

# Financial Markets and Products

## FRM一级培训讲义-强化班

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*101% Contribution Breeds Professionalism*



## Topic Weightings in FRM Part I

| Session NO.     | Content                        | Weightings |
|-----------------|--------------------------------|------------|
| Study Session 1 | Foundations of Risk Management | 20         |
| Study Session 2 | Quantitative Analysis          | 20         |
| Study Session 3 | Financial Markets and Products | 30         |
| Study Session 4 | Valuation and Risk Models      | 30         |

# Framework

- Bond Market
  - Interest Rates
  - Treasury Market
  - Corporate Bond
- Derivatives Market
  - Introduction of Derivatives Market
  - Forward and Futures
  - Swaps
  - Options Markets
- MBS
- Financial Institutions
  - Banks
  - Insurance Companies
  - Fund Management

# Bond Market

## Topic 1: Interest Rates

1. Market Rate
2. Compounding
3. Spot Rate and Forward Rate

# Market Rate

## ➤ Common Market Rate

### ● Treasury Rates

- ✓ The rates an investor earns on Treasury bills and Treasury bonds.
- ✓ Treasury rates are risk-free rates in the sense that it is considered highly unlikely that the government of a developed country will default on debt issued in its own currency.
- ✓ The Treasury rate is usually not adopted as risk-free rate, because it is usually artificially low, mainly due to the following two reasons:
  - Regulation generally does not require Banks to retain capital for their Treasury positions.
  - In some countries (such as the United States), Treasury yields get preferential tax treatment.

# Market Rate

## ➤ Common Market Rate

### ● LIBOR

- ✓ LIBOR are compiled from the estimated unsecured borrowing costs of 18 highly rated global banks.

### ● Repo Rates

- ✓ In a repurchase agreement, the difference between selling price (today) and the repurchased price (tomorrow or later) is called the repo rate.

### ● SOFR

- ✓ There are plans to begin phasing out Libor and replace it with a rate based on actual transactions. U.S. has proposed the use of the repo-based Secured Overnight Financing Rate (SOFR)

## ➤ Risk-Free Rate

- The risk-free rate at which derivatives are priced is determined from overnight interbank rates using overnight indexed swaps.

# Compounding

## ➤ Compounding Frequencies

- Suppose we have an account where the simple interest is added in each year and then that money also earns interest.

- Assuming

$R_C$  is the rate of interest with continuous compounding.

$R_m$  is the rate of interest with discrete compounding (m per annum)

T is the number of years.

$$FV = PV \left( 1 + \frac{R_m}{m} \right)^{mT}$$

$$FV = PV e^{R_C T}$$

$$PV \left( 1 + \frac{R_1}{m_1} \right)^{m_1 T} = PV \left( 1 + \frac{R_2}{m_2} \right)^{m_2 T}$$

$$PV \left( 1 + \frac{R_m}{m} \right)^{mT} = PV e^{R_C T}$$

# Spot Rate and Forward Rate

## ➤ Spot Rate

- A t-period spot rate, or **zero rate**, is the interest rate earned when cash is received at just one future time.

## ● Forward rates

- ✓ Interest rates corresponding to a future period implied by the spot curve.

$$(1 + R_1)^{T_1}(1 + F)^{(T_2 - T_1)} = (1 + R_2)^{T_2}$$

$$e^{R_1 T_1} \times e^{F(T_2 - T_1)} = e^{R_2 T_2} \rightarrow F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$



# Bond Market

## Topic 2: Treasury Market

### 1. Treasury Instruments

# Treasury Instruments

## ➤ Treasury Bills

- A short-term debt obligation with a maturity of one year or less.
- Interest rate is expressed on a discount basis.

## ➤ Treasury Notes and Treasury Bonds

- Bond with a maturity of more than one year. Bonds which typically have maturities between one to ten years are called Treasury Notes. But to keep the terminology simple, we will refer to all coupon-bearing Treasury instruments as Treasury Bonds.
- Both make interest payments semi-annually.
- **Quoted Price:**
  - ✓ Dollars and thirty-seconds of a dollar with face value of \$100

## ➤ Treasury STRIPS

- C-Strips and P-Strips

# Treasury Instruments

## ➤ Clean Price

- The price of a coupon bond not including any accrued interest. Immediately following each coupon payment, the clean price will equal the dirty price.

## ➤ Dirty Price

- A bond pricing quote referring to the price of a coupon bond that includes the present value of all future cash flows, including interest accruing on the next coupon payment.

$$\text{dirty price} = \text{clean price} + \text{accrued interest}$$

## ➤ Accrued Interest and Day Count Conventions

- Treasury bonds: actual/ actual
- Corporate and municipal bonds: 30/360
- Money market instruments (Treasury bills): actual/360

# Treasury Instruments

## ➤ Example

Suppose a 1000 par value US **corporate bond** pays a semi-annual 10 percent coupon on January 1 and July 1. Assume that it is now April 1, 2005, and the bond matures on July 1, 2015. Compute the invoice (full) price of this bond if the required annual yield is 8 percent. Compute the flat (clean) price of the above bond.

| Time        | Mar 1st | Apr 1st | May 1st | June 1st | July 1st |
|-------------|---------|---------|---------|----------|----------|
| dirty price | 1155.30 | 1162.87 | 1170.50 | 1178.18  | 1185.90  |
| clean price | 1138.63 | 1137.87 | 1137.17 | 1136.51  | 1135.90  |

# Bond Market

## Topic 3: Corporate Bond

1. Bond Indentures
2. Classification of Bonds
3. Bond Risk

# Bond Indentures

## ➤ Bond Indenture

- Contract contains corporate bond issuer promises and investors' rights.
- Made out to corporate trustee, who represents bondholders' interests.

## ➤ Corporate Trustee

- A financial institution that looks after the interests of the bondholders and ensures that the issuer complies with the indentures.
- Its duties are specified in the indentures and the trustee is under no obligation to exceed those duties.
- For example, sometimes the indenture specified that trustee can rely on the issuer for information, so that, it is not required to conduct its own investigations.

# **Classification of Bonds**

## ➤ **Interest Rate**

- **Fixed-Rate Bonds**
- **Floating-Rate Bonds**
- **Zero-Coupon Bonds**

## ➤ **Collateral**

- Mortgage Bonds
- Collateral Trust Bonds
- Equipment Trust Certificates
- Debenture Bonds (including Subordinated Debentures)
- Guaranteed Bonds

## ➤ **High-Yield Bond**

# Bond Risk

## ➤ Event Risk

- There are many events in the market that can adversely affect bonds, such as natural disasters. This type of risk is called event risk. One important type of event risk is the risk of a large increase in leverage.

## ➤ Credit Risk

- **Credit Default Risk:** Risk that a bond issuer will be unable to meet its financial obligations.
- **Credit Spread Risk:** Risk of financial loss resulting from changes in the level of credit spreads.



# Exercise 1



- Each of the following is true about the corporate trustee in a corporate bond issuance **except:**
- A. The trustee is paid by bondholders.
  - B. The trustee acts in a fiduciary capacity for investors who own the bond issue.
  - C. The trustee must, at the time of issue, authenticate the bonds issued (i.e., keep track of all the bonds sold) and make sure that they do not exceed the principal amount authorized by the indenture.
  - D. If a corporate issuer fails to pay interest or principal, the trustee may declare a default and take such action as may be necessary to protect the rights of bondholders.
- Answer: A

# Derivatives Market

## Topic 1: Introduction of Derivatives Market

1. Introduction of Derivatives
2. OTC and Exchange Market
3. Central Counterparty

# Introduction of Derivatives

## ➤ Derivatives

- An instrument whose value depends on the values of other more basic underlying assets.

## ➤ Basic Types of Derivatives

### ● Forward and Futures

- ✓ Agreement to buy/sell asset at future time for certain price.
- ✓ **Forward**: traded in the over-the-counter (OTC) market.
- ✓ **Futures**: Standardized and trades on an exchange.

### ● Swap

- ✓ A series of forward contracts.
- ✓ Exchange cash flows on period settlement dates.

### ● Option

- ✓ Gives holder the right (but not obligation) to buy/sell at a certain price.

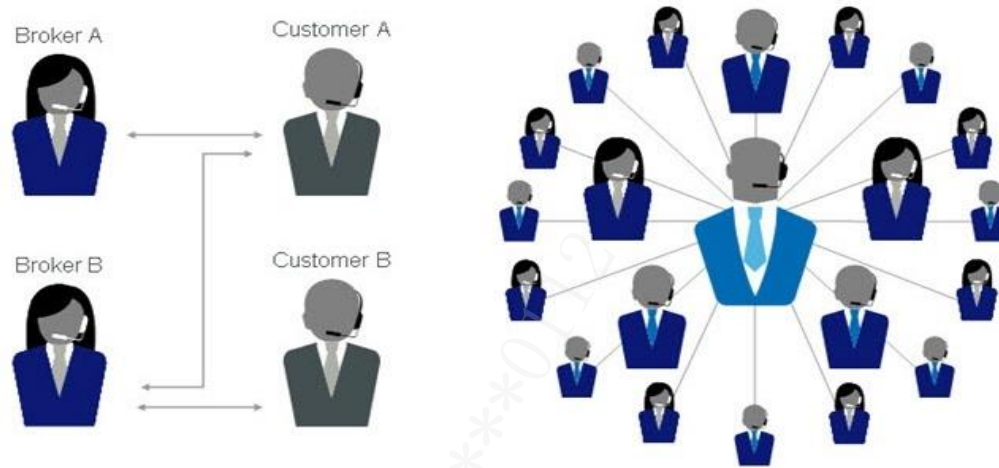
# Introduction of Derivatives

## ➤ Linear and Non-Linear Derivatives

- Derivatives can be divided into linear and nonlinear categories.
- The payoff of linear derivatives is linearly related to the value of the underlying assets. For example, forward contracts are linear derivatives.
- Options, on the other hand, are nonlinear derivatives, that is, there is a non-linear relationship between the payoff of the option and the value of the underlying asset.

# OTC and Exchange Market

## ➤ Over-the-Counter and Exchange Traded



| Over-the-Counter                       | Exchange-Traded              |
|--|------------------------------|
| Customized                             | Standardized                 |
| Trade with counterparty (Default Risk) | Backed by a clearing house   |
| Not trade in a central location        | Trade in a physical exchange |
| Unregulated                            | Regulated                    |
| Trading volume: large                  | Trading volume: small        |

# OTC and Exchange Market

## ➤ Exchange Market

- An exchange market is a market where investors trade standardized contracts made by exchanges.
- Today, exchanges clear all trades between members through so-called **central counterparties (CCPs)**.
- ✓ Exchanges (through their CCPS) act as counterparties to all members.
- ✓ Another advantage of using a CCP is that it is easier for exchange members to close out positions.
- Another measure to protect members from losses is **netting**. Netting is an operation in which short and long positions in a particular contract can offset each other.

# OTC and Exchange Market

## ➤ Exchange Market

- The exchange requires members to protect themselves by providing **margin**. Margin refers to the cash or assets transferred from one trader to another for protection against counterparty default.
- ✓ **Variation Margin**
- ✓ **Initial Margin**
- In addition, members are required to submit a **default fund** as a loss protection.
- ✓ If the initial margin is not sufficient to cover a member's losses during a default, the member's default fund contributions will be used to cover the difference. If these funds remain insufficient, they are replenished by the default funds of other members.

# OTC and Exchange Market

## ➤ Exchange Market

### ● Maintenance Margin

- ✓ So far, we've been talking about margin accounts between CCPs and their members. However, if a retail trader contacts a broker to trade, that trader will be required to provide margin to the broker.
- ✓ Margin accounts between retail traders and brokers differ from those between CCPs and their members. It generally contains provisions for maintenance margin. In accordance with the general rules of maintenance margin, if the balance of the margin account falls below the maintenance margin level, the trader must provide additional margin to restore the account to the initial margin level. If the trader does not provide additional margin, the broker enters a reverse trade on behalf of the trader to close out the position.



# Central Counterparty

## ➤ Operation of CCPs

- Variable margin payments are made daily to reflect changes in the value of each member's portfolio.
- When a member defaults, the exchange usually holds an auction, inviting other members to bid for the transaction.
- CCPs may choose to tear up deals. This involves the immediate close out of transactions between a member and the defaulting party at a price that causes some loss to the non-defaulting party.
- The initial margin to be paid by each member is calculated using historical data. However, if the default member's initial margin is insufficient to cover the loss, the default fund of the default member needs to be used to replenish. If that is not enough, the contributions from other members are used.

# **Central Counterparty**

## ➤ **Advantages and Disadvantages of CCPs**

### ● **Advantages of OTC Central Clearing**

- ✓ Easy exit
- ✓ Loss mutualization
- ✓ Standard loss management mechanism (margin, netting, default resolution)
- ✓ Increased liquidity
- ✓ Formulation of standard documents for OTC derivatives transactions.

### ● **Disadvantages of OTC Central Clearing**

- ✓ Moral hazard
- ✓ Adverse selection
- ✓ Procyclicality
- ✓ Credit risk faced by members based on default funds contribution

# Derivatives Market

## Topic 2: Forward and Futures

1. Forward Rate Agreement
2. Futures Market
3. Forward and Futures Prices
4. Interest Rate Futures
5. Hedging Strategies using Futures
6. Foreign Exchange Markets

# Forward Rate Agreement

## ➤ Forward Rate Agreement

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period.
- The buyer locks in a borrowing rate, and the seller locks in a lending rate.
- **Settlement:** The interest payment of FRA is normally paid at the end of the period. However, an FRA is usually settled at the beginning of the period covered by the FRA by convention. The payoff for the party who pays fixed and receives floating or the other side of the transaction is:

$$\frac{(R - R_K)\tau L}{1 + R\tau} \quad \text{or} \quad \frac{(R_K - R)\tau L}{1 + R\tau}$$

Where  $R$  is the realized floating rate,  $R_K$  is the fixed rate,  $L$  is the principal and  $\tau$  is the length of the time horizon.

# Forward Rate Agreement

## ➤ Valuation

- The valuation for the party who pays fixed and receives floating or the other side of the transaction is:

$$PV \left( \frac{(R_F - R_K)\tau L}{1 + R_F\tau} \right)$$

Or

$$PV \left( \frac{(R_K - R_F)\tau L}{1 + R_F\tau} \right)$$

Where  $R_F$  is the forward rate and PV denotes the present value from the beginning of the period to today.

# Futures Market

## ➤ Operation of Exchanges

- The number of contracts that exist at any time is called **open interest**. This is the number of net long contracts held by members, which is equal to the number of net short contracts held by members.
- The number of contracts traded in a day is called **trading volume**. If many traders close their positions, the volume of the day may be greater than the open interest. It can also happen if there is a large amount of intraday trading.

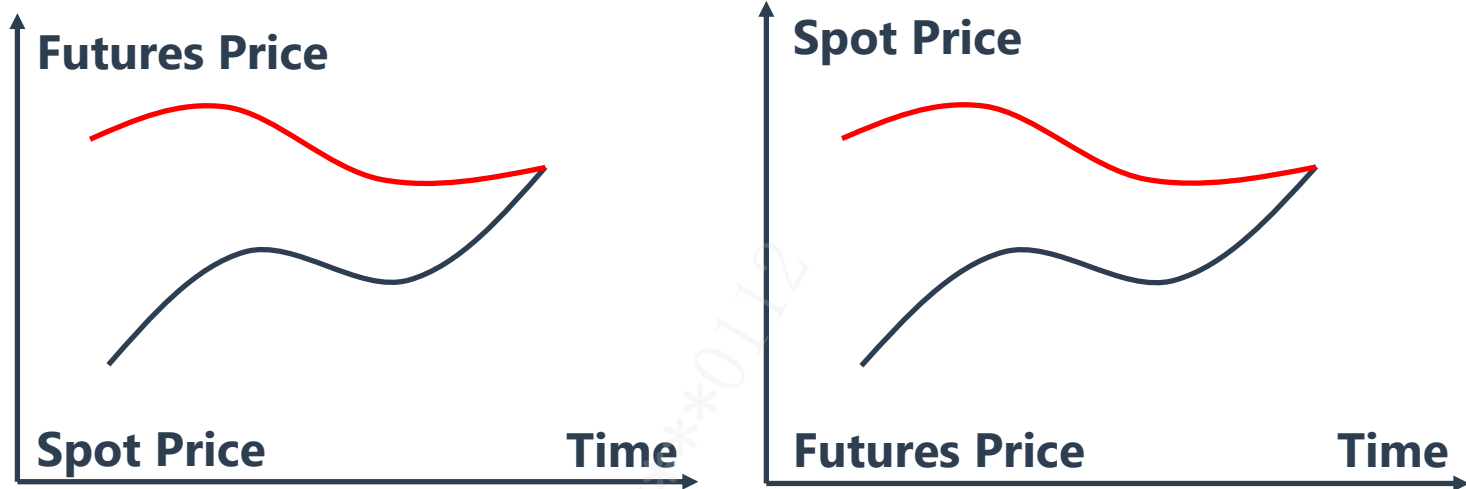
# Futures Market

## ➤ Convergence of Futures and Spot Prices

- As the delivery period approaches, the futures price converges to the spot price. If the futures price is higher than the spot price during the delivery period, the trader has an obvious arbitrage opportunity, which can be realized by:
  - ✓ Shorting futures,
  - ✓ Buying the asset, and
  - ✓ Making the delivery.
- Such arbitrage opportunities do not last long because traders take advantage of them. In addition, if the futures price is lower than the spot price during delivery, those who want access to the underlying assets will find it profitable to take long futures positions and wait for delivery. When they do so, futures prices will rise toward spot prices.

# Futures Market

## ➤ Normal and Inverted Futures Market



- If the futures price increases as time to maturity increases, the futures curve is said to be **normal**, or in **Contango**.
- If the future price declines as maturity increases, the futures curve is said to be **inverted**, or in **Backwardation**.
- Some assets have patterns that are partly normal and partly inverted.



# Futures Market

## ➤ Trading Order Types

### ● Market Order

- ✓ A request that a trade be carried out immediately at the best price available in the market.

### ● Limit Order

- ✓ This order specifies a particular price, the order can be executed only at this price or at one more favorable to the investor.

### ● Stop Order/Stop-Loss Order

- ✓ Also specifies a particular price. The order is executed at the best available price once a bid or offer is made at that particular price or a less-favorable price.

# Futures Market

## ➤ Trading Order Types

### ● Stop-Limit Order

- ✓ Combination of stop & limit order. Order becomes limit order as soon as a bid/offer is made at a price equal to/less favorable than the stop price.

### ● Market-if-Touch Order/Board Order

- ✓ Executed at the best available price after a trade occurs at a specified price/at a price more favorable than the specified price. It is designed to ensure profits are taken if sufficiently favorable price movements occur.

### ● Discretionary Order/Market-not-Held Order

- ✓ Is traded as a market order except that execution may be delayed at the broker's discretion in an attempt to get a better price.

### ● Fill-or-Kill Order

- ✓ Must be executed immediately on receipt or not at all.

# Futures Market

## ➤ Futures vs. Forward

| Forward   | Futures   |
|---|---|
| Trade over-the-counter (OTC)  | Trade on an exchange  |
| Not standardized  | Standardized contracts  |
| One specified delivery date   | Range of delivery dates   |
| <u>Settled at contract's end</u>  | <u>Settled daily</u>  |
| <u>Delivery or final cash settlement usually occurs</u>                         | <u>Contract usually closed out prior to maturity</u>                                |
| <u>Reduces basis risk</u> due to tailored specifications but <u>less liquid</u> | <u>High liquidity</u> due to standardized specifications but <u>more basis risk</u> |
| <u>Default risk is present</u>  | <u>Guaranteed by clearinghouse</u>  |
| No margin deposit required  | <u>Margin required and adjusted</u>   |

# Forward and Futures Prices

## ➤ Assumptions of Pricing: No Arbitrage Principle

$$F = S(1 + R)^T$$

$$F > S(1 + R)^T$$

*Now:*

Borrow  $S$  to buy a unit of asset, enter into a forward contract to short the asset for  $F$  in time  $T$ ;

*T later:*

- Sell asset at  $F$  and repay the loan for  $S(1 + R)^T$
- Gain a profit of

$$F - S(1 + R)^T$$

$$F < S(1 + R)^T$$

*Now:*

Short sale  $S$  and invest in a bank, enter into a forward contract to buy the asset for  $F$  in time  $T$ ;

*T later:*

- Get  $S(1 + R)^T$  from the bank and buy the asset at  $F$  to close short position.
- Gain a profit of

$$S(1 + R)^T - F$$

# Forward and Futures Prices

## ➤ Forward Price for a Financial Asset that Provides no Income

$$F = S(1 + R)^T$$

- **Example:** Consider a forward contract to sell a non-dividend-paying stock in 3 months. The current stock price is \$40 and the 3-month risk-free rate (annually compounded) is 2.5% per year. The forward price:

$$F = 40(1 + 0.025)^{0.25} = 40.25$$

## ➤ Forward Price for a Financial Asset that Paying a Known Cash Income

$$F = (S - I)(1 + R)^T$$

- **Example:** Consider a 10-month forward contract on a bond paying a USD 2 coupon in 3 months and in 9 months. Assume the  $r_f$  for all maturities is 6% per year and the cash price of the bond is USD 107.

$$\frac{2}{1.06^{0.25}} + \frac{2}{1.06^{0.75}} = 3.8856 \quad F = (107 - 3.8856) \times 1.06^{\frac{10}{12}} = 108.2450$$

# Forward and Futures Prices

## ➤ Forward Price for a Financial Asset that Provides a Known Yield

$$F = S \left( \frac{1 + R}{1 + Q} \right)^T$$

- **Example:** Consider an asset expected to provide a 2.5% yield per year over the next three years. The risk-free rate is 3% per year and the current spot price of the asset is USD 30. The forward price (USD) is

$$F = 30 \left( \frac{1 + 3\%}{1 + 2.5\%} \right)^3 = 30.44$$

## ➤ Forward Price for Stock Index

- **Example:** Consider an index of 2,000, the  $r_f$  is 4% per year and the dividend yield is 2% per year. The futures price with a maturity of six months is

$$F = 2,000 \times \left( \frac{1.04}{1.02} \right)^{0.5} = 2019.5127$$

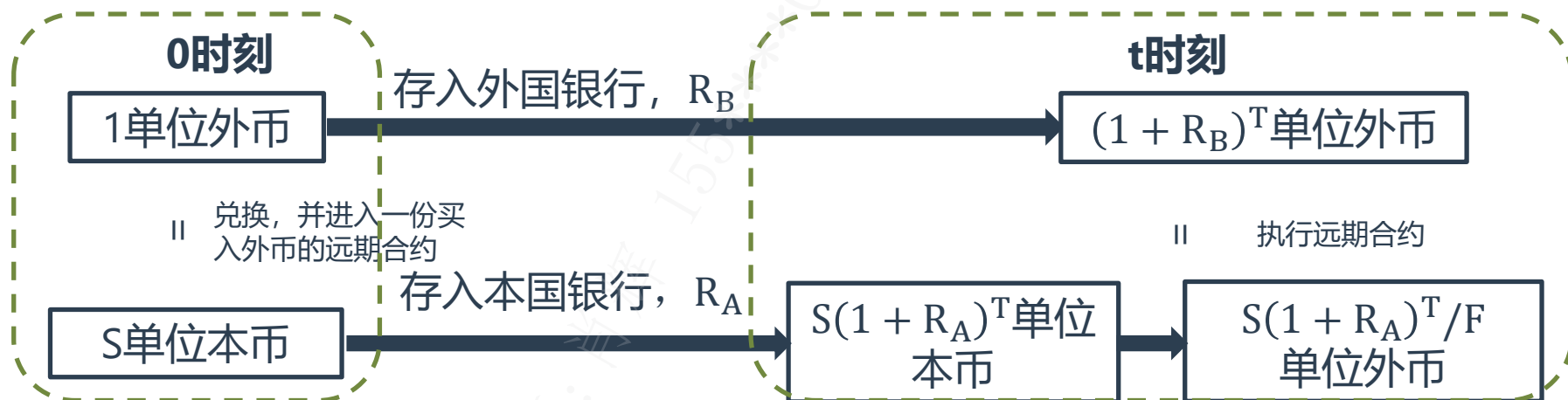
# Forward and Futures Prices

## ➤ Foreign Exchange Forward/Futures

### ● Interest Rate Parity

$$F = S \left( \frac{1 + R_A}{1 + R_B} \right)^T$$

Where B/A represents the exchange rate as the number of A per B.



# Forward and Futures Prices

## ➤ Forward Price for a Commodity Asset with a Lease Rate

$$F = S \left( \frac{1 + R}{1 + I} \right)^T$$

- **Example:** Assume that the spot price of gold is \$1,250, the lease rate is 2.5%, and the 6-month risk-free rate is 4% (with annual compounding). The 6-month futures price is given by:

$$1,250 \times \left( \frac{1.04}{1.025} \right)^{0.5} = 1,259.1131$$

## ➤ Forward Price for a Commodity with Storage Cost & Convenience Yield

$$F = (S + U) \left( \frac{1 + R}{1 + Y} \right)^T$$



# Forward and Futures Prices

## ➤ Forward Price for a Commodity with Storage Cost & Convenience Yield

- **Example:** The spot price of oil is USD 65 per barrel, and the convenience yield is 15%. The storage cost for six months has a present value of USD 3 per barrel, and the risk-free rate is 2% per year. the 6-month futures price satisfies

$$(65 + 3) \times \left( \frac{1.02}{1.15} \right)^{0.5} = 64.0413$$

# Forward and Futures Prices

## ➤ Forward Price vs. Value of a Forward Contract

- The value of a forward contract is quite different from the forward price. When forward contracts for financial assets are first entered, the value of the forward contracts themselves is zero. Over time, however, asset prices change and the value of forward contracts can become positive or negative.
- While the value of the contract changes, the price at which the asset will eventually be bought or sold remains the same as the original forward price.

$$\text{Value of Long Forward Contract} = \frac{F - K}{(1 + R)^T}$$

# Interest Rate Futures

## ➤ T-Bond Futures

- The **Treasury bond futures contract** allows the party with the short position to choose which particular bond with a maturity more than 15 years on the first day of the delivery month and is not callable within 15 years from that day to deliver.
- When a particular bond is delivered, a parameter known as conversion factor defines the price received for the bond by the party with the short position.
- Specially, the cash received by the short position is:

$$\text{Cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

- **Cheapest-to-Deliver Bond**

$$\text{Cost} = \text{quoted bond price} - (\text{QFP} \times \text{CF})$$

# Interest Rate Futures

## ➤ T-Bond Futures

- **Example:** Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is \$95.75 (this is the **quoted futures price**). Determine which bond is the cheapest-to-deliver.

| Bond | Quoted Bond Price | Conversion Factor | Cost |
|------|-------------------|-------------------|------|
| 1    | 99                | 1.01              | 2.29 |
| 2    | 125               | 1.24              | 6.27 |
| 3    | 103               | 1.06              | 1.51 |
| 4    | 115               | 1.14              | 5.85 |

# Interest Rate Futures

## ➤ Eurodollar Futures

- One of the most popular interest rate futures in the United States is the three-month Eurodollar futures contract traded by the CME Group.
- A three-month Eurodollar futures contract is a futures contract on the interest that will be paid (by someone who borrows at the Eurodollar interest rate) on \$1 million for a future three-month period.
- A final settlement price is used to determine final transfers between those with long and short positions. It is USD 100 - R, where R is the Libor fixing for 90-day USD borrowings. For example, if the USD 90-day Libor fixing is 2.5%, the final settlement price of the corresponding Eurodollar futures contract would be USD 97.50 (= 100 - 2.5).
- 1 basis point move in the futures quote corresponds to a gain/loss of \$25 per contract.

# Interest Rate Futures

## ➤ Eurodollar Futures

### ● Eurodollar Futures vs. FRA

- ✓ With the same underlying and the same maturity, They should be the same if interest rates are perfectly predictable.
- ✓  $\rho(S, r) < 0$ , Futures price is lower than forward price.
- ✓ For short maturities, the differences are small enough to be ignored.

### ● Convexity Adjustment

$$\text{Forward Rate} = \text{Futures rate} - \frac{1}{2} \sigma^2 T(T + 0.25)$$

- ✓  $\sigma$  is the standard deviation of the change in the short-term interest rate in one year.
- ✓  $T$  is time to maturity of futures contract.
- ✓  $T+0.25$  is time to maturity of the rate underlying the futures contract.

# Hedging Strategies using Futures

## ➤ Basis Risk

- The **basis** is the difference between the price of the futures contract and the spot price of the underlying asset.

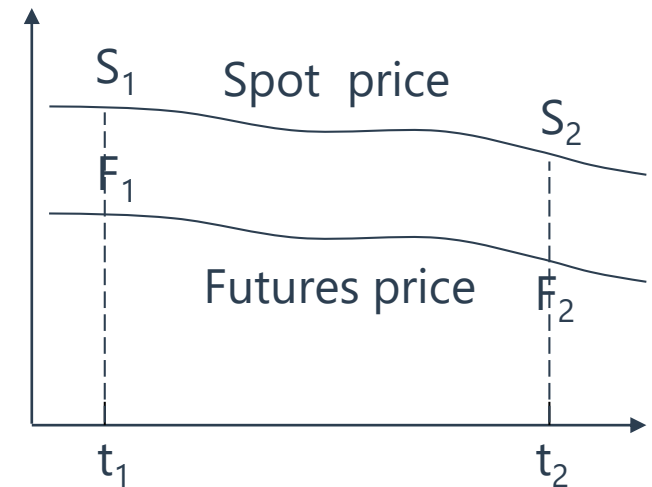
$$\text{Basis} = \text{spot price} - \text{futures price}$$

- **Long the basis** refers to a set of positions that consists of a short futures position and a long cash position. Position that are long the basis benefit when the basis is strengthening.
- **Short the basis** refers to a set of positions that consists of a long futures position and a short cash position. Positions that are short the basis benefit when the basis is weakening.

# Hedging Strategies using Futures

## ➤ Basis Risk

- Futures contract often does not track exactly with the underlying commodity. **Basis risk** is the risk (to the hedger) created by the uncertainty in the basis.
- The hedging risk is the uncertainty associated with  $b_2$ :
  - ✓ Different asset
  - ✓ Different maturity
- **Cross hedging** occurs when the assets underlying the futures contract and the asset whose price is being hedged are different.





# Hedging Strategies using Futures

## ➤ Short Hedge and Long Hedge

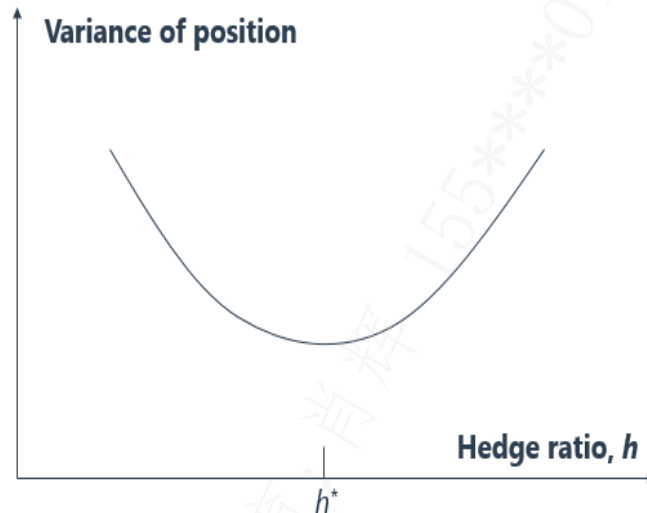
- A **short hedge** involves a short position in futures contracts. A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future.
- A **long hedge** involves a long position in a futures contract. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

### ● Minimum Variance Hedge Ratio

- ✓ The minimum variance hedge ratio depends on the relationship between changes in the spot price and changes in the futures price. By using it, we can form a hedged position with minimum variance.



Dependent of variance of hedger's position on the hedge ratio

$$h^* = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

- **Optimal Number of Futures Contracts**

$$N^* = \frac{h^* Q_A}{Q_F}$$

- ✓  $Q_A$ : Size of position being hedged (units)

- ✓  $Q_F$ : Size of one futures contract (units)

- ✓  $N^*$ : Optimal number of futures contracts for hedging

- **Tailing the Hedge**

- ✓ When futures contracts are used for hedging, there is daily settlement and series of one-day hedges. Tailing the hedge can deal with this case when making hedging decision.

$$N^* = \frac{\hat{\rho} \hat{\sigma}_S S}{\hat{\sigma}_F F} \frac{Q_A}{Q_F} = \frac{h^* \times V_A}{V_F}$$

- ✓  $\hat{\sigma}_S, \hat{\sigma}_F$  is the standard deviation of the one-day return,  $\hat{\rho}$  is the correlation between the one-day spot return and the futures return.

# Hedging Strategies using Futures

## ➤ Hedging with **Futures Contract**

- Example:** An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging. Each heating oil contract traded by the CME Group is on 42,000 gallons of heating oil.

| Month<br><i>i</i> | Change in<br>heating oil futures<br>price per gallon<br>(= $\Delta F$ ) | Change in<br>jet fuel<br>price per gallon<br>(= $\Delta S$ ) |
|-------------------|---|--|
| 1                 | 0.021   | 0.029  |
| 2                 | 0.035   | 0.020  |
| 3                 | -0.046  | -0.044   |
| 4                 | 0.001   | 0.008  |
| 5                 | 0.044   | 0.026  |
| 6                 | -0.029  | -0.019   |
| 7                 | -0.026  | -0.010   |
| 8                 | -0.029  | -0.007   |
| 9                 | 0.048   | 0.043  |
| 10                | -0.006  | 0.011  |
| 11                | -0.036  | -0.036   |
| 12                | -0.011  | -0.018   |
| 13                | 0.019   | 0.009  |
| 14                | -0.027  | -0.032   |
| 15                | 0.029   | 0.023  |

|             | $\Delta F$ | $\Delta S$ |
|-------------|------------|------------|
| Std.        | 0.031      | 0.026      |
| Correlation | 0.928      |            |

$$\begin{aligned}
 HR &= \rho_{S,F} \frac{\sigma_S}{\sigma_F} \\
 &= 0.928 \times \frac{0.026}{0.031} = 0.778
 \end{aligned}$$

$$N = 0.778 \times \frac{2000000}{42000} = 37.03$$

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

### ● Hedging with Stock Index Futures

$$\begin{aligned}\text{number of contracts} &= \beta_{\text{portfolio}} \times \frac{\text{portfolio value}}{\text{value of futures contract}} \\ &= \beta_{\text{portfolio}} \times \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\end{aligned}$$

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

### ● Hedging with Interest Rate Futures

- ✓ The number of contracts required to hedge against an uncertain change in the yield given by  $\Delta y$ , is given by:

$$N^* = \frac{PD_P}{FD_F} = \frac{DV01_P}{DV01_F}$$

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

### ● Example 1

- ✓ You are a portfolio manager with a \$20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 1,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. **Identify whether a long or short hedge is appropriate**, and determine the number of S&P 500 contracts you need to implement the hedge.

$$\text{Short } 1.4 \times \frac{\$20,000,000}{1,150 \times 250} \approx 97 \text{ contracts}$$

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

### ● Example 2

- ✓ Suppose we have a well-diversified \$100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the 3-month S&P 500 Index is 1,080. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. Demonstrate how to adjust the portfolio's beta:

$$\text{number of contracts} = (0 - 1.2) \times \frac{100,000,000}{1,080 \times 250} = -444.44$$

# Hedging Strategies using Futures

## ➤ Hedging with Futures Contract

### ● Example 3

- ✓ There is a portfolio of \$100 million with a 6-month hedging horizon. And the 6-month T-bond contract is quoted at 105-09, and the contract size is \$100,000. The duration of the portfolio is 15, and the duration of the futures contract is 17. Outline the appropriate hedge for small changes in yield.

$$N = -\frac{P \times D_P}{F \times D_F} = -\frac{100,000,000 \times 15}{105,281.25 \times 17} \approx -838$$



# Hedging Strategies using Futures

## ➤ Creating Long-Term Hedges

- Sometimes hedgers are faced with a lack of liquid futures contracts for the hedging strategy. A hedger can solve this problem by following a strategy called **stack and roll**. This involves
  - ✓ Implementing a short-term futures hedging,
  - ✓ Close the hedge out before the delivery date and replace it with another short-term futures,
  - ✓ Close the new hedge out before the delivery period, replace it with another short-term futures hedge, and so on.
- Hedgers face multiple risks because there is some uncertainty to the difference between the futures prices every time an old contract is closed out while a new one is created.

## Exercise 1



- A European firm needs to hedge the Mexican pesos in six months, but peso futures are not liquid. So the firm decided to hedge its exposure by buying futures contract on USD. The standard deviation of pesos against the Euros over a six-month period is 18%, while the standard deviation of USD/EUR futures price over a six-month period is 10%. If the correlation coefficient between pesos and dollars is 0.65, calculate the optimal hedge ratio.
- A. 0.15
  - B. 0.36
  - C. 1.17
  - D. 2.77
- Answer: C

# Foreign Exchange Quotes

## ➤ Quotes

- Currency pairs are typically indicated as **XXXYYY or XXX/YYY** (with XXX as the base currency and YYY as the quote currency). It shows how much quoted currency is required to buy a unit of base currency.
- **Bid-Ask Spread:** A bid-ask spread is the amount by which the ask price exceeds the bid price for an asset in the market.

## ➤ Outright Transaction

- A forward foreign exchange transaction, where two parties agree on an exchange at some future date, is termed an **outright transaction or a forward outright transaction.**

## ➤ FX Swap

- FX swap refers to buying (selling) a foreign currency in the spot market and then selling (buying) in the forward market.

# FX Risk

## ➤ Transaction Risk

- Risk related to receivables and payables.
- Transaction risk can be hedged with outright forward transactions.
- FX swaps are useful when a company has the foreign currency it will use to buy in the future but wants to earn interest in its own currency.

## ➤ Translation Risk

- Risk arises from assets and liabilities denominated in a foreign currency. These must be valued in a firm's domestic currency when financial statements are produced. Translation risk does not directly affect a company's cash flows, but it can have a big effect on its reported earnings.

## ➤ Economic Risk

- Risk that an enterprise's future cash flow will be affected by exchange rate movements.
- Economic risks are harder to quantify, but possible currency movements should be taken into account when making key strategic decisions.

# **FX Risk**

## ➤ **Factors that Determine Exchange Rates**

- Balance of Payments and Trade Flows
- Inflation
- Monetary Policy

## ➤ **Nominal and Real Rates**

- Nominal interest rate is usually quoted in the market and indicates the return a currency will receive.
- Real interest rates are adjusted for inflation.

$$R_{\text{real}} = \frac{1 + R_{\text{nom}}}{1 + R_{\text{infl}}} - 1 \quad R_{\text{real}} = R_{\text{nom}} - R_{\text{infl}}$$

## ➤ **Covered Interest Parity**

$$F = S \frac{(1 + R_{\text{YYY}})^T}{(1 + R_{\text{XXX}})^T}$$

# Derivatives Market

## Topic 3: Swaps

1. Interest Rate Swaps
2. Currency Swaps

# Interest Rate Swaps

## ➤ Comparative Advantage Argument

|                         | Fixed | Floating   | Comparative Advantage |
|-------------------------|-------|------------|-----------------------|
| <b>BetterCreditCorp</b> | 4%    | Libor + 1% | Fixed Market          |
| <b>WorseCreditCorp</b>  | 6%    | Libor + 2% | Floating Market       |



| Net Borrowing Rates (i.e., including swap) |       |              |
|--|-------|--------------|
|  | Fixed | Floating     |
| <b>BetterCreditCorp</b>                    |       | Libor + 0.5% |
| <b>WorseCreditCorp</b>                     | 5.5%  |              |

# Interest Rate Swaps

## ➤ Valuation

- A swap's value is zero at the time it is entered. At later times, it may have a positive or negative value because of changes in interest rates.
- Consider a swap that was entered some time ago and now has two years to maturity. Suppose that a fixed rate of 3% is received and Libor is paid every three months on a principal of USD 100 million. We also suppose the swap rate for a new two-year swap is 1.96%. And it is therefore reasonable to assume that a swap where 1.96% is paid and Libor is received is worth zero today.



# Interest Rate Swaps

## ➤ Valuation

- For valuation purposes, we can imagine a trader taking two positions.
- ✓ A two-year swap where a fixed rate of 3% is received and three-month Libor is paid on a principal of USD 100 million.
- ✓ A two-year swap where a fixed rate of 1.96% is paid and three-month Libor is received on a principal of USD 100 million.
- These two swaps net out to a position where 1.04% is received. Now suppose that the two-year risk-free rate is 2% (compounded quarterly) for all maturities. The (USD) value of the position is therefore:

$$\sum_{t=1}^8 \left( \frac{260,000}{1.005^t} \right) = 2,033,969$$

- As the value of the second swap is zero. It follows that the value of the first swap is USD 2,033,969.

# Interest Rate Swaps

## ➤ Valuation

- Large financial institutions are market makers for interest rate swaps. The bid quote is the rate the institution is willing to pay to receive Libor, whereas the ask quote is the rate the firm is willing to receive to pay Libor.
- Note that the swap rate is the average of the bid and ask quotes.
- It is sometimes necessary to interpolate between quotes to value a swap.
- For example, if a swap has a remaining life of 2.5 years and the quotes for a 2-year swap and a 3-year swap is 2.96% and 3.075% respectively, one could assume that a 2.5-year swap where the fixed rate is 3.0175% ( $= 0.5 \times (2.96\% + 3.075\%)$ ) has a value of zero.

# Interest Rate Swaps

## ➤ Valuation using Forward Rates

- The procedure is as follows:
  1. Assume that forward Libor rates will be realized.
  2. Calculate the net cash flows that will be exchanged.
  3. Discount the net cash flows at risk-free rates.



# Currency Swaps

## ➤ Currency Swaps

- Fixed-for-fixed currency swap
- One difference between a currency swap and an interest rate swap is that the principal amount is exchanged in a currency swap. Specifically, the principal amount is in the opposite direction to the interest rate payment at the beginning of the swap, and in the same direction as the interest rate payment at the end of the swap.
- Currency swaps can be used to swap liabilities and assets in a similar way to interest rate swaps.



# Currency Swaps

## ➤ Valuation

- Consider a currency swap with a remaining life of 3 years in which Interest at 4.5% in USD on a principal of USD 9 Million is exchanged for Interest at 2.5% on a principal of 7 million Euros.
- Suppose just after the last exchange of payments:
  - ✓ The risk-free interest rate in USD is 4.2% for all maturities,
  - ✓ The risk-free interest rate in euros is 3.0% for all maturities, and
  - ✓ The exchange rate (USD per euro) is 1.2.

$$\frac{405,000}{1.042} + \frac{405,000}{1.042^2} + \frac{9,405,000}{1.042^3} = 9,074,644$$

$$\frac{175,000}{1.03} + \frac{175,000}{1.03^2} + \frac{7,175,000}{1.03^3} = 6,900,999$$

$$\text{Value in USD} = 9,074,644 - 6,900,999 \times 1.2 = 793,446$$

# Currency Swaps

## ➤ Valuation using **Forward Exchange Rates**

- The procedure is as follows:
  1. Assume that forward exchange rates will be realized.
  2. Calculate the net cash flows that will be exchanged.
  3. Discount the net cash flows at risk-free rates.

# Exercise 1



- ABC corp. entered into a swap agreement over a 2-year period on August 9, 2014, with which it received a 4% fixed rate and paid LIBOR plus 1.2% on a notional amount of USD 6.5 million. Payments were to be made every 6 months. The table below displays the actual **annual 6-month** LIBOR rates over the 2-year period:

| Date               | 6-month LIBOR |
|--------------------|---------------|
| Aug 9, 2014        | 3.11%         |
| Feb 9, 2015        | 1.76%         |
| Aug 9, 2015        | 0.84%         |
| Feb 9, 2016        | <b>0.39%</b>  |
| <u>Aug 9, 2016</u> | 0.58%         |

Assuming no default, how much did corp. receive on August 9, 2016?

- A. USD 72,150
- B. USD 78,325
- C. USD 117,325
- D. USD 156,650

➤ **Answer: B**

## Exercise 2



- You are required to estimate the value of an overnight indexed swap that has three years left in its life and involves paying a fixed rate of 5% at the end of each quarter and receiving the rate implied by the overnight rate when it is compounded day-by-day during the quarter. The notional principal is USD 20 million. The current quote for a three-year overnight index swap is bid 3.80, ask 3.88. The risk-free rate is **3.6% for all maturities.** All rate are compounded quarterly.
- A. -679,591
  - B. -656,938**
  - C. -634,285
  - D. -633.819



## Exercise 2



➤ **Solution: B**

- The swap rate is the average of 3.8 and 3.88, or 3.84%.

$$V = \sum_{t=1}^{12} - \frac{(20,000,000 \times (5\% - 3.84\%) \times 0.25)}{\left(1 + \frac{3.6\%}{4}\right)^t} = \sum_{t=1}^{12} \frac{-58,000}{\left(1 + \frac{3.6\%}{4}\right)^t}$$
$$= -656,938$$

# Derivatives Market

## Topic 4: Options Market

1. Properties of Stock Options
2. Trading Strategies
3. Exotic Options

# Properties of Stock Options

## ➤ Basics

- Call and Put Options
- European and American Option

## ➤ Moneyiness

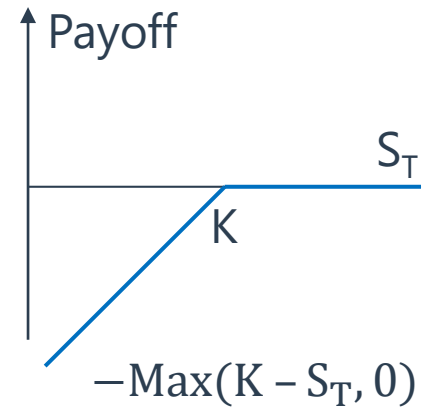
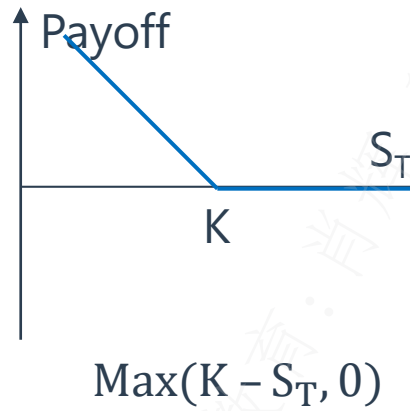
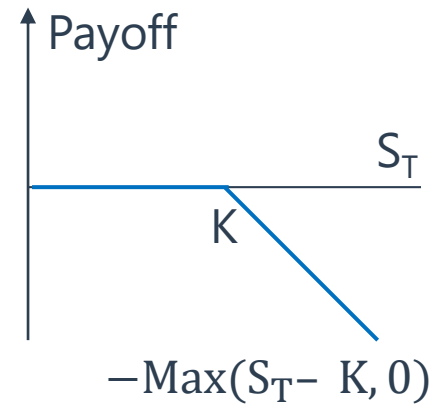
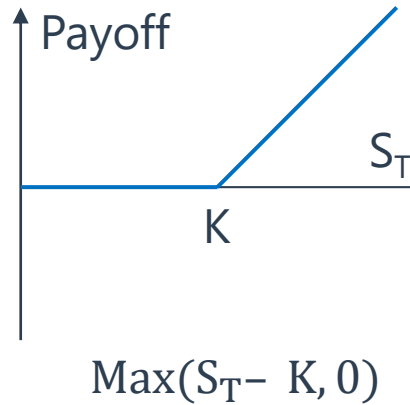
- **In the money:** Immediate exercise would generate a positive payoff
- **At the money:** Immediate exercise would generate no payoff
- **Out of the money:** Immediate exercise would result in a loss

## ➤ Intrinsic Value and Time Value

- **Intrinsic Value:** The amount that it is in the money, and zero otherwise
- ✓ Intrinsic value of call option:  $C = \max [S - X, 0]$
- ✓ Intrinsic value of put option:  $P = \max [X - S, 0]$
- **Time Value:** The difference between the price of an option (called its premium) and its intrinsic value is due to its time value.

# Calls and Puts

## ➤ Payoffs of Options



# Properties of Options

## ➤ Call Options

| Call Option            | Upper Bounds | Lower Bounds                      |
|------------------------|--------------|-----------------------------------|
| European (No Dividend) | $S_0$        | $\max(S_0 - PV(K), 0)$            |
| European (Dividend)    | $S_0$        | $\max(S_0 - PV(K) - PV(Divs), 0)$ |
| American (No Dividend) | $S_0$        | $\max(S_0 - PV(K), 0)$            |
| American (Dividend)    | $S_0$        | 视红利情况而定                           |

## ➤ Put Options

| Put Option             | Upper Bounds | Lower Bounds                      |
|------------------------|--------------|-----------------------------------|
| European (No Dividend) | $PV(K)$      | $\max(PV(K) - S_0, 0)$            |
| European (Dividend)    | $PV(K)$      | $\max(PV(K) + PV(Divs) - S_0, 0)$ |
| American (No Dividend) | $K$          | $\max(K - S_0, 0)$                |
| American (Dividend)    | $K$          | 视红利情况而定                           |

## ➤ Put-Call Parity $\text{European Call Price} + PV(K) + PV(Divs) = \text{European Put Price} + S$

# Properties of Options



- What will be the lower bound for the price of a three-month European put option on a non-dividend-paying stock if the current stock price is USD 22, the strike price is USD 25, and the risk-free rate is 6% per year (annually compounded)?

- The lower bound (USD) is

$$\max(\text{PV}(K) - S_0, 0) = \frac{25}{(1 + 0.06)^{0.25}} - 22 = 2.64$$

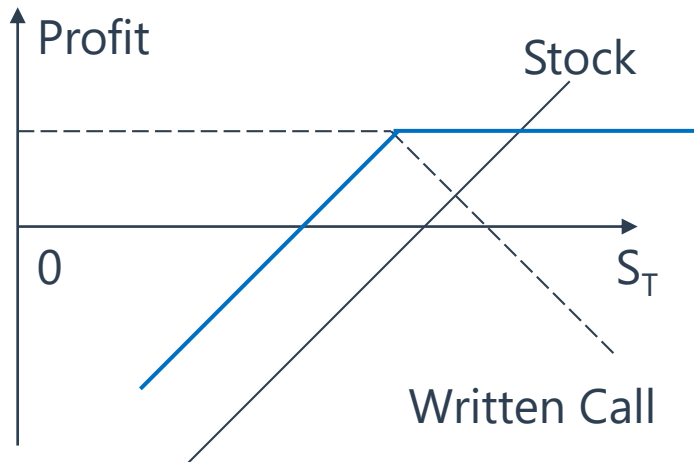
- The current price of a non-dividend-paying stock is USD 29 and the price of a four-month call option on the stock with a strike price of USD 30 is USD 2. The risk-free rate is 4% per annum (annually compounded). What is the price of a four-month put option on the stock with a strike price of USD 30? Assume no arbitrage opportunities exist.

- By put-call parity :  $\text{Put} = \text{Call} + \text{PV}(K) - S$

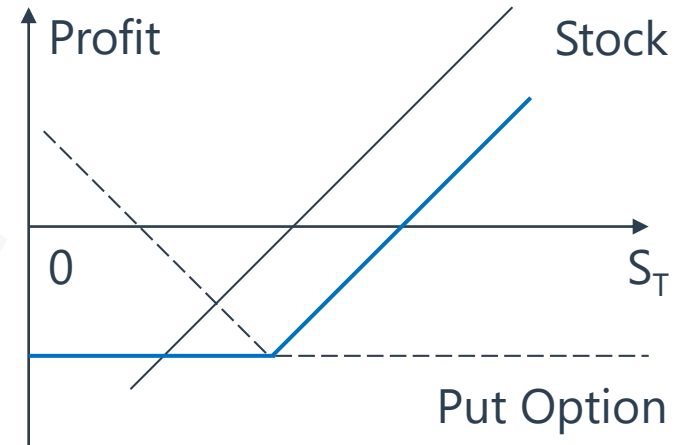
- The put price(USD) is thus given by:  $2 + \frac{30}{1.04^{1/3}} - 29 = 2.61$

# Simple Strategies

## ➤ Covered Call and Protective Put



$$\text{Covered Call} = -C + S$$



$$\text{Protective Put} = S + P$$

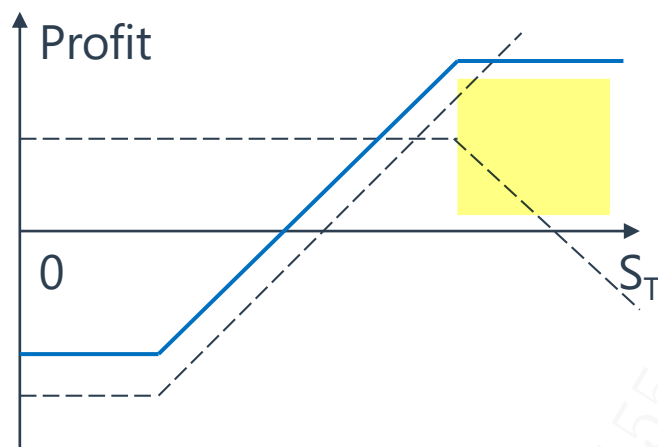
## ➤ **Principal Protected Notes(PPN)**

- A PPN is structured as a zero-coupon bond and an option with a payoff that is linked to an underlying asset, index, or benchmark.
- It guarantees a minimum return equal to the investor's initial investment (the principal amount), regardless of the performance of the underlying assets.

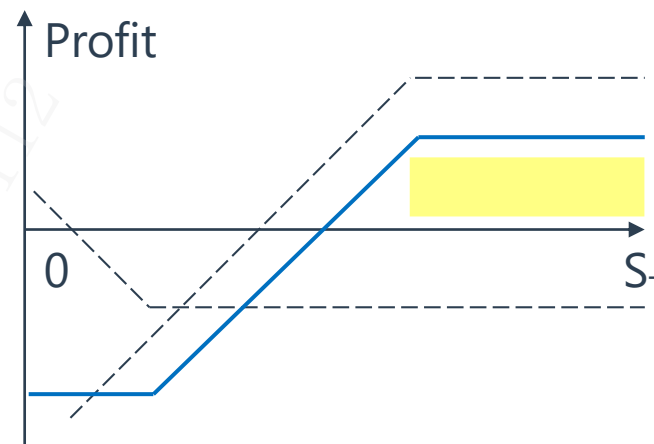
# Spread Strategies

## ➤ Bull Spread

- Vertical spread, outlook is bullish



**Bull Call Spread**



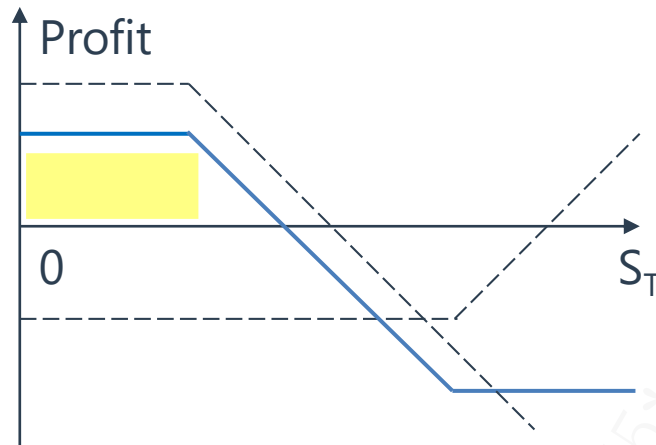
**Bull Put Spread**



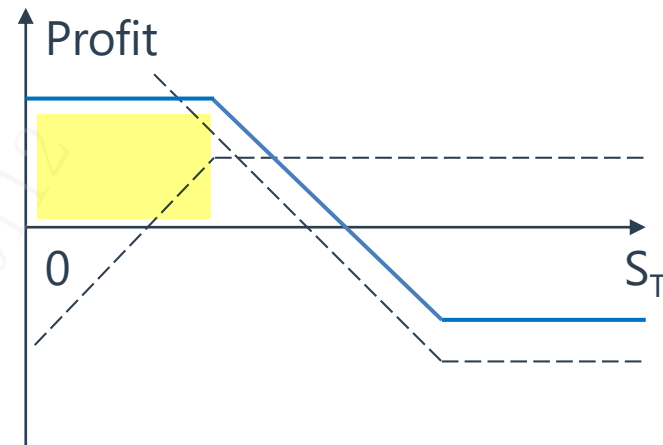
# Spread Strategies

## ➤ Bear Spread

- Vertical spread, outlook is bearish



**Bear Call Spread**



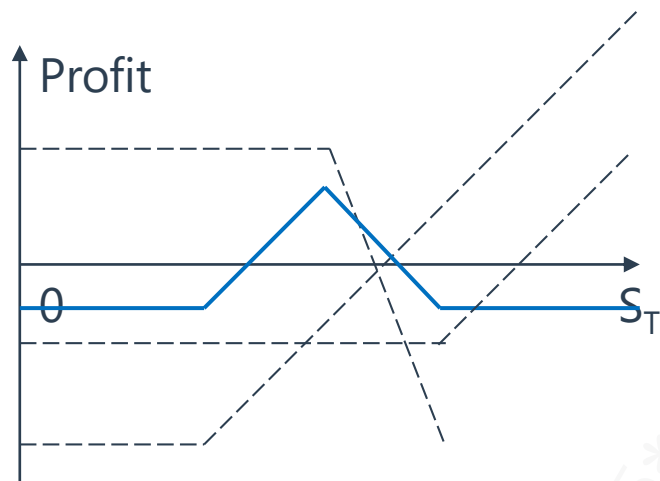
**Bear Put Spread**

## ➤ Box Spread

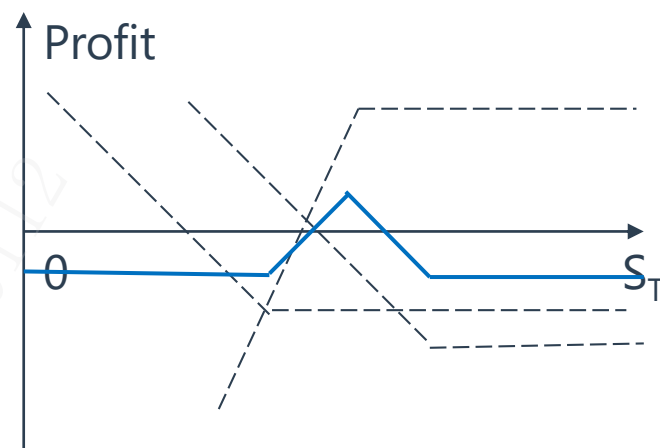
- A box spread is a combination of a bull call spread with strike prices  $K_1$  and  $K_2$  and a bear put spread with the same two strike prices.
- The payoff from a box spread is always  $K_2 - K_1$ .

# Spread Strategies

## ➤ Butterfly Spread



Butterfly Spread



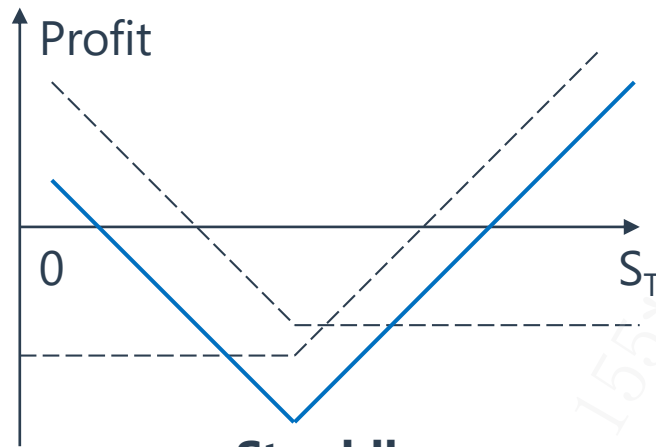
Butterfly Spread

- Expects low volatility
- Capped risk

# Combination Strategies

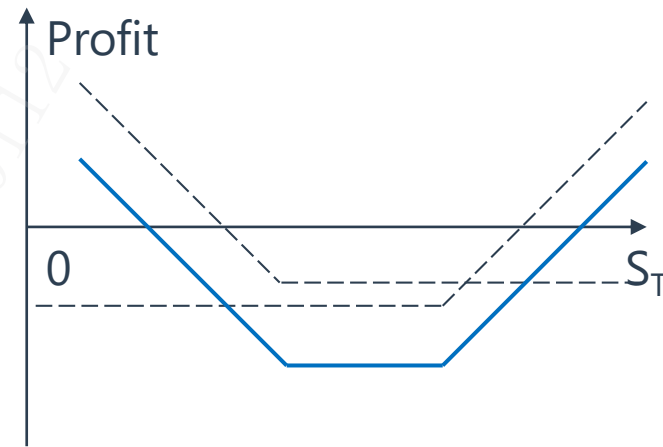
## ➤ Straddle and Strangle

- A Combination is an option trading strategy that involves taking a position in both calls and puts on the same stock.



**Straddle**

- A call and a put
- Same strike price
- Direction neutral
- Wants volatility



**Strangle**

- A call and a put
- Different strike price
- Like straddle, but cheaper
- **Wants volatility**

# Trading Strategies involving Options

## ➤ Conclusion

| <b>Simple Strategy</b>      | <b>A share and an option</b>          |
|-----------------------------|---------------------------------------|
| <b>Spread Strategy</b>      | <b>Both are call, or both are put</b> |
| Bull                        | 2 different K                         |
| Bear                        | 2 different K                         |
| butterfly                   | 3 different K                         |
| Calendar                    | 2 different T                         |
| <b>Combination Strategy</b> | <b>Call and put</b>                   |
| Straddle & Strangle         | Wants volatility                      |

# Exotic Options

## ➤ Gap Options

- The payoff from a call option is  $S_T - K_1$ , if  $S_T \geq K_2$
- The payoff from a put option is  $K_1 - S_T$ , if  $S_T \leq K_2$

## ➤ Forward Start Options

- A forward start option is an advance purchase of a put or call option that will become active at some specified future time. It is essentially a forward on an option.

# Exotic Options

## ➤ Compound Options

- Options on options
- A call on a call, a put on a call, a call on a put, and a put on a put
- If both options are exercised, the total premium will be more than the premium on a single option

## ➤ Chooser Option

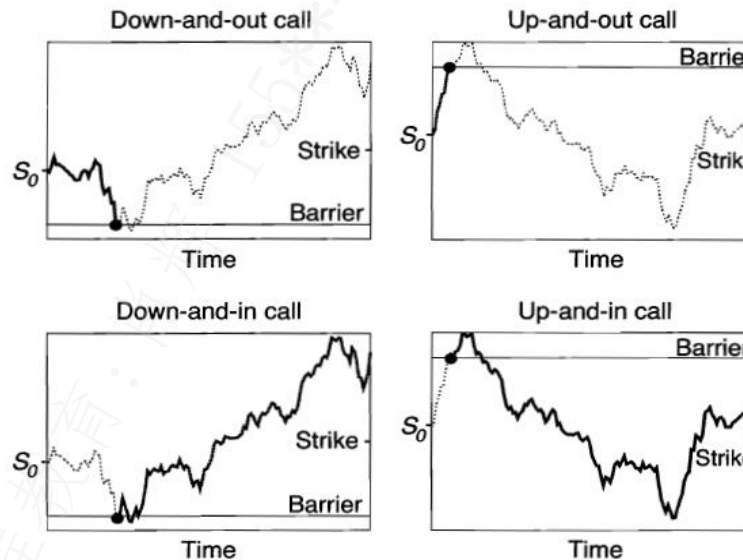
- After a specified period of time, the holder can choose whether the option is a call or a put.

$$\max( c, p )$$

# Exotic Options

## ➤ Barrier Options

- Payoffs and existence depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.
- A **knock-out** option ceases to exist when the underlying asset price reaches a certain barrier while a **knock-in** option comes into existence only when the underlying asset price reaches a barrier.



# Exotic Options

## ➤ Binary Options/Digital Options

- **Cash-or-Nothing**

- ✓ Pays some fixed amount of cash if the option expires in-the-money.

- **Asset-or-Nothing**

- ✓ Pays the value of the underlying security.

- A regular European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call.

- A regular European put option is equivalent to a long position in a cash-or-nothing put and a short position in an asset-or-nothing put.



# Exotic Options

## ➤ Lookback Options

- Payoffs depend on maximum or minimum price of the underlying asset
- With **floating strike** and with **fixed strike**.

$$\text{Call}_{\text{floating strike}} = \text{Max}(S_T - S_{\min}, 0) \quad \text{Put}_{\text{floating strike}} = \text{Max}(S_{\max} - S_T, 0)$$

$$\text{Call}_{\text{fixed strike}} = \text{Max}(S_{\max} - k, 0) \quad \text{Put}_{\text{fixed strike}} = \text{Max}(K - S_{\min}, 0)$$

## ➤ Asian Options

- Payoff depends on arithmetic average of the underlying asset price
- **Average price option** and **average strike option**.

$$\text{Call}_{\text{average price}} = \text{Max}(S_{\text{avg}} - K, 0) \quad \text{Put}_{\text{average price}} = \text{Max}(K - S_{\text{avg}}, 0)$$

$$\text{Call}_{\text{average strike}} = \text{Max}(S_T - S_{\text{avg}}, 0) \quad \text{Put}_{\text{average strike}} = \text{Max}(S_{\text{avg}} - S_T, 0)$$

# Exotic Options

## ➤ Volatility and Variance Swap

### ● Volatility Swap

- ✓ Exchanging of volatility based on a notional principal
- ✓ Payments based on pre-specified volatility and realized volatility.

### ● Variance Swap

- ✓ Exchanging pre-specified fixed variance rate for realized variance rate

## ➤ Static Options Replication

- This technique involves searching for a portfolio of actively traded options (regular options) that approximately replicates the exotic option. Shorting this position provides the hedge.



# MBS

## Topic 1: MBS

1. Mortgages
2. Mortgage-Backed Securities

# Mortgages

## ➤ Monthly Payments

- The amortization statement shows the monthly principal and interest payments on the mortgage (assuming the principal is not repaid in advance).
- For example
  - ✓ A 30-year U.S. mortgage where the fixed rate is 6% with monthly compounding. If the amount borrowed is USD 300,000.
  - ✓ Payments of USD 1798.65 per month therefore fully amortize (i.e., repay) borrowings of USD 300,000 over 30 years.

# Mortgages

| Month | End-of-Month Interest Payment | End-of-Month Principal payment | Balance at End-of-Month |
|-------|-------------------------------|--------------------------------|-------------------------|
| 0     |                               |                                | 300,000.00              |
| 1     | 1,500.00                      | 298.65                         | 299,701.35              |
| 2     | 1,498.51                      | 300.14                         | 299,401.20              |
| 3     | 1,497.01                      | 301.65                         | 299,099.56              |
| 4     | 1,495.50                      | 303.15                         | 298,796.40              |
| 5     | 1,493.98                      | 304.67                         | 298,491.73              |
| ...   | ...                           | ...                            | ...                     |
| 356   | 44.30                         | 1,754.35                       | 7,105.57                |
| 357   | 35.53                         | 1,763.12                       | 5,342.44                |
| 358   | 26.71                         | 1,771.94                       | 3,570.50                |
| 359   | 17.85                         | 1,780.80                       | 1,789.70                |
| 360   | 8.95                          | 1,789.70                       | 0.00                    |

In the first month

- ✓ The interest on the mortgage is

$$0.005 \times \text{USD } 300,000 = \text{USD } 1,500.00$$

- ✓ Repayment of principal on the mortgage is

$$\text{USD } 1,798.65 - \text{USD } 1,500.00 = \text{USD } 298.65$$

# Mortgages

- Calculate the principal outstanding by **discounting the remaining cash flow**.
- ✓ the relationship between the amount borrowed  $A$ , the interest rate  $R$  (compounded monthly), and the monthly payment  $X$  is

$$\frac{X}{R/12} \left[ 1 - \frac{1}{(1 + R/12)^{12T}} \right] = A$$

where  $T$  years is the life of the mortgage

# Mortgages

## ➤ Mortgage portfolios (mortgage pools)

### ● WAC

- ✓ The weighted average coupon (WAC) is the weighted average interest rate of the mortgage pool and the weighting allocated to each mortgage is proportional to the principal outstanding.

### ● WAM

- ✓ The weighted-average maturity (WAM) is similarly calculated as the weighted-average of the number of months to maturity, with the weighting of each mortgage proportional to the principal outstanding

$$WAC = \sum_{i=1}^n w_i c_i \qquad WAM = \sum_{i=1}^n w_i L_i$$

- ✓ Where  $n$  is the number of mortgages in the pool,  $c_i$  is the coupon for the  $i$ th mortgage,  $L_i$  is the remaining life of the  $i$ th mortgage.

# Mortgages

- **SMM**

- ✓ One-month mortality (SMM) is the percentage of principal outstanding prepaid in a given month.

- **CPR**

- ✓ A constant prepayment rate (also known as conditional prepayment rate) is an annualized SMM.

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$



# Mortgage-Backed Securities

## ➤ Agency Mortgage-Backed Securities (MBSs)

- The simplest agency mortgage-backed securities are pass-through securities(MPS), where all investors in a pool receive the same return.
- Trading of Pass-Throughs
- ✓ Prepayment risk
- ✓ Pass-through agency securities trade as **specified pools** and **to-be-announced (TBAs)**.

# Mortgage-Backed Securities

## ➤ Agency Mortgage-Backed Securities (MBSs)

### ● Dollar Roll

- ✓ Involves selling TBA in one settlement month and buying a similar TBA in the next. A dollar roll is similar (in some respects) to a repo. But there are two important differences:
- Securities purchased in the second month may be different from those offered in the first month. The other party to the transaction may sell back the same security, but may also deliver a security with a less prepayment nature.
- Dollar roll transactions involve the originator losing one month's interest payments from the pool with the specified coupon, while the other party gains one month's interest.

# Mortgage-Backed Securities

## ➤ Agency Mortgage-Backed Securities (MBSs)

- Value of the roll

$$A - B + C - D$$

- ✓ A: sale price of the pool for the first month (including accrued interest, accrued interest is calculated at 30 days per month).
- ✓ B: second month purchase pool price (including accrued interest).
- ✓ C: interest on one month's sales.
- ✓ D: coupon and principal repayment for the capital pool sold in the first month.

# Mortgage-Backed Securities

## ➤ For example

- Suppose a 4.5% pool with a face value of \$1 million was sold at \$101.50 in August and bought back at \$101.00 in September.
- The sales revenue of the first month can be invested at 0.1% of that month.
- The pool is not sold, then the interest and principal payments on the pool during the month are going to be 0.4% of face value.
- What is the value of the roll?(The payment date is the twelfth of the month for both months )

## ➤ Answer

Accrued interest is USD1,500  $(=(12/30) \times (0.045/12) \times 1,000,000)$

A= USD 1,016,500 and B=USD1,011,500

C= \$1,016.5  $(1,016,500 \times 0.1\%)$

D= \$4000.0  $(1,000,000 \times 0.4\%)$

The value of the roll (USD) is  $1,016,500 - 1,011,500 + 1,016.5 - 4000.0 = 2016.5$

# Mortgage-Backed Securities

## ➤ Other Agency Products

- Collateralized Mortgage Obligation(CMO)
- In a CMO, a type of security is created that takes on different amounts of prepayment risk. These classes are called **tranches**.
- For example, suppose that there are Tranches A, B, and C with the following properties
  - ✓ Tranche A investors finance 40% of the MBS principal
  - ✓ Tranche B investors finance 30% of the MBs principal, and
  - ✓ Tranche C investors finance the remaining 30% of the MBS principal.

# Mortgage-Backed Securities

## ➤ Other Agency Products

- Interest-only securities (IOs) and principal-only securities (POs)

- ✓ IOs and POs also called **stripped MBSs**.
- ✓ All the interest payments from a mortgage pool go to the IOs, while all the principal payments go to the POs. Both IOs and POs are risky instruments.
- ✓ As prepayments increase, a POs becomes more valuable because cash flows are received earlier than expected. In contrast, IOs becomes less valuable because it pays less interest overall. When prepayments go down, the reverse happens.

# Mortgage-Backed Securities

## ➤ Evaluating the MBS

- The first step in evaluating the MBS pool is to develop a prepayment model.
- Two variables that future prepayments may depend on are as follows
  - ✓ the level of interest rates
  - ✓ Prices
- Monte Carlo simulation is used to evaluate MBS. This includes the following steps

# Mortgage-Backed Securities

## ➤ **Option-adjusted Spread (OAS)**

- the excess of the expected return provided by a fixed-income instrument over the risk-free return adjusted to account for embedded options.
- Complete the following steps
  1. Make an initial estimate of the OAS.
  2. Carry out a Monte Carlo simulation using discount rates equal to the Treasury rate plus the current estimate of the OAS.
  3. Compare the obtained price with the market price.
  4. If the market price is higher than the simulated price, the OAS estimate is reduced. If the market price is lower than the simulated price, increase the OAS estimate.
  5. Continue to change the OAS estimate until the simulated price equals the market price.



# Financial Institutions

## Topic 1: Financial Institutions

1. Banks
2. Insurance Companies
3. Fund Management



# Banks

## ➤ Service Features

### ● Originate-to-Distribute Model

#### ✓ Securitization

- ✓ By securitizing the loans it gets them off the balance sheet and frees up funds to enable it to make more loans. It also frees up capital that can be used to cover risks being take elsewhere in the bank. A bank can earn a further fee if it services the loan after it has been sold.
- ✓ Banks may relax their mortgage lending standards and the credit quality of the instruments being originated may decline sharply.



# Banks

## ➤ Service Features

### ● Investment Banking

- ✓ **Private Placement:** large institutional investors
- ✓ **Public Offering:** general public
  - Best Efforts
  - Firm Commitment
- ✓ **Initial Public Offering (IPO)**
  - Issues shares is not publicly traded
  - Dutch auction approach



# Banks

## ➤ Essentials of Management

### ● Conflicts of Interest Problem

#### ✓ Main Problems

- Between Security Trading and Investment Banking
- Between Commercial Banking and Investment Banking

#### ✓ Recommend Solutions - Internal Barriers (Chinese Walls)

- Prohibit the transfer of information from one part of the bank to another



# Banks

## ➤ Essentials of Management

### ● Three Main Types of Risk Facing Banks

- ✓ Market Risk
- ✓ Credit Risk
- ✓ Operational Risk

### ● Banking Book and Trading Book

- ✓ **Banking Book:** Includes loans made to corporations and individuals
- ✓ **Trading Book:** Includes all assets & liabilities bank has as a result of trading operations



# Banks

## ➤ Essentials of Management

- **Capital Management**

- ✓ **Regulatory Capital**

- Requirement for central bank regulators

- ✓ **Economic Capital**

- Own management requirement

- **Deposit Insurance**

- ✓ Guaranty programs introduced by government regulators

- ✓ Insure depositors against losses up to a certain level

- ✓ Moral hazard problem

# Insurance Companies

## ➤ Service Features

- **Life Insurance**
- ✓ **Mortality tables**
- ✓ **Break-even Premium:** calculated by finding the value of premium by equating PV of expected premium to PV of the expected payout.
- **Basic Risks**
- ✓ **Mortality Risk:** living not as long as expected
  - Adversely affects most types of life insurance contracts
  - Increase profitability of annuity contracts
- ✓ **Longevity Risk:** living longer
  - Increases the profitability of most life insurance contracts
  - Adversely affects the profitability of most types of annuity contracts
- ✓ **Hedging:** reinsurance, longevity derivative contract (longevity bond)

# Insurance Companies

| Age (Years) | Males                              |                      |                 | Females                            |                      |                 |
|-------------|------------------------------------|----------------------|-----------------|------------------------------------|----------------------|-----------------|
|             | Probability of Death within 1 Year | Survival Probability | Life Expectancy | Probability of Death within 1 Year | Survival Probability | Life Expectancy |
| 30          | 0.001498                           | 0.97520              | 47.86           | 0.000673                           | 0.98641              | 52.06           |
| 31          | 0.001536                           | 0.97373              | 46.93           | 0.000710                           | 0.98575              | 51.10           |
| 32          | 0.001576                           | 0.97224              | 46.00           | 0.000753                           | 0.98505              | 50.13           |
| 33          | 0.001616                           | 0.97071              | 45.07           | 0.000805                           | 0.98431              | 49.17           |
| ...         | ...                                | ...                  | ...             | ...                                | ...                  | ...             |
| 50          | 0.004987                           | 0.92913              | 29.64           | 0.003189                           | 0.95794              | 33.24           |
| 51          | 0.005473                           | 0.92449              | 28.79           | 0.003488                           | 0.95488              | 32.34           |
| 52          | 0.005997                           | 0.91943              | 27.94           | 0.003795                           | 0.95155              | 31.45           |
| 53          | 0.006560                           | 0.91392              | 27.11           | 0.004105                           | 0.94794              | 30.57           |
| ...         | ...                                | ...                  | ...             | ...                                | ...                  | ...             |
| 70          | 0.023380                           | 0.73427              | 14.32           | 0.015612                           | 0.82818              | 16.53           |
| 71          | 0.025549                           | 0.71710              | 13.66           | 0.017275                           | 0.81525              | 15.78           |
| 72          | 0.027885                           | 0.69878              | 13.00           | 0.019047                           | 0.80117              | 15.05           |
| 73          | 0.030374                           | 0.67930              | 12.36           | 0.020909                           | 0.78591              | 14.34           |



# Insurance Companies



- What is the minimum USD annual premium that an insurance company should charge for a two-year term life insurance policy with face value of USD 1 million when the policyholder is a woman aged 71? Assume an interest rate of 3% compounded annually.
- A. 18,153
  - B. 17,874
  - C. 17,996
  - D. 17,767

# Insurance Companies

## ➤ Correct Answer: B

- The probability of a payout in the first year (time 0.5years) is 0.017275.  
The probability of a payout in the second year (time 1.5 years) is

$$(1-0.017275) \times 0.019047 = 0.018718$$

- The PV of the expected cost of the policy is therefore

$$17,275 / (1.03^{0.5}) + 18,718 / (1.03^{1.5}) = 34,928$$

- The first premium is at time zero
- The second premium, at time one year, has a probability of  $1 - 0.017275 = 0.982725$  of being made. If the premium is  $X$ , the expected present value is  $X + 0.982725X / 1.03 = 1.954102X$ . The minimum premium is given by solving  $1.954102X = 34,928$
- It is 17,874

# Insurance Companies

## ➤ Service Features

- **Property-Casualty Insurance**

- ✓ **Loss Ratio**

- Payouts/Premiums

- ✓ **Expense Ratio**

- Expenses/Premiums

- ✓ **Combined Ratio**

- Loss Ratio + Expense Ratio

- ✓ **Combined Ratio after Dividends**

- Combined Ratio + Dividend Yield

- ✓ **Operating Ratio**

- Combined Ratio after Dividend – Investment Income

# Insurance Companies

## ➤ Example

### ● Operating Ratio for a Property-Casualty Insurance Company

|                                |             |
|--------------------------------|-------------|
| Loss Ratio                     | 70%         |
| Expense Ratio                  | <u>26%</u>  |
| Combined Ratio                 | 96%         |
| Dividends                      | <u>1%</u>   |
| Combined Ratio After Dividends | 97%         |
| Investment Income              | <u>(2%)</u> |
| Operating Ratio                | 95%         |

# Insurance Companies

## ➤ Service Features

### ● Pension Plan

#### ✓ Defined Benefit Plan

- Pension that employee will receive is defined by the plan.
- All contributions are pooled and to retirees are made out of the pool.
- Significant risks on employers because they are ultimately responsible for paying the promised benefits.

#### ✓ Defined Contribution Plan

- Contributions are invested on behalf of the employee.
- An account is set up for each employee and the pension is calculated only from the funds contributed to that account.
- If the performance of the plan's investment is less than anticipated, the employee bears the cost.

# Insurance Companies

## ➤ Essentials of Management

### ● Risks facing Insurance Companies

#### ✓ Moral Hazard

- Risk that existence of insurance will cause the policyholder to behave differently than he or she would without the insurance.
- **Solution:** Aligning interest of policyholders more closely with those of insurance company: Deductible; Co-Insurance Provision; Policy Limit.

#### ✓ Adverse Selection

- Risk arises when company cannot distinguish between good and bad risks and offers the same price. This will inadvertently attracts more of the bad risks.
- **Solution:** Try to find out as much as possible about the policyholder before committing itself.

# Fund Management

## ➤ Mutual Fund

- **Open-End Funds:** Shares in fund can be bought/sold back at any time.

### ✓ NAV

- Market value of the portfolio/Number of Shares Outstanding

- **Closed-End Funds:** Have a fixed number of shares outstanding.

### ✓ NAV

- Price at which the shares of the fund are trading
- Market value of the fund's portfolio/Number of Shares Outstanding

## ➤ Exchange-Traded Funds

- Investors can give up shares they hold and receive the block of securities or they can deposit new block of securities and receive new shares.
- Some or all of the shares in the ETF are then traded on a stock exchange.
- No appreciable difference between trading price and fair market value

# Fund Management

## ➤ Undesirable Trading Behavior in Mutual Fund and ETF Market

- Late Trading (not permitted by SEC)
- Market Timing (not illegal)
- Front Running (illegal)
- Directed Brokerage (frown upon by regulators)



# Fund Management

## ➤ Mutual Funds vs Hedge Funds

|                       | Mutual Fund                 | Hedge Fund   |
|-----------------------|-----------------------------|--|
| Client Group          | Relatively Small Investors  | Wealthy Individuals/Large Investors                |
| Redemption Provisions | Can Redeem on Any Given Day | Lock-Up Period                                     |
| Regulatory            | Stringent                   | Less   |
| NAV                   | At Least once a Day         | No Such Requirement                                |
| Strategies            | Must Disclose               | Don't Disclose Everything                          |
| Leverage              | Constrained                 | Only Limited by How Much Banks are Willing to Lend |
| Fee Structure         | Management                  | Management + Incentive                             |

# Fund Management



- What is the expected fee to a hedge fund if the fund uses a standard 2 and 20 incentive fee structure with an investment that has a 35% probability of making 55% and a 65% probability of losing 45%?

- A. 5.71%
- B. 6.12%
- C. 3.78%
- D. 5.28%

- **Answer: A**

The hedge fund could potentially earn fees of 12.6% [2% (flat fee) +  $0.20 \times 53\%$  (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.71% computed as follows:  $(0.35 \times 12.6\%) + (0.65 \times 2\%) = 5.71\%$

# Fund Management

## ➤ Hedge Fund Strategies

### ● Long/Short Equity

- ✓ The hedge fund manager identifies a set of stocks that are considered to be undervalued by the market and a set that are considered to be overvalued.

### ● Dedicated Short

- ✓ Look exclusively for overvalued companies and sell them short

### ● Fixed Income Arbitrage

- ✓ Buy bonds that seem relatively cheap while shorting those that are relatively expensive.

### ● Convertible Arbitrage

- ✓ They hedge risks related to a company's share price, credit spreads and interest rates.



# Fund Management

## ➤ Hedge Fund Strategies

- **Distressed Securities**

- ✓ Managers of funds are searching for debt that is undervalued by the market.

- **Merger Arbitrage**

- ✓ Cash Deals
- ✓ Share-for-Share Exchanges

- **Global Macro**

- ✓ Reflect global macroeconomic trends.

- **Managed Futures**

- ✓ Attempt to predict future movements in commodity prices

- **Emerging Markets**

- ✓ Specialize in investments associated with developing countries.

## 问题反馈

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    - ✓ 所在班级（eg.2205FRM一级长线无忧班）
    - ✓ 问题所在科目（若未知科目，请提供章节、知识点）和页码
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