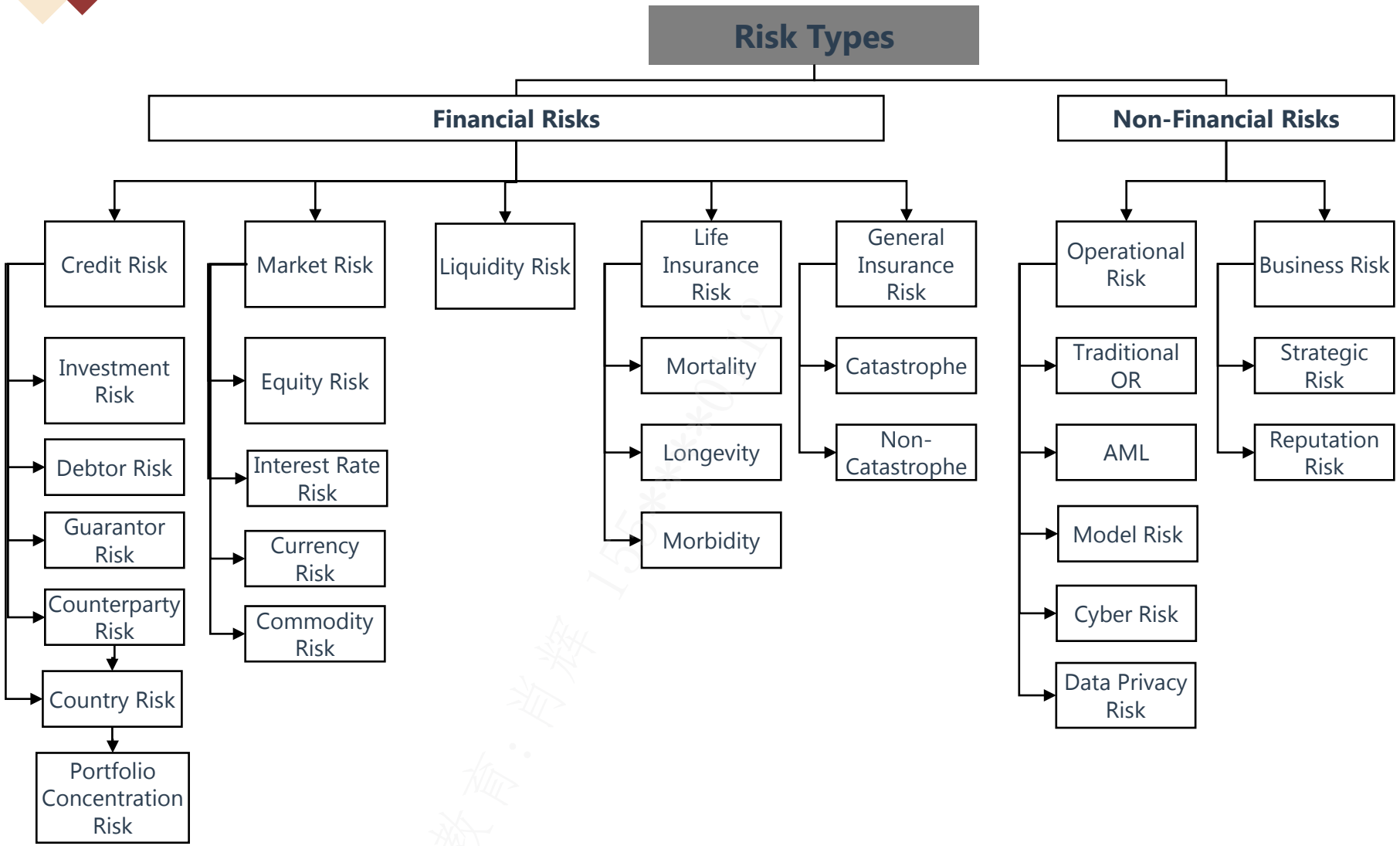
FRM一级核心知识点

风险管理基础

Basic Risk Types


Risk Types
Financial Risks
Non-Financial Risks
Credit Risk
Investment Risk
Debtor Risk
Guarantor Risk
Counterparty Risk
Country Risk
Portfolio Concentration Risk
Market Risk
Equity Risk
Interest Rate Risk
Currency Risk
Commodity Risk
Liquidity Risk
Life Insurance Risk
Mortality
Longevity
Morbidity
General Insurance Risk
Catastrophe
Non-Catastrophe
Operational Risk
Traditional OR
AML
Model Risk
Cyber Risk
Data Privacy Risk
Business Risk
Strategic Risk
Reputation Risk

Typology of Risk Exposures

- **Market risk** is the risk that changes in financial market prices and rates will reduce the value of a security or a portfolio.
  - Equity price risk
  - Interest rate risk
    - ✓ Trading risk
    - ✓ Gap risk (the risk that arises in the balance sheet of an institution)
  - Currency (Foreign exchange) risk
  - Commodity price risk
- **Market risk is driven by:**
  - General market risk (systematic risk)
  - Specific risk (idiosyncratic risk)

Typology of Risk Exposures

- **Credit Risk:** the risk of an economic loss from the failure of a counterparty to fulfil its contractual obligations, or from the increased risk of default during the term of the transaction.
- **Credit risk is driven by:**
  - Probability of default of the obligor or counterparty
  - Exposure amount at the time of default
  - Loss given default
- **Credit risk type**
  - Default risk
  - Bankruptcy risk
  - Downgrade risk
  - Settlement (Herstatt) risk

Typology of Risk Exposures

- **Liquidity Risk**
  - **Funding Liquidity Risk:** a firm's ability to raise the necessary cash to roll over its debt; to meet the cash requirements, the margin requirements, and the collateral requirements; and to satisfy the need of capital withdrawals.
  - **Market (Trading) Liquidity Risk:** an institution will not be able to execute a transaction at the prevailing market price because there is temporarily no appetite for the deal on the other side of the market.
    - ✓ Endogenous vs. Exogenous

Typology of Risk Exposures

- **Operational Risk** refers to potential losses resulting from inadequate or failed internal processes, people, and systems or from external events.
- It includes legal risk, but excludes business, strategic and reputational risk.
  - Human factor risk
    - ✓ ***Human factor risk*** is a special form of operational risk. Such as pushing the wrong button on a computer.
  - Technology risk
    - ✓ ***Technology risk***, principally computer systems risk, also falls into the operational risk category.

Typology of Risk Exposures

- **AML Risk**
  - Anti money laundry and financing for terrorism
- **Cyber Risk**
  - The risk of hackers stealing and destroying data and compromising systems
- **Data Privacy Risk**
- **Model risk**
  - The risk of potential indirect costs of relying on models

Typology of Risk Exposures

- **Business Risk:** lies at the heart of any business and includes all the usual worries of firms, such as customer demand, pricing decisions, supplier negotiations, competition, and managing product innovation.
- **Strategic Risk:** involves making large, long-term decisions about the firm's direction, often accompanied by major investments of capital, human resources, and management reputation.
- **Reputation Risk:** is the danger that a firm will suffer a sudden fall in its market standing or brand with economic consequences.

Risk Appetite

True Risk Governance

- **Risk Appetite statement - ”a written articulation of the aggregate level and types of risk that a firm will accept or avoid in order to achieve its business objectives.”**
  - The board must characterize an appropriate “ risk appetite” for the firm.
  - Be connected to a firm’s overall business strategy and capital plan.
  - Clear communication throughout the firm of the firm's risk appetite and risk position.
  - Effective risk management program should be consistent with fundamental strategic and risk appetite choices.
  - Risk appetites can be expressed in a number of ways, including quantitative and qualitative statements.

Determining A Bank's Risk Appetite

- **A bank's optimal credit rating approaches**
  - A bank targets a certain credit rating.
    - ✓ The optimal rating of a bank is generally not the highest rating.
    - ✓ Banks with very different strategies, or liability and asset structures, could end up having very different credit ratings, and different attitudes towards risk.
    - ✓ A bank with more deposit franchise and with more relationship lending is likely to prefer a higher rating than an institution that is engaged in more transactional activities.
  - A bank targets a certain PD depending on the bank's business model.

Determining A Bank's Risk Appetite

- **Taking social costs into account**
  - To maximise value for banks' shareholders ⇒ generating systemic risk to society.
  - Regulators therefore impose restrictions (capital).
  - Banks have to choose their level of risk subject to external constraints.

Determining A Bank's Risk Appetite

➢ **Risk Appetite Statement**

Our guiding principle is to practice sound risk management, supported by strong capital and funding positions, as we pursue our client-focused strategy. In defining our risk appetite, we take into consideration our vision, values, and strategy, along with our risk capacity (defined by regulatory constraints). It defines how we conduct business, which is to be consistent with the following objectives:

- Safeguarding our reputation and brand.
- Doing the right thing for our clients/stakeholders.
- Engaging in client-oriented businesses that we understand.
- Maintaining a balance between risk and returns.
- Retaining a prudent attitude towards tail and event risk.
- Meeting regulatory expectations and/or identifying and having plans in place to address any issues in a timely manner.
- Achieving/ maintaining an AA rating.

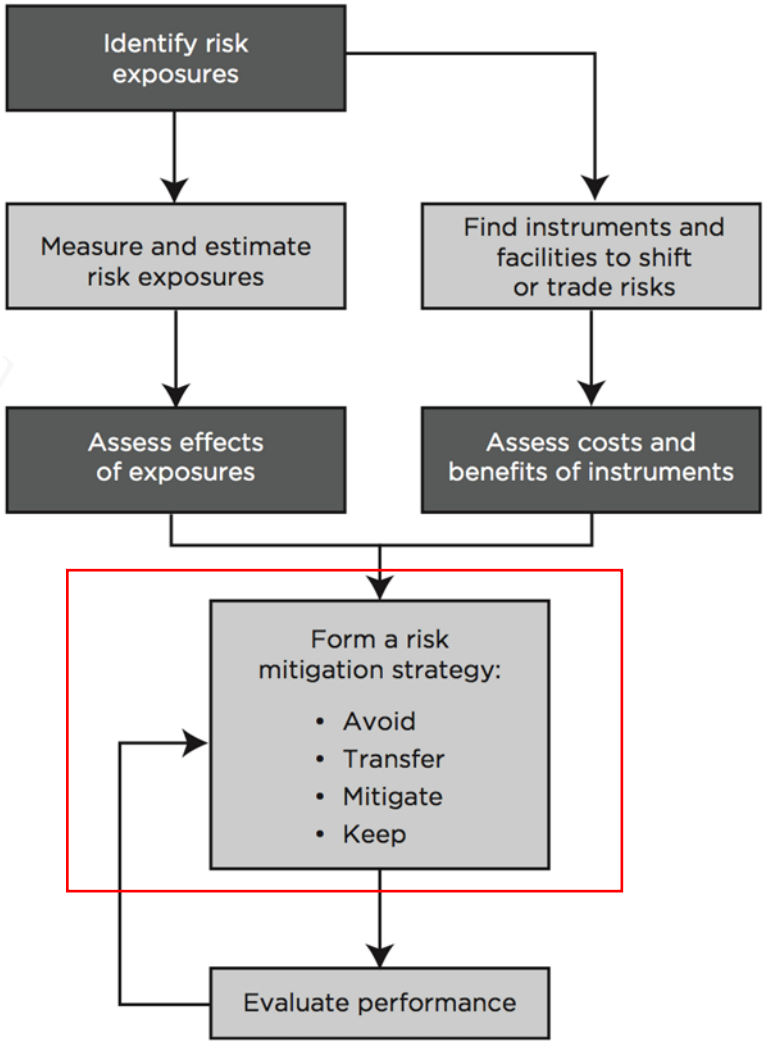
Risk Management Process

Three Lines of Defense

- **First line:** business line that generates, owns, and manages risk;
- **Second line:** risk managers that specialize in risk management and day-to-day oversight;
- **Third line:** periodic independent oversight and assurance, such as internal audit.

Risk Management Process

- 1. Identify the risk
- 2. Analyze and measure risk
- 3. Assess the effects of all risk
- 4. Manage the risk using different kind of tools



Management of Risk

- **Strategy selection:**

1. Avoid
2. Retain
3. Mitigate
4. Transfer

EL, UL, EVT, RAROC

Expected Loss(EL)

EL is the average loss a position taker might expected to incur from a position or portfolio. It can be estimated by historical data of a period of time.

It can be treated as a predictable expense rather than a risk or uncertainty.

- EL=EAD×LGD×PD for credit risk
- EL can be calculated by the distribution of loss for market risk and operation risk.

Unexpected Loss(UL)

UL is the surprising loss that above the EL in bad days.

- Value at risk (VaR) uses the loss distribution to estimate losses at a given level of likelihood(confidence).
- UL= VaR - EL

Extreme Value - Tail Risk

From the crisis lesson, we focus on the tail risk beyond confidence level which cannot be explained by VaR.

- Extreme value theory(EVT) focuses on tail distribution to understand the black swans.
- Expected shortfall is defined as the average of the VaR numbers that exceed the VaR at that tail probability.

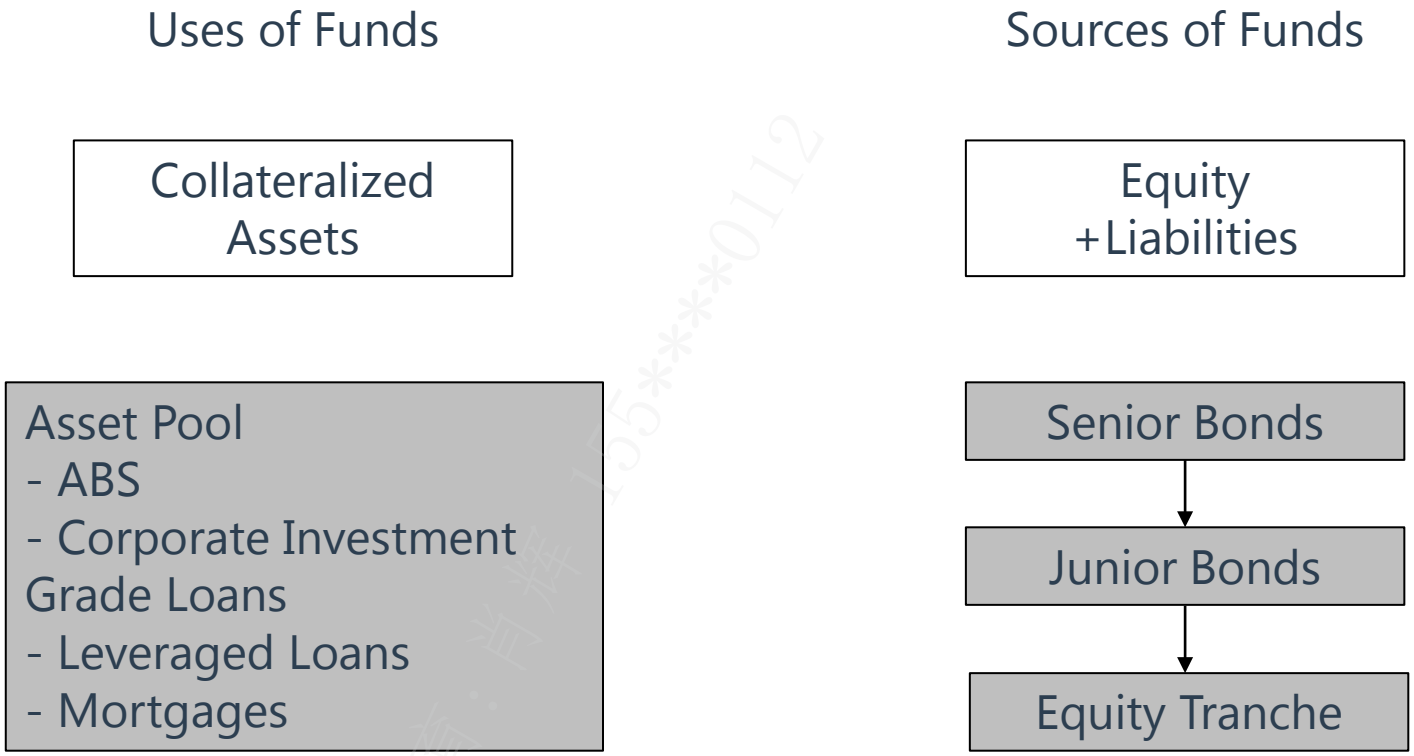
Risk adjusted return on capital (RAROC)

- RAROC = reward/risk
- Reward: after-tax risk-adjusted expected return
- Risk: economic capital

Securitization

The Mechanics of Securitization

SPV (Special Purpose Vehicle)



From Buy-and-hold to Originate-to-distribute (OTD)

- Originators benefited from greater capital efficiency (by Basel capital adequacy requirements) and enhanced funding opportunities, as well as lower earnings volatility.
- Investors benefited from a wider array of investments, allowing them to diversify their portfolios and better synchronize their risk/return profiles with their goals and preferences.
- Borrowers benefited from the expansion of available credit and product options, as well as from the lower borrowing costs resulting from these benefits.

Issues Addressed in Securitization

- **Securitization amplified systemic risk during the crisis by allowing massive leverage and risk concentration in the financial sector.**
  - Loan origination: compensation was tied to high loan volumes and high commission mortgages.
  - Securitization: the risk embedded was not transparent for investors.
  - Credit rating: overreliance on the accuracy and transparency of credit ratings.
  - Risk management: poor risk management in many segment (e.g., assessment, stress test).

Scenario Analysis

Scenario Analysis

- Scenario analysis must be unfolded over several quarters.
- Scenario analysis drives a series of interlinked factors covering a variety of risks.
- The risk variables are not static.
- Banks can adjust their capital planning with the scenario's results.
- Imposing a standard set of scenarios on the biggest banks allows regulators to see systemic effects and to compare bank risk exposures.

Scenario Analysis

Advantages	ERM View
No need to consider risk frequency beyond "plausibility".	Difficult to gauge probability of events; does not lead to the quantification of risk.
Scenarios can take the form of transparent and intuitive narratives.	Unfolding scenarios can become complex with many choices.
Challenges firms to imagine the worst and gauge the effects.	Firms may not stretch their imaginations (e.g., scenarios might underestimate the impact of an extreme loss event or omit important risk exposures).
Can allow firms to focus on their key exposures, key risk types, and the ways in which risk develops over time.	Only a limited number of scenarios can be fully developed-are they the right ones?

Scenario Analysis

Advantages	ERM View
Allows firms to identify warning signals and build contingency plans.	Are they the right warnings and plans, given the scenario selection challenge?
Does not depend on historical data; can be based around either historical events or forward-looking hypothetical events.	The scenarios chosen are often prompted by the last major crisis; imaginative future scenarios may be dismissed as improbable.
Firms can make scenario analysis as sophisticated or straightforward as they like, outside regulator defined programs.	Scenario analyses vary in terms of quality and sophistication. Their credibility and assumptions can be difficult to assess.
Stress test results can influence risk appetite, risk limits, and capital adequacy.	Usefulness depends on accuracy, comprehensiveness, and the forward-looking qualities of the firm's stress test program.

Financial products in the Crisis

The Panics

1. Asset-backed Commercial Paper

- **What is ABCP?**
- **Why ABCP Becoming Prevalent?**
  - More transparent ⇒ lowering funding costs.
  - Save on regulatory capital.
- **ABCP Run**
  - Lenders are unwilling to refinance CP when it comes due.
  - Programs were more likely to experience a run if they had high credit risk or high liquidity risk.

The Panics

2. Money Market Mutual Funds

- **A Chain Effect**
  - MMFs saw the values of their stakes decline when ABCP yields rose ⇒ were forced to sell their underlying assets ⇒ further downward pressure on asset classes held by many MMFs.
- **The Lehman bankruptcy was a major shock to MMFs.**
  - It led to run on many MMFs.
  - Investors moved to government-only MMFs.

The Panics

3. Repo

- Repo: A firm borrows funds by selling a collateral asset today and promising to repurchase it at a later date.
- Overnight repos: banks have to roll over their funding on a daily basis.
- Repo is the shadow-banking equivalent of a deposit market.
- Haircuts continued a steady rise throughout 2007-2008.
- Following the Lehman failure, the haircut rose by an additional 20% to 100%.

➢ **Leading up to the crisis, any reduction in funding liquidity could thus lead to significant stress for the financial system.**

➢ **Subprime crisis turned into the collapse of global financial institutions.**

The Role of Financial Intermediaries in the Crisis

The Role of Financial Intermediaries

1. Banks

- “Originate and distribute” model: banks repackaged loans and passed them on to various other financial investors.
- Collateralized Debt Obligations (CDOs).
- Tranches: senior, equity, mezzanine tranches.
- Credit Default Swaps (CDS): buyers of these tranches can protect themselves by purchasing CDS contracts insuring against the default.

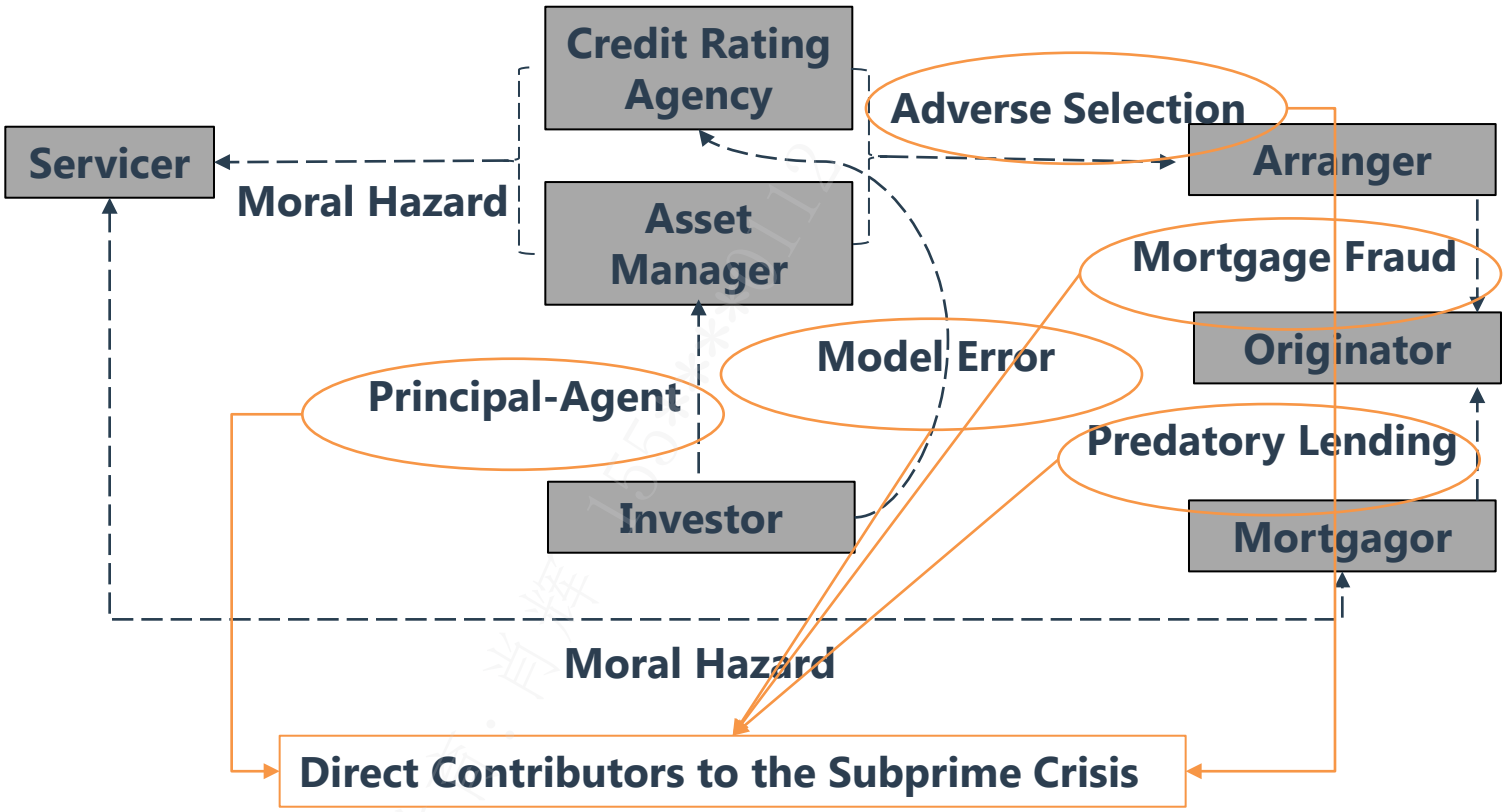
The Role of Financial Intermediaries

2. Rating Agencies

- CDO trust partners would pay credit rating agencies to rate CDOs.
- They could structure the payment waterfalls and associated liabilities to obtain a high percentage of AAA-rated bonds.
- The data problem for rating agencies.
- More favorable ratings: higher rating fees, high return.
- Rating agencies collected higher fees for structured products than corporate bond.

The Role of Financial Intermediaries

3. Other participants



Foundation of Quantitative Methods

Foundation of Quantitative Methods

- **Expected return / mean / μ**

$$E(R) = \sum_{i=1}^{n} P_i \times R_i = P_1R_1 + P_2R_2 + \cdots + P_nR_n$$

- **Variance / risk / σ^2**

$$VAR = \sigma^2 = \sum_{i=1}^{n} [R_i - E(R)]^2 P_i$$

- **Standard deviation / volatility / σ**
- **Covariance / Cov**

$$COV(X, Y) = E[(X - E(X))(Y - E(Y))]$$

- **Correlation / ρ** $\rho_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$

Foundation of Quantitative Methods

- **Put $100 on an asset with 3 results in the next year:**
  - 10% probability for earning $20
  - 40% probability for making nothing
  - 50% probability for lost $1
- **For this kind of investment, we can calculate as followings:**

E(R)=[20 × 10%+0 × 40%+(−1) × 50%] ÷ 100=1.5%

$$\mathrm{VAR} = (\frac{20}{100} - 1.5\%)^2 \times 10\% + \left(\frac{0}{100} - 1.5\%\right)^2 \times 40\% + \left(-\frac{1}{100} - 1.5\%\right)^2 \times 50\% = 0.003825$$

σ=$\sqrt{0.003825} = 6.18\%$

Note that σ is nearly 4 times to E(R).

Foundation of Quantitative Methods

- **Expected return and volatility of a two-asset portfolio**

$$R_P = W_A R_A + W_B R_B$$

$$\sigma_P^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2\rho W_A W_B \sigma_A \sigma_B$$

When $\rho_{AB} = 1$,

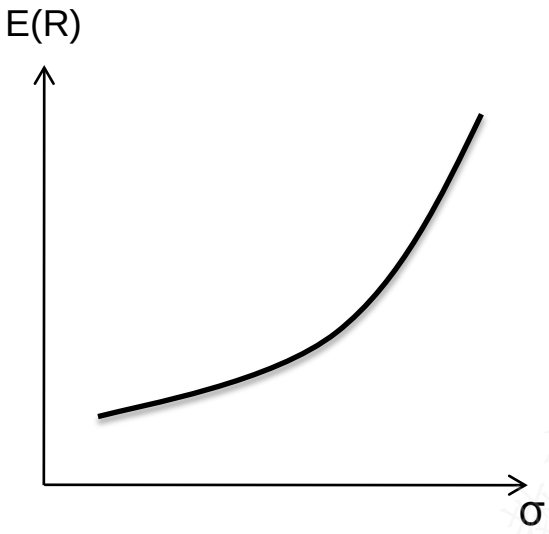
$$\sigma_P = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_A W_B \sigma_A \sigma_B} = W_A\sigma_A + W_B\sigma_B$$

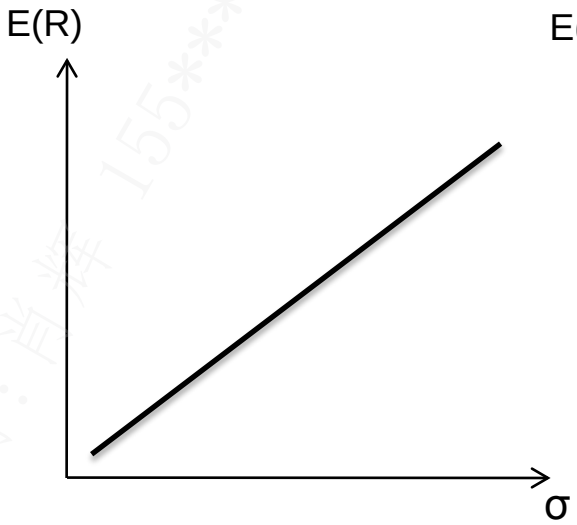
When $-1 < \rho_{AB} < 1$, $\sigma_P < W_A\sigma_A + W_B\sigma_B$

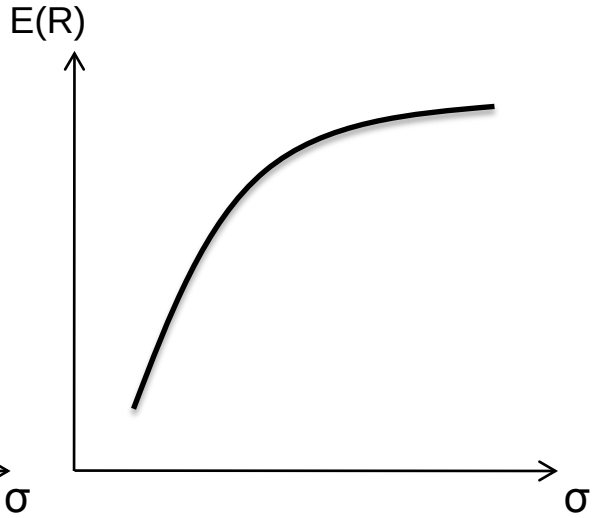
Introduction and Assumption of MPT

Markowitz Portfolio Theory

- **A rational person**
  - Risk aversion
  - Risk neutral
  - Risk preference



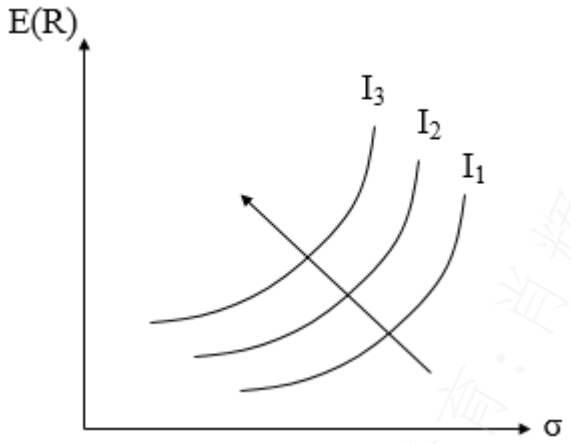




Markowitz Portfolio Theory

- **Mean Variance Model**
  - Return: E(R)
  - Risk: σ
- **Indifferent curve**

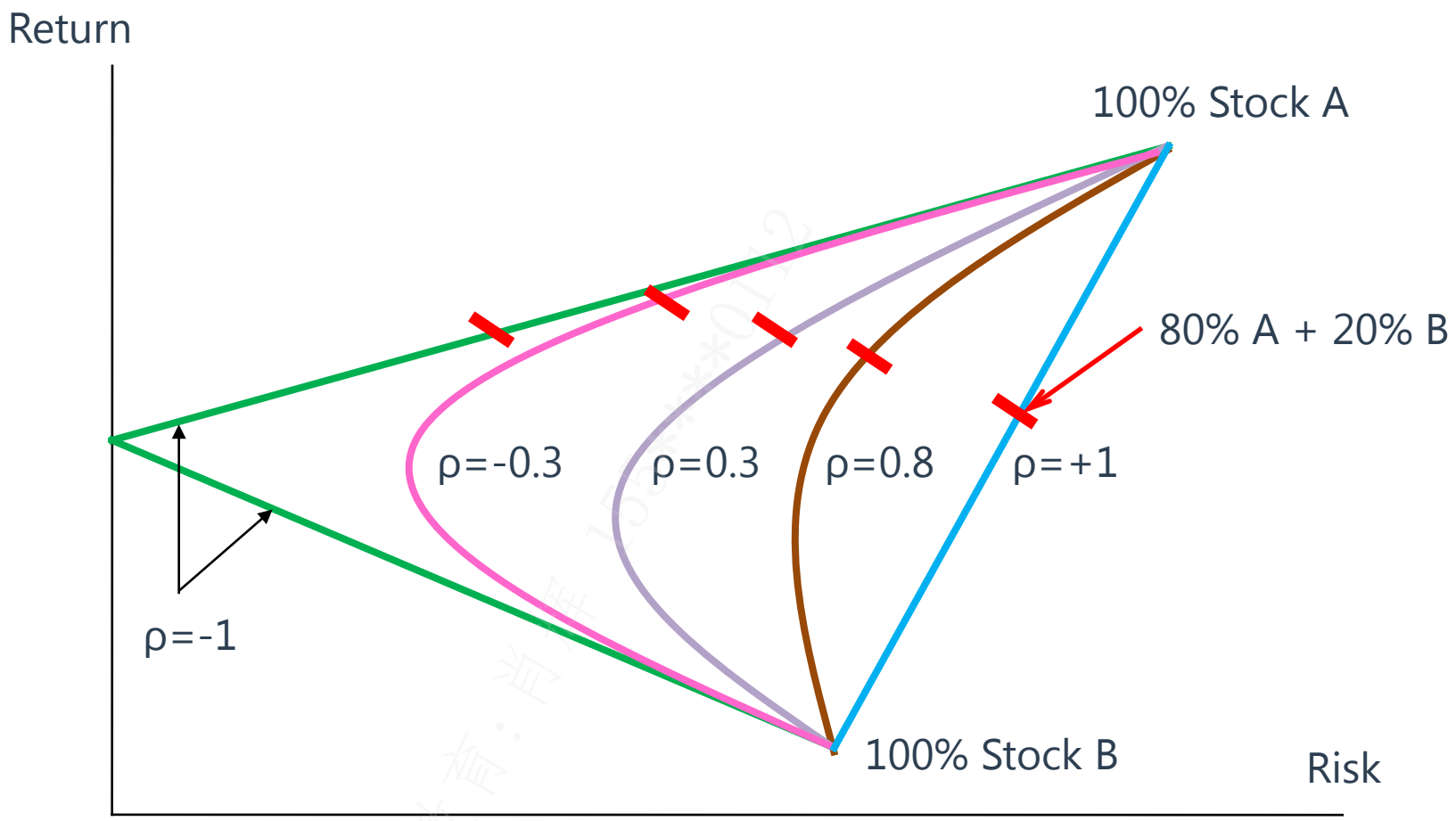
Harry M. Markowitz （1927-）
1990年诺贝尔经济学奖获得者



Markowitz Portfolio Theory

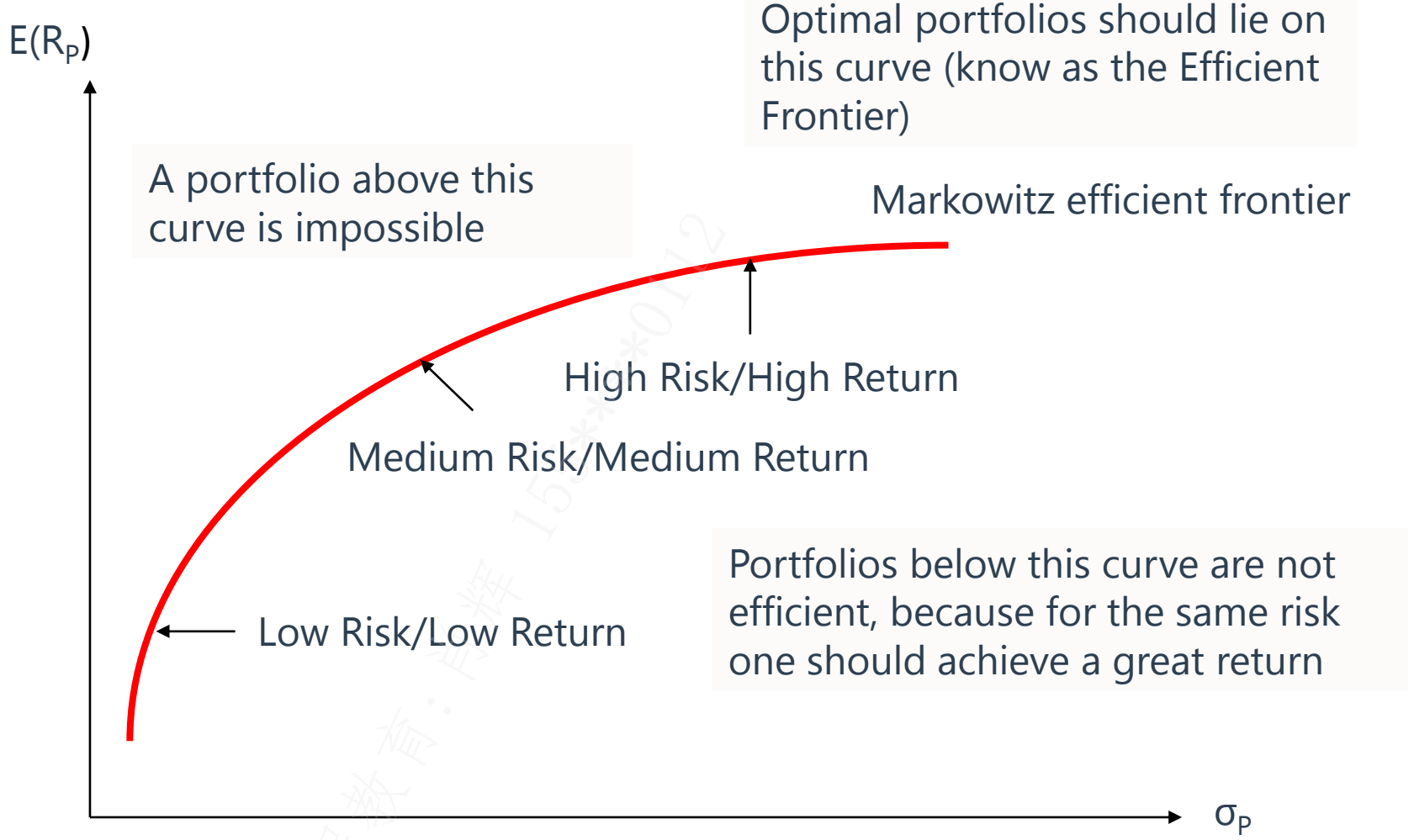
- **Assumptions about a Markowitz investor**
  - No transaction costs
  - Assets are infinitely divisible
  - The absence of personal income tax
  - An individual cannot affect the price of a stock by his trading
  - Investors make decisions solely in terms of returns and standard deviation of the returns
  - Unlimited short sales are allowed
  - Unlimited lending and borrowing at the riskless rate
  - All investors have identical expectations: μ σ ρ
  - All assets are marketable

Effects of Correlation on Diversification Benefits



Markowitz Efficient Frontier and Minimum Variance Portfolio

Markowitz Efficient Frontier



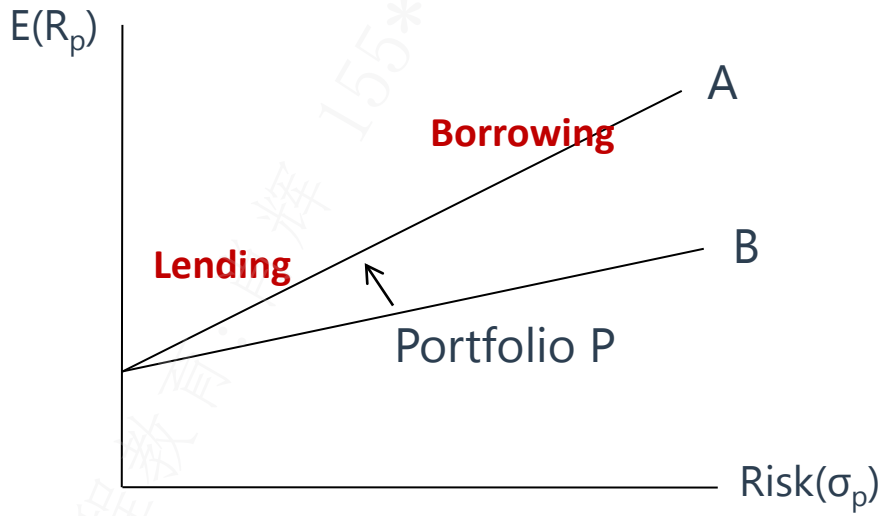
Minimum Variance Portfolio

- **Definition: the portfolio with the smallest variance among all possible portfolios on a portfolio possibilities curve.**
- **The shape of the portfolio possibilities curve is best described in two pieces:**
  - The portion of the portfolio possibility curve that lies above the minimum variance portfolio is concave.
  - The portion of the portfolio possibility curve that lies below the minimum variance portfolio is convex.

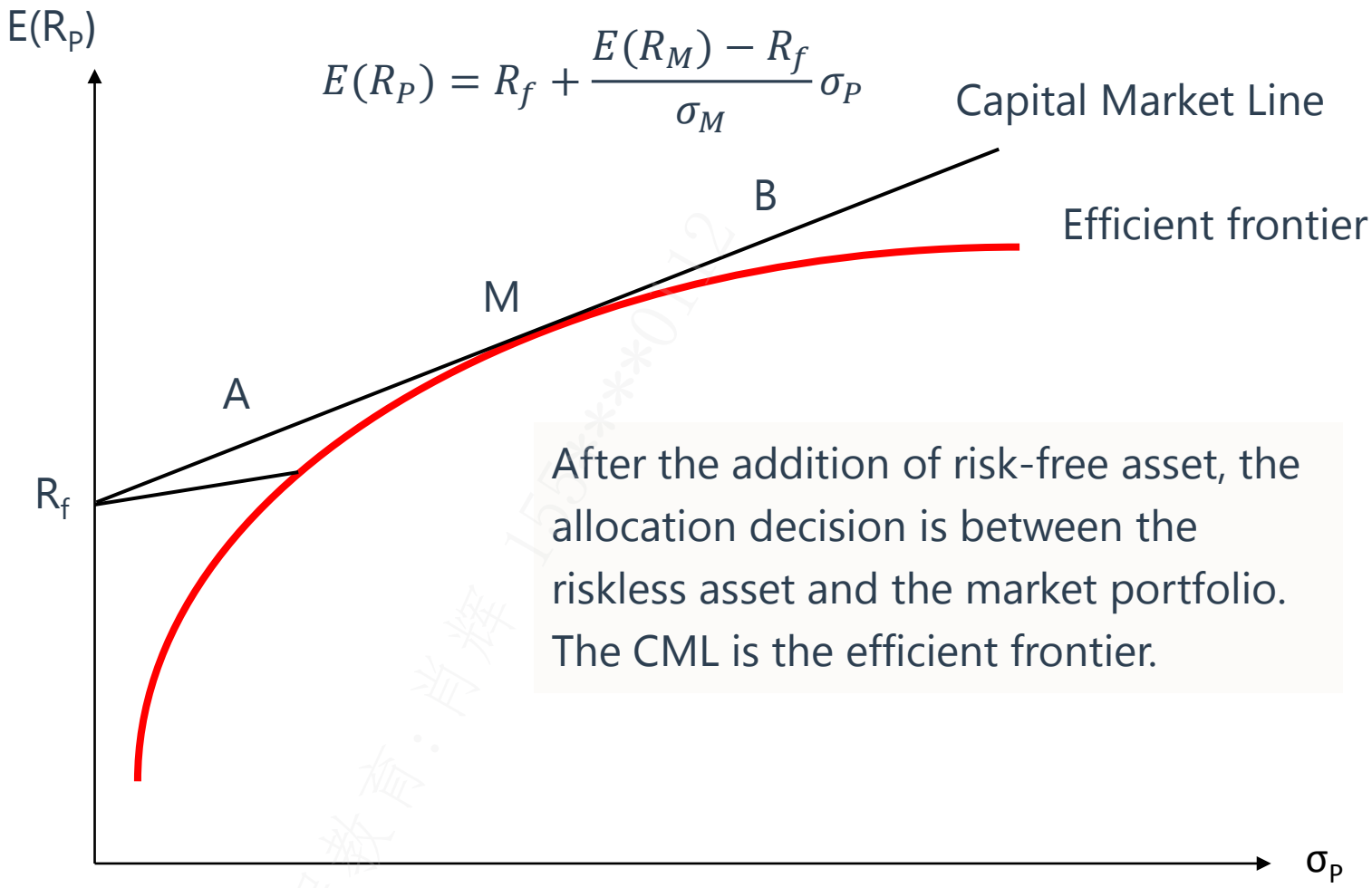
CML and SML

Capital Market Line (CML)

- The point of tangency – Portfolio P, is known as the market portfolio. When one can lent or borrow money use riskless rate, investor will hold a combination of the market portfolio and the risk-free asset.
- Risk-averse investors will create lower risk portfolios by lending (i.e., investing in the risk-free asset). More risk-tolerant investors will increase portfolio return by borrowing at the risk-free rate.

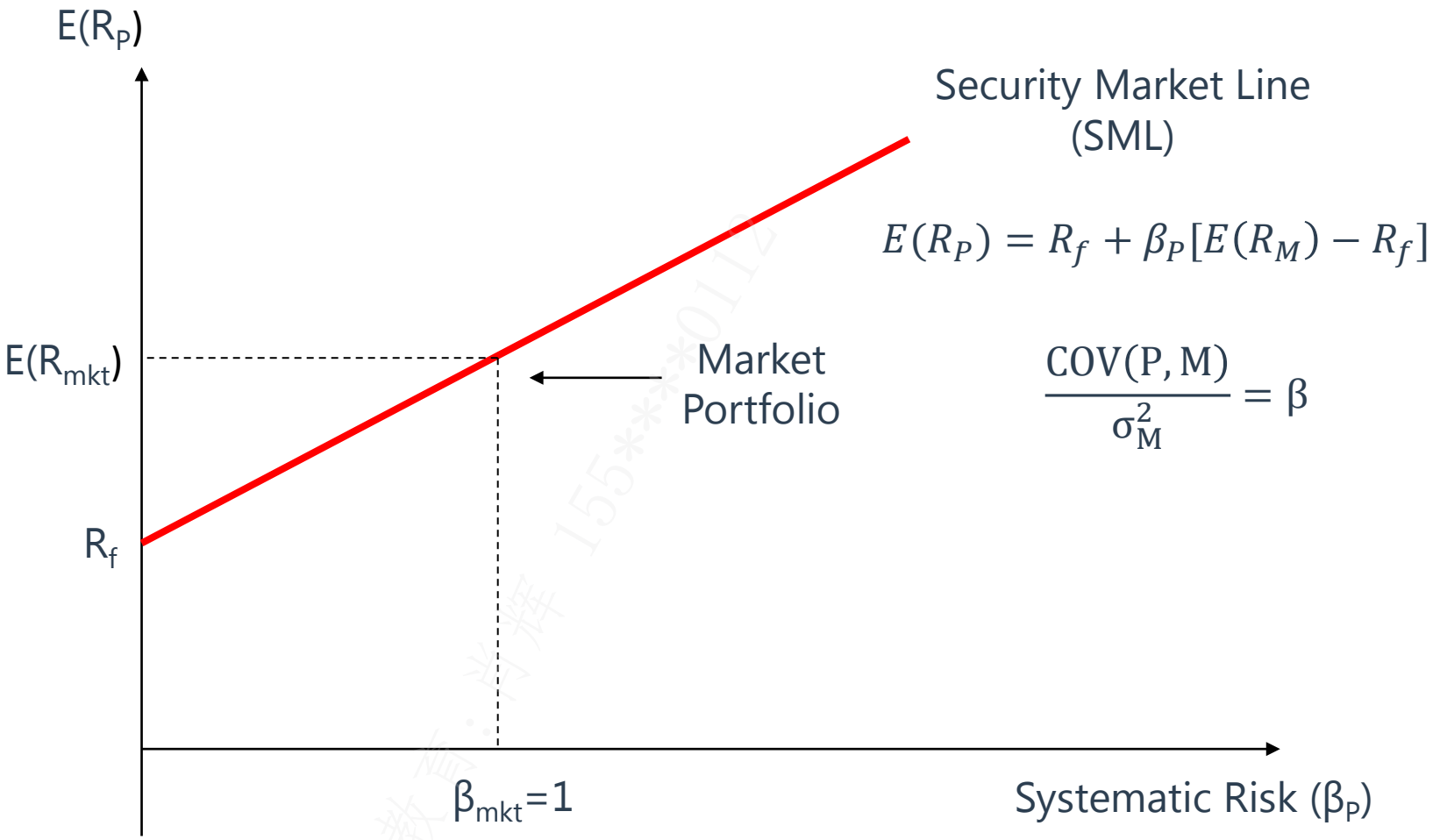


Capital Market Line (CML)



After the addition of risk-free asset, the allocation decision is between the riskless asset and the market portfolio. The CML is the efficient frontier.

Security Market Line (SML)



$$E(R_P) = R_f + \beta_P[E(R_M) - R_f]$$

$$\frac{\mathrm{COV(P,M)}}{\sigma_{\mathrm{M}}^2} = \beta$$

Comparison of CML and SML

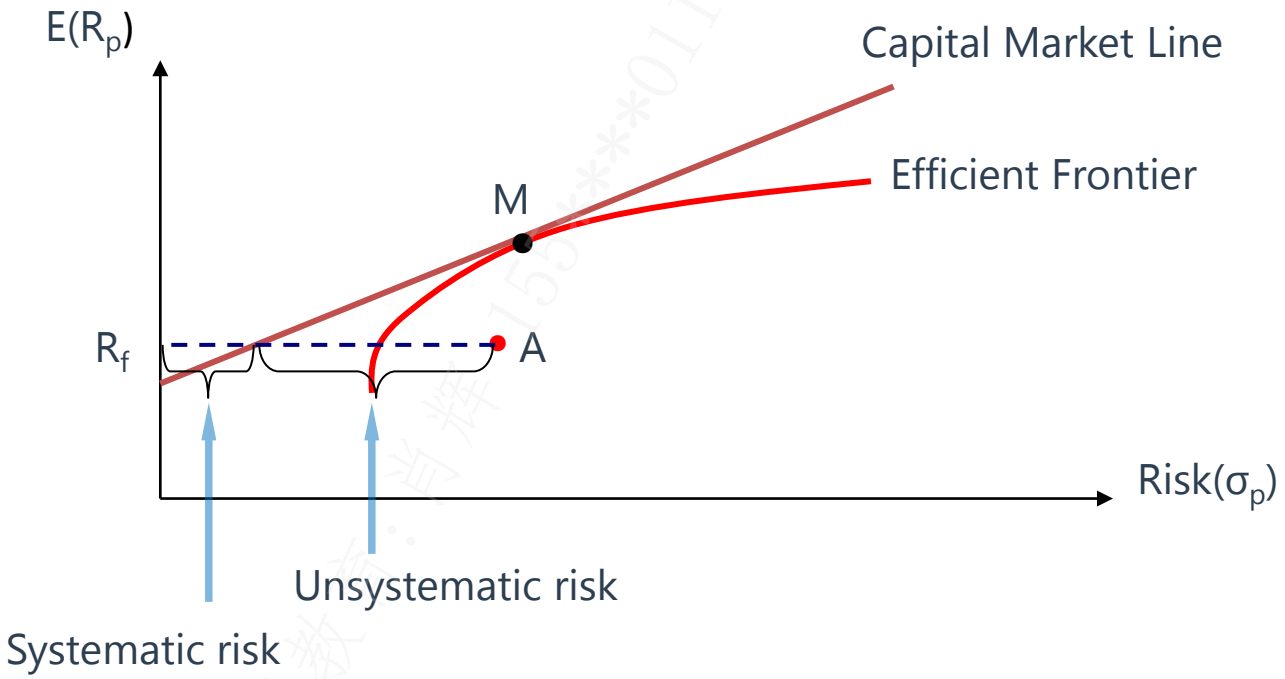
Differences	SML	CML
Measure of Risk	Uses systematic risk.	Uses standard deviation.
Application	Tool used to determine the appropriate expected returns for securities.	Tool used to determine the appropriate asset allocation (percentages allocated to the risk-free asset and to the market portfolio) for the investor.
Definition	Graph of the CAPM.	Graph of the efficient frontier.
Slope	Market risk premium.	Market portfolio Sharpe Ratio.

Systematic and Unsystematic Risk

Systematic and Unsystematic Risk

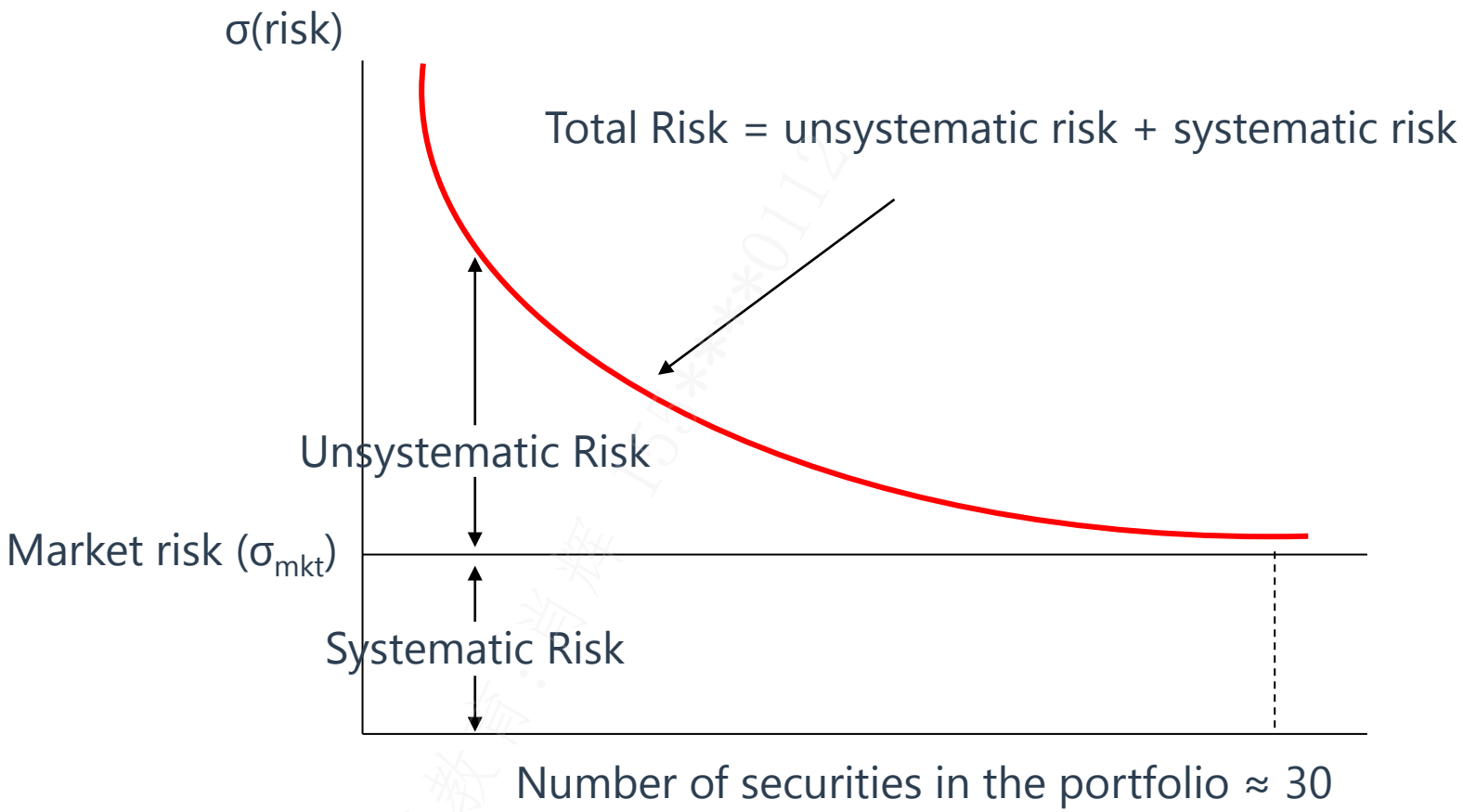
Total risk= systematic risk+ unsystematic risk

Systematic risk is the only important ingredient in determining expected returns and that nonsystematic risk plays no role



Systematic and Unsystematic Risk

Risk vs. Number of Portfolio Assets



Capital Asset Pricing Model (CAPM)

Capital Asset Pricing Model (CAPM)

- **Assumptions about CAPM model**
  - Access to information for all market participants, meaning that all information is freely available and instantly absorbed.
  - All market participants have the same expectations.
  - All market participants make their investment decisions based on the mean and variance of returns.
  - No transaction costs, taxes, or other frictions.
  - Allocations can be made in an investment of any partial amount (i.e., perfect divisibility).
  - All participants can borrow and lend at a common risk-free interest rate.
  - Any individual investor's allocation decision cannot change the market prices.

Capital Asset Pricing Model (CAPM)

- $E(R_P) = R_f + \beta_P[E(R_M) - R_f]$
- $\beta_P = \frac{Cov(P,M)}{\sigma_M^2} = \rho_{P,M}\frac{\sigma_P}{\sigma_M}$
  - $E(R_P)$: expected return on risky asset
  - R_f : risk-free rate
  - $E(R_M)$ - R_f : market portfolio risk premium
  - β_P : systematical risk of asset P
  - $\beta_P \times [E(R_M) - R_f]$: beta-adjusted market risk premium

Capital Asset Pricing Model (CAPM)

- **The expected return on the market is 15%, the risk-free rate is 8%, and the beta for stock A is 1.2. Compute the rate of return that would be expected (required) on this stock.**
  - Answer: $\mathrm{E}(\mathrm{R_A}) = 0.08 + 1.2 \times (0.15 - 0.08) = 0.164$
  - Note: $\beta_A > 1 \Rightarrow \mathrm{E}(\mathrm{R_A}) > \mathrm{E}(\mathrm{R_m})$
- The expected return on the market is 15%, the risk-free rate is 8%, and the beta for stock B is 0.8. Compute the rate of return that would be expected (required) on this stock.
  - Answer: $\mathrm{E}(\mathrm{RB}) = 0.08 + 0.8 \times (0.15 - 0.08) = 0.136$
  - Note: $\beta_B < 1 \Rightarrow \mathrm{E}(\mathrm{R_B}) < \mathrm{E}(\mathrm{R_m})$

Performance Measurement Indicators

Sharpe Ratio

- Measures the ratio of the average rate of return $E(R_P)$, in excess of the risk-free rate R_F, to the absolute risk $\sigma(R_P)$.

$$SR = \frac{E(R_P) - R_F}{\sigma(R_P)}$$

  - Widely used for measuring portfolio performance that are not very diversified.
  - A better method for measuring historical performance.
  - Suitable for evaluating the performance of a portfolio that represents an individual's total investment.

Treynor Ratio

- Treynor ratio is equal to the risk premium divided by beta (systematic risk)

$$TR = \frac{E(R_P) - R_F}{\beta_P}$$

  - More appropriate for comparing well-diversified portfolios and a more forward-looking measure.

Sortino Ratio

- MAR (minimum accepted rate of return) is the return below which the investor does not wish to drop.
- Sortino ratio measures the ratio of the average rate of return $E(R_P)$, in excess of MAR (minimum acceptable return), to the semi-standard deviation, which considers only data points that represent a loss.

$$Sortino\ Ratio = \frac{E(R_P) - MAR}{\sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(R_{Pt} - MAR)^2}} \quad (R_{Pt} < MAR)$$

  - Where N is the number of observed losses.
  - The Sortino ratio is more relevant than the Sharpe ratio when the return distribution is skewed to the left.

Jensen's Alpha

- Jensen's alpha is the asset's excess return over the return predicted by the CAPM.

$$E(R_P) - R_F = \alpha_P + \beta_P[E(R_M) - R_F]$$

  - Most appropriate for comparing portfolios that have the same beta and can be used to rank portfolios within peer groups.

Tracking Error

- The tracking error (TE) measures the difference between a portfolio's returns and those of a benchmark. The first way to calculate TE is:

$$R_P - R_B$$

- Another way to measure TE is to calculate the standard deviation of the differences in the portfolio and the benchmark returns over time. It is also called the tracking error volatility (TEV).

$$TE(TEV) = \sigma(R_P - R_B)$$

- In most cases, the mean of tracking error is zero, so the calculation will become (N is the number of return periods measured):

$$\sqrt{\frac{\sum(R_P - R_B)^2}{N-1}}$$

Information Ratio

- The information ratio measures the ratio of the residual return of the portfolio compared with its residual risk (tracking error).

$$IR = \frac{E(R_P) - E(R_B)}{\sigma(R_P - R_B)} = \frac{\alpha_P}{\sigma(\alpha_P)}$$

  - To check that the risk taken by the manager, in deviating from the benchmark, is sufficiently rewarded.

Single-Factor Model

Single-Factor Model

- In a single-factor model, uncertainty in asset returns has two sources: a common or macroeconomic factor, and firm-specific events.
- The factor model states that the actual return on firm i will equal its initially expected return plus a (zero expected value) random amount attributable to unanticipated economy-wide events, plus another (zero expected value) random amount attributable to firm–specific events.
- $R_i = E(R_i) + \beta_i F + e_i$
  - F is the deviation of the common factor from its expected value;
  - β_i is the sensitivity of firm i to that factor;
  - e_i is the firm-specific disturbance;
  - $E(R_i)$ is the expected excess return on stock i;
  - The nonsystematic components of returns (e_i), are assumed to be uncorrelated among themselves and uncorrelated with the factor F.

Single-Factor Model

- The expected value of e_i for any well-diversified portfolio is zero, and its variance also is effectively zero. We conclude that for a well-diversified portfolio, for all practical purposes
- $R_i = E(R_i) + \beta_i F$

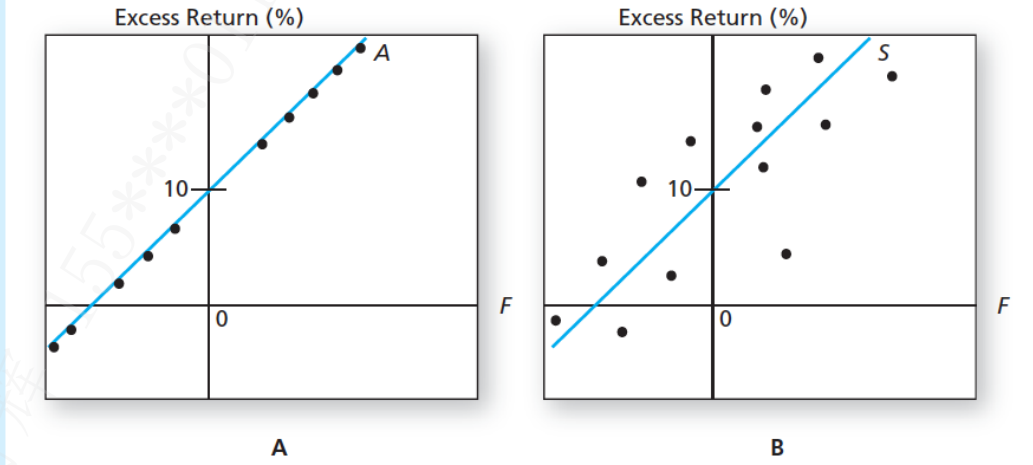


Figure 10.1 Excess returns as a function of the systematic factor. ***Panel A,*** Well-diversified portfolio *A*. ***Panel B,*** Single stock (*S*).

Multifactor Models

Multifactor Models

- Multifactor Model
  - Models that allow for several factors can provide better descriptions of security returns.
  - Multifactor models of security returns can be used to measure and manage exposure to each of **many economy-wide factors such as business-cycle risk, interest or inflation rate risk, energy price risk**, and so on.
  - Factor models are tools that allow us to **describe and quantify the different factors that affect the rate of return** on a security during any time period.

Multifactor Models

- The equation for multifactor model for stock i
- $\mathrm{R_i} = \mathrm{E(R_i)} + \beta_{\mathrm{i,GDP}}\mathrm{GDP} + \beta_{\mathrm{i,IR}}\mathrm{IR} + \mathrm{e_i}$
  - R_i = return on stock i
  - $E(R_i)$ = expected excess rate of return for stock i
  - $\beta_{i,GDP}$ = GDP factor beta for stock i
  - $\beta_{i,IR}$ = interest rate factor beta for stock i
  - GDP = deviation of GDP factor from its expected value
  - IR = deviation of interest rate factor from its expected value
  - e_i = firm-specific return for stock i

Fama-French Three-Factor Model

Fama-French Three-Factor Model

- The Fama-French three-factor model incorporates the following systematic factors:
- $R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + e_{it}$
  - SMB = Small minus big (the return of a portfolio of small stocks – return on a portfolio of large stocks)
  - HML = High minus low (the return of a portfolio of stocks with a high book-to-market ratio – return on a portfolio of stocks with a low book-to-market ratio)
- F&F have observed: firms with high ratios of book-to-market value are more likely to be in financial distress and that small stocks may be more sensitive to changes in business conditions. Thus, these variables may capture sensitivity to risk factors in the macro economy.

Introduction and Assumptions of APT

Arbitrage Pricing Theory

- Arbitrage pricing theory (APT) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient.
- **Assumptions:**
  - Asset returns can be explained by systemic factors that affect all securities.
  - By using diversification, investors can eliminate specific(or idiosyncratic) risk from their portfolios.
  - There are no arbitrage opportunities among well-diversified portfolios. If any arbitrage opportunities were to exist, investors would exploit them away.

Arbitrage, Risk Arbitrage, and Equilibrium

- CAPM's argument: If a security is mispriced, then investors will tilt their portfolios toward the underpriced and away from the overpriced securities, each by a relatively small dollar amount.
- APT's implication: a few investors who identify an arbitrage opportunity will mobilize large dollar amounts and quickly restore equilibrium.
- Arbitrageur often refers to a professional searching for mispriced securities in specific areas such as merger-target stocks.

Comparison of CAPM and APT

	CAPM	APT
Assumptions	All investors should hold some combination of the market portfolio and the risk-free asset. To control risk, less risk averse investors simply hold more of the market portfolio and less of the risk-free asset.	APT gives no special role to the market portfolio, and is far more flexible than CAPM. Asset returns follow a multifactor process, allowing investors to manage several risk factors, rather than just one.
Conclusions	The risk of the investor's portfolio is determined solely by the resulting portfolio beta.	Investor's unique circumstances may drive the investor to hold portfolios titled away from the market portfolio in order to hedge or speculate on multiple risk factors.

Risk premiums

- **The risk premiums are derived as follows:**
  - Create factor portfolios. Each factor portfolio is a well-diversified portfolio that has a beta equal to one for a single risk factor, and betas equal to zero on the remaining factors.
  - Derive returns for each factor portfolio. For instance, define $E(R_i)$ as the expected return on Factor Portfolio i.
  - Calculate risk premiums for each factor portfolio.

Arbitrage Pricing Theory Application

Arbitrage Pricing Theory

Example

- Northeast Airlines has a GDP beta of 1.2 and an interest rate beta of -0.3. Suppose the risk premium for one unit of exposure to GDP risk is 6%, while the risk premium for one unit of exposure to interest rate risk is -7%. If the risk-free rate is 4%, what the total risk premium for Northeast Airlines?

Explanation

- $\mathrm{E(R)} = \mathrm{R_f} + \mathrm{GDP} \times \mathrm{RP_{GDP}} + \mathrm{IR} \times \mathrm{RP_{IR}}$
- $\mathrm{E(R)} = 4\% + 1.2 \times 6\% + (-0.3) \times (-7\%) = 13.3\%$

Empirical Work Preference

- APT approach is more popular than CAPM in empirical works for the following reasons:
  - CAPM is a special case of APT. CAPM is a one-factor model while APT is a multi-factor model.
  - APT better helps facilitate risk analyses than CAPM. Consider the case of a portfolio with n = 50 different equities:
    - ✓ CAPM requires a full volatility and correlation matrix, which means there will be a total of $50 + \frac{50^2-50}{2}$=1275 calculations.
    - ✓ With a multi-factor model, for example, if the 50 equities are categorized into 3 factors, the number of calculations will be $3 \times 50 + \frac{3^2-3}{2}$ =153.