times 1100 \times 200 = 660000$

② Section Conv. : take $L_0 = 550$, $P_0 = 550 + 200 - 1 = 749$

Set $P = 720$, $L = 720 - 200 + 1 = 521$

$$S = \frac{1100}{521} = 3$$

$$2S\,MUL_P + 3SP = 2 \times 3 \times 3620 + 3 \times 3 \times 720$$
$$= 28200$$

③ DFT : $P \geq 1100 + 200 - 1 = 1299$

$$2MUL_{1344} + 3 \times 1344 = 2 \times 8252 + 4032 = 20536$$

∴ Using FFT and then IFFT is best way.

Total number of real multiplications = 20536.

(b) $len(y[n]) = 20$

① Directly computing : $3NM = 3 \times 1100 \times 20 = 66000$

② Section Conv. : $L_0 = 105$, $P_0 = 105 + 20 - 1 = 124$,

Set $P = 120$, $L = 120 - 20 + 1 = 101$

$$S = \frac{1100}{101} = 11$$

$$2S\,MUL_P + 3SP = 2 \times 11 \times 380 + 3 \times 11 \times 120$$
$$= 12320$$

③ DFT: $P \geq 1100 + 20 - 1 = 1119$

$$2MUL_{1152} + 3 \times 1152 = 2 \times 7088 + 3 \times 1152 = 17632$$

Section Convolution is the best way.

① $P=120,\ L=101,\ S=11,$ comp. $=12320$

② $P=144,\ L=125,\ S=9,$ comp. $=2\times9\times436+3\times9\times144=11736$

③ $P=96,\ L=77,\ S=15,$ comp $=2\times15\times280+3\times15\times96=12720$

④ $P=72,\ L=53,\ S=21,$ comp $=2\times21\times164+3\times21\times72=11424$

⑤ $P=168,\ L=149,\ S=8,$ comp $=2\times8\times680+3\times8\times168=14912$

$\therefore$ Need 11736 real multiplications

(c) $\text{len}(y[n])=7$

① Directly computing : $3\times1100\times7=23100$

② Section ConV. : $L_0=25,\ P_0=25+7-1=31$

(2-1) $P=32,\ L=26,\ S=43,$ comp $=2\times43\times72+3\times43\times32=10320$

(2-2) $P=28,\ L=22,\ S=50,$ comp $=2\times50\times64+3\times50\times28=10600$

(2-3) $P=36,\ L=30,\ S=37,$ comp $=2\times37\times64+3\times37\times36=8732$

(2-4) $P=24,\ L=18,\ S=62,$ comp $=2\times62\times28+3\times62\times24=7936$

(2-5) $P=16,\ L=10,\ S=110,$ comp $=2\times110\times20+3\times110\times16=9680$

$\therefore$ the best way is using Section Convolution

Total number of real multiplications $=7936$

(d) $len(y[n]) = 2$

① Directly computing : $3 \times 1100 \times 2 = 6600$

② Section conv. : $L_0 = 2$, $P = 3$

(2-1) $P = 4$, $L = 3$, $S = 367$, comp. $= 2 \times 367 \times 0 + 3 \times 367 \times 4$
$= 4404$

(2-2) $P = 2$, $L = 1$, $S = 1100$, comp. $= 0 + 3 \times 1100 \times 2 = 6600$

(2-3) $P = 6$, $L = 5$, $S = 220$, comp. $= 2 \times 220 \times 4 + 3 \times 220 \times 6 = 5720$

(2-4) $P = 8$, $L = 7$, $S = 158$, comp. $= 2 \times 158 \times 4 + 3 \times 158 \times 8 = 5056$

∴ The best way is using section convolution.

Total number of real multiplication is 4404 #

Extra Question : $MUL_{121} = ?$

$MUL_{121} = 11\, MUL_{11} + 11\, MUL_{11} + 3 \times 10 \times 10$

$= 11 \times 40 + 11 \times 40 + 300$

$= 1180$ #