FRM一级核心知识点

金融市场与产品

Market Rate

➢ **Common Market Rate**

  ● **Treasury Rates**

    ✓ The rates an investor earns on Treasury bills and Treasury bonds.

    ✓ Treasury rates are <u>risk-free rates</u> in the sense that it is considered highly unlikely that the government of a developed country will default on debt issued in its own currency.

  ● **LIBOR**

    ✓ LIBOR are <u>compiled from the estimated unsecured borrowing costs of 18 highly rated global banks.</u>

  ● **SOFR**

    ✓ There are plans to begin <u>phasing out Libor</u> and <u>replace it with a rate based on actual transactions. </u>U.S. has proposed the use of the <u>repo-based Secured Overnight Financing Rate (SOFR).</u>

Market Rate

➢ **Common Market Rate**

- **Repo Rates**
  - ✓ In a repurchase agreement, the difference between selling price (today) and the repurchased price (tomorrow or later) is called the repo rate.

➢ **Risk-Free Rate**

- The risk-free rate at which <u>derivatives are priced</u> is determined from <u>overnight interbank rates using overnight indexed swaps</u>. The <u>Treasury rate</u> is usually not adopted, because it is usually <u>artificially low</u>, mainly due to the following two reasons:
  - ✓ <u>Regulation generally does not require Banks to retain capital for their Treasury positions.</u>
  - ✓ <u>In some countries (such as the United States), Treasury yields get preferential tax treatment.</u>

◆ Compounding

> **Compounding Frequencies**

- Suppose we have an account where the simple interest is added in each year and then that money also earns interest.

- Assuming

  R_C is the rate of interest with continuous compounding.

  R_m is the rate of interest with discrete compounding (m per annum)

  T is the number of years.

$$FV = PV\left(1 + \frac{R_m}{m}\right)^{mT} \qquad\qquad FV = PVe^{R_CT}$$

$$PV\left(1 + \frac{R_1}{m_1}\right)^{m_1T} = PV\left(1 + \frac{R_2}{m_2}\right)^{m_2T} \qquad PV\left(1 + \frac{R_m}{m}\right)^{mT} = PVe^{R_CT}$$

◆ Spot Rate and Forward Rate

➢ **Spot Rate**

- A t-period spot rate, or **zero rate**, is <u>the interest rate earned when cash is received at just one future time.</u>

- **Determining Zero Rates with Bootstrap Method**
  - ✓ Working forward and <u>fitting the zero rates to progressively longer maturity instruments</u>. The other coupons can be determined by <u>interpolation from the rates that have already been determined.</u>
  - ✓ **Example:** Given the 0.5, 1.0 and 1.5 years zero rate are 2.0%, 2.3%, 2.5% respectively, the 2 year zero rate of a 2 year bond with a par value of $100 and a market value of $98.82 which pays (semi-annual) coupon at the rate of 2% per year can be calculated as follows:

$$\frac{1}{1+0.02/2} + \frac{1}{(1+0.023/2)^2} + \frac{1}{(1+0.025/2)^3} + \frac{101}{(1+R/2)^4} = 98.82$$

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◆ Spot Rate and Forward Rate

- **Forward rates**
  - ✓ Interest rates <u>corresponding to a future period</u> implied by the spot curve.

$$(1 + R_1)^{T_1}(1 + F)^{(T_2 - T_1)} = (1 + R_2)^{T_2}$$

$$e^{R_1 T_1} \times e^{F(T_2 - T_1)} = e^{R_2 T_2} \rightarrow F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

- ➢ **Par Rate**
  - Let's say there is a bond with a maturity of T years. When the coupon is 0, this is a zero coupon bond. This bond is issued at a discount. If the coupon rate goes up, the value of the bond goes up. And <u>when the coupon rate is at some particular value, the value of the bond is exactly equal to the par value, which is called the par rate.</u>

◆ Spot Rate and Forward Rate

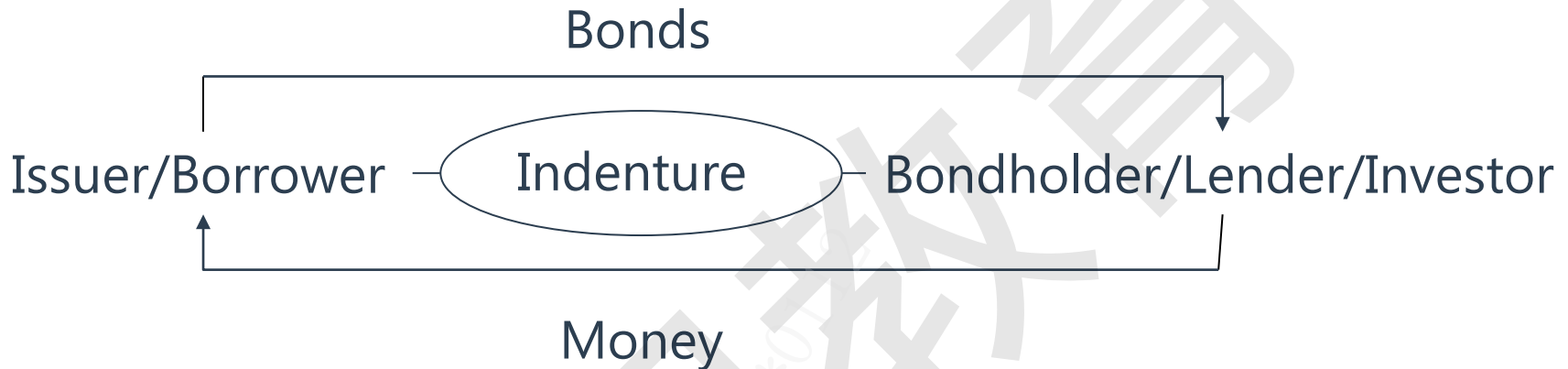| Spot Rates and Forward Rates		
Maturity (year)	**Spot Rate**	**Forward Rate**
0.5	0.94%	0.94%
1	1.37%	1.79%
1.5	1.82%	2.73%
2	2.51%	4.58%
2.5	3.08%	5.37%
3	3.87%	7.87%
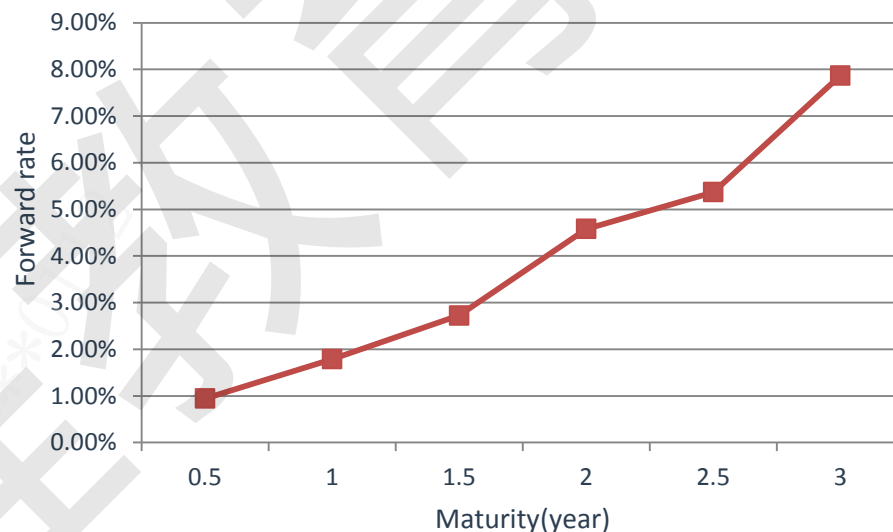
$$\left(1 + \frac{0.94\%}{2}\right)\left(1 + \frac{F_{0.5,1}}{2}\right) = \left(1 + \frac{1.37\%}{2}\right)^2$$

$$F_{0.5,1} = 1.79\%$$

Introduction of Bond Markets

Introduction of Bond Markets

➢ **What is a bond?**

Bonds

Issuer/Borrower ── (Indenture) ── Bondholder/Lender/Investor

Money

➢ **Characteristics of Bonds**

- Coupon Rate

- Face Value

- Maturity

- Yield to Maturity (YTM)

➢ **Annuity and Perpetual Bond**

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◆ **Introduction of Bond Markets**

➢ **How to determine the price of a bond?**

  ● **Principle**

$$P = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \cdots + \frac{C_T}{(1+y)^T} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

  ✓ Where C_t = the cash flow (coupon or principal) in period t

  ✓ t = number of periods to each payment (e.g., half years; a quarter years)

  ✓ T = the number of periods to maturity

  ✓ y = the discounting rate per period

Treasury Instruments

◆ Treasury Instruments

➢ **Treasury Bills**

 - A short-term debt obligation with a maturity of <u>one year or less.</u>
 - Interest rate is expressed on a <u>discount basis</u>.

➢ **Treasury Notes and Treasury Bonds**

 - Bond with a maturity of <u>more than one year</u>. Bonds which typically have maturities between one to ten years are called Treasury Notes. <u>But to keep the terminology simple, we will refer to all coupon-bearing Treasury instruments as Treasury Bonds.</u>
 - Both make interest payments <u>semi-annually</u>.
 - **Quoted Price：**
   - ✓ <u>Dollars and thirty-seconds</u> of a dollar with face value of $100

➢ **Treasury STRIPS**

 - C-Strips and P-Strips

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Treasury Instruments

➤ **Clean Price and Dirty Price**

Prices for 6.0% coupon bond with Jan 1st, 2019 maturity
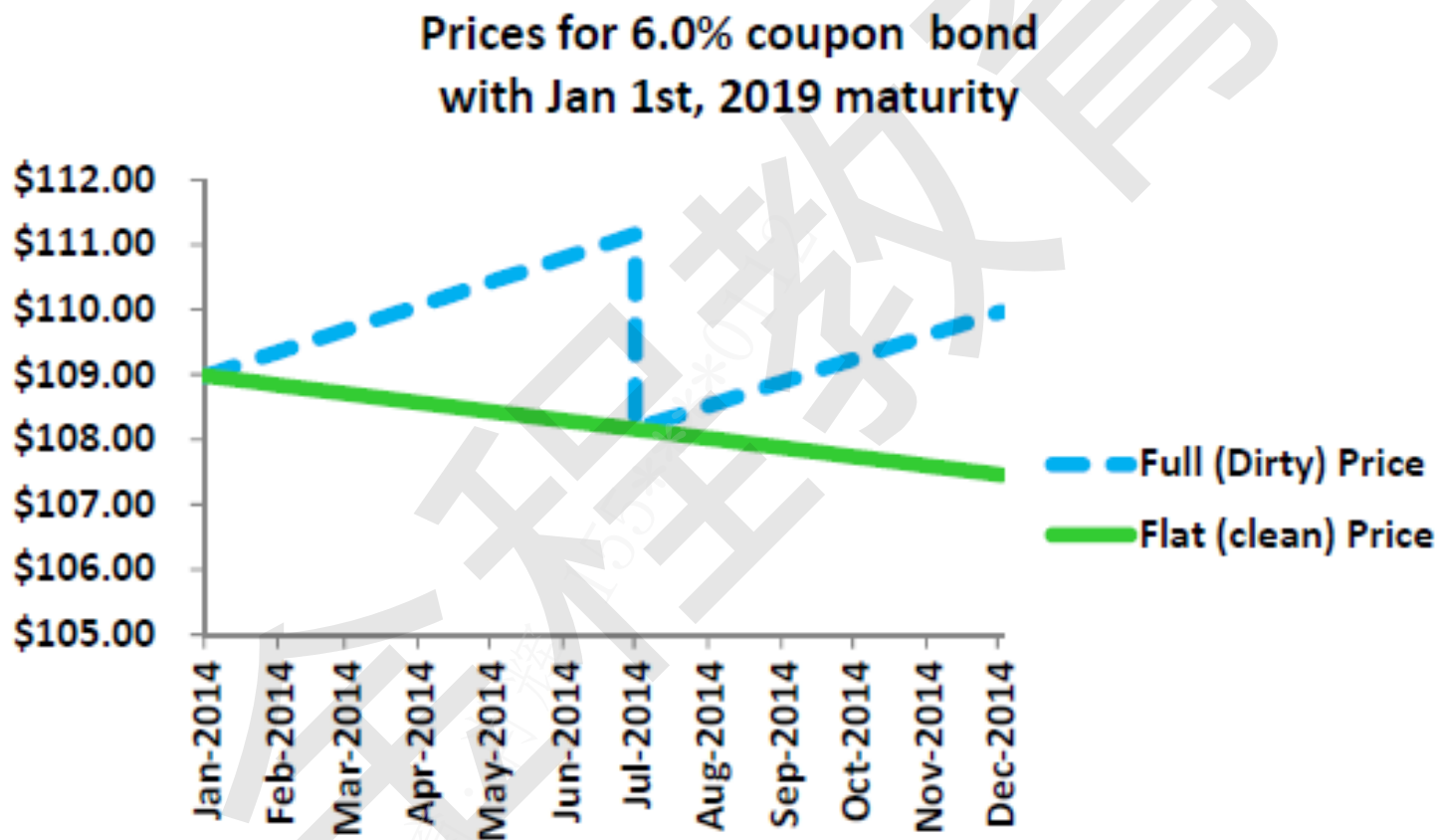
Treasury Instruments

➢ **Clean Price**

- The price of a coupon bond not including any accrued interest. Immediately following each coupon payment, the clean price will equal the dirty price.

➢ **Dirty Price**

- A bond pricing quote referring to the price of a coupon bond that includes the present value of all future cash flows, including interest accruing on the next coupon payment.

➢ **Accrued Interest and Day Count Conventions**

- Treasury bonds: actual/actual
- Corporate and municipal bonds: 30/360
- Money market instruments (Treasury bills): actual/360

dirty price = clean price + accrued interest

Treasury Instruments

➢ **Example**

Suppose a 1000 par value US corporate bond pays a semi-annual 10 percent coupon on January 1 and July 1. Assume that it is now April 1, 2005, and the bond matures on July 1, 2015. Compute the invoice (full) price of this bond if the required annual yield is 8 percent. Compute the flat (clean) price of the above bond.

Time	Mar 1st	Apr 1st	May 1st	June 1st	July 1st
dirty price	1155.30	1162.87	1170.50	1178.18	1185.90
clean price	1138.63	1137.87	1137.17	1136.51	1135.90

◆ Bond Indentures

➢ **Bond Indenture**
- Contract contains corporate bond issuer promises and investors' rights.
- Made out to corporate trustee, who represents bondholders' interests.

➢ **Corporate Trustee**
- A <u>financial institution</u> that <u>looks after the interests of the bondholders and ensures that the issuer complies with the indentures.</u>
- Its duties are <u>specified in the indentures</u> and the trustee is <u>under no obligation to exceed those duties</u>.
- For example, sometimes the indenture specified that trustee can rely on the issuer for information, so that, it is not required to conduct its own investigations.

OTC and Exchange Market

◆ OTC and Exchange Market

➢ **Over-the-Counter and Exchange Traded**
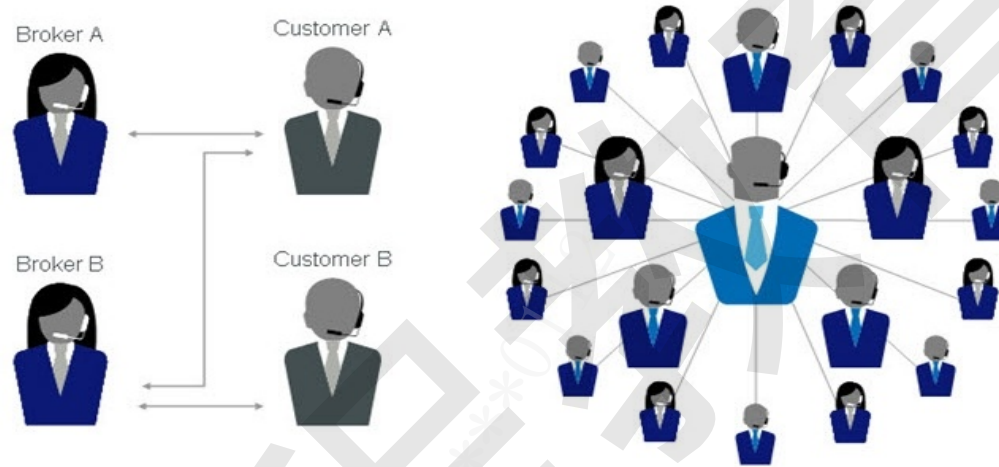
Over-the-Counter	Exchange-Traded
Customized	Standardized
Trade with counterparty (Default Risk)	Backed by a clearing house
Not trade in a central location	Trade in a physical exchange
Unregulated	Regulated
Trading volume: large	Trading volume: small

◆ OTC and Exchange Market

➢ **Exchange Market**

- The exchange requires members to protect themselves by providing **margin**. Margin refers to the cash or assets transferred from one trader to another for protection against counterparty default.

✓ **Variation Margin**

✓ **Initial Margin**

- In addition, members are required to submit a **default fund** as a loss protection.

✓ If the <u>initial margin is not sufficient</u> to cover a member's losses during a default, the <u>member's default fund contributions</u> will be used to cover the difference. If these funds remain insufficient, they are replenished by the <u>default funds of other members</u>.

◆ OTC and Exchange Market

➢ **Variation Margin**

● Trades made on an exchange are not settled at maturity. Instead, it <u>marks to market</u> day by day.

● Each day, the <u>losing member pays the exchange CCP an amount equal to the loss, while the profitable member receives an amount equal to the gain from the exchange CCP</u>. <u>These payments are called variation margin and usually occur daily</u>.

● Marking to market has another important benefit: it makes closing out contracts easier.

◆ OTC and Exchange Market

➤ **OTC Market**

● An OTC market is a market in which participants <u>directly contact</u> each other (or possibly by using a broker as an intermediary) to trade.

● OTC market participants can be divided into <u>end users</u> and <u>dealers</u>. End-users include companies, fund managers and other financial institutions that use derivatives to manage risk or gain specific exposures. <u>Dealers are large financial institutions that provide bid and ask quotes for common derivatives trades</u>.

● <u>Interest rate derivatives</u> are the <u>most popular derivatives</u> in the OTC market. Most interest rate derivatives are <u>interest rate swaps</u>.

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◆ OTC and Exchange Market

➢ **OTC Market**

- Before the 2007-08 credit crisis, over-the-counter markets were largely unregulated. <u>Several new rules have been introduced since the financial crisis.</u> Here are some important rules:

- ✓ In the US, <u>standardized OTC derivatives traded between dealers must be traded on swap execution facilities</u>. This is similar to exchanges or market participants who publish bid and ask prices.

- ✓ <u>Central counterparties (CCPS) must be used for standardized trading between dealers.</u>

- ✓ <u>All transactions must be reported to a central registry.</u>

Forward Rate Agreement

Forward Rate Agreement

➢ **Forward Rate Agreement**

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period.

- The buyer locks in a borrowing rate, and the seller locks in a lending rate.

- **Settlement:** The interest payment of FRA is normally paid at the end of the period. However, an FRA is usually settled at the beginning of the period covered by the FRA by convention. The payoff for the party who pays fixed and receives floating or the other side of the transaction is:

$$\frac{(R - R_K)\tau L}{1 + R\tau} \quad \text{or} \quad \frac{(R_K - R)\tau L}{1 + R\tau}$$

Where R is the realized floating rate, R_K is the fixed rate, L is the principal and τ is the length of the time horizon.

◆ Forward Rate Agreement

➢ **Valuation**

● The valuation for the party who pays fixed and receives floating or the other side of the transaction is:

$$PV\left(\frac{(R_F - R_K)\tau L}{1 + R_F\tau}\right)$$

Or

$$PV\left(\frac{(R_K - R_F)\tau L}{1 + R_F\tau}\right)$$

Where R_F is the forward rate and PV denotes the present value from the beginning of the period to today.

Futures Market

◆ Futures Market

➢ **Operation of Exchanges**

- The <u>number of contracts that exist at any time</u> is called **open interest**. This is the number of net long contracts held by members, which is equal to the number of net short contracts held by members.

- The <u>number of contracts traded in a day</u> is called **trading volume**. <u>If many traders close their positions, the volume of the day may be greater than the open interest. It can also happen if there is a large amount of intraday trading.</u>

Futures Market

➢ **Specification of Contracts**

● **Underlying Asset**

  ✓ As far as financial assets are concerned, the definition of underlying assets is usually simple. For example, the underlying asset of a contract traded on the S&P 500 by CME group is $250 times the index. When the asset is a commodity (e.g., cotton, orange juice), the exchange specifies a grade (quality).

● **Contract Size**

  ✓ Treasury bond Futures has a face value of $100,000;

  ✓ S&P 500 Futures contract is index×$250 (multiplier of 250);

  ✓ Eurodollar futures contract has a face value of $1 million.

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◆ Futures Market

➢ **Specification of Contracts**

● **Delivery Choose**

✓ Whenever there is a choice about what to deliver, where to deliver, and when to deliver, the short party (the deliverer) almost always has the right to choose.

● **Settlement Price**

✓ The daily settlement price is the <u>futures price at the close of trading</u> and is <u>used to determine the daily settlement</u>.

● **Price Quotes**

✓ The exchange specifies the quote convention it will use and the minimum price change for each contract.

Futures Market

➢ **Specification of Contracts**

● **Price Limit**

✓ For most contracts, exchanges set limits on <u>how much a futures price can move in a day</u>. If a price rises or falls more than the limit price, trading usually <u>stops</u> for the day.

✓ The purpose of price limits is to <u>prevent large price fluctuations caused by speculation</u>. However, these price restrictions may also <u>hamper the determination of true market prices</u> if limit moves arise from new information reaching the market.

● **Position Limits**

✓ Position limits are <u>limits on the size of positions that speculators can hold</u>. The aim is to <u>prevent speculators from exerting undue influence on the market</u>.

Futures Market

Asset	Corn (No. 2 Yellow..)
Contract Size	5000 bushels
Delivery Arrangement	Toledo, St. Louis
Delivery Months	Dec, Mar, May, Jul, Sep
Price Quotes	1/4 cent/bushel ($12.50/contract)
Price limits and position limits	Daily Price Limit: Thirty cent ($0.30) per bushel ($1,500/contract) above or below the previous day's settlement price. No limit in the spot month

| Asset	S&P 500 Index
Contract Size	$250 x S&P 500 Futures Price
Delivery Arrangement	Cash settlement
Delivery Months	Mar, Jun, Sep, Dec
Price Quotes	0.05 index points = $12.50
Price limits and position limits	20,000 net long or short in all contract months combined

◆ Futures Market

➤ **Delivery Mechanics**

- The delivery process <u>begins when a member with a short position issues a notice of intent to deliver to exchange CCP</u>. This notice indicates how many contracts will be delivered.

- The standard procedure is for <u>the exchange to allocate delivery notices to the member with the longest net long position</u>.

- In theory, futures contracts could be designed to be <u>settled in cash. CME group's S&P 500 index futures contracts are settled in cash. CME group's popular Eurodollar futures contract is also settled in cash.</u>

48. Normal and Inverted Futures Market

◆ Normal and Inverted Futures Market

➢ **Normal and Inverted Futures Market**
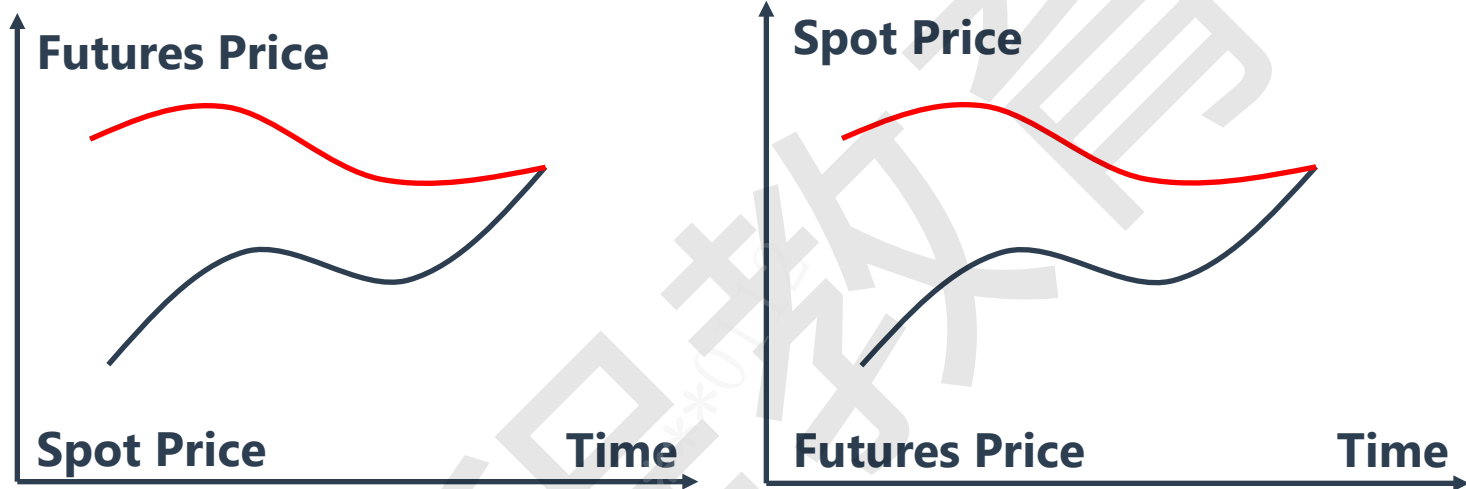
- If the <u>futures price increases as time to maturity increases</u>, the futures curve is said to be **normal**, or in **Contango**.

- If the <u>future price declines as maturity increases</u>, the futures curve is said to be **inverted**, or in **Backwardation**.

- Some assets have patterns that are <u>partly normal and partly inverted</u>.

Forward and Futures Prices

Forward and Futures Prices

➢ **Background Knowledge - Short Selling**

● Orders to sell securities that the seller does not own.

● **Example**

Purchase of Shares	
April: Purchase 500 shares at $120	-$60,000
June: Receive dividends $1 per share	$500
July: Sell 500 shares at $100	$50,000
Net Profit	**-$9,500**

Short Sale of Shares	
April: Borrow 500 shares and sell at $120	$60,000
June: Pay dividends $1 per share	-$500
July: buy 500 shares at $100, replace and close	-$50,000
Net Profit	**$9,500**

◆ Forward and Futures Prices

➢ **Assumptions of Pricing: No Arbitrage Principle**

$$F = S(1 + R)^T$$

$F > S(1 + R)^T$	$F < S(1 + R)^T$
Now: Borrow S to buy a unit of asset, enter into a forward contract to short the asset for F in time T;	*Now:* Short sale S and invest in a bank, enter into a forward contract to buy the asset for F in time T;
T later: ■ Sell asset at F and repay the loan for $S(1 + R)^T$ ■ Gain a profit of $$F - S(1 + R)^T$$	*T later:* ■ Get $S(1 + R)^T$ from the bank and buy the asset at F to close short position. ■ Gain a profit of $$S(1 + R)^T - F$$

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◆ Forward and Futures Prices

➤ **Forward Price for a Financial Asset that Provides no Income**

$$F = S(1 + R)^T$$

- **Example:** Consider a forward contract to sell a non-dividend-paying stock in 3 months. The current stock price is $40 and the 3-month risk-free rate (annually compounded) is 2.5% per year. The forward price:

$$F = 40(1 + 0.025)^{0.25} = 40.25$$

➤ **Forward Price for a Financial Asset that Paying a Known Cash Income**

$$F = (S - I)(1 + R)^T$$

- **Example:** Consider a 10-month forward contract on a bond paying a USD 2 coupon in 3 months and in 9 months. Assume the r_f for all maturities is 6% per year and the cash price of the bond is USD 107.

$$\frac{2}{1.06^{0.25}} + \frac{2}{1.06^{0.75}} = 3.8856 \qquad F = (107 - 3.8856) \times 1.06^{\frac{10}{12}} = 108.2450$$

◆ Forward and Futures Prices

➤ **Forward Price for a Financial Asset that Provides a Known Yield**

$$F = S\left(\frac{1+R}{1+Q}\right)^T$$

- **Example:** Consider an asset expected to provide a 2.5% yield per year over the next three years. The risk-free rate is 3% per year and the current spot price of the asset is USD 30. The forward price (USD) is

$$F = 30\left(\frac{1+3\%}{1+2.5\%}\right)^3 = 30.44$$

➤ **Forward Price for Stock Index**

- **Example:** Consider an index of 2,000, the r_f is 4% per year and the dividend yield is 2% per year. The futures price with a maturity of six months is

$$F = 2{,}000 \times \left(\frac{1.04}{1.02}\right)^{0.5} = 2019.5127$$

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◆ Forward and Futures Prices

➢ **Forward Price for a Commodity Asset with a Lease Rate**

$$F = S\left(\frac{1+R}{1+l}\right)^{T}$$

● **Example:** Assume that the spot price of gold is $1,250, the lease rate is 2.5%, and the 6-month risk-free rate is 4% (with annual compounding). The 6-month futures price is given by:

$$1,250 \times \left(\frac{1.04}{1.025}\right)^{0.5} = 1,259.1131$$

➢ **Forward Price for a Commodity with Storage Cost & Convenience Yield**

$$F = (S+U)\left(\frac{1+R}{1+Y}\right)^{T}$$

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Forward and Futures Prices

➢ **Forward vs. Futures**

- If <u>risk-free rate is constant or if they change in a perfectly predictable way</u>, then forward price should <u>equal</u> futures price.

- Correlation of underlying asset with interest rates

- ✓ <u>Strongly positive: futures > forward</u>

- ✓ <u>Strongly negative: futures < forward</u>

- Contract Life

- ✓ Short: negligible

- ✓ Long: can be significant.

Value of a Forward Contract

◆ Value of a Forward Contract

➢ **Value of a Forward Contract**

- The value of a forward contract is quite different from the forward price. When forward contracts for financial assets are <u>first entered</u>, the value of the forward contracts themselves is <u>zero</u>. Over time, however, asset prices change and the value of forward contracts can become positive or negative.

- <u>While the value of the contract changes, the price at which the asset will eventually be bought or sold remains the same as the original forward price.</u>

$$\text{Value of Long Forward Contract} = \frac{F - K}{(1 + R)^T}$$

T-Bond Futures

T-Bond Futures

➢ **T-Bond Futures**

- The Treasury bond futures contract allows the party with the short position to choose which particular bond with <u>a maturity more than 15 years on the first day of the delivery month and is not callable within 15 years from that day to deliver.</u>

- When a particular bond is delivered, a parameter known as <u>conversion factor</u> defines the price received for the bond by the party with the short position.

- Specially, the cash received by the short position is:

$$\text{Cash received} = (QFP \times CF) + AI$$

- **Cheapest-to-Deliver Bond**

$$\text{Cost} = \text{quoted bond price} - (QFP \times CF)$$

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◆ T-Bond Futures

➢ **T-Bond Futures**

● **Example:** Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is $95.75 (this is the quoted futures price). Determine which bond is the cheapest-to-deliver.

Bond	Quoted Bond Price	Conversion Factor	Cost
1	99	1.01	2.29
2	125	1.24	6.27
3	103	1.06	1.51
4	115	1.14	5.85

Eurodollar
Futures

◆ Eurodollar Futures

➢ **Eurodollar Futures**

- One of the most popular interest rate futures in the United States is the three-month Eurodollar futures contract traded by the CME Group.

- A three-month Eurodollar futures contract is a futures contract on the interest that will be paid (by someone who borrows at the Eurodollar interest rate) on $1 million for a future three-month period.

- A final settlement price is used to determine final transfers between those with long and short positions. It is USD 100 - R, where R is the Libor fixing for 90-day USD borrowings. For example, if the USD 90-day Libor fixing is 2.5%, the final settlement price of the corresponding Eurodollar futures contract would be USD 97.50 (= 100 - 2.5).

- 1 basis point move in the futures quote corresponds to a gain/loss of $25 per contract.

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Eurodollar Futures

➢ **Eurodollar Futures**

● **Eurodollar Futures vs. FRA**

✓ With the same underlying and the same maturity, They should be the same if interest rates are perfectly predictable.

✓ ρ(S, r) < 0, Futures price is lower than forward price.

✓ For short maturities, the differences are small enough to be ignored.

● **Convexity Adjustment**

$$\text{Forward Rate} = \text{Futures rate} - \frac{1}{2}\sigma^2 T(T + 0.25)$$

✓ σ is the standard deviation of the change in the short-term interest rate in one year.

✓ T is time to maturity of futures contract.

✓ T+0.25 is time to maturity of the rate underlying the futures contract.

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◆ Eurodollar Futures

➢ **Eurodollar Futures**

● **Convexity Adjustment**

✓ **Example:** Consider a situation where σ = 0.012 and calculate the forward rate when the 8-year Eurodollar futures price quote is 94.

Maturity of Futures (years)	Convexity adjustments (bps)
2	3.2
4	12.2
6	27.0
8	47.5
10	73.8

✓ When this formula is applied, both the forward and futures rate should be expressed using the actual/actual with continuous compounding.

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Hedging Strategies using Futures

◆ Hedging Strategies using Futures

➢ **Basis Risk**

- The **basis** is the <u>difference between the price of the futures contract and the spot price of the underlying asset</u>.

$$\text{Basis} = \text{spot price} - \text{futures price}$$

- **Long the basis** refers to a set of positions that consists of <u>a short futures position and a long cash position. Position that are long the basis benefit when the basis is strengthening.</u>

- **Short the basis** refers to a set of positions that consists of <u>a long futures position and a short cash position. Positions that are short the basis benefit when the basis is weakening.</u>

Hedging Strategies using Futures

➢ **Basis Risk**

● Futures contract often does not track exactly with the underlying commodity. **Basis risk** is the risk (to the hedger) created by the uncertainty in the basis.

● The hedging risk is the uncertainty associated with b_2:

✓ Different asset

✓ Different maturity

● **Cross hedging** occurs when the assets underlying the futures contract and the asset whose price is being hedged are different.
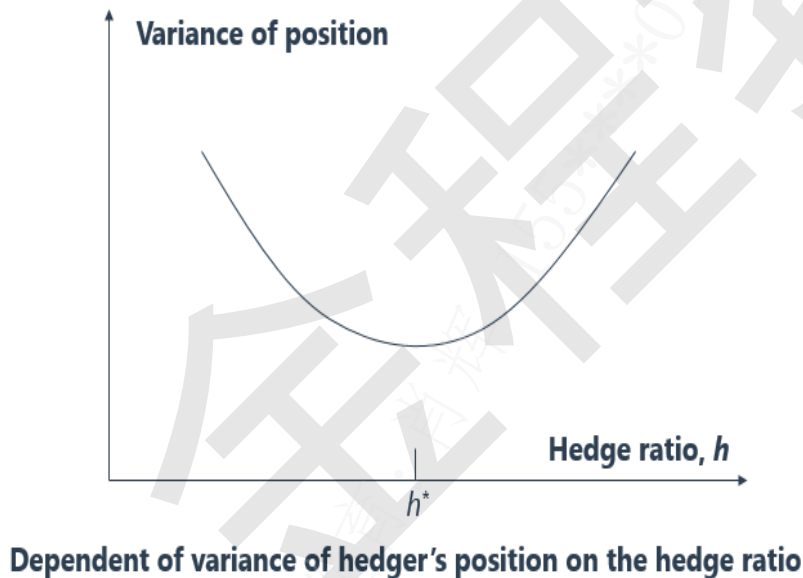

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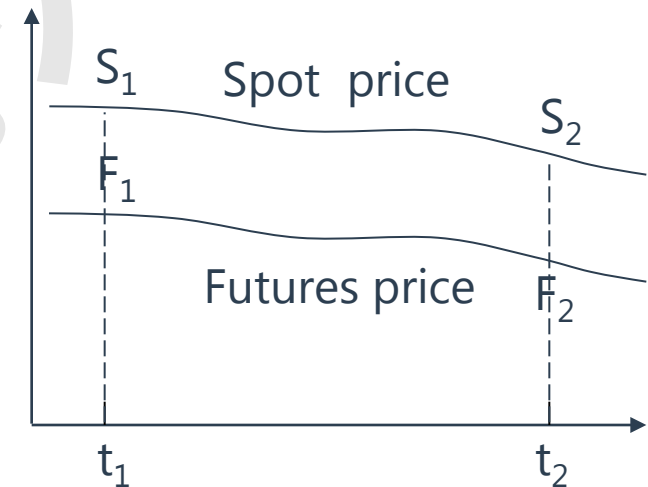
◆ Hedging Strategies using Futures

➢ **Hedging with Futures Contract**

● **Minimum Variance Hedge Ratio**

✓ The minimum variance hedge ratio depends on the relationship between changes in the spot price and changes in the futures price. By using it, we can <u>form a hedged position with minimum variance</u>.

Variance of position

Hedge ratio, *h*

h^*

Dependent of variance of hedger's position on the hedge ratio

$$h^* = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

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Hedging Strategies using Futures

➢ **Hedging with Futures Contract**

- **Optimal Number of Futures Contracts**

$$N^* = \frac{h^* Q_A}{Q_F}$$

- ✓ Q_A: Size of position being hedged (units)

- ✓ Q_F: Size of one futures contract (units)

- ✓ N^*: Optimal number of futures contracts for hedging

- **Tailing the Hedge**

- ✓ When futures contracts are used for hedging, there is <u>daily settlement and series of one-day hedges</u>. Tailing the hedge can deal with this case when making hedging decision.

$$N^* = \frac{\hat{\rho} \hat{\sigma}_S S}{\hat{\sigma}_F F} \frac{Q_A}{Q_F} = \frac{h^* \times V_A}{V_F}$$

- ✓ <u>$\hat{\sigma}_S, \hat{\sigma}_F$ is the standard deviation of the one-day return, $\hat{\rho}$ is the correlation between the one-day spot return and the futures return.</u>

◆ Hedging Strategies using Futures

➢ **Hedging with Futures Contract**

● **Effectiveness of Hedge**

✓ Measures how much variance can be reduced by implementing the optimal hedge. We can use coefficient of determination (R^2) term to evaluate the effectiveness.

✓ Note that for this simple linear regression, the R^2 measure is also the square of the correlation coefficient (ρ^2) between spot and futures prices.

Regressing ΔSpot on ΔForward

Δ Spot

Δ Forward
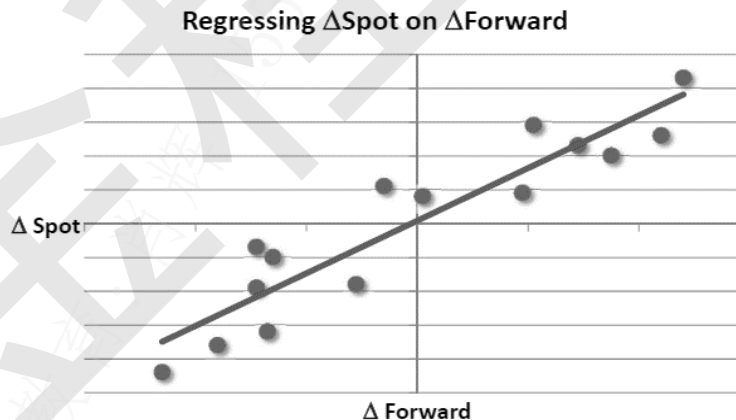
◆ Hedging Strategies using Futures

➢ **Hedging with Futures Contract**

● **Example:** An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging. Each heating oil contract traded by the CME Group is on 42,000 gallons of heating oil.

Month i	Change in heating oil futures price per gallon ($= \Delta F$)	Change in jet fuel price per gallon ($= \Delta S$)
1	0.021	0.029
2	0.035	0.020
3	−0.046	−0.044
4	0.001	0.008
5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.011	−0.018
13	0.019	0.009
14	−0.027	−0.032
15	0.029	0.023

	ΔF	ΔS
Std.	0.031	0.026
Correlation		0.928

$$HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

$$=0.928 \times \frac{0.026}{0.031} = 0.778$$

$$N = 0.778 \times \frac{2000000}{42000} = 37.03$$

◆ Hedging Strategies using Futures

➢ **Hedging with Futures Contract**

● **Hedging with Stock Index Futures**

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$
$$= \beta_{\text{portfolio}} \times \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}$$

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

● **Hedging with Interest Rate Futures**

✓ The number of contracts required to hedge against an uncertain change in the yield given by Δy, is given by:

$$N^* = \frac{\text{PD}_p}{\text{FD}_F} = \frac{\text{DV01}_P}{\text{DV01}_F}$$

Foreign Exchange

Foreign Exchange Quotes

➢ **Quotes**
- Currency pairs are typically indicated as **XXXYYY or XXX/YYY** (with <u>XXX as the base currency and YYY as the quote currency</u>). It shows how much quoted currency is required to buy a unit of base currency.
- **Bid-Ask Spread:** A bid-ask spread is the <u>amount by which the ask price exceeds the bid price for an asset in the market.</u>

➢ **Spot Market**
- <u>In the case of exchange rate between the U.S. dollar and the British pound, the U.S. dollar is the quote currency. This is also the case when the U.S. dollar is quoted with the euro, the Australian dollar, and the New Zealand dollar.</u>
- <u>In most other cases, the U.S. dollar is the base currency</u> and the other currency is the quote currency.

◆ Foreign Exchange Quotes

➢ **Forward Market**

- Forward rates are quoted with the <u>same base currency as spot exchange rates.</u> They are usually shown as <u>points that are multiplied by 1/10,000 and then added to the spot quote</u>.

➢ **Futures Market**

- Foreign exchange futures are traded on exchanges all around the world. The CME group trades many different futures contracts on rates between U.S. dollar and other currencies. <u>These are always quoted with USD as the quote currency.</u>

◆ Outright (Forward) vs. Swap

➢ **Outright Transaction**

- A forward foreign exchange transaction, where <u>two parties agree on an exchange at some future date</u>, is termed an **outright transaction or a forward outright transaction.**

➢ **FX Swap**

- FX swap refers to <u>buying (selling) a foreign currency in the spot market and then selling (buying) in the forward market.</u>

- While an FX swap involves the <u>exchange of currency on two different dates</u>, a **currency swap** involves the <u>exchange of principal and a stream of interest rate payments in one currency for principal and a stream of interest payments in another currency.</u>

◆ Covered & Uncovered Interest Parity

➢ **Covered Interest Parity**

● <u>Concerns forward exchange rates</u> and is expected to hold well because it depends on the <u>arbitrage argument</u>.

$$F = S \frac{(1 + R_{YYY})^T}{(1 + R_{XXX})^T}$$

➢ **Uncovered Interest Parity**

● Uncovered interest parity is an <u>argument related to the exchange rate itself, which is just one of many interacting factors that determine the exchange rate movements.</u> It argues that <u>investors should earn the same interest rate in all currencies when taking into account expected exchange rate movements.</u>

Interest Rate Swaps

◆Interest Rate Swaps

➢ **Interest Rate Swaps**

- An agreement to <u>swap a fixed interest rate for Libor.</u>

- For example, Party A may agree to pay interest to Party B at a fixed rate of 6% per annum for five years, with a notional principal of USD 100 million. Meanwhile, Party B may agree to pay the three-month Libor.

- In an interest rate swap, <u>notional principal is never actually exchanged</u>.

- **Confirmations:** <u>Agreements in OTC derivatives transactions</u> are known as confirmations (also known as a confirm).

- Interest rate swaps are popular products because <u>they can be used to transform assets and liabilities.</u>

- In practice, if swap participants can not get together directly, a swap trader working for a financial institution could act as <u>intermediary</u> by standing between the participants.

Interest Rate Swaps

➢ **Comparative Advantage Argument**

	Fixed	Floating	Comparative Advantage
BetterCreditCorp	4%	Libor +1%	Fixed Market
WorseCreditCorp	6%	Libor + 2%	Floating Market

4% fixed ← **Better Credit Corp** ← 3.5% fixed ← **Worse Credit Corp** → Libor + 2%
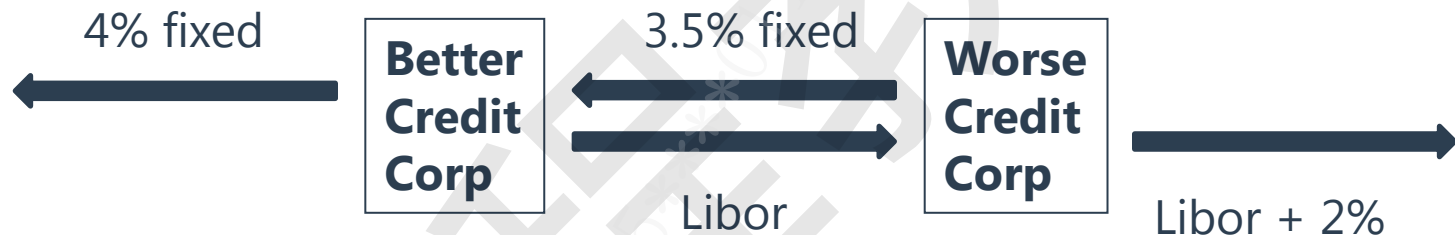
Libor

Net Borrowing Rates (i.e., including swap)		
	Fixed	**Floating**
BetterCreditCorp		Libor + 0.5%
WorseCreditCorp	5.5%	

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◆ Interest Rate Swaps

➤ **Valuation**

- A swap's value is zero at the time it is entered. At later times, it may have a positive or negative value because of changes in interest rates.

- Consider a swap that was entered some time ago and now has two years to maturity. Suppose that a fixed rate of 3% is received and Libor is paid every three months on a principal of USD 100 million. We also suppose the swap rate for a new two-year swap is 1.96%. And it is therefore reasonable to assume that a swap where 1.96% is paid and Libor is received is worth zero today.

◆ Interest Rate Swaps

➢ **Valuation**

- For valuation purposes, we can imagine a trader taking two positions.

- ✓ A two-year swap where a fixed rate of 3% is received and three-month Libor is paid on a principal of USD 100 million.

- ✓ A two-year swap where a fixed rate of 1.96% is paid and three-month Libor is received on a principal of USD 100 million.

- These two swaps net out to a position where 1.04% is received. Now suppose that the two-year risk-free rate is 2% (compounded quarterly) for all maturities. The (USD) value of the position is therefore:

$$\sum_{t=1}^{8} \left(\frac{260,000}{1,005^t} \right) = 2.033,969$$

- As the value of the second swap is zero. It follows that the value of the first swap is USD 2,033,969.

Currency Swaps

◆ Currency Swaps

➢ **Currency Swaps**

- Fixed-for-fixed currency swap

- One difference between a currency swap and an interest rate swap is that the principal amount is exchanged in a currency swap. Specifically, the principal amount is in the opposite direction to the interest rate payment at the beginning of the swap, and in the same direction as the interest rate payment at the end of the swap.

- Currency swaps can be used to swap liabilities and assets in a similar way to interest rate swaps.

◆ Currency Swaps

➤ **Valuation**

- The two sets of cash flows in a swap are referred to as legs.

- Currency swaps are often designed so that their value is close to zero when initially negotiated. The remaining cash flow can then be evaluated by considering each leg individually.

- Suppose a fixed-for-fixed swap involves exchanging the principal and interest of currency X for the principal and interest of currency Y, and requires the valuation in currency X. The valuation process is as follows.

- ✓ Value the remaining currency X cash flows in currency X terms.
- ✓ Value the remaining currency Y cash flows in currency Y terms.
- ✓ Convert the value of the currency Y cash flows to currency X at the current exchange rate.

◆ Currency Swaps

➢ **Valuation**

- Consider a currency swap with a remaining life of 3 years in which Interest at 4.5% in USD on a principal of USD 9 Million is exchanged for Interest at 2.5% on a principal of 7 million Euros.

- Suppose just after the last exchange of payments:

✓ The risk-free interest rate in USD is 4.2% for all maturities,

✓ The risk-free interest rate in euros is 3.0% for all maturities, and

✓ The exchange rate (USD per euro) is 1.2.

$$\frac{405,000}{1.042} + \frac{405,000}{1.042^2} + \frac{9,405,000}{1.042^3} = 9,074,644$$

$$\frac{175,000}{1.03} + \frac{175,000}{1.03^2} + \frac{7,175,000}{1.03^3} = 6,900,999$$

$$\text{Value in USD} = 9,074,644 - 6,900,999 \times 1.2 = 793,446$$

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◆ Calls and Puts

➤ **Option Contract**

- Options involve paying a <u>premium</u> for the <u>right</u> to buy (or sell) an asset at a future price..

➤ **Long/Short; Call/Put**

- An option to <u>buy an asset</u> at a particular price is termed a <u>call option</u>.

| Long call	Right to buy	
Short call		Obligation to sell

- An option to <u>sell an asset</u> at a particular price is termed a <u>put option</u>.

Long put	Right to sell	
Short put		Obligation to buy

- The seller is sometimes referred to as the <u>writer of the option</u>.

- The price specified in the contract is known as <u>strike price/exercise price</u>.
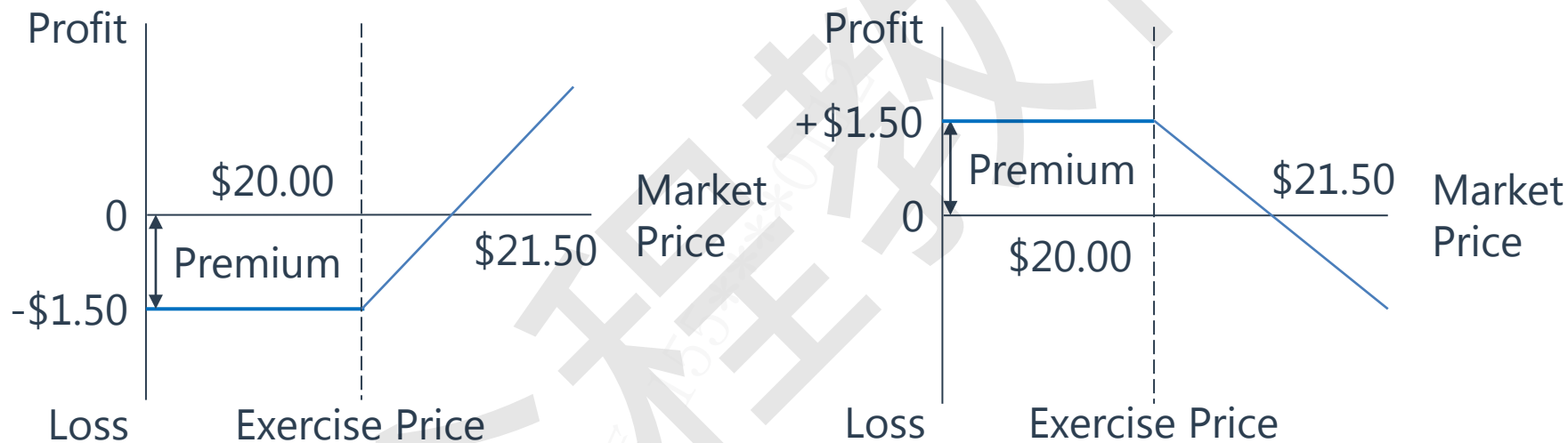
◆ Calls and Puts

➢ **Payoffs of Options**

$$Max(S_T - K, 0)$$

$$-Max(S_T - K, 0)$$

$$Max(K - S_T, 0)$$

$$-Max(K - S_T, 0)$$
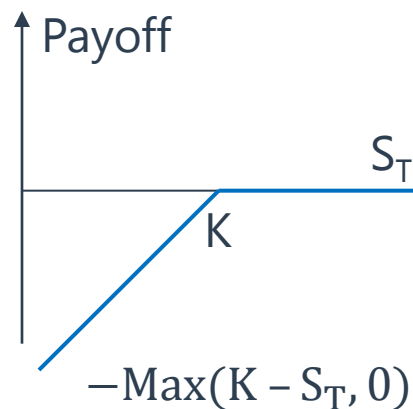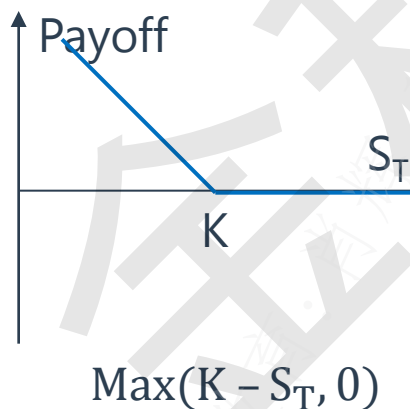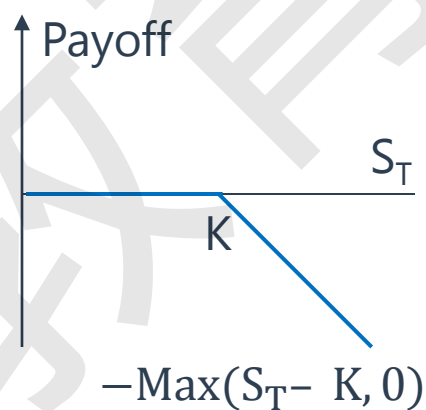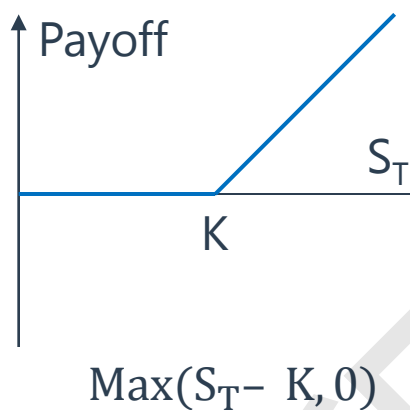
◆ Calls and Puts

➢ **Payoffs of Options**

- While the <u>payoff</u> from a European option is <u>related to the asset price at maturity</u>, the so-called <u>intrinsic value</u> is based on the <u>current asset price</u> (S_0).

- **Intrinsic value** measures the <u>value of an option if it can only be exercised immediately.</u>

- ✓ The <u>intrinsic value of a call option</u> is $Max(S_0 - K, 0)$;

- ✓ The <u>intrinsic value of a put option</u> is $Max(K - S_0, 0)$.

- **Time Value**: The difference between the price of an option (called its premium) and its intrinsic value is due to its time value.

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Calls and Puts

> **Profits from Call Option**

◆ Calls and Puts

➤ **Profits from Put Option**

◆ Calls and Puts

➢ **European Options**

- A European call (or put) option gives the buyer the right to buy (or sell) an asset at a specified price <u>on a specified date</u>.

➢ **American Options**

- An American call (or put) option gives the buyer the right to buy (or sell) an asset at a specified price <u>at any time prior to and during a specified date.</u>

- <u>In some cases</u>, the American option <u>should never be exercised</u> in advance, so the price must <u>be the same as the European option.</u>

➢ **Early Exercise Features of American Call and Put Options**

- From a mathematical standpoint, <u>it's never optimal to execute an early exercise on an American call option on a non-dividend paying stock.</u> However, it can be optimal to execute an early exercise on American put.

◆ Calls and Puts

➢ **Moneyness**

- An option which would give a <u>positive payoff if exercised today</u> is referred to as **in-the-money**.

- If it would give a <u>negative payoff</u> then it is referred to as **out-of-the money.**

- An option that **is at-the-money** would give a <u>payoff of zero</u>.

Moneyness	Call option	Put Option
In-the-money	S>K	S<K
At-the-money	S = K	S = K
Out-the-money	S<K	S>K

Margin Requirements

Margin Requirements

➢ **Margin Requirements of Exchange-Traded Options**

- When call and put options with <u>maturities less than 9 months</u> are purchased, the option price must be paid in full. <u>Investors are not allowed to buy these options on margin</u> because options already contain substantial leverage and buying on margin would raise this leverage to an unacceptable level. For options with maturities <u>greater than 9 months</u> investors can <u>buy on margin, borrowing up to 25% of the option value</u>.

- <u>A trader who writes option is required to maintain funds in a margin account</u>.

Margin Requirements

➢ **Margin Requirements of Exchange-Traded Options**

● **Writing Naked Options**

✓ The margin requirement by the CBOE for a **written naked call option** is the greater of the following two calculations:

1. A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount by which the option is out of the money.

2. A total of 100% of the proceeds plus 10% of the underlying share price.

✓ For a **written naked put option**, it is the greater of

1. A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount by which the option is out of the money.

2. A total of 100% of the proceeds plus 10% of the exercise price.

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Margin Requirements

➤ **Margin Requirements of Exchange-Traded Options**

● **Example**

An investor writes four naked call option contracts on a stock. The option price is $5, the strike price is $40, and the stock price is $38. Because the option is $2 out of the money, the first calculation gives:

$$400 \times (5 + 20\% \times 38 - 2) = \$4,240$$

The second calculation gives

$$400 \times (5 + 10\% \times 38) = \$3,520$$

The initial margin requirement is therefore $4,240. Note that, if the option had been a put, it would be $2 in the money and the margin requirement would be

$$400 \times (5 + 20\% \times 38) = \$5,040$$

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Properties of Options

◆ Six Factors

➢ **Six Factors that Affect Option's Price**

Factor	European call	European put	American call	American put
S	+	—	+	—
K	—	+	—	+
T	?	?	+	+
σ	+	+	+	+
r	+	—	+	—
D	—	+	—	+

◆ Upper and Lower Bounds for Option Prices

➤ **Call Options**

Call Option	Upper Bounds	Lower Bounds
European (No Dividend)	S_0	$\max(S_0 - PV(K), 0)$
European (Dividend)	S_0	$\max(S_0 - PV(K) - PV(Divs), 0)$
American (No Dividend)	S_0	$\max(S_0 - PV(K), 0)$
American (Dividend)	S_0	视红利情况而定

➤ **Put Options**

Put Option	Upper Bounds	Lower Bounds
European (No Dividend)	$PV(K)$	$\max(PV(K) - S_0, 0)$
European (Dividend)	$PV(K)$	$\max(PV(K) + PV(Divs) - S_0, 0)$
American (No Dividend)	K	$\max(K - S_0, 0)$
American (Dividend)	K	视红利情况而定

◆ Upper and Lower Bounds for Option Prices

➤ **Example**

- Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%(continuously compounding).

$$P > \max(0, K - S_0) = \max(0, 65 - 63) = \$2$$

$$p > \max(0, PV(K) - S_0) = \max(0, 65\, e^{-5\%/3} - 63) = \$0.92$$

- Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%%(continuously compounding).

$$C > \max(0, S_0 - PV(K)) = \max(0, 68 - 65e^{-5\%/4}) = \$3.81$$

$$c > \max(0, S_0 - PV(K)) = \max(0, 68 - 65e^{-5\%/4}) = \$3.81$$

◆ Put-Call Parity

> **Put-Call Parity**

- Put-call parity describes the price relationship between a European call option and a European put option with the same strike price and time to maturity.

- ✓ Portfolio A: European call option plus an amount of cash equal to $PV(K) + PV(Divs)$,

- ✓ Portfolio C: European put option plus one share.

- ✓ At maturity:

  Value of Portfolio A = Value of Portfolio C = $\max(S_T, K) + FV(Divs)$

  $$\boxed{\text{European Call Price} + PV(K) + PV(Divs) = \text{European Put Price} + S}$$

- There is <u>no exact price relationship</u> between an American call option and an American put option.

◆ Simple Strategies

➢ **Covered Call and Protective Put**
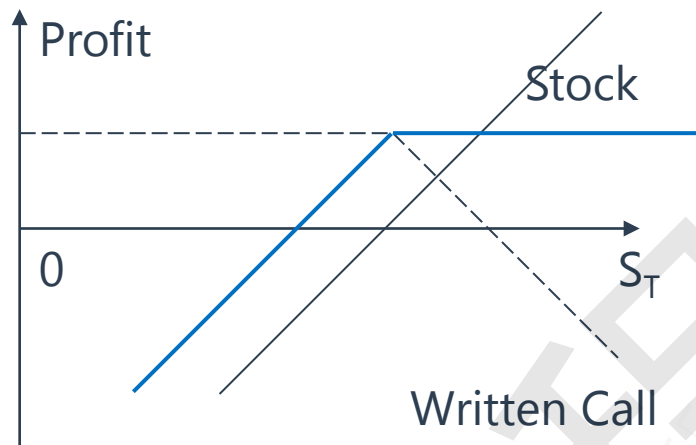
Covered Call = -C + S

- Income Strategy
- Outlook is neutral to bullish
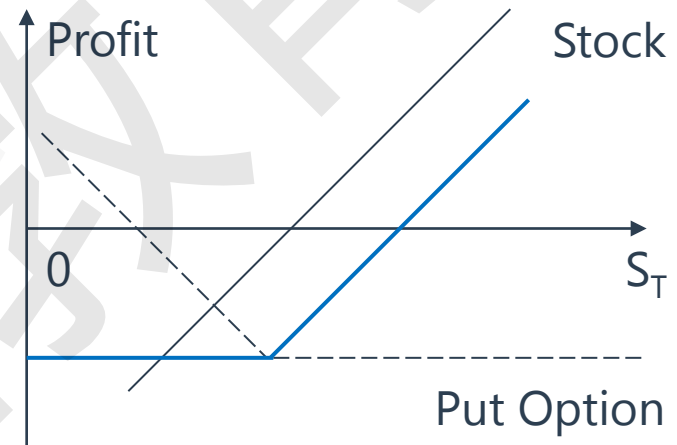
Protective Put = S + P

- Insurance Strategy

◆ Simple Strategies

➢ **Principal Protected Notes(PPN)**

- A PPN is structured as a <u>zero-coupon bond</u> and <u>an option</u> with a payoff that is linked to an underlying asset, index, or benchmark.

- It <u>guarantees a minimum return equal to the investor's initial investment (the principal amount)</u>, regardless of the performance of the underlying assets.

- Example: A three-year zero-coupon bond that will pay USD 10,000 in three years; and A three-year call option on Portfolio B, which is currently worth USD 10,000 with a strike price of USD 10,000.

- <u>Full participation PPNs</u> (i.e., <u>the owner receives 100% of the upside</u>) are <u>only possible for portfolios that provide an income.</u>

Spread
Strategies

◆ **Spread Strategies**

➢ **Bull and Bear Spread**
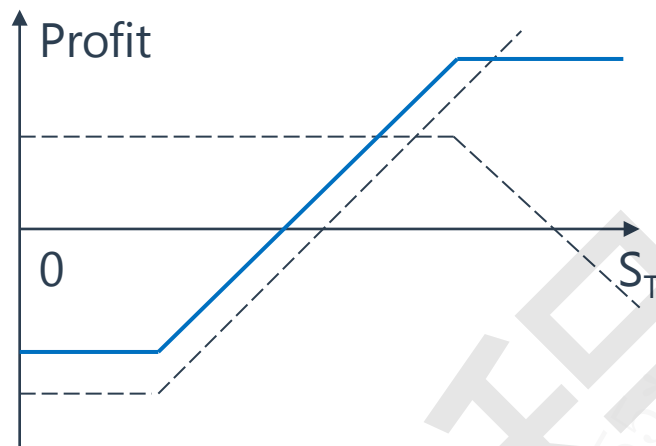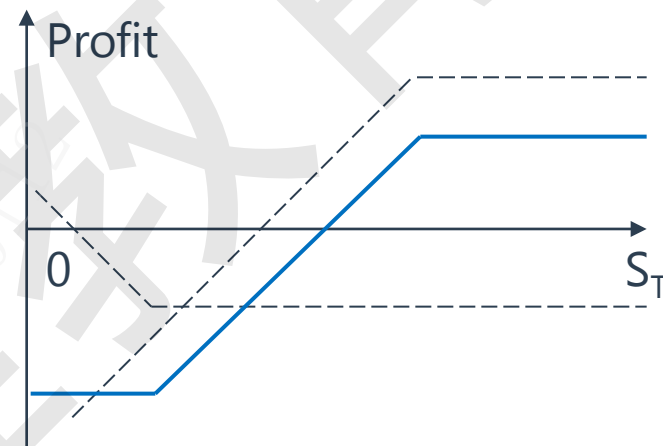
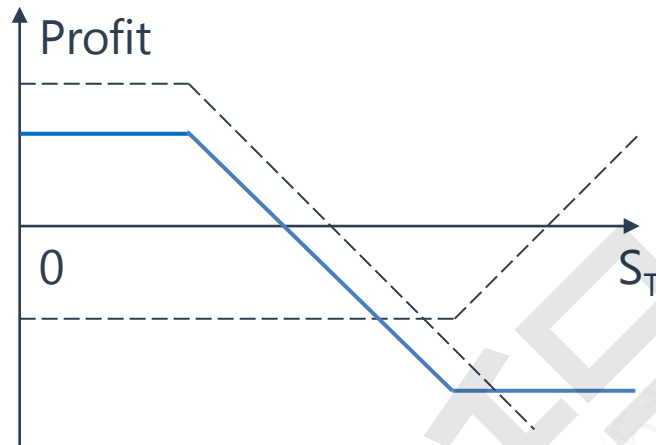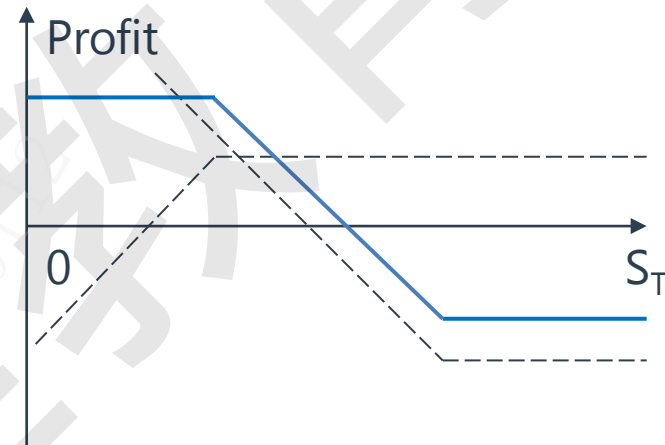**Bull Call Spread**　　　　　**Bull Put Spread**

- Vertical spread
- Outlook is <u>bullish</u>

Spread Strategies
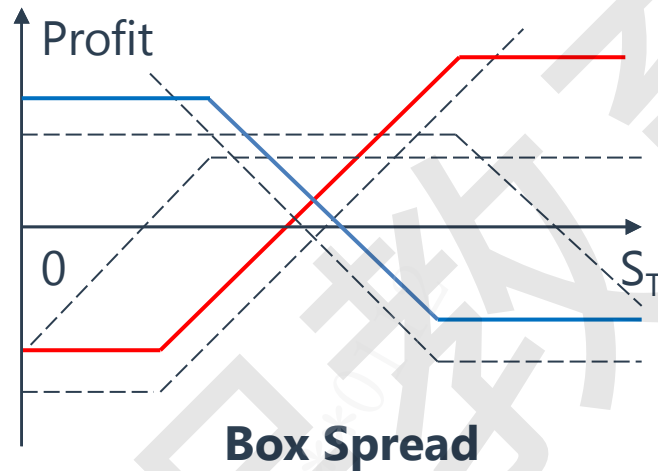
➤ **Bull and Bear Spread**

Bear Call Spread

Bear Put Spread

- Vertical spread
- Outlook is <u>bearish</u>
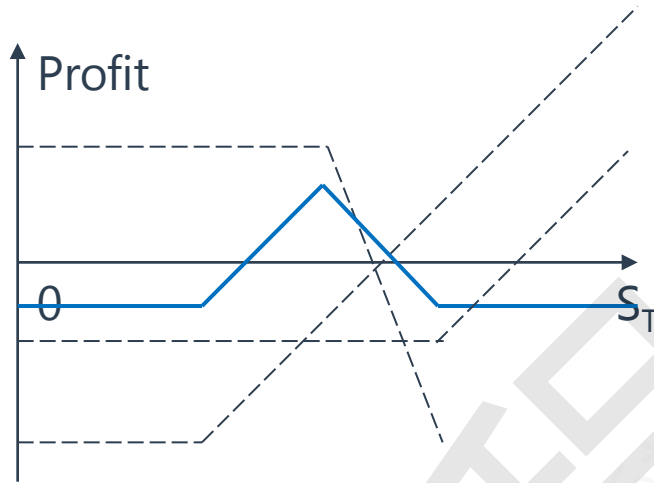
◆ Spread Strategies

➢ **Box Spread**

Box Spread

- A box spread is a combination of a <u>bull call spread</u> with strike prices K_1 and K_2 and a <u>bear put spread</u> with the same two strike prices.
- The <u>payoff from a box spread is always $K_2 - K_1$</u>. The value of a box spread is therefore always the present value of this payoff or $(K_2 - K_1)e^{-rT}$.
- If the market price of the box spread is too low, it is profitable to buy the box. If the market price of the box spread is too high, it is profitable to sell the box.

◆ Spread Stategies
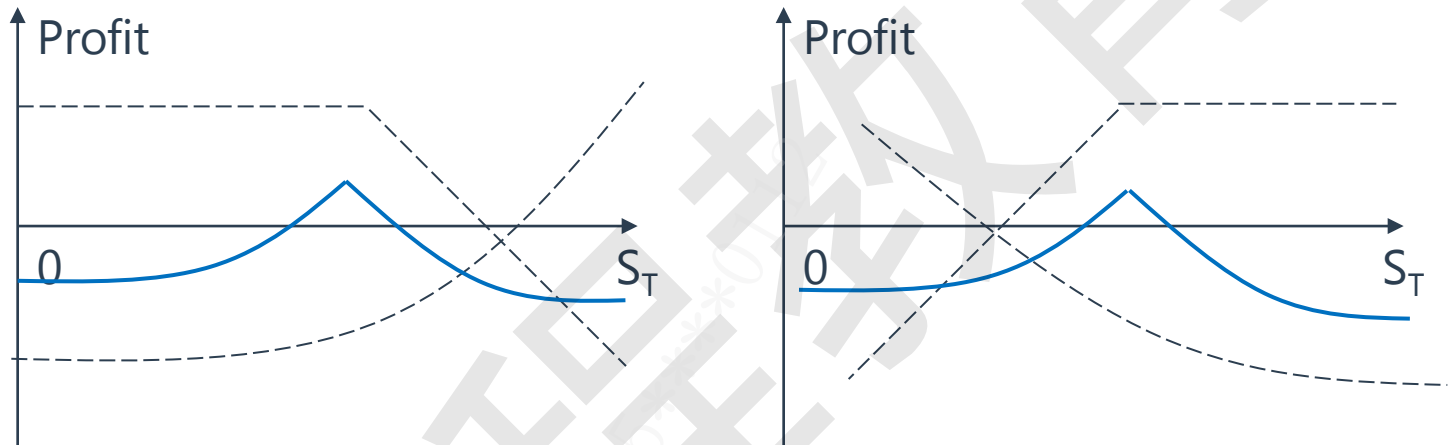
➢ **Butterfly Spread**
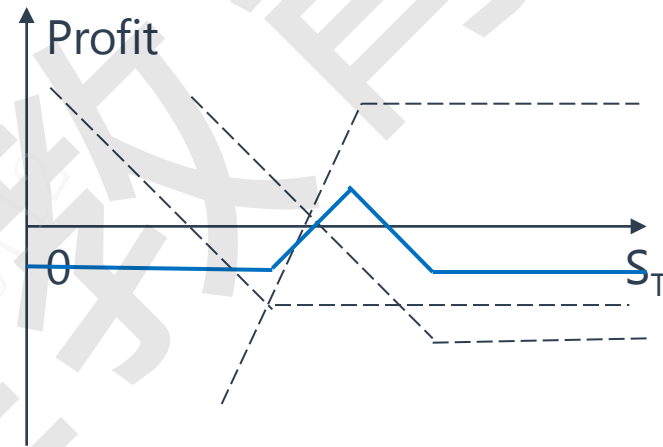
Butterfly Spread **Butterfly Spread**

- Expects <u>low volatility</u>
- <u>Capped risk</u>
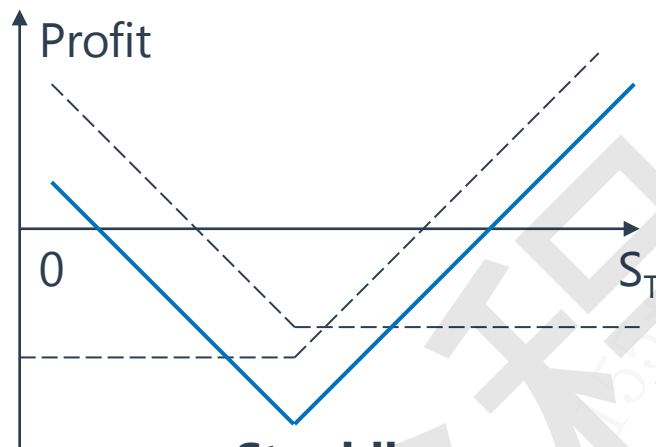
◆ Spread Strategies

➢ **Calendar Spread**

Calendar Spread

Combination Strategies
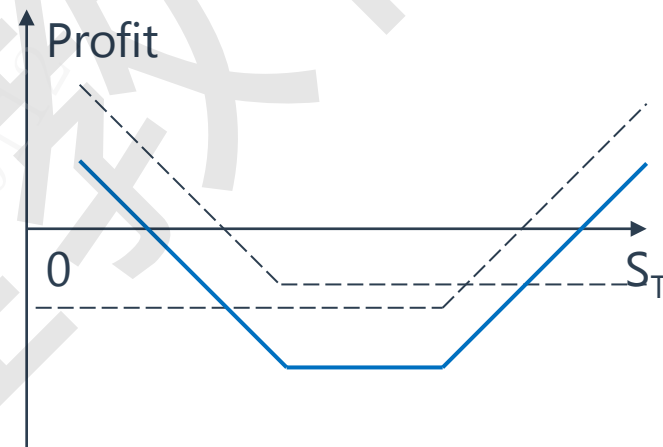
◆ Combination Strategies

➢ **Straddle and Strangle**

● A Combination is an option trading strategy that involves taking a position in both calls and puts on the same stock.

Straddle

- A call and a put
- Same strike price
- Direction neutral
- Wants volatility

Strangle

- A call and a put
- Different strike price
- Like straddle, but cheaper

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Exotic Options

◆ Exotic Options

➢ **Standard and Nonstandard American Options**

- Options with non-standard properties are called exotic options (or simply exotics).

- In a standard American option, exercise can take place at any time during the life of the option and the exercise price is always the same.

- The American options that are traded in the over-the-counter market sometimes have nonstandard features.

- ✓ Early exercise may be <u>restricted to certain dates</u> – **Bermudan Option**.

- ✓ <u>Early exercise may be allowed during only part of the life of the option-lock-out period</u>: Employee stock options.

- ✓ The <u>strike price</u> may change during the life of the option.

◆ Exotic Options

➢ **Gap Call Option & Gap Put Option**

Gap call
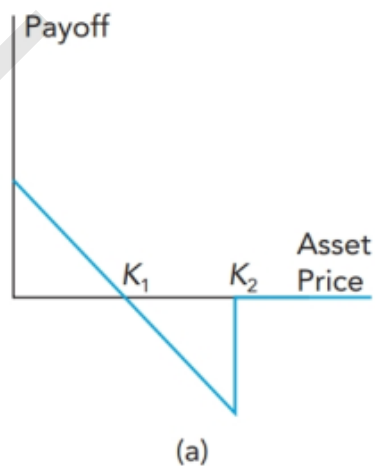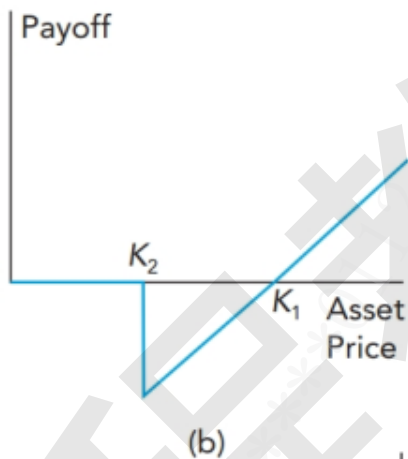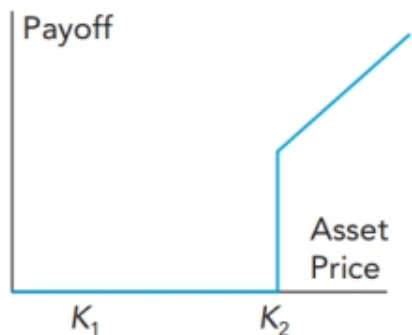
Gap put
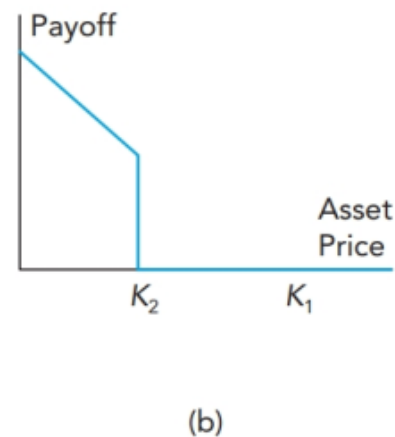
Exotic Options

➢ **Compound Options**

- Compound options are <u>options on options</u>. There are four main types of compound options:

  ✓ <u>a call on a call</u>

  ✓ <u>a put on a call</u>

  ✓ <u>a call on a put</u>

  ✓ <u>a put on a put</u>

- The advantages of compound options are that they allow for <u>large leverage</u>.

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◆ Exotic Options

➢ **Forward Start Options**

● A forward start option is an <u>advance purchase of a put or call option that will become active at some specified future time. It is essentially a forward on an option</u>.

● Employee stock options can be forward start options if an employer promises that they will be granted on future dates

➢ **Chooser Option**

● A chooser option has the feature that <u>after a specified period of time, the holder can choose whether the option is a call or a put</u>.

● Suppose that the time when the choice is made is T_1. The value of the chooser option at this time is: $\boxed{\max(c, p) = c + \max(0, p - c)}$

● If both options are European options maturing at time T_2. The value of the chooser option is: $\boxed{c + \max(0, PV(K) - S_1)}$

Exotic Options

- **Barrier Options**

  - When a knock-out option approaches a barrier, an increase in volatility may lower the price because it increases the likelihood that the barrier will be hit.

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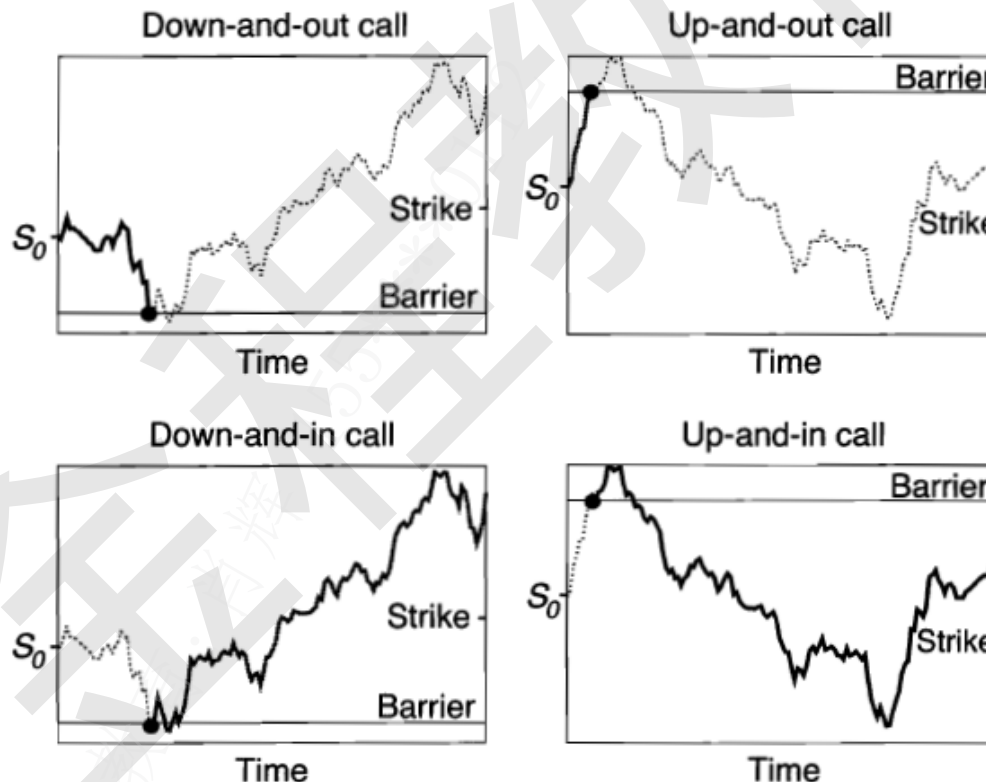
◆ Exotic Options

➢ **Binary Options/Digital Option**

● Binary options and gap options, their payoff is discontinuous.

Cash-or-nothing Call   Cash-or-nothing Put

Asset-or-nothing Call   Asset-or-nothing Put

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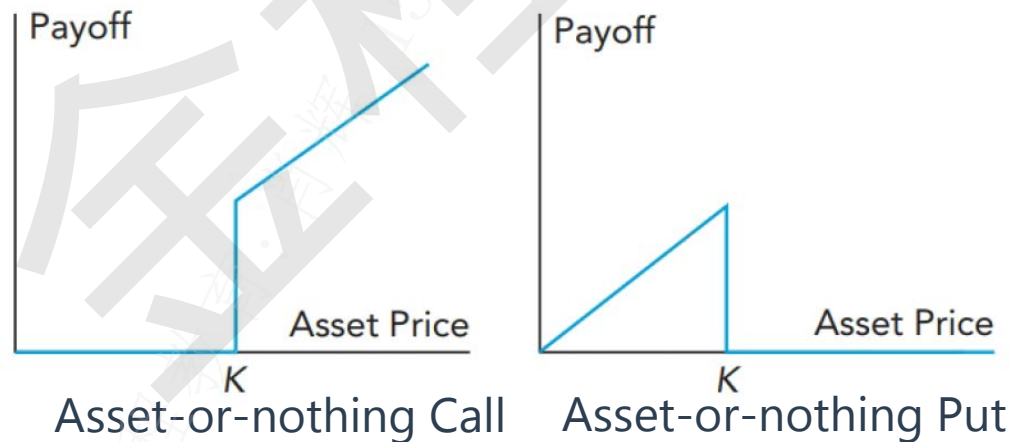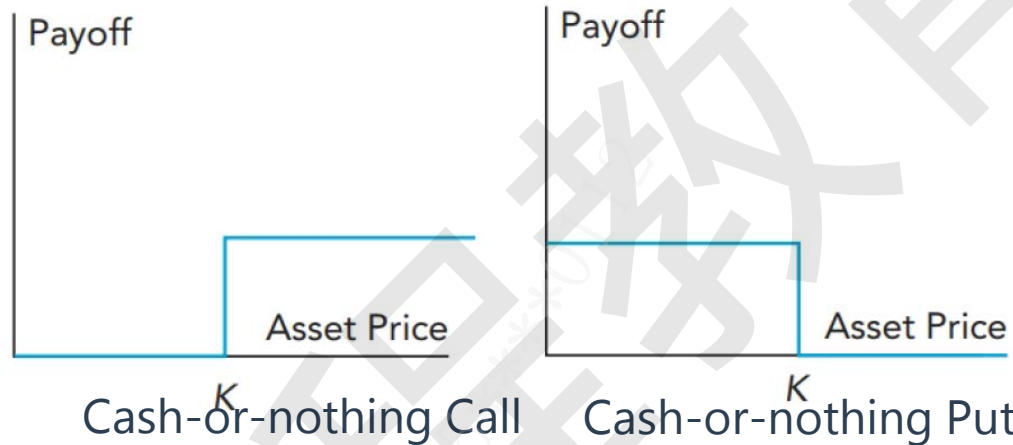
◆ Exotic Options

➢ **Lookback Options**

- The payoff from lookback options depend on the maximum or minimum asset price reached during the life of the option. There exist two kinds of lookback options: <u>with floating strike and with fixed strike</u>.

Floating lookback call payoff $= \text{Max}(\text{Final Asset Price} - \text{Minimum Price}, 0)$

Floating lookback put payoff $= \text{Max}(\text{Maximum Price} - \text{Final Asset Price}, 0)$

Fixed lookback call payoff $= \text{Max}(S_{\text{max}} - K, 0)$

Fixed lookback put payoff $= \text{Max}(K - S_{\text{min}}, 0)$

◆ Exotic Options

➤ **Asian Options**

- Options where the <u>payoff depends on the arithmetic average of the price of the underlying asset</u> during the life of the option. There are two types of Asian Options: <u>average price option and average strike option</u>.

- $Call_{average\ price} = Max(S_{ave} - K, 0)$ $Put_{average\ price} = Max(K - S_{ave}, 0)$

- $Call_{average\ strike} = Max(S_T - S_{ave}, 0)$ $Put_{average\ strike} = Max(S_{ave} - S_T, 0)$

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Exotic Options

➢ **Asset-Exchange Options**

  ● In an asset-exchange option, the holder has the right to exchange one asset for another.

  ● Asset-exchange options are closely related to options where a trader will receive the more valuable of two assets (i.e., Asset A and Asset B).

➢ **Basket Options**

  ● A basket option is an option on a portfolio of assets.

  ● Basket options can be appropriate hedging instruments for firms seeking to reduce costs by hedging their aggregate exposure to several assets with a single trade.

  ● Basket options are dependent on the correlation between the returns from the assets in the basket .

Exotic Options

➢ **Volatility and Variance Swaps**

- **Volatility Swap**

✓ Exchanging of volatility based on a national principal.

✓ Payments base on <u>pre-specified volatility</u> and <u>historical volatility</u>.

- **Variance Swap**

✓ Exchanging <u>pre-specified</u> fixed variance rate for <u>realized variance rate</u>

✓ <u>Easier to price and hedge than volatility swap since they can be replicated using a collection of regular calls and puts.</u>

➢ **Static Options Replication**

- The key principle behind this process is that two portfolios must have the same value at a certain boundary (a function of asset price and time) and at all internal points (the points that are likely to be reached before reaching the boundary).

- So if we can find a normal options portfolio with roughly the same value as the exotic options on the boundary, the exotic options can hedge by shorting the portfolio.

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Mortgages

➢ **U.S. Mortgage Market**

● Residential mortgages usually last 15 or 30 years. Their <u>interest rates can be fixed or variable</u>.

● Variable rate mortgages are called <u>adjustable rate mortgages (ARMs)</u>. In <u>ARM, the interest rate is usually fixed for several years and then indexed to the interest rate index.</u>

✓ ARMs are less risky for lenders than fixed-rate mortgages and more risky for borrowers. For this reason, the initial interest rate on a mortgage is usually lower than on a comparable fixed-rate mortgage.

● <u>Fixed-rate mortgages in the U.S. is that borrowers have an American-style option to repay their outstanding mortgage balances.</u> This is called the borrower's <u>prepayment option</u>.

Mortgages

➤ **Monthly Payments**

- The amortization statement shows the monthly principal and interest payments on the mortgage (assuming the principal is not repaid in advance).

- For example

- ✓ A 30-year U.S. mortgage where the fixed rate is 6% with monthly compounding. If the amount borrowed is USD 300,000.

- ✓ Payments of USD 1798.65 per month therefore fully amortize (i.e., repay) borrowings of USD 300,000 over 30 years.

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Mortgages

Month	End-of-Month Interest Payment	End-of-Month Principal payment	Balance at End-of-Month
0			300,000.00
1	1,500.00	298.65	299,701.35
2	1,498.51	300.14	299,401.20
3	1,497.01	301.65	299,099.56
4	1,495.50	303.15	298,796.40
5	1,493.98	304.67	298,491.73
...	...	...	...
356	44.30	1,754.35	7,105.57
357	35.53	1,763.12	5,342.44
358	26.71	1,771.94	3,570.50
359	17.85	1,780.80	1,789.70
360	8.95	1,789.70	0.00

In the first month

✓ The interest on the mortgage is

$$0.005 \times USD\ 300,000 = USD\ 1,500.00$$

✓ Repayment of principal on the mortgage is

$$USD\ 1798.65 - USD\ 1,500.00 = USD\ 298.65$$

◆ Mortgages

- Calculate the principal outstanding by **discounting the remaining cash flow**.

✓ the relationship between the amount borrowed A, the interest rate *R* (compounded monthly), and the monthly payment X is

$$\frac{X}{R/12}\left[1 - \frac{1}{(1 + R/12)^{12T}}\right] = A$$

where T years is the life of the mortgage

◆ **Mortgages**

> **Mortgage portfolios (mortgage pools)**
>
> - **WAC**
>
> ✓ The weighted average coupon (WAC) is the weighted average interest rate of the mortgage pool and the weighting allocated to each mortgage is proportional to the principal outstanding.
>
> - **WAM**
>
> ✓ The weighted-average maturity(WAM) is similarly calculated as the weighted-average of the number of months to maturity, with the weighting of each mortgage proportional to the principal outstanding
>
> $$WAC = \sum_{i=1}^{n} w_i c_i \qquad\qquad WAM = \sum_{i=1}^{n} w_i L_i$$
>
> ✓ Where n is the number of mortgages in the pool, c_i is the coupon for the ith mortgage, L_i is the remaining life of the ith mortgage.

◆ **Mortgages**

- **SMM**
- ✓ One-month mortality (SMM) is the percentage of principal outstanding prepaid in a given month.
- **CPR**
- ✓ A constant prepayment rate (also known as conditional prepayment rate) is an annualized SMM.

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

Mortgage-Backed Securities

Mortgage-Backed Securities

➢ **Agency Mortgage-Backed Securities (MBSS)**

- The simplest agency mortgage-backed securities are pass-through securities(MPS), where all investors in a pool receive the same return.

- Trading of Pass-Throughs

- ✓ Prepayment risk

- ✓ Pass-through agency securities trade as **specified pools** and **to-be-announced (TBAs)**.

Mortgage-Backed Securities

- **Specified pools market**

✓ In the specified pools market, the buyer and seller agree to trade a specific number of pools at a specific price.

- **TBA market**

✓ In the TBA market, the buyer and seller agree on: Issuer , Maturity , Coupon , Price per USD 100 of par value , Par value , Settlement month and so on.

✓ The TBA market is a forward market that attracts more trades than specified pools market .

✓ Seller has what are called the cheapest-to-deliver option.

Mortgage-Backed Securities

➢ **Dollar Roll**

● Involves selling TBA in one settlement month and buying a similar TBA in the next.

● A dollar roll is similar (in some respects) to a repo. But there are two important differences:

✓ Securities purchased in the second month may be different from those offered in the first month. The other party to the transaction may sell back the same security, but may also deliver a security with a less prepayment nature.

✓ Dollar roll transactions involve the originator losing one month's interest payments from the pool with the specified coupon, while the other party gains one month's interest.

Mortgage-Backed Securities

➤ **Value of the roll**

A-B + C-D

- A: sale price of the pool for the first month (including accrued interest, accrued interest is calculated at 30 days per month).

- B: second month purchase pool price (including accrued interest).

- C: interest on one month's sales.

- D: coupon and principal repayment for the capital pool sold in the first month.

Mortgage-Backed Securities

➤ **Other Agency Products**

- Collateralized Mortgage Obligation(CMO)

- In a CMO, a type of security is created that takes on different amounts of prepayment risk. These classes are called **tranches**.

- For example, suppose that there are Tranches A, B, and C with the following properties

✓ Tranche A investors finance 40% of the MBS principal

✓ Tranche B investors finance 30% of the MBs principal, and

✓ Tranche C investors finance the remaining 30% of the MBS principal.

Mortgage-Backed Securities

➢ **Other Agency Products**

● **Interest-only securities(IOs) and principal-only securities (POs)**

✓ IOs and POs also called **stripped MBSs**.

✓ All the interest payments from a mortgage pool go to the IOs , while all the principal payments go to the POs. Both IOs and POs are risky instruments.

✓ As prepayments increase, a POs becomes more valuable because cash flows are received earlier than expected. In contrast, IOs becomes less valuable because it pays less interest overall. When prepayments go down, the reverse happens.

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Bank Regulation

◆ Bank Regulation

➢ **Capital**

- **Equity capital** is sometimes called <u>going concern capital</u> because it absorbs losses while the bank is operating.

- **Debt capital** is called <u>gone concern capital</u> because it is only affected by losses after a bank has failed.

➢ **Regulatory Capital and Economic Capital**

- **Regulatory capital** is the <u>minimum capital that regulators require banks to keep</u>.

- **Economic capital** is the bank's <u>own estimate of its capital requirements</u>. A common goal in calculating economic capital is to <u>maintain a high credit rating.</u>

- In both cases, capital can be considered funds that can be used to <u>absorb unexpected losses</u>.

Bank Regulation

➢ **Trading Book versus Banking Book**

- **Trading book** consists of assets and liabilities <u>held for trading purposes.</u>

- **Banking book** is made up of assets and liabilities that are expected to be <u>held to maturity</u>.

- Items in the <u>trading book</u> are subject to <u>market risk capital requirement,</u> while items in the <u>banking book</u> are subject to <u>credit risk capital requirement</u>. These calculations are quite different.

➢ **Basel Requirement**

- **1988, Basel I Requirement**

✓ Require regulators in all signatory countries to <u>calculate capital requirements in the same manner</u>. Requirements were designed to <u>cover losses arising from default on loans and derivatives contracts</u>.

Bank Regulation

> **Basel Requirement**

- **1998, Market Risk Amendment**

✓ A modification to Basel I which require banks to <u>keep capital for both market risk and credit risk.</u>

- **1999, Basel II**

✓ <u>Revise the procedure for calculating credit risk capital and introduced a capital requirement for operational risk.</u> The total requirement was the sum of amounts for <u>credit risk, market risk, and operational risk.</u>

- **Basel II.5**

✓ The 2007-2008 crisis led to several bank failures and bailouts.

✓ Global bank regulators determined that the rules for <u>calculating market risk capital were inadequate and the rules were revised in Basel II.5.</u>

◆ Bank Regulation

➢ **Basel Requirement**

- **Basel III**

- The latest set of regulations includes a <u>large increase in the amount of equity capital requirement</u> are altogether called Basel Ill.

- Meanwhile, <u>the rules for market risk have been revised again</u> with the **Fundamental Review of the Trading Book**.

- As a result of the <u>liquidity problems</u> encountered during the crisis, Basel III sets two liquidity ratios that Banks must adhere to, the <u>liquidity coverage ratio and the net stable funding ratio.</u>

Types of Life Insurance

Life Insurance

➢ **Types of Life Insurance Policies**

● **Whole Life Insurance**

✓ Provides insurance for the <u>whole life of the policyholder</u>. The policyholder makes <u>regular monthly or annual payments</u> until he or she dies. At that time, the face value of the policy is paid to the designated beneficiary.

● **Term Life Insurance**

✓ Term life insurance <u>lasts a specified number of years. If the Policyholder dies during the life of the policy, there is a payout equal to the face value of the policy. Otherwise there is no payout.</u>

◆ Life Insurance

➢ **Types of Life Insurance Policies**

- **Endowment Life Insurance**

  ✓ A type of term life insurance where <u>there is always a payout at a pre-specified contract maturity. If the policyholder dies during the life of the policy, the payout occurs at the time of the policyholder's death. Otherwise, it occurs at the end of the life of the policy. Sometimes the payout is also made when the policyholder has a critical illness.</u>

- **Group Life Insurance**

  ✓ Typically <u>purchased by companies for their employees</u>. <u>The premiums may be paid entirely by the company or shared between the company and its employees.</u>

◆ Life Insurance

➢ **Types of Life Insurance Policies**

● **Annuity Contracts**

✓ While life insurance converts regular payments into a lump sum, <u>annuity contracts convert a lump sum into regular payments</u>. Typically, the <u>payments</u> in an annuity contract <u>last for the rest of the policyholder's life</u>. In some cases, the annuity starts as soon as a lump sum is deposited with the insurance company. In the case of deferred annuities, it starts several years later.

Valuation of Term Life Insurance

Life Insurance

➢ Mortality Tables

● Mortality tables are the key to valuing life insurance contracts.

Age (Years)	Males			Females		
	Probability of Death within 1 Year	Survival Probability	Life Expectancy	Probability of Death within 1 Year	Survival Probability	Life Expectancy
30	0.001498	0.97520	47.86	0.000673	0.98641	52.06
31	0.001536	0.97373	46.93	0.000710	0.98575	51.10
32	0.001576	0.97224	46.00	0.000753	0.98505	50.13
33	0.001616	0.97071	45.07	0.000805	0.98431	49.17
…	…	…	…	…	…	…
50	0.004987	0.92913	29.64	0.003189	0.95794	33.24
51	0.005473	0.92449	28.79	0.003488	0.95488	32.34
52	0.005997	0.91943	27.94	0.003795	0.95155	31.45
53	0.006560	0.91392	27.11	0.004105	0.94794	30.57
…	…	…	…	…	…	…

Life Insurance

➢ **Valuation of Term Life Insurance Contracts**

● **Example**

✓ Consider a three-year term life insurance contract for a 50-year-old woman. Assuming that deaths always occur halfway through a year, the discount rate is 5% (compounded annually). According to the above mortality table, what is the break-even premium of the $1,000,000 life insurance provided by the insurance company for the woman?

✓ Probability that she will die in the first year is 0.003189

✓ Probability that she will die during the second year is $(1 - 0.003189) \times 0.003488 = 0.003477$

✓ Probability that she will die during the third year is $(1 - 0.003189) \times (1 - 0.003488) \times 0.003795 = 0.003770$

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Life Insurance

➢ **Valuation of Term Life Insurance Contracts**

● **PV of Expected Payout per Dollar of Face Value**

Time (Years)	Expected Payout	Discount Factor	PV of Expected Payout
0.5	0.003189	0.9759	0.003112
1.5	0.003477	0.9294	0.003232
2.5	0.003770	0.8852	0.003337
Total			**0.009681**

● **PV of Expected Premiums**

Time (Years)	Probability of Receiving Premium	Discount Factor	PV of Expected Premiums
1	1.000000	1.000000	X
2	0.996811	0.952381	0.949344X
3	0.993334	0.907029	0.900983X
Total			**2.850327X**

Life Insurance

➢ **Valuation of Term Life Insurance Contracts**

- The **breakeven premium** is calculated by <u>equating the present value of expected premiums with the present value of the expected payout</u>:

$$2.850327X = 0009681$$

- This gives $X = 0.003396$. The breakeven cost of three-year term insurance for a 50-year old female is therefore 0.003396 per dollar of face value.

- For a policy with a face value of USD 1 million the breakeven premium is therefore $0.003396 \times 1,000,000 = $ USD 3,396 per year.

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◆ Life Insurance

➢ **Risks in Life Insurance**

● **Mortality Risk**

✓ The risk that wars, epidemics such as AIDS, or pandemics such as Spanish flu will lead to people <u>living not as long as expected</u>.

✓ This <u>adversely affects the payouts on most types of life insurance contracts</u> since the insured amount has to be paid earlier than expected.

✓ In <u>annuity</u> business, if people die sooner than expected, the contract will prove to be <u>less expensive</u> for the insurance company.

◆ Life Insurance

➢ **Risks in Life Insurance**

● **Longevity Risk**

✓ The risk that advances in medical sciences and lifestyle changes will lead to people living longer.

✓ Increases in longevity increases the profitability of most life insurance contracts since final payout is either delayed or less likely to happen.

✓ In annuity business, if people live longer than expected, they receive the annuity for longer and thus make the contract more expensive for the insurance company.

Property and Casualty Insurance

◆ Property and Casualty Insurance

➢ **Key Indicators**

● **Loss Ratio**

✓ Ratio of <u>payouts made to premiums</u> earned in a year.

● **Expense Ratio**

✓ Ratio of <u>expenses to premiums</u> earned in a year.

✓ <u>The two major sources of expenses are loss adjustment expenses and selling expenses</u>.

● **Combined Ratio**

✓ <u>Sum of the loss ratio and the expense</u>.

✓ Sometimes a small dividend is paid to policyholders. Ratios taking it into account is referred to as the <u>combined ratio after dividends</u>.

● **Operating Ratio**

✓ Take <u>investment income</u> into account.

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Property and Casualty Insurance

➢ **Key Indicators**

  ● **Operating Ratio for a Property-Casualty Insurance Company**

Loss Ratio	70%
Expense Ratio	<u>26%</u>
Combined Ratio	96%
Dividends	<u>1%</u>
Combined Ratio After Dividends	97%
Investment Income	<u>(2%)</u>
Operating Ratio	95%

Hedge Funds Market

◆ Hedge Funds Market

➤ **Key Differences between Hedge Funds and Mutual Funds**

- **Mutual funds**, which are called "unit trusts" in some countries, serve the needs of <u>relatively small investors</u>, while **hedge funds** seek to attract funds from <u>wealthy individuals and large investors such as pension funds</u>.

- **Hedge funds** are subject to much <u>less regulation</u> than mutual funds because they accept funds only from financially sophisticated individuals and organizations.

- **Mutual funds or ETFs** <u>allow investors to redeem</u> their shares on any given day. **Hedge funds** may have a <u>lock-up period</u> during which they cannot be redeemed.

◆ Hedge Funds Market

➢ **Key Differences between Hedge Funds and Mutual Funds**

- The <u>NAV</u> of a **mutual fund or ETF** <u>must be calculated and reported at least once a day</u>. **Hedge funds**, by contrast, have <u>no such requirement and report much less frequently</u>.

- **Mutual funds and ETFs** <u>must disclose their investment strategies</u>. **Hedge funds** typically follow a <u>proprietary strategy</u>. They give potential clients some information to explain their value proposition, but they <u>don't disclose everything</u>. Moreover, they are <u>not obliged to stick to one strategy</u>.

Hedge Funds Market

➢ **Key Differences between Hedge Funds and Mutual Funds**

- **Mutual funds and ETFs** can be <u>constrained in their use of leverage</u>. **Hedge funds** are <u>only limited by how much Banks are willing to lend to them.</u>

- **Hedge funds** charge <u>incentive fees and management fees</u>. A typical hedge fund fee is <u>2 plus 20%.</u> This means that if net profit is positive, investors are charged 2% of the value of their investment per year along with 20% of the profits.

Hedge Funds Market

➢ **Fee Structure**

● **Example: Calculate the Return on a Hedge Fund Investment**

Suppose that an investment is divided equally between two funds, A and B. Both funds charge 2 plus 20%. In the first year, Fund A earns 20% while Fund B earns −10%.

✓ Average return on investment before fees:
$$0.5 \times 20\% + 0.5 \times (-10\%) = 5\%$$

✓ Fees paid to fund A:
$$2\% + (20\% - 2\%) \times 20\% = 5.6\%$$

✓ Fees paid to fund B: 2%

✓ Average fee paid on the investment in the hedge funds:
$$0.5 \times 5.6\% + 0.5 \times 2\% = 3.8\%$$

✓ The investor is left with a 1.2% return.

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