FRM一级核心知识点

估值与风险模型

101% Contribution Breeds Professionalism

Basic Valuation Method

Basic Valuation Method

➢ **How to determine the price of a bond?**

$$P=\frac{C_1}{1+S_1}+\frac{C_2}{(1+S_2)^2}+\cdots+\frac{C_T}{(1+S_T)^T}=\sum_{t=1}^{T}\frac{C_t}{(1+y)^t}$$

- Suppose that a 2-year Treasury bond with a principal of $100 provides coupons at the rate of 6% per annum semiannually. The Treasury zero rates are below, compute the bond price:

Treasury Zero Rates Maturity (Years)	Zero Rate (%) (continuously Compounded)
0.5	5.0
1	5.8
1.5	6.4
2.0	6.8

$$P=3e^{-0.05\times 0.5}+3e^{-0.058\times 1}+3e^{-0.064\times 1.5}+103e^{-0.068\times 2.0}=98.39$$

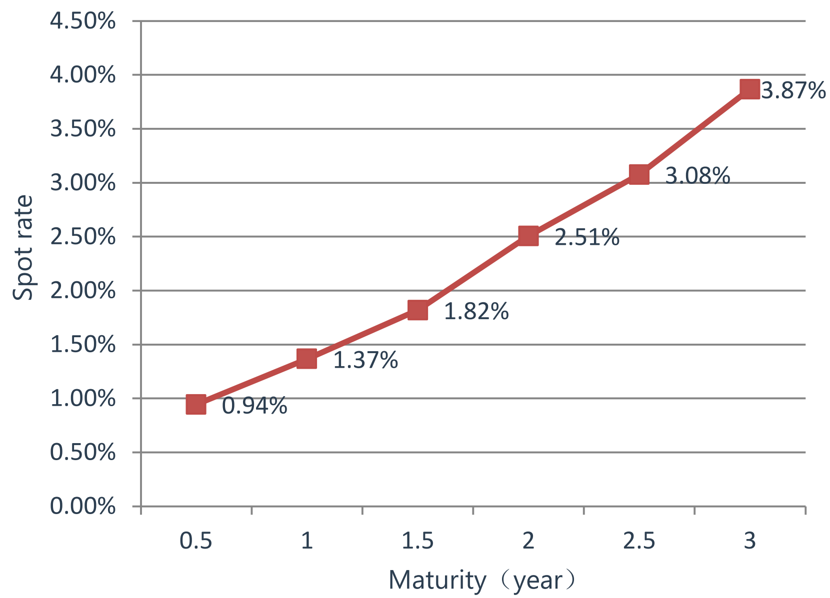
Discount Factor

➢ Discount Factor

- The discount factor, d(t), for a term of (t) years, gives the present value of one unit of currency ($1) to be received at the end of that term.

$$\boxed{d(t) = \left(1 + \frac{r(t)}{m}\right)^{-mt} \qquad \text{or} \qquad d(t) = e^{-r(t)t}}$$

STRIPS Prices and Discount Factors			
Maturity	**STRIPS Price**	**Discount Factor**	**Spot Rate**
0.5	99.5303	0.995303	0.94%
1	98.6478	0.986478	1.37%
1.5	97.3214	0.973214	1.82%
2	95.1423	0.951423	2.51%
2.5	92.6547	0.926547	3.08%
3	89.1474	0.891474	3.87%



Basic Valuation Method

➢ **Example**

The table below shows selected T-bond prices for semiannual coupon, $100 face value bonds. Calculate discount factors for these given bond prices.

Bond	Coupon	Maturity	Price
1	4%	0.5	101-23
2	6.3%	1	104-22+

$$\text{Bond 1:}\left(100+100\times\frac{4\%}{2}\right)\times d(0.5)=101+\frac{23}{32}$$

$$d(0.5)=0.9972$$

$$\text{Bond 2:}\left(100\times\frac{6.3\%}{2}\right)\times d(0.5)+\left(100+100\times\frac{6.3\%}{2}\right)\times d(1)=104+\frac{22.5}{32}$$

$$d(1)=0.9845$$

Basic Valuation Method

➢ **Pricing Bond using Discount Factors, Spot Rates, or Forward Rates**

- Assume a 1-year treasury bond that pays a 8% semi-annual coupon.

Maturity	Spot Rate (%)	Discount Factor	6 Month Forward Rate (%)
0.50	0.94	0.995303	0.94
1.00	1.37	0.986478	1.79
1.50	1.82	0.973214	2.73
2.00	2.51	0.951336	4.58
2.50	3.08	0.926547	5.37

Basic Valuation Method

➢ **Pricing Bond using Discount Factors, Spot Rates, or Forward Rates**

- Calculate the Bond Price using Discount Factors

$$\text{Price} = (\$4 \times 0.995303) + (\$104 \times 0.986478) = \$106.57$$

- Calculate the Bond Price using Spot Rates

$$\text{Price} = \frac{\$4}{1+\frac{0.94\%}{2}} + \frac{\$104}{\left(1+\frac{1.37\%}{2}\right)^2} = \$106.57$$

- Calculate the Bond Price using Forward Rates

$$\text{Price} = \frac{\$4}{1+\frac{0.94\%}{2}} + \frac{\$104}{\left(1+\frac{0.94\%}{2}\right)\times\left(1+\frac{1.79\%}{2}\right)} = \$106.57$$

Replication Method

Replication Method

➢ **Law of one price:**

- Absent confounding factors (e.g., liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for the same price. While the law of one price is intuitively reasonable, its justification rests on a stronger foundation. It turns out that a deviation from the law of one price implies the existence of an arbitrage opportunity, that is, a trade that generates profits without any chance of losing money.
- When mispricing happens, arbitrage may not occur because factors other than promised cash flows are occasionally considered in the way the instruments are priced, such as tax treatment and liquidity.

Replication Method

- **Example**

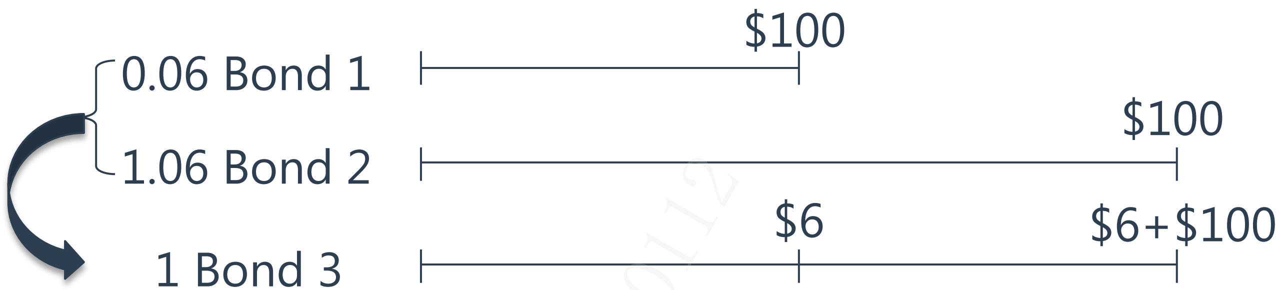
Three bond yields and prices are shown below

	Maturity	YTM	Coupon	Price (% of par)
1	1 year	5%	0%	95.238
2	2 years	6%	0%	99.00
3	2 years	6%	6%	100

The 2-year spot rate is 6.2%. Is there an arbitrage opportunity using these three bonds? If so, describe the trades necessary to exploit the arbitrage opportunity?

Replication Method

- **Example**



$$0.06B_1 + 1.06B_2 = B_3$$

$$0.06 \times 95.238 + 1.06 \times 99.00 = 110.6543$$

Bonds 3 is undervalued, so we buy bond 3.

Bond 1 and Bond 2 are overvalued, then we sell them together.

Replication Method

- **Example**

Time = 0		1 year		2 years	
-1,000,000.00	(cost of 2-year, 6% coupon bonds)	+60,000	(coupon)	+1,060,000	(coupon)
+57,142.80	(proceeds 1-year, 0% coupon bonds)	-60,000	(maturity)		
+1,049,400.00	(proceed 2-year, 0% coupon bonds)			-1,060,000	
+106,542.80	Net	0		0	(maturity)

Bond Return

Bond Return

➢ **Gross Realized Returns**

$$R_{t,t+1} = \frac{P_{t+1} + c - P_t}{P_t}$$

- If we want to look at the return over a longer period, we must consider the investment of the coupon as well.
- Suppose that the initial purchase price of a bond is $98, and the purchase occurred immediately after a coupon payment date. It earns $1.75 coupon every six months. A year later the price is 98.7. Assuming that the coupon can be invested at an annualized rate of 2.2% (semi-annual compounding), the gross realized return is:

$$\frac{98.7 - 98 + 1.75 + 1.75 \times (1 + 1.1\%)}{98} = 0.0431$$

Bond Return

Net Realized Returns

- Incorporates funding cost

$$R_{t,t+1} = \frac{P_{t+1} + c - B_{\text{funded price}}}{P_t}$$

- Continue the example, assuming that the funds to buy are financed at 3% per annum (semi-annual compounding), the net realized return is:

$$\frac{98.7 - 98 \times (1 + 1.5\%)^2 + 1.75 + 1.75 \times (1 + 1.1\%)}{98} = 0.0128$$

Bond Return

- **Yield to Maturity**
  - The YTM of a bond is the **single discount rate** at which **all cash flows** of the bond are **discounted** and **summed up** to the **market price**.
  - The YTM can be viewed as the realized return on the bond assuming:
    - ✓ All cash flows are reinvested at the YTM.
    - ✓ The bond is held to maturity.
  - **Example:** Suppose a bond pays \$40 every six months for four years and a final payment of \$1,000 at maturity in four years. If the price is \$850, calculate the YTM.
    - ✓ **Answer:** The YTM is the y that solves the following equation:

$$\$850=\frac{\$40}{\left(1+\frac{y}{2}\right)^{1}}+\frac{\$40}{\left(1+\frac{y}{2}\right)^{2}}+\cdots+\frac{\$40+\$1000}{\left(1+\frac{y}{2}\right)^{8}}$$

FV=\$1000; PV=-850; PMT=40; N=8; CPT→I/Y=6.46 →YTM=12.92%

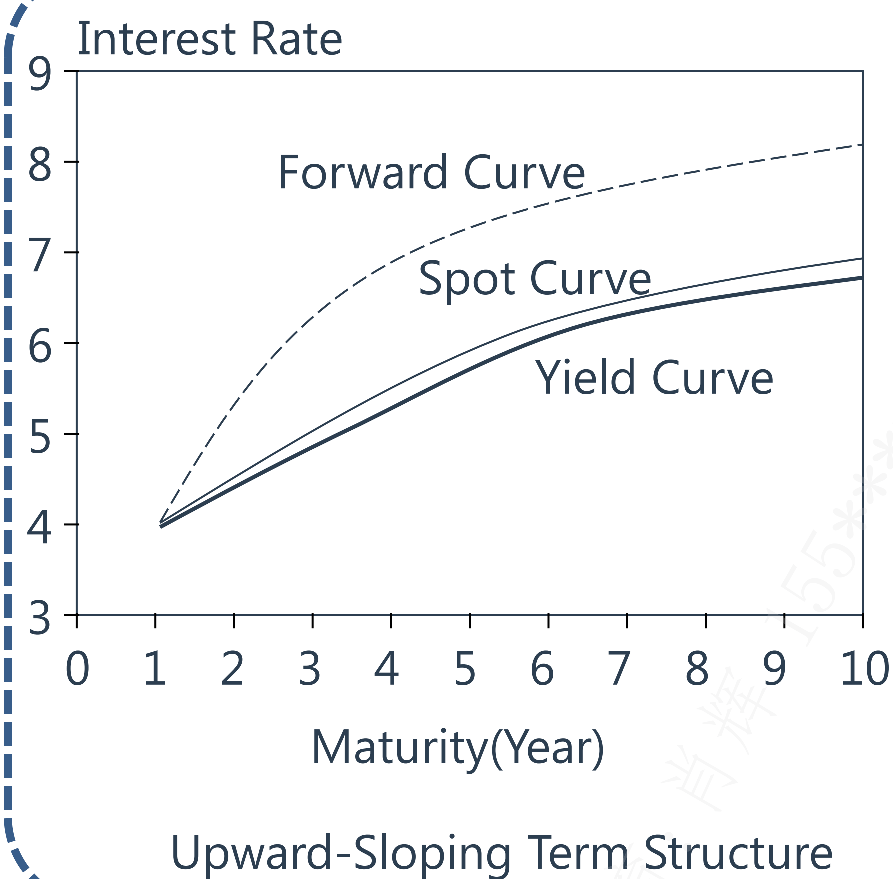
Bond Return

Relationship between Spot Rates and YTM

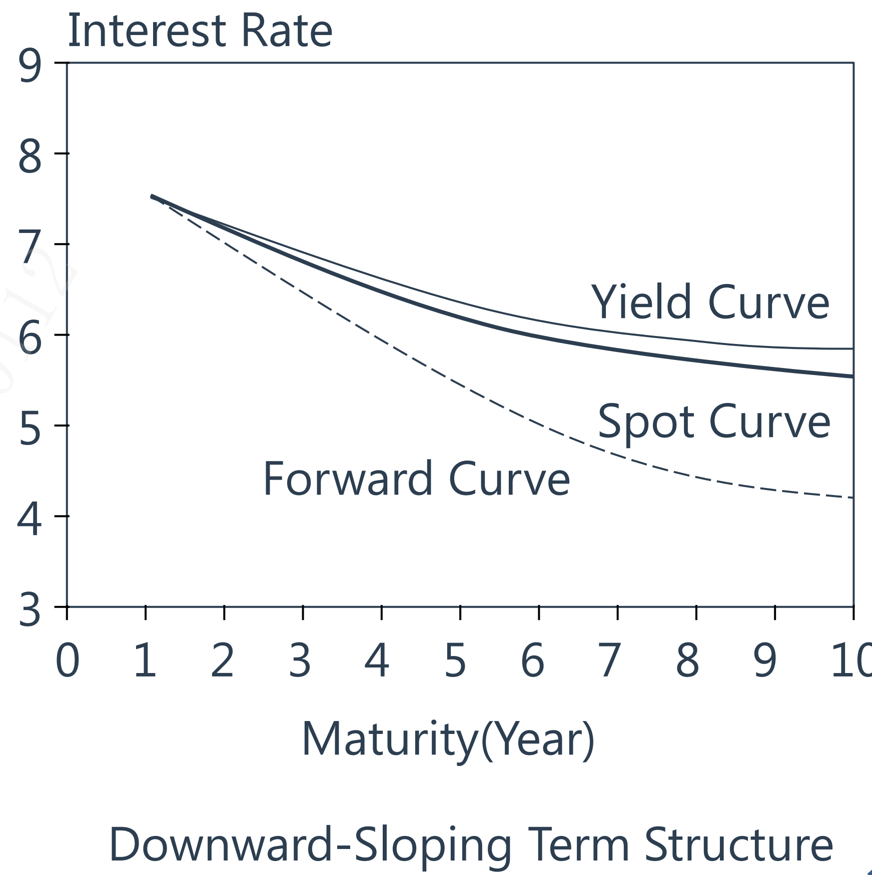
$$P = \frac{CF_1}{1+YTM} + \frac{CF_2}{(1+YTM)^2} + \cdots + \frac{CF_n}{(1+YTM)^n}$$
$$= \frac{CF_1}{1+R_1} + \frac{CF_2}{(1+R_2)^2} + \cdots + \frac{CF_n}{(1+R_n)^n}$$

- YTM is a kind of average of all the spot rates.
- When there is a **flat term structure** of spot rates, the yield must equal those spot rates.
- When the term structure of spot rates is **upward-sloping**, the yield of the two-year bond will be below the two-year spot rate (use two-year bond as example).
- When the term structure of spot rates is **downward-sloping**, the yield of the two-year bond will be above the two-year spot rate.

Bond Return



Interest Rate
9
8
7
6
5
4
3
0 1 2 3 4 5 6 7 8 9 10
Forward Curve
Spot Curve
Yield Curve
Maturity(Year)
Upward-Sloping Term Structure



Interest Rate
9
8
7
6
5
4
3
0 1 2 3 4 5 6 7 8 9 10
Yield Curve
Spot Curve
Forward Curve
Maturity(Year)
Downward-Sloping Term Structure

Bond Return

➢ **Coupon Effect**

- The fact that correctly priced bonds with the **same maturity** but **different coupons** have **different yields to maturity** is called the **coupon effect**.
- As the **coupon rises**, the **average time** it takes bondholders to recover their cash flows **falls.** Therefore, the spot rates for the **early payment** dates is becoming more important in **determining the yield to maturity**.
  - ✓ **Upward-sloping trend**, the **yield to maturity falls as the coupon rises**.
  - ✓ **Downward-sloping trend**, the **yield to maturity rises as the coupon rises.**

P&L Components

P&L Components

➢ **Decomposition of P&L**

- The bond's profit and loss consists of both:
  - ✓ **Price appreciation (or depreciation)**; a.k.a., capital gain or loss.
  - ✓ **Cash-carry**: cash flows such as **coupon** payments.
- **Price appreciation** can be decomposed into three components:

① **Carry-roll-down**: the price change due to the **passage of time** where rates move as expected but with no change in the spread.

◆The most common assumption when the carry roll-down is calculated is that forward rates are realized (i.e., the forward rate for a future period remains unchanged as we move through time).

P&L Components

- **Decomposition of P&L**
  - ② **Rate change**: the price effect of **rates changing** from the intermediate term structure to the term structure that actually prevails at time t+1.
  - ③ The price appreciation due to a **spread change** is the price effect due to the bond's individual spread changing from s(t) to s(t+1).

P&L Components

- **Example**

	Start period	**2010 1-1**	**2011 1-1**	**2012 1-1**	**price**	**P&L**
Pricing date: 2010-1-1; annual coupon=1						
Initial forwards	Term structure	2%	3%	4%	93.0229	
	spreads	0.5%	0.5%	0.5%		
Pricing date: 2011-1-1; annual coupon=1						
Carry-roll-down	Term structure		3%	4%	94.3485	+1.3256 Carry-Roll-Down: 2.3256
	spreads		0.5%	0.5%		
Rate change	Term structure		2%	3%	96.1800	+1.8315
	spreads		0.5%	0.5%		
Spread change	Term structure		2%	3%	95.2577	-0.9223
	spreads		1%	1%		

P&L Components

$$P = \frac{1}{1+2.5\%} + \frac{1}{(1+2.5\%)(1+3.5\%)} + \frac{101}{(1+2.5\%)(1+3.5\%)(1+4.5\%)} = 93.0229$$

$$P_{\text{carry-roll-down}} = \frac{1}{1+3.5\%} + \frac{101}{(1+3.5\%)(1+4.5\%)} = 94.3485$$

$$P_{\text{rate change}} = \frac{1}{1+2.5\%} + \frac{101}{(1+2.5\%)(1+3.5\%)} = 96.18$$

$$P_{\text{spread change}} = \frac{1}{1+3\%} + \frac{101}{(1+3\%)(1+4\%)} = 95.2577$$

Parallel Term Structure Shifts

Parallel Term Structure Shifts

- **Macaulay Duration**
  - Average period of cash flow returning weighted by discounted cash flow.

$$\boxed{\text{Mac. D} = \sum_{t=1}^{T}\left(\frac{PV(CF_t)}{P}\times t\right) = \sum_{t=1}^{T}(w_t \times t)}$$

- **Example**

If a bond has a present value of USD 93.06 with a cash flow in one year providing a present value of USD 5.45 and a cash flow in two years providing a present value of USD 87.60, the Macaulay duration would be:

$$\frac{5.45}{93.06}\times 1 + \frac{87.60}{93.06}\times 2 = 1.9414$$

- For a **plain bond**, the Macaulay duration is **less than** or equal to its maturity.
- For a **zero coupon bond**, the Macaulay duration **equals** to its maturity.

Parallel Term Structure Shifts

Modified Duration and Dollar Duration

- The value of the duration depends on the **compounding frequency** of the yield.

$$\boxed{\text{Modified Duration(MD)} = \frac{\Delta P/P}{\Delta y} = \frac{\text{Macaulay Duration}}{1 + y/m}}$$

- The approximate duration relationship of bond is:

$$\Delta P = -MD \times P \times \Delta y$$

Where P is bond price; MD is duration; Δy is yield change and ΔP is price change.

- If the yield is measured with **continuous compounding**

$$\text{Macaulay Duration} = \text{Modified Duration}$$

- **Dollar duration** is another measure which is the **product** of the **modified duration** and the **bond price**.

$$DD = MD \times P = \Delta P/\Delta y$$

Parallel Term Structure Shifts

- **DV01**
  - The DV01 is the **absolute value** of the price change of a bond from one basis point change in yield.

$$\mathrm{DV01} = -\frac{\Delta \mathrm{P}}{\Delta \mathrm{r}}$$

$$\mathrm{DV01} = \mathrm{MD} \times \mathrm{Bond\ Value} \times 0.0001$$

- **Example**
  - Macaulay duration =1.9414
  - Bond value=93.06
  - ✓ This indicates that a 1 bps change in the continuously compounded yield to give rise to a price change of:

$$-1.9414 \times 93.06 \times 0.0001 = -0.01807$$

Parallel Term Structure Shifts

- **Limitations of Duration**
  - Duration provides a <u>good approximation</u> when there's <u>small parallel shift in the interest rate term structure</u>. However, it will provide a <u>poor approximation</u> if there's <u>non-parallel shift or the change is large</u>.
- **Convexity**
  - A measure of the <u>non-linear relationship.</u>

$$\text{Convexity} = \sum_{t=1}^{T} (w_t \times t^2)$$

  - When rates are expressed with compounding m times per year:

$$\text{Modified Convexity} = \frac{\text{Convexity}}{\left(1 + \frac{y}{m}\right)^2}$$

Risk Metrics

➢ Price Approximation

$$P(y_0+\Delta y)=P(y_0)+\underbrace{f'(y_0)}_{\text{Dollar Duration}}\Delta y+\frac{1}{2}\underbrace{f''(y_0)}_{\text{Dollar Convexity}}(\Delta y)^2+\cdots$$

- The actual, exact price: $P=f(y_0+\Delta y)$
- The duration estimate: $P=P_0-D\times P_0\times\Delta y$
- The duration and convexity estimate:

$$P=P_0-D\times P_0\times\Delta y+\frac{1}{2}\times C\times P_0\times(\Delta y)^2$$

Parallel Term Structure Shifts

Duration and Convexity Analysis

- The effect of parallel shifts of interest rate term structure can be more accurate by adding convexity analysis to the analysis of duration.
- If C is convexity, the approximate price change can be refined to:

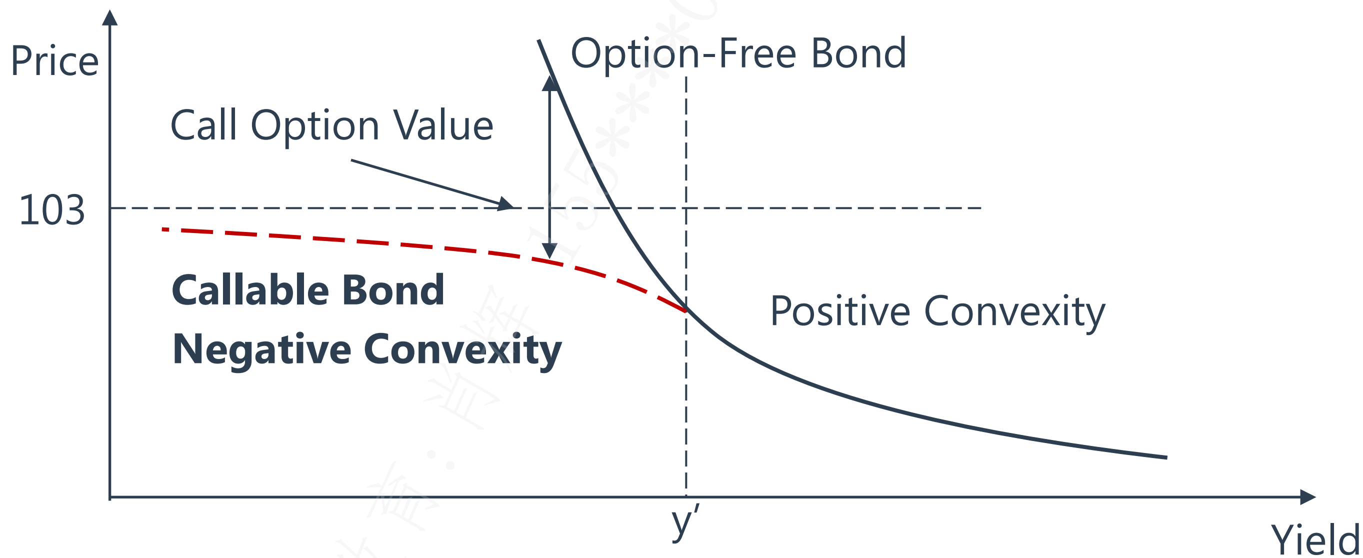
$$\boxed{\Delta P = -DP\Delta y + \frac{1}{2}CP(\Delta y)^2}$$

- This approximation allows relatively large parallel shifts to be considered.

Parallel Term Structure Shifts

➢ **Negative Convexity**

- A **callable bond** gives the issuer the right to redeem all or part of the bond before the specified maturity date.
- Most **mortgage bonds** are negatively convex, and **callable bonds** usually exhibit negative convexity at lower yields.



Parallel Term Structure Shifts

- **Effective Duration and Effective Convexity**
  - In a vanilla bond (without embedded options) we can typically use modified and effective interchangeably. When the bond contains embedded options, we prefer **effective duration**:

$$\boxed{D^E = -\frac{\Delta P/P}{\Delta r} = \frac{P_- - P_+}{2P_0 \Delta y}}$$

✓ Where

✓ P_0 = initial observed bond price

✓ Δy = change in required yield (in decimal form)

  - **Effective convexity**: an approximate measure of convexity

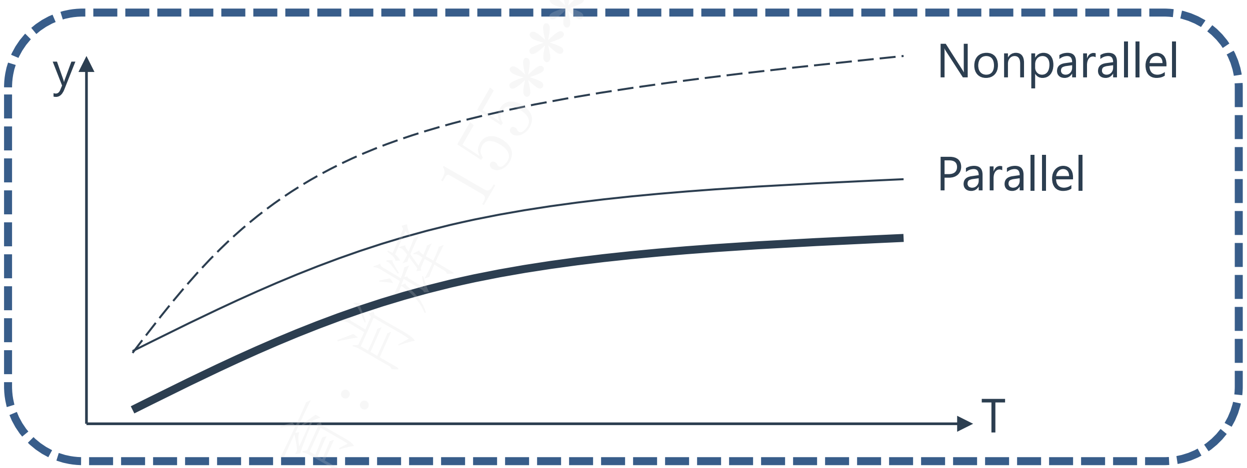
$$\boxed{C^E = \frac{D_- - D_+}{\Delta y} = \frac{P_- + P_+ - 2P_0}{P_0 \Delta y^2}}$$

Non-Parallel Term Structure Shifts

Non-Parallel Term Structure Shifts

Modeling Non-Parallel Term Structure Shifts

- In practice, there are many different types of non-parallel shifts. Sometimes short-term rates move down while long-term rates move up, or vice versa. Occasionally, short and long-term interest rates move in one direction, while medium-term rates move in the other direction



Non-Parallel Term Structure Shifts

- **Principal Components Analysis**
  - A statistical technique called principal components analysis can be used to understand term structure changes in historical data.
  - Principal component analysis can analyze the effects of multiple factors and estimate their relative importance in describing movements in the term structure.
  - This technique looks at the daily changes in interest rates corresponding to various maturities and identifies factors that have the following characteristics:

① These factors are **uncorrelated**;

② Daily changes in term structure are linear combinations of the factors;

③ The **first two or three** factors account for the **majority** of the observed daily movements.

Non-Parallel Term Structure Shifts

Rate Maturity	Factor							
	1	2	3	4	5	6	7	8
1 year	−0.129	0.384	0.778	−0.479	−0.009	0.003	−0.017	−0.007
2 year	−0.258	0.490	0.081	0.583	0.588	0.000	0.025	0.001
3 year	−0.326	0.426	−0.119	0.241	−0.727	−0.292	0.150	0.062
5 year	−0.410	0.203	−0.317	−0.216	−0.068	0.574	−0.548	−0.096
7 year	−0.434	−0.008	−0.294	−0.397	0.221	0.099	0.705	0.110
10 year	−0.414	−0.193	−0.104	−0.204	0.211	−0.707	−0.338	−0.284
20 year	−0.385	−0.397	0.234	0.161	−0.023	0.019	−0.169	0.764
30 year	−0.367	−0.440	0.348	0.317	−0.164	0.275	0.193	−0.557

- The change in the jth rate has the form:

$$\sum_{i=1}^{8} a_i f_{ij}$$

- f_{ij} is the factor loading for the ith factor and the jth rate
- a_i are referred to as factor scores

Non-Parallel Term Structure Shifts

Factors							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
14.15	4.91	2.44	1.59	1.09	0.85	0.78	0.68

- For our data, the total variance is

$$14.15^2 + 4.91^2 + \cdots + 0.68^2 = 235.77$$

  - The first factor accounts for 84.9% $(= 14.15^2/235.77)$
  - The first two factors account for:

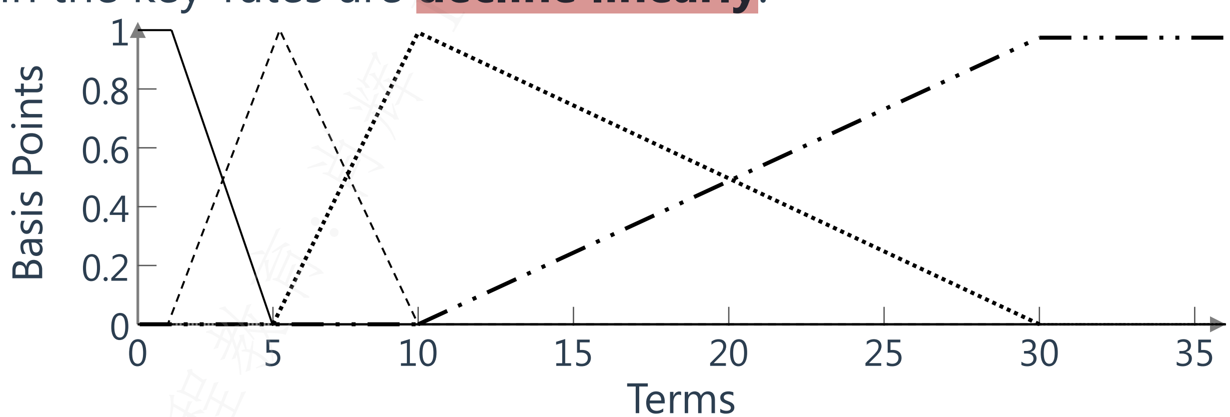
$$\frac{14.15^2 + 4.91^2}{235.77} = 95.14\%$$

  - The first three factors account for 97.66% of the variance.

$$\frac{14.15^2 + 4.91^2 + 2.44^2}{235.77} = 97.66\%$$

Non-Parallel Term Structure Shifts

- **Key-Rate Exposure**
  - **Key-Rate Shifts Assumption**
  - ✓ The crucial assumption of the key rate approach is that all rates can be determined as a function of a relatively small number of key rates. A few rates along the term-structure are picked which are representative of the curve.
  - ✓ The rate of a given maturity is affected **solely** by its **closest key-rate**.
  - ✓ Shifts in the key-rates are **decline linearly**.



Non-Parallel Term Structure Shifts

- **Key-Rate Exposure**
  - **Key-Rate Shifts Metrics**
    - ✓ **Key Rate '01s**: which is the key rate equivalent of DV01.
    - ✓ **Key Rate Duration**: which is the key rate equivalent of durations.

$$DV01_{key} = -0.0001 \times \frac{\Delta P}{\Delta y} \qquad D_{key} = -\frac{1}{P} \cdot \frac{\Delta P}{\Delta y}$$

  - **Example:** Calculate 30-year key rate 01 and key rate duration applying a one basis point shift.

	Initial Curve	2-year shift	5-year shift	10-year shift	30-year shift
Value	25.11584	25.11681	25.11984	25.13984	25.01254

$$\text{Key Rate } 01_{30} = -0.0001 \times \frac{25.01254 - 25.11584}{0.01\%} = 0.1033$$

$$\text{Key Rate Duration}_{30} = -\frac{(25.01254 - 25.11584)}{25.11584 \times 0.01\%} = 41.1294$$

Non-Parallel Term Structure Shifts

➢ **Forward Bucket Shift**

- Each forward bucket 01 is computed by shifting the **forward rates** in that bucket by one basis point.

Term	Cash Flow	Forward Rates (%)			
		Current	0-2 shift	2-5 shift	Shift All
0.5	1.06	1.012	1.022	1.012	1.022
1.0	1.06	1.248	1.258	1.248	1.258
1.5	1.06	1.412	1.422	1.412	1.422
2.0	1.06	1.652	1.662	1.652	1.662
2.5	1.06	1.945	1.945	1.955	1.955
3.0	1.06	2.288	2.288	2.298	2.298
3.5	1.06	2.614	2.614	2.624	2.624
4.0	1.06	2.846	2.846	2.856	2.856
4.5	1.06	3.121	3.121	3.131	3.131
5.0	101.06	3.321	3.321	3.331	3.331
Present Value		99.9955	99.9760	99.9679	99.9483
'01			0.0196	0.0276	0.0472

Non-Parallel Term Structure Shifts

- **Estimating Portfolio Volatility**
  - Regulators require Banks to consider ten different KR01s when analyzing the risk of their portfolios (including 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 10-year, 15-year, 20-year, and 30-year spot rates).
  - Banks don't need to make all the KR01s zero, but they need to use **KR01 exposure** and **standard deviation** of the ten rates to estimate risk measures such as VaR or Expected Shortfall.
  - The formula for the standard deviation of the change in value of the portfolio in one day is:

$$\sigma_P = \sqrt{\left(\sum_{i=1}^{10}\sum_{j=1}^{10}\rho_{ij}\sigma_i\sigma_j \times KR01_i \times KR01_j\right)}$$

σ_i is the volatility of the daily movement in rate i (measured in bps) and ρ_{ij} is the correlation between the daily movements in rate i and j.

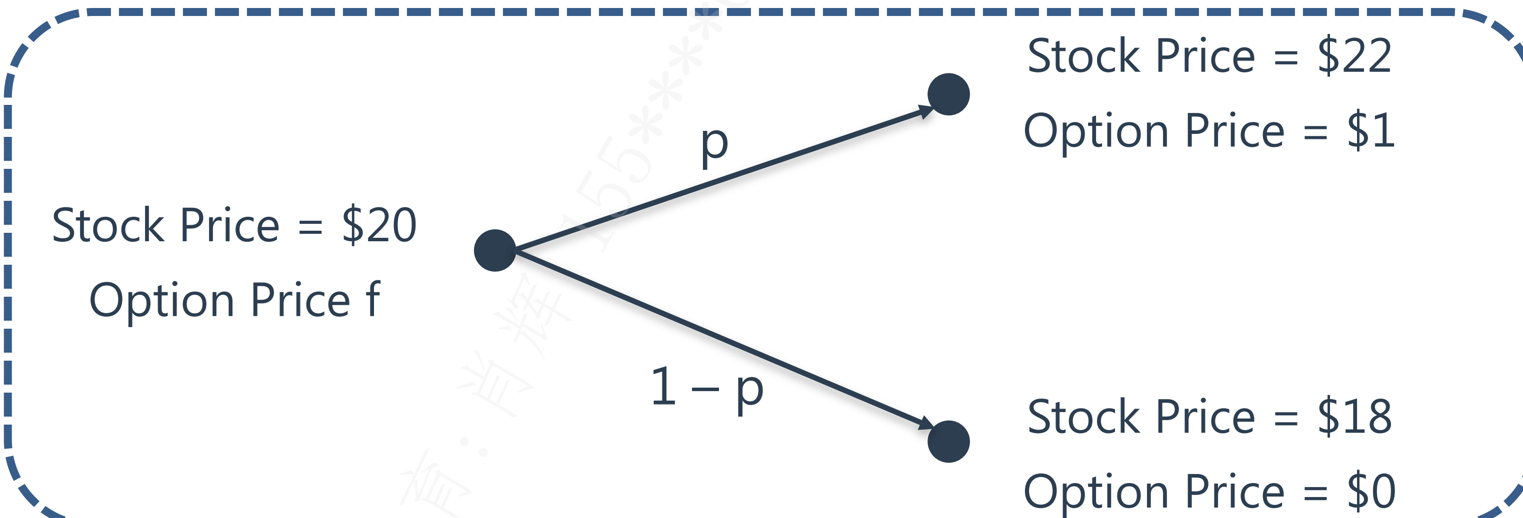
One-Step Trees

One-Step Trees

- **Risk-Neutral Valuation**
  - We define risk-neutral world as one where investors do not adjust their expected return based on risk, so the expected return on all assets is risk-free interest rate.
  - The **risk-neutral valuation** principle states that if we assume we are in a risk-neutral world, we can get a fair price for a derivative.

One-Step Trees

- **Example**
  - The stock price of ABC company is \$20 presently. The stock will go up to 22 or down to 18 three months later. What is the European call price of this stock three months from now? Suppose the strike price is $K = \$21$, continuously compounded risk-free rate is 12%.



One-Step Trees

- **Solution 1**
  - Long Δ stock, short 1 call.
  - In a risk-neutral world: $22\Delta - 1 = 18\Delta$
  - $\Delta = 0.25$
  - Portfolio's value at $t_0 = 20\Delta - f$
  - Portfolio's value at $t_1 = 22\Delta - 1$

$$(20 \times 0.25 - f)e^{0.12\times0.25} = 22 \times 0.25 - 1$$

  - $f = 0.633$
- **Solution 2**

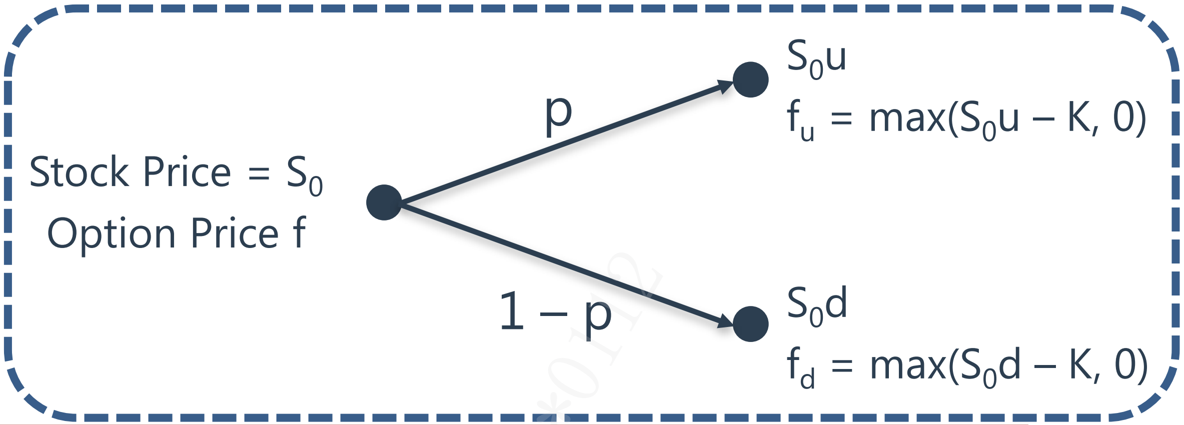
$$22P + 18(1 - P) = 20e^{0.12\times0.25}$$

$$P = 0.6523$$

$$f = \big(1 \times p + 0 \times (1 - p)\big)e^{-0.12\times0.25} = 0.633$$

One-Step Trees

➢ **Generalization**



- Long Δ stocks and short 1 call option to hedge the risk.
- $S_0u\Delta - f_u = S_0d\Delta - f_d$, to eliminate risks in both cases.

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d}$$

$$f = (pf_u + (1-p)f_d)e^{-r\Delta t}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \qquad u = e^{\sigma\sqrt{\Delta t}} \qquad d = e^{-\sigma\sqrt{\Delta t}}$$

Multi-Step Trees

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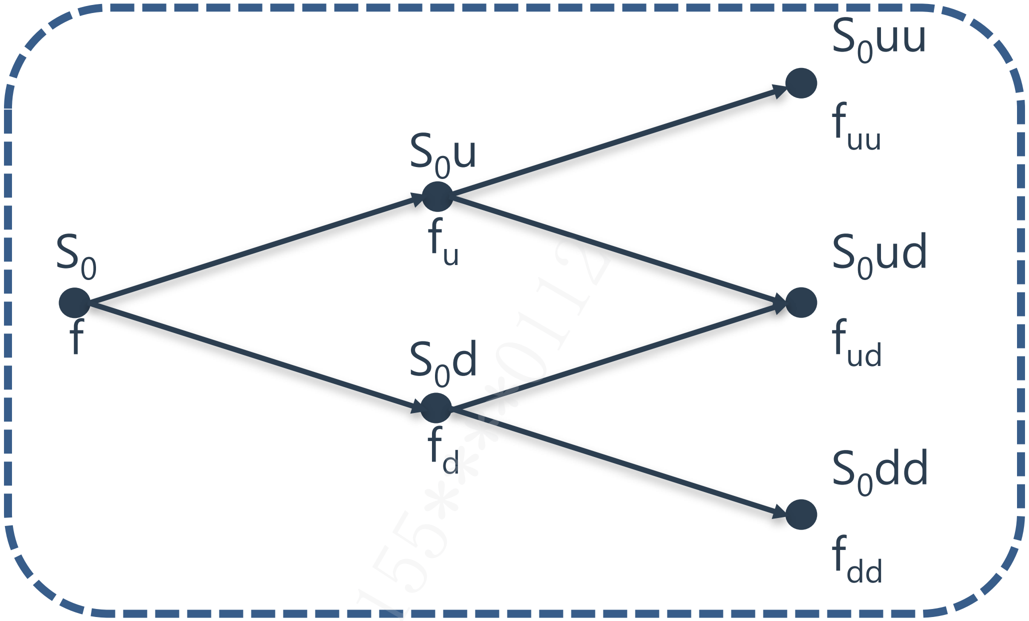
Multi-Step Trees

- **European Options**



$$p = \frac{e^{r\Delta t} - d}{u - d}$$

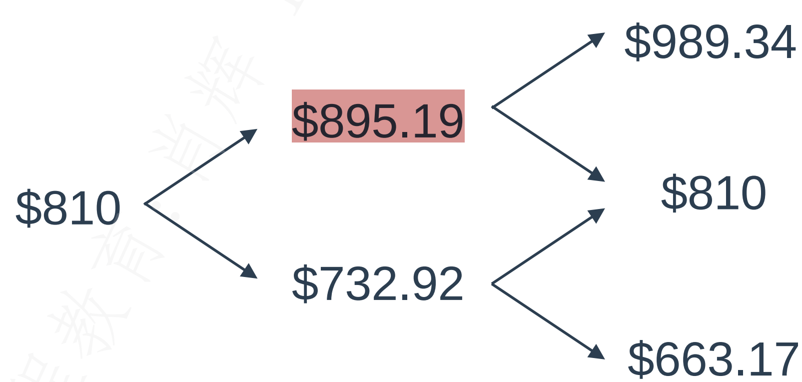
$$f = e^{-2r\Delta t}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$

Multi-Step Trees

- **Example: Two-step European Call Option with Up and Down Informed by Volatility**

Asset	Strike	Time	Volatility	Riskless	Div. Yield
\$810	\$800	0.5	20%	5%	2%

u	d	p
1.1052	0.9048	0.5126

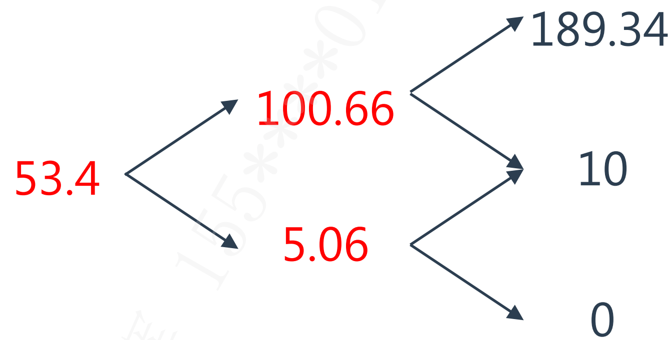


Multi-Step Trees

➢ **Example: Two-step European Call Option with Up and Down Informed by Volatility**

$$u = e^{\sigma\sqrt{t}} = e^{20\% \times \sqrt{0.25}} = 1.1052 \quad d = 0.9048$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} = \frac{e^{(5\% - 2\%) \times 0.25} - 0.9048}{1.1052 - 0.9048} = 0.5126$$



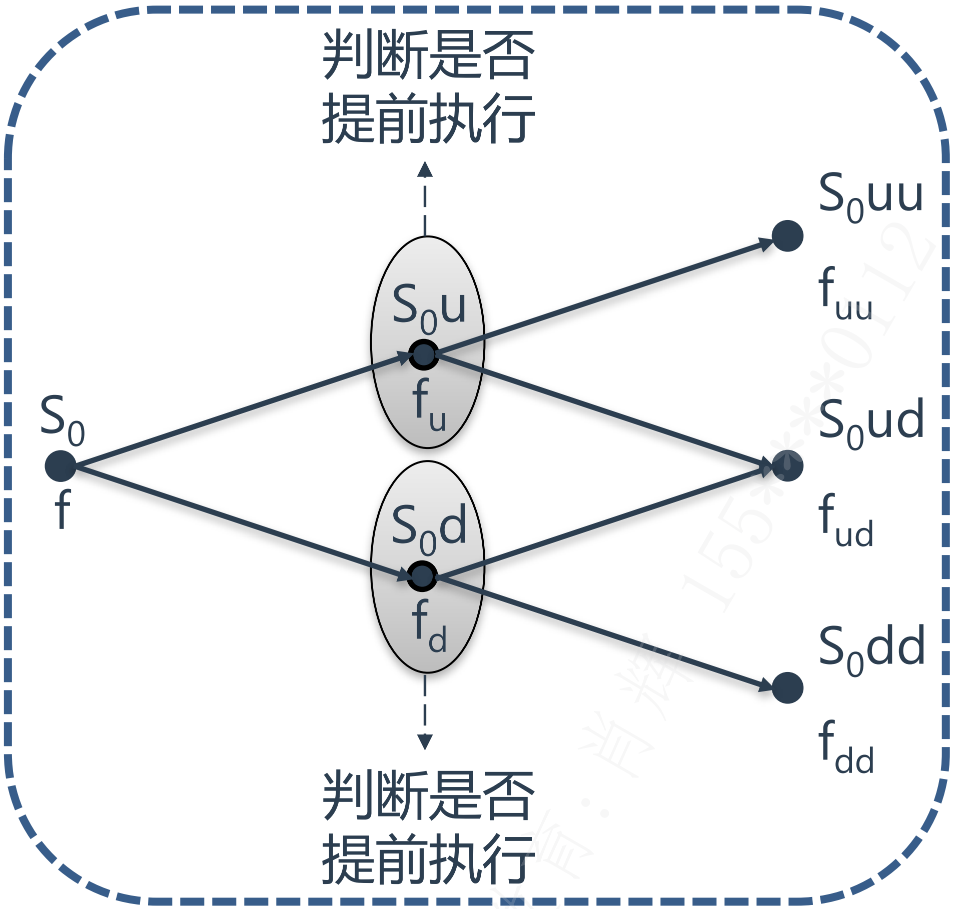
$$(189.34 \times 0.5126 + 10 \times (1 - 0.5126))e^{-0.05 \times 0.25} = 100.66$$

$$\big(10 \times 0.5126 + 0 \times (1 - 0.5126)\big)e^{-0.05 \times 0.25} = 5.06$$

$$\big(100.66 \times 0.5126 + 5.06 \times (1 - 0.5126)\big)e^{-0.05 \times 0.25} = 53.4$$

Multi-Step Trees

➢ **American Options**



- Make sure that the option value at each node is no less than the intrinsic value.

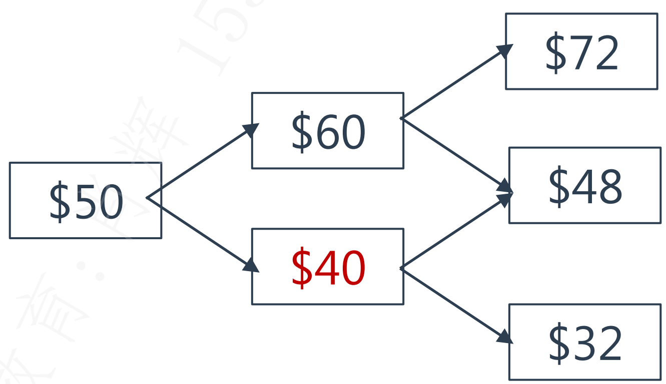
Multi-Step Trees

➢ **Example: American Put Option with price jump of +/- 20%**

Asset	Strike	Time	Riskless	Div. Yield
$50	$52	2	5%	0%

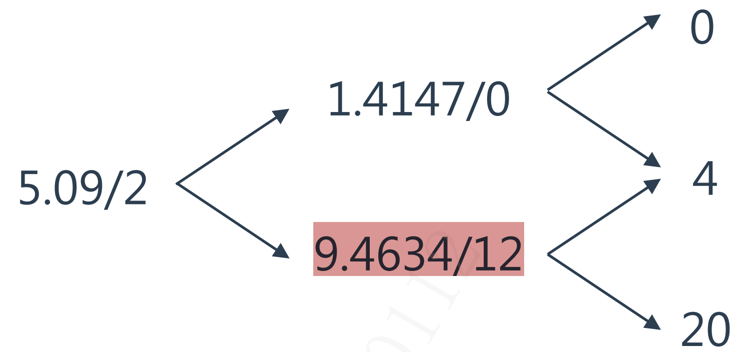
⇩

u	d	p
1.2	0.8	0.6282



Multi-Step Trees

- **Example: American Put Option with price jump of +/- 20%**



$$\left(0 \times 0.6282 + 4 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 1.4147$$

$$\left(4 \times 0.6282 + 20 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 9.4634$$

$$\left(1.4147 \times 0.6282 + 12 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 5.09$$

Other Assets

- **Options on Stocks with Dividends**

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

  - Everything else about the tree is the same as before.

- **Options on Stock Indices**
  - Usually, index provides a dividend yield. Therefore, the valuation should involve the modification as above.

- **Options on Currencies**
  - Currency can be considered as an asset providing a yield.

- **Options on Futures**
  - It costs noting to enter into a futures contract and we can treat a futures contract like a stock paying a dividend yield of r. Therefore, we get:

$$p = \frac{1 - d}{u - d}$$

BSM

Model Assumptions

➢ **Assumptions**

- The stock price follows the process with μ and σ constant.
- There are no transaction costs or taxes. All securities are perfectly divisible.
- There are no dividends during the life of the derivative.
- There are no riskless arbitrage opportunities.
- Security trading is continuous.
- The risk-free rate of interest, r, is constant and the same for all maturities.
- The options being considered cannot be exercised early.

Stock Price Movement

➢ Lognormal Assumption

- Geometric Brownian motion $dS_t = uS_t dt + \sigma S_t dZ_t$
- Ito's Lemma

$$df(t,S) = \left(\frac{\partial f}{\partial t} + uS_t\left(\frac{\partial f}{\partial S}\right) + \frac{1}{2}\sigma^2 S_t^2\left(\frac{\partial^2 f}{\partial S^2}\right)\right)dt + \sigma S_t\left(\frac{\partial f}{\partial S}\right)dZ_t$$

- As the stock price follows a Geometric Brownian Motion, if we define $G = \ln S$, using Ito's lemma, we have:

$$dG = \left(u - \frac{1}{2}\sigma^2\right)dt + \sigma dZ$$

- A variable has a lognormal distribution if the natural logarithm of the variable is normally distributed. While the equation above shows that $\ln S_T$ is normally distributed. The stock price is lognormally distributed.

Pricing Formulas

➢ **The Price of European Options**

$$\text{call} = S_0N(d_1) - Ke^{-rT}N(d_2)$$

$$\text{put} = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$

$$d_{1,2} = \frac{\ln(S_0/K) + \left(r \pm \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

- $N(d_1)$ is the delta of call
- $N(d_2)$ is the probability of call exercise
- $1 - N(d_2)$ is the probability of put exercise

Other Assets

- **Options on Stocks with Dividends**

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$d_{1,2} = \frac{\ln(S_0/K) + (r - q \pm \sigma^2/2)T}{\sigma\sqrt{T}}$$

- **Options on Currencies**
  - Behaves like a <u>stock paying a dividend yield at the foreign risk-free rate.</u>
- **Options on Futures**
  - Behaves like a <u>stock paying a dividend yield at the domestic risk-free rate.</u>

Delta Hedging

Delta Hedging

- **Delta**
  - The delta of an option, Δ, is defined as the rate of change of the option price with respect to the price of the underlying asset.
  - Delta is the slope of the curve that relates the option price to the underlying asset price.
- **Option Delta according to the BSM Model**
  - European call option

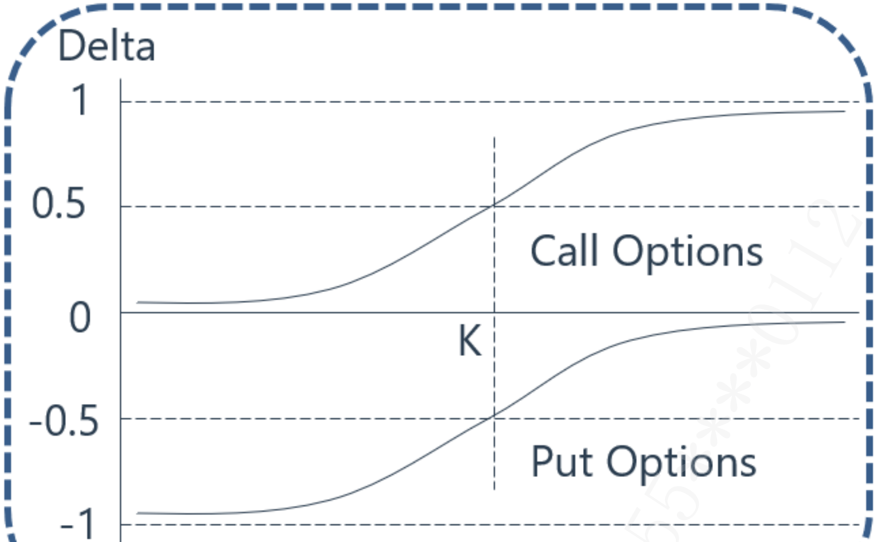
$$\boxed{\Delta(\text{call}) = N(d_1)}$$

  - European put option

$$\boxed{\Delta(\text{put}) = N(d_1) - 1}$$

Delta Hedging

➢ Characteristics of Delta



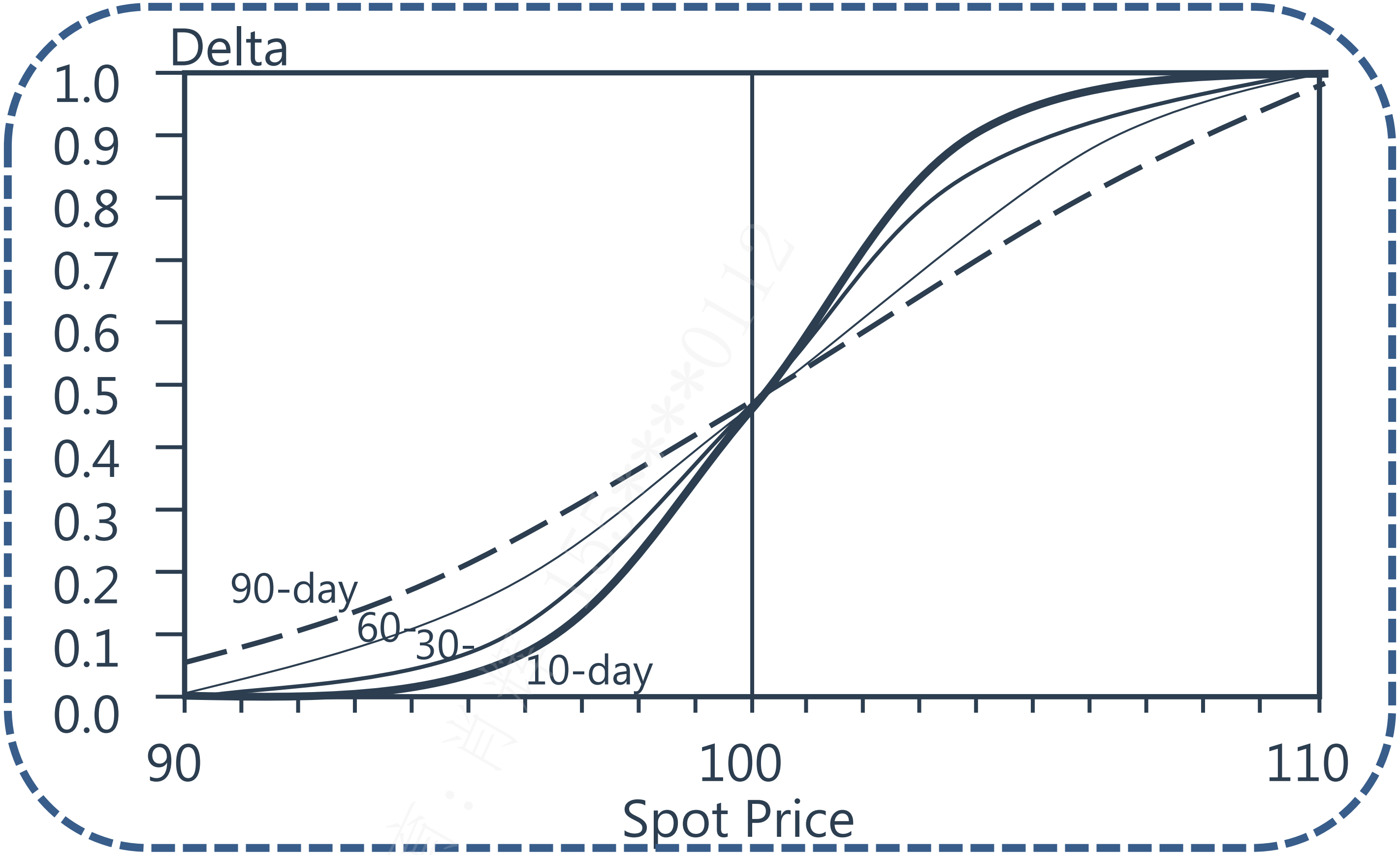
Call and Put Option Delta

Long Call	$\Delta > 0$
Short Call	$\Delta < 0$
Long Put	$\Delta < 0$
Short Put	$\Delta > 0$

- **Call** option Δ range from **0 to 1.**
- **Put** option Δ range from **-1 to 0.**

Delta Hedging

➤ **Delta (cont'd)**



- When t → T, Delta is unstable.

Delta Hedging

- **Delta Hedging Example**
  - A position with a delta of zero is called a delta neutral position.
  - Consider a European call option with a stock price of $102, a strike price of $105, a risk-free interest rate of 5%, a volatility of 20%, and a one-year maturity. In this case:

$$d_1 = \frac{\ln(102/105) + (5\% + 20\%^2/2)}{20\%} = 0.2051$$

$$\Delta = N(0.2051) = 0.5812$$

  - Therefore, the trader with a short option position of 1 million has a delta:

$$(-1{,}000{,}000) \times 0.5812 = -581{,}200$$

  - The trader should buy 581,200 shares to hedge the position.
  - A position is delta neutral only instantaneously (for a very short period of time). To maintain a delta neutral position, the trader must re-balance the portfolio.

Gamma Hedging

Gamma Hedging

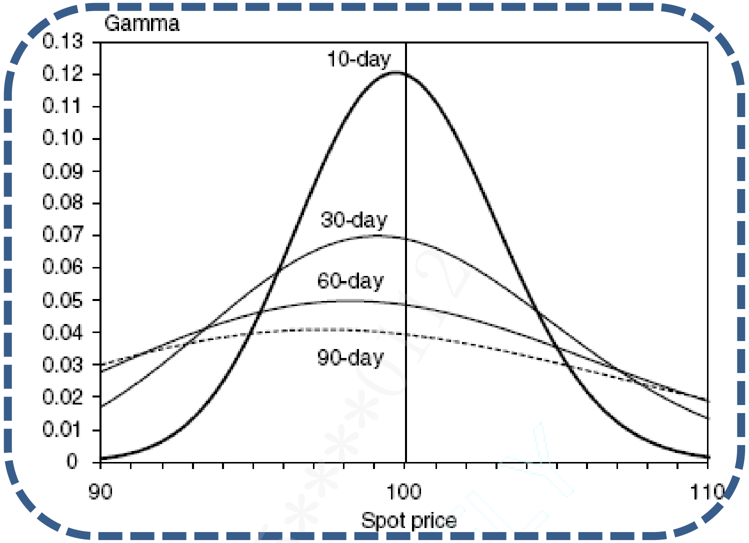
➢ **Gamma**

- The Gamma of options on an underlying asset is the rate of change of the option's delta with respect to the price of the underlying asset. That is to measure the stability of delta.
- Gamma is used to correct the hedging error associated with delta-neutral positions by providing added protection against large movements in the underlying asset's price. If gamma is highly negative or highly positive, delta is very sensitive to price of the underlying asset.
- Gamma is the same for call and put options.
- At-the-money options have the largest gamma
- When $t \rightarrow T$， Gamma ↑.

Long Call	Short Call	Long Put	Short Put
$\Gamma > 0$	$\Gamma < 0$	$\Gamma > 0$	$\Gamma < 0$

Gamma Hedging

➢ Gamma



➢ Option Gamma according to the BSM Model

- Gamma of a call (or put) option on a non-dividend paying stock:

$$\text{gamma} = \frac{N'(d_1)}{S\sigma\sqrt{T}}$$

Gamma Hedging

- **Gamma Hedging**
  - **Gamma Neutral Positions**: hedge against larger changes in stock price.
  - **Example:** Suppose that a portfolio is delta neutral and has a gamma of -3,000. The delta and gamma of a particular traded call option are 0.62 and 1.50, respectively. Create a gamma-neutral position.

✓ Buy 3,000/1.5 =2,000 options

✓ Then sold $2{,}000 \times 0.62 = 1{,}240$ shares of the underlying position.

	Delta 0	Gamma -3000
Buy 2000 options (3000/1.5 = 2000)	2000 × 0.62= 1240	0
Sell 1240 stocks	0	0

✓ You can also create a gamma needed position, e.g., gamma of -6,000

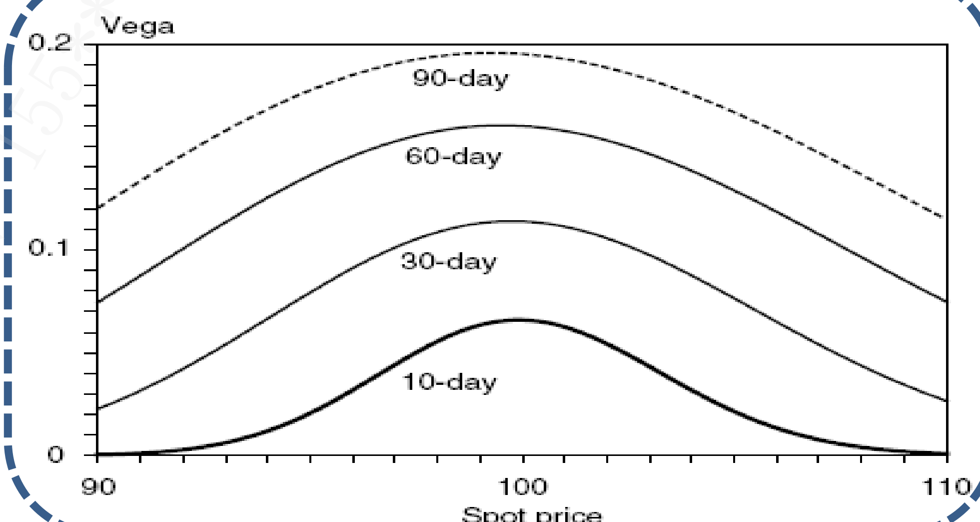
Vega Hedging

Vega Hedging

Vega

- Vega is the rate of change of the value of the option with respect to the volatility of the underlying asset. The volatility here is actually the implied volatility.
- At-the-money options are the most sensitive to volatility.
- Vega of a call is equal to the Vega of a put.
- When T ↑ , Vega ↑.

Long Call	Vega > 0
Short Call	Vega < 0
Long Put	Vega > 0
Short Put	Vega < 0



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Vega Hedging

➢ **Vega Hedging**

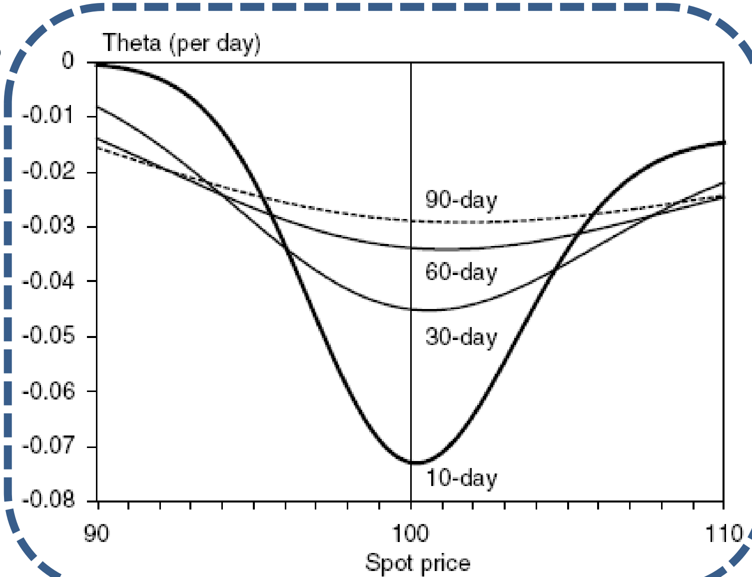
- Hedging Vega is not as easy as hedging delta. Vega can only be adjusted by taking a position in another derivative dependent on the same asset.
- Example: Given a delta-neutral portfolio which has a Vega of USD-300. An option on the same asset can be traded that has a delta of USD 2.5 and a Vega of USD 2. A trader can hedge Vega by buying 150 of these options. However, an amount of delta equal to USD 375 (2.5×150) is introduced into the portfolio. The trader must therefore short 375 shares to maintain delta neutrality.
- We can make both gamma and Vega zero. This can be done with two options.

Other Sensitivity Measures

Theta

➢ **Theta**

- Theta is the rate of change of the value of the option with respect to the passage of time with all else remaining the same.
- Theta is sometimes referred to as the time decay. As time to maturity decreases with all else remaining the same, the option tends to become less valuable, so theta is usually negative for an long option position.
- For long position, theta<0, means option lose value as time goes by.
- Short-term at the money option has a greatest negative theta.
- Deep ITM European put option, $\theta > 0$.
- Theta is not a risk factor.



Theta

➤ **Relationship between Delta, Theta, and Gamma**

- It follows that the value of f of a portfolio of such derivatives also satisfies the differential equation

$$rf = theta + rS_0 \times delta + \frac{1}{2}\sigma^2 {S_0}^2 \times gamma$$

- Where

✓ r = the risk-neutral risk-free rate of interest

✓ f = the value of the option

✓ θ = the option theta

✓ S_0 = the price of the underlying stock

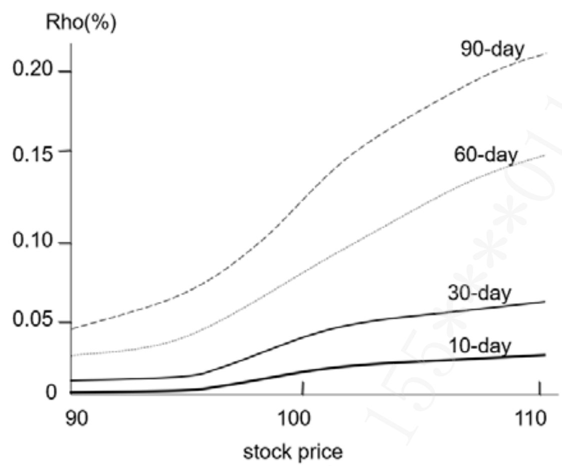
✓ Δ = the option delta

✓ σ^2 = the variance of the underlying stock

✓ Γ = the option gamma

Rho and Portfolio

- **Rho**
  - Rho is the sensitivity to the interest rates.
  - The impact on option prices when interest rates change is relatively small. The influence of interest rates is generally not a major concern.



$$\rho_C = \frac{\partial c}{\partial r} = Ke^{-rT}TN(d_2)$$

$$\rho_p = \frac{\partial p}{\partial r} = -Ke^{-rT}TN(-d_2)$$

- **Portfolio**
  - Any of the Greek letters for a portfolio of derivatives dependent on the same asset can be calculated as the weighted sum of the Greek letters for each component.
  - **Forward Delta:** 1 or e^{-qT}
  - **Futures Delta:** e^{rT} or $e^{(r-q)T}$

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Portfolio Insurance

Portfolio Insurance

- A portfolio manager is often interested in acquiring a put option on his or her portfolio. This provides protection against market declines while preserving the potential for a gain if the market does well. One approach is to buy put options on a market index such as the S&P 500. An alternative is to create the options synthetically.
- Creating an option synthetically involves maintaining a position in the underlying asset (or futures on the underlying asset) so that the delta of the position is equal to the delta of the required option. The position necessary to create an option synthetically is the reverse of that necessary to hedge it.

Measures of Financial Risk

Coherent Risk Measure

- **Features of Coherent Risk Measures**
  - **Monotonicity**
    - ✓ If one portfolio consistently produces worse results than another, it should have a higher risk metric.
  - **Subadditivity**
    - ✓ For any two portfolios A and B, the risk measure for the portfolio formed by combining A and B should not be greater than the sum of the risk measures for portfolio A and B.
  - **Homogeneity**
    - ✓ Changing the size of the portfolio by multiplying the total amount of assets by λ results in a measure of risk multiplied by λ.
  - **Translation Invariance**
    - ✓ If an amount of cash K is added to a portfolio, its risk measure should decrease by K.

Mean-Variance Framework

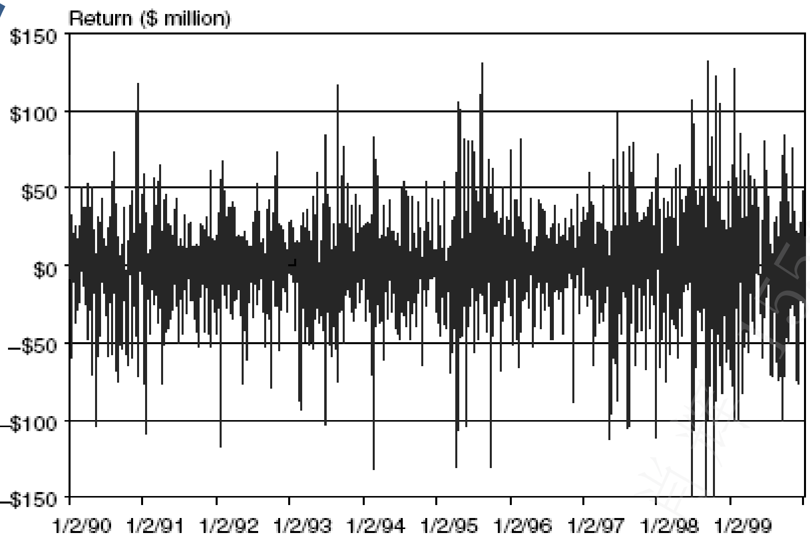
Mean-Variance Framework

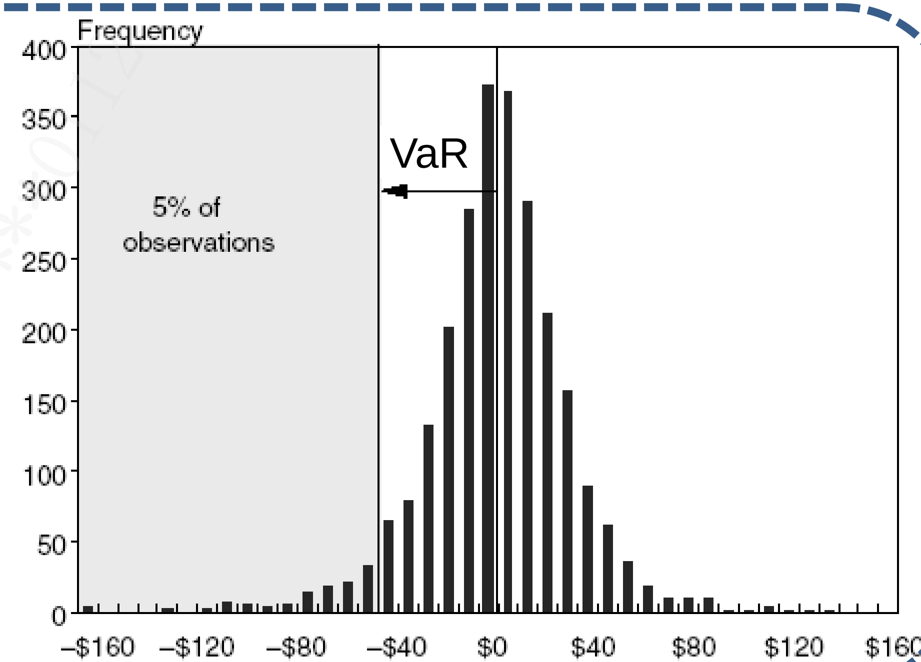
- In the mean-variance framework, we model financial risk in terms of the mean and variance (or standard deviation) of P/L (or returns). As a related convenience, we assume the daily P/L or returns obey a normal distribution.
- If our distribution is skewed or has heavier tails – as is typically the case with financial returns – then the normality assumption is inappropriate and the mean-variance framework can produce misleading estimates of risk. The use of the standard deviation as a risk measurement is not appropriate for non-normal distributions.

Value at Risk

➢ **Value at Risk**

- VaR is the maximum loss over a target horizon and for a given confidence level.





Value at Risk

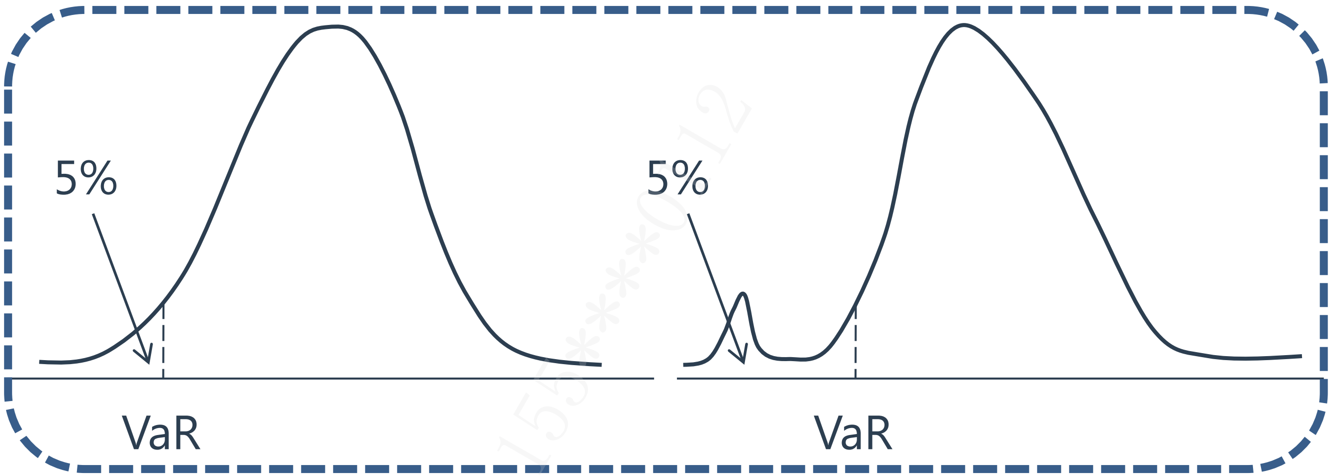
➢ **Limitations of VaR**

- The VaR only tells us the most we can lose if a tail event does not occur. (e.g., it tells us the most we can lose 95% of the time); if a tail event does occur, we can expect to lose more than the VaR, but the VaR itself gives us no indication of how much that might be.
- VaR is not subadditive. We can only make the VaR subadditive by imposing the severe restriction that the P/L distribution is elliptically distributed. A single counter example: A portfolio with two identical bonds, A and B. Each defaults with probability 4%, and we get a loss of 100 if default occurs, and a loss of 0 if no default occurs. The 95% VaR violates subadditivity.

Expected Shortfall

➢ Expected Shortfall

- A risk measure take account of expected losses beyond the VaR level which is also called conditional VaR or tail loss.



- Given the previous example where losses of $10 million, $6 million, and $3 million have probabilities of 4%, 8%, and 88% respectively. The 95% VaR is USD 6 million. The 95% expected shortfall is 9.2
- The expected shortfall is coherent.

Calculating and Applying VaR

Delta-Normal Model

- **Basic Measurement of VaR**
  - Given an expected return other than zero, VaR can be measured as:

$$\boxed{\text{VaR}(X\%) = |E(R) - Z_{X\%} \times \sigma| \qquad \text{VaR}(\$) = |E(R) - Z_{X\%} \times \sigma| \times V}$$

  - Example: given an expected return for a $1,000,000 portfolio, the expected 1-year portfolio return and standard deviation are 0.00124 and 0.0321, respectively. Calculate the 1-year VaR at 1% significance level.

$$\text{VaR} = |E(R) - Z_{X\%} \times \sigma| \times V = |0.00124 - 2.33 \times 0.0321| \times 1{,}000{,}000 = 73{,}553$$

- **Square Root Rule**
  - Usually, the time horizon is chosen as one day.
  - If fluctuations in a stochastic process from one period to the next are independent (i.e., there are no serial correlations or other dependencies).
  - In the longer term, the assumption is that

$$\text{VaR}(T, X) = \sqrt{T} \times \text{VaR}(1, X), \quad \text{ES}(T, X) = \sqrt{T} \times \text{ES}(1, X)$$

Here, $\text{VaR}(T, X)$ and $\text{ES}(T, X)$ are the value at risk and expected shortfall (respectively) for a time horizon of T days and confidence level of X.

Delta-Normal Model

➢ **Delta-Normal Approximation**

- The linear approximation is assumed and the underlying factor is assumed to follow a normal distribution. It is not good for derivatives with extreme nonlinearities (MBS, Fixed-income securities with embedded option).

$$\mathrm{VaR}(dP) = |-D \times P| \times \mathrm{VaR}(dy) \quad \mathrm{VaR}(df) = |\Delta| \times \mathrm{VaR}(dS)$$

➢ **Delta-Gamma Approximation**

$$\mathrm{VaR}(dP) = |-D \times P| \times \mathrm{VaR}(dy) - \frac{1}{2} \times C \times P \times \mathrm{VaR}(dy)^2$$

$$\mathrm{VaR}(df) = |\Delta| \times \mathrm{VaR}(dS) - \frac{1}{2} \times \Gamma \times \mathrm{VaR}(dS)^2$$

Historical Simulation

- **Example**
  - The portfolio under consideration is assumed to mainly depend on stock price, an exchange rate, an interest rate, and a credit spread.
  - **Stock prices and exchange rates** are examples of risk factors where the **percentage change** in the past is used to define a percentage change in the future.
  - **Interest rates and credit spreads** are examples of risk factors where the **actual change** in the past is used to define an actual change in the future.

Historical Simulation

- **Example**
  - From the data we've collected, we can create 500 scenarios and generate portfolio values for each of the scenarios and then calculate losses.
  - We sort the losses from the largest to the smallest.
  - Suppose we are interested in the VaR and expected shortfall with a one-day horizon and a 99% confidence level. The VaR can be calculated as the fifth worst loss. The expected shortfall is usually calculated as the average of the four losses that are worse than the VaR level.

Historical Simulation

Scenario Number	Loss (USD millions)
210	7.8
195	6.5
2	4.6
23	4.3
48	3.9
367	3.7
235	3.5
..	..
..	..
..	..

Monte Carlo Simulation

- **Procedure**
  - Suppose that we **assume** the change in risk factors has a multivariate **normal distribution.**
  - **Evaluate** the portfolio value **today** using the current values of the risk factors.
  - Sample once from the multivariate normal probability distribution for the **change in risk factors**.
  - Use the sampled values of the change to determine the values of the risk factors at the end of the time period (usually one day).
  - **Revalue the portfolio** using these risk factor values.
  - Subtract the value of the portfolio from the current value to determine the **loss**.
  - **Repeat** steps 2 to 5 many times to determine a **loss distribution**.
  - Suppose there is a total of 1000 simulation trials. The 99% VaR for the period considered will be the tenth worst loss.(1000*1%=10)

Monte Carlo Simulation

- **Characteristics**
  - Monte Carlo simulations generate scenarios by taking random samples from a hypothetical distribution of the risk factors (rather than using historical data). It applies to both linear and non-linear portfolios.
  - Any distribution can be assumed for the risk factors.
  - Monte Carlo simulation is computationally intensive and therefore quite slow.

Measuring and Monitoring Volatility

Volatility Measurement

➢ **Volatility Measurement**

- Denote the return on day i by r_i. Assuming the asset provides no income:

$$r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$$

- The usual formula for calculating standard deviations from sample data would give the volatility estimated for day n from the return on the m previous days as

$$\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(r_{n-i} - \bar{r})^2}$$

- $\bar{r}$ is the average return over the m previous days.

Volatility Measurement

Volatility Measurement

- In risk management, we usually simplify the formula in two ways: We substitute m for m-1, and we assume that $\bar{r} = 0$
- The two simplifications lead to the formula

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} r_{n-i}^2$$

- Each day, the forecast is updated by adding the most recent day and dropping the furthest day.

Current Volatility Estimation

- **EWMA**
  - Places exponentially declining weights on historical data.

$$\boxed{\sigma_n^2 = (1-\lambda)r_{n-1}^2 + \lambda\sigma_{n-1}^2}$$

  - We can also apply exponentially decreasing weights when using historical simulations.

Current Volatility Estimation

GARCH

- Places exponentially declining weights on historical data.

$$\boxed{\sigma_n^2 = \alpha r_{n-1}^2 + \beta\sigma_{n-1}^2 + \gamma V_L}$$

- Here, V_L is the long run mean variance rate. The parameters α, β, and γ are the weights. Because the weights must sum to one:

$$\alpha + \beta \le 1, \text{and}$$

$$\gamma = (1 - \alpha - \beta)$$

- Persistence: In GARCH (1,1), the sum of the alpha (α) and beta (β) parameters is called persistence. GARCH (1, 1) is unstable if the persistence > 1. A persistence of 1.0 implies no mean reversion. A persistence of less than 1.0 implies "reversion to the mean", where a lower persistence implies greater reversion to the mean.

Current Volatility Estimation

- **GARCH**
  - EWMA is a special case of GARCH.
  - Example:

$$\sigma_n^2 = 0.2 + 0.2\mu_{n-1}^2 + 0.7\sigma_{n-1}^2$$

  - The omega is 0.2 but don't mistake omega for the long-run variance. Omega is the product of gamma and the long-run variance. So, if alpha + beta = 0.9, then gamma must be 0.1. Given that omega is 0.2, we know that the long-run variance must be 2.

Correlation

Correlation Estimation

- The EWMA/GARCH model to update the covariance of regression X and regression Y is as follows:

$$cov_n = \lambda cov_{n-1} + (1-\lambda) x_{n-1} y_{n-1}$$

$$cov_n = \omega + \beta cov_{n-1} + \alpha x_{n-1} y_{n-1}$$

- If we use EWMA/GARCH to estimate the standard deviation of the rate of return, we can estimate the correlation coefficient.

$$\rho = \frac{Cov_n}{\sigma_{xn} \times \sigma_{yn}}$$

Rating Scales

Rating Scales

- **Long-Term Ratings**
  - Ratings for <u>bonds</u> are termed long-term ratings.

Investment Grade Ratings			
Interpretation	**S&P/Fitch**	**Moody**	**Interpretation**
Highest rating. Extremely strong capacity to meet obligations.	AAA	Aaa	Highest quality, with minimal credit risk.
Capacity to meet its financial obligation is very strong.	AA+	Aa1	High quality and subject to very low credit risk.
	AA	Aa2	
	AA-	Aa3	
Capacity to meet obligation still strong but susceptible to adverse changes in economic conditions.	A+	A1	Considered upper-medium grade and subject to low credit risk.
	A	A2	
	A-	A3	
Exhibits adequate protection parameters.	BBB+	Baa1	Subject to moderate credit risk.
	BBB	Baa2	
	BBB-	Baa3	

Rating Scales

➢ Long-Term Ratings

Speculative Grade Ratings			
Interpretation	**S&P/Fitch**	**Moody**	**Interpretation**
Less vulnerable to non-payment than other speculative issues but faces major ongoing uncertainties.	BB+	Ba1	Judged to have speculative elements and are subject to substantial credit risk.
	BB	Ba2	
	BB-	Ba3	
More vulnerable to non-payment than “BB” but has current capacity to meet financial obligation.	B+	B1	Considered speculative and are subject to high credit risk.
	B	B2	
	B-	B3	
Vulnerable to non-payment.	CCC+	Caa1	In poor standing.
	CCC	Caa2	
	CCC-	Caa3	
Highly vulnerable to non-payment.	CC/C	Ca	Highly speculative, likely to default.
In payment default.	D	C	Lowest rated bonds – typically in default, with little prospect for recovery.

Rating Scales

Short-Term Ratings

- Ratings for money market instruments are termed short-term ratings.

Moody's	S&P	Fitch
Investment Grade		
P-1	A-1+	F1+
	A-1	F1
P-2	A-2	F2
P-3	A-3	F3
Non-Investment Grade		
NP	B	B
	C	C
	D	D

Rating Process

Rating Process

- **Rating Process**
  - An instrument is usually first rated when it is issued.
  - Includes historical and projected financial information, industry and/or economic data, peer comparisons, and details of planned financing. Analysis is also based on qualitative factors, such as the institutional or governance framework. Rating is reviewed periodically.
  - A meeting with management is commonly undertaken.
  - The fee for rating is paid by the firm being rated.
- **Impact of Economic Cycle**
  - A **through-the-cycle rating** tries to capture a company's average creditworthiness over several years and should not be unduly influenced by ups and downs in overall economic conditions.
  - A **point-in-time rating** is designed to provide the best current estimate of future default probabilities.

Rating Process

- **Impact of Industry**
  - In the past, Banks with a given rating have indeed defaulted at a higher rate than non-financial companies with the same rating.
  - There has been less agreement among different rating agencies for banks than for other companies.
- **Impact of Geography**
  - Moody's, S&P, and Fitch are based in the United States and much of the information they report is based on U.S. data. Consequently, it is sometimes difficult to determine whether ratings for non-U.S. firms are in line with those of U.S. firms.

Rating Process

- **Impact of Rating Changes**
  - It is possible that when a rating is moved down, new information is being provided to the market so that both the stock and bond prices decline while credit default swaps spreads increase.
  - The stock and bond markets reacted **strongly** to the **downgrade**.
  - ✓ This is especially true when the downgrade is from investment grade to non-investment grade.
  - However, the market's reaction to upgrades is much less pronounced.
  - Generally, credit default swap changes seem to anticipate rating changes. Indeed, they found credit spread changes provide helpful information in estimating the probability of negative credit rating changes.

Alternative to Ratings

Alternative to Ratings

➢ Other Models alternative to Rating

- Organizations such as KMV and Kamakura use models to estimate default probabilities and provide the output from these models to clients.
- The balance sheet summarizes a firm's capital structure, where the value of assets (A) on the left-hand side must equal the value of the sum of debt (D) and equity(E).
- Equity prices, which are a key input to their models, are continually changing to reflect the latest information.

Alternative to Ratings

- The default happens if the value of the assets falls below the face value of the debt repayment that is required at that time. If V is the value of the assets and D is the face value of the debt, the firm defaults when V<D. The value of the equity at the future time is

$$\max(V-D,0)$$

- Value of Equity=Call on Firm= $VN(d_1) - De^{-rT}N(d_2)$
- The firm defaults if the option is not exercised.

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Internal Ratings

Internal Ratings

➢ Internal Ratings

- Banks and other financial institutions develop their own internal rating systems. Their ratings are usually based on several factors. In general, each factor is scored, and a weighted average score is calculated to determine the overall final rating.
- Like external ratings, internal ratings can be either through-the-cycle or point-in-time. There is a tendency for them to be point-in-time, but through-the-cycle ratings may be more relevant for relatively long-term lending commitments.
- Banks must back-test their procedures for calculating internal ratings.

Internal Ratings

- **Altman's Z-score**
  - This is used to come up with a rule for distinguishing between those firms that default from those that do not.

$$Z=1.21X_1+1.40X_2+3.30X_3+0.6X_4+0.999X_5$$

- ✓ X_1 is working capital/total assets,
- ✓ X_2 is retained earnings/total assets,
- ✓ X_3 is earnings before interest and taxes to total assets,
- ✓ X_4 is market value of equity to book value of total liabilities,
- ✓ X_5 is sales/total assets, and z is a number

  - A Z-score **above 3** indicated that the firm was **unlikely to default**. As the **Z-score was lowered**, the **probability of default increased** to the point where a firm with a **Z-score below 1.8** had a **very high probability of defaulting.**

Hazard Rates

Hazard Rates

➢ **Using Hazard Rate to Calculate Unconditional Default Probability**

- Hazard Rate is the rate at which default are happening. We can use it to calculate unconditional default probabilities.
- Suppose that $\bar{\lambda}$ is the average hazard rate between time zero and time t, the unconditional default probability between time zero and time t is:

$$\boxed{1 - e^{-\bar{\lambda}t}}$$

- The unconditional probability of default between time t_1 and t_2 is:

$$\boxed{e^{-\bar{\lambda}_1 t_1} - e^{-\bar{\lambda}_2 t_2}}$$

- Where $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are the average hazard rates between today and time t_1 and t_2 (respectively).

Hazard Rates

- **Example**
  - Suppose that the hazard rate is constant at 1% per year. The default probability by the end of the second year is:

$$1 - e^{-0.01\times 2} = 0.0198$$

  - Unconditional probability of a default occurring during the third year is:

$$e^{-0.01\times 2} - e^{-0.01\times 3} = 0.0098$$

  - Conditional probability of defaulting in the third year, given that it has survived until the end of the second year is:

$$\frac{0.0098}{1 - 0.0198} = 0.00995$$

Evaluation of Country Risk

Evaluation of Risk

- **GDP Growth Rates**
  - An important consideration in assessing country risk is how a country will respond to economic cycles.
  - During economic downturns, GDP in developing countries tends to decline more than GDP in developed countries.
- **Political risk**
  - Risk that changes in governments, government decisions, or the way governments operate will significantly affect the profitability of a business.
  - ✓ Democracies or authoritarian governments.
  - ✓ Corruption.
  - ✓ Violence.
  - ✓ Nationalization or expropriation.

Evaluation of Risk

- **Legal Risk**
  - Risk of losses due to inadequacies or biases in a country's legal system. Property rights and contract enforcement are important aspects of a legal system.
- **The Economy**
  - It is also important to assess the country's competitive advantages and the extent to which its economy is diversified.

Composite Risk Measure

- **Political Risk Service (PRS)**
  - Uses 22 measures of political, financial and economic risk to calculate its index. Individual companies can customize the PRS forecasting model to their own projects or exposures.
- **Euromoney**
  - Euromoney's score is based on a survey of 400 economist.
- **The Economist**
  - The Economist develops country risk scores internally based on currency risk, sovereign debt risk, and banking risk.
- **The World Bank**
  - Provides country risk data measuring corruption, government effectiveness, political stability, regulatory quality, the rule of law, and accountability.

Sovereign Credit Risk

Sovereign Credit Risk

- **Foreign Currency Defaults**
  - The risk for the issuing country is that it cannot simply print more money to repay its debts.
- **Local Currency Defaults**
  - Some countries have defaulted on debt issued in their own currency as well as on debt denominated in foreign currency.
  - Reasons:
  - ✓ In the decades before 1971, currencies had to be backed by gold reserves.
  - ✓ Members of the European Union use the euro as their domestic currency.
  - ✓ Printing more money debases the currency and leads to inflation.

Sovereign Credit Risk

➢ **Impact of a Default**

- The modern consequences of a default by a sovereign nation include:

✓ A loss of reputation and an increased difficulty in raising funds for several years.

✓ A lack of investors willing to buy the debt and equity of corporations in the country.

✓ An economic downturn.

✓ Political instability because people have lost confidence in their leaders.

✓ A default has a negative impact on GDP growth, has a negative impact on the country's credit rating for many years, can hurt exports, and can make the banking system of the defaulting country more vulnerable.

Sovereign Credit Risk

- **Sovereign Credit Ratings**
  - Rating agencies consider several factors when rating countries:
  - ✓ Commitment to social security.
  - ✓ The tax base.
  - ✓ Political risk.
  - ✓ Implicit Guarantees.

Sovereign Credit Risk

Credit Spread

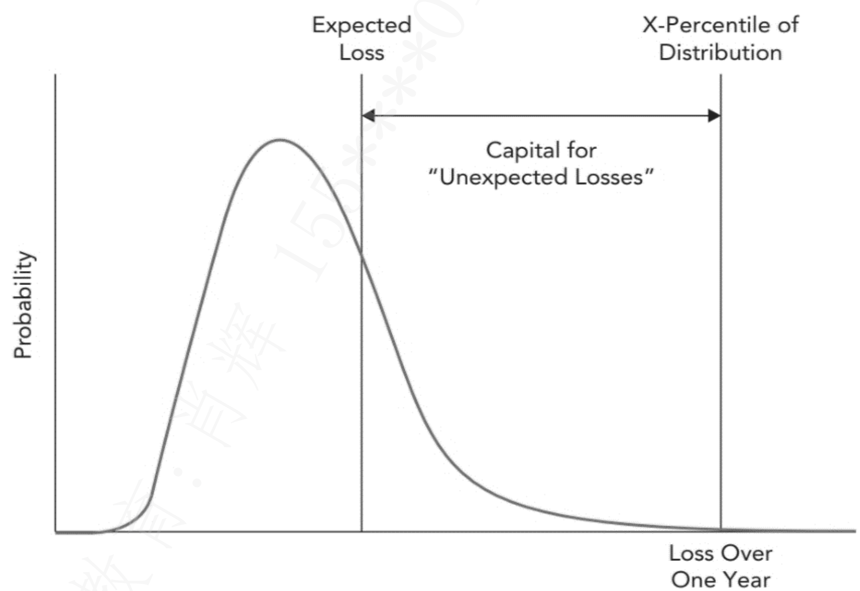
- The credit spread for sovereign debt in a given currency is the excess interest paid over the risk-free rate in that currency.
- credit spreads can provide additional information on the ability of a country to repay its debt. One reason for this is that credit spreads are more granular than ratings.
- Credit spreads also have the advantage of being able to adapt to new information more quickly than ratings. However, they are also more unstable.
- One source of data on credit spreads is the credit default swap market.

Model for Determining Capital

Model for Determining Capital

Capital Model

- Expected losses refer to the losses that banks take into account when setting lending rates.
- The bank's capital is a buffer against unexpected loss (i.e., the actual loss in a given year above the expected loss).



Three Drivers

- **Probability of Default (PD)**
  - Likelihood that a borrower will default within a specified time horizon.
  - **Credit migrations** or discrete changes in credit quality (such as those due to ratings changes) are crucial, since they influence the term structure of default probability.
- **Exposure at Default (EAD)**
  - Amount of money lender can lose in the event of a borrower's default.
- **Loss given Default (LGD)**
  - The amount of creditor loss in the event of a default.

Recovery Rates

Recovery Rates

- The recovery rate of a bond is usually defined as the value of the bond shortly after its default and expressed as a percentage of its face value. The loss given default (which provides the same information) is the percentage recovery rate subtracted from 100%.
- Recovery rate is negatively correlated with default rates.
- Bonds vary according to their seniority and to the extent to which there is collateral.

$$\mathrm{RR} = \frac{\text{recovery}}{\text{exposure}} = 1 - \frac{\text{LGD}}{\text{exposure}}$$

- The expected loss of a loan over a certain period of time is

$$\text{Probability of Default} \times \text{Loss Given Default}$$

$$\text{Probability of Default} \times (1 - \text{Recovery Rate})$$

Models of Measuring Credit Risk

Mean and Standard Deviation of Credit Losses

➢ **Mean and Standard Deviation**

- Assume: L_i is the amount borrowed in the ith loan; p_i is the probability of default for the ith loan; R_i is the recovery rate.

$$\mu_i = p_i \times L_i(1 - R_i) + (1 - p_i) \times 0 = p_i L_i (1 - R_i)$$

$$\sigma_i = \sqrt{p_i - p_i^2}(L_i(1 - R_i))$$

- Assume all loans have the same principal L, all recovery rates are the same and equal to R, all default probabilities are the same and equal to p. The standard deviation of the loss is then the same for all i.

$$\sigma_P^2 = n\sigma^2 + n(n-1)\rho\sigma^2$$

- The standard deviation of the loss from the loan portfolio as a percentage of its size

$$\alpha = \frac{\sigma_P}{nL} = \frac{\sigma\sqrt{1 + (n-1)\rho}}{L\sqrt{n}}$$

Mean and Standard Deviation of Credit Losses

- **Example**
  - Suppose a bank has a portfolio with 50,000 loans, and each loan is USD 1 million and has a 2% probability of default in a year. The recovery rate is 30%. Assume $\rho = 0.1$, then:
  - ✓ The standard deviation of the loss is

$$\sigma = \sqrt{p_i - p_i^2}(L_i(1 - R_i))$$

$$= \sqrt{(0.02 - 0.0004)} \times 1 \times 0.7 = 0.098$$

  - ✓ The standard deviation of the loss from the loan portfolio as a percentage of its size is

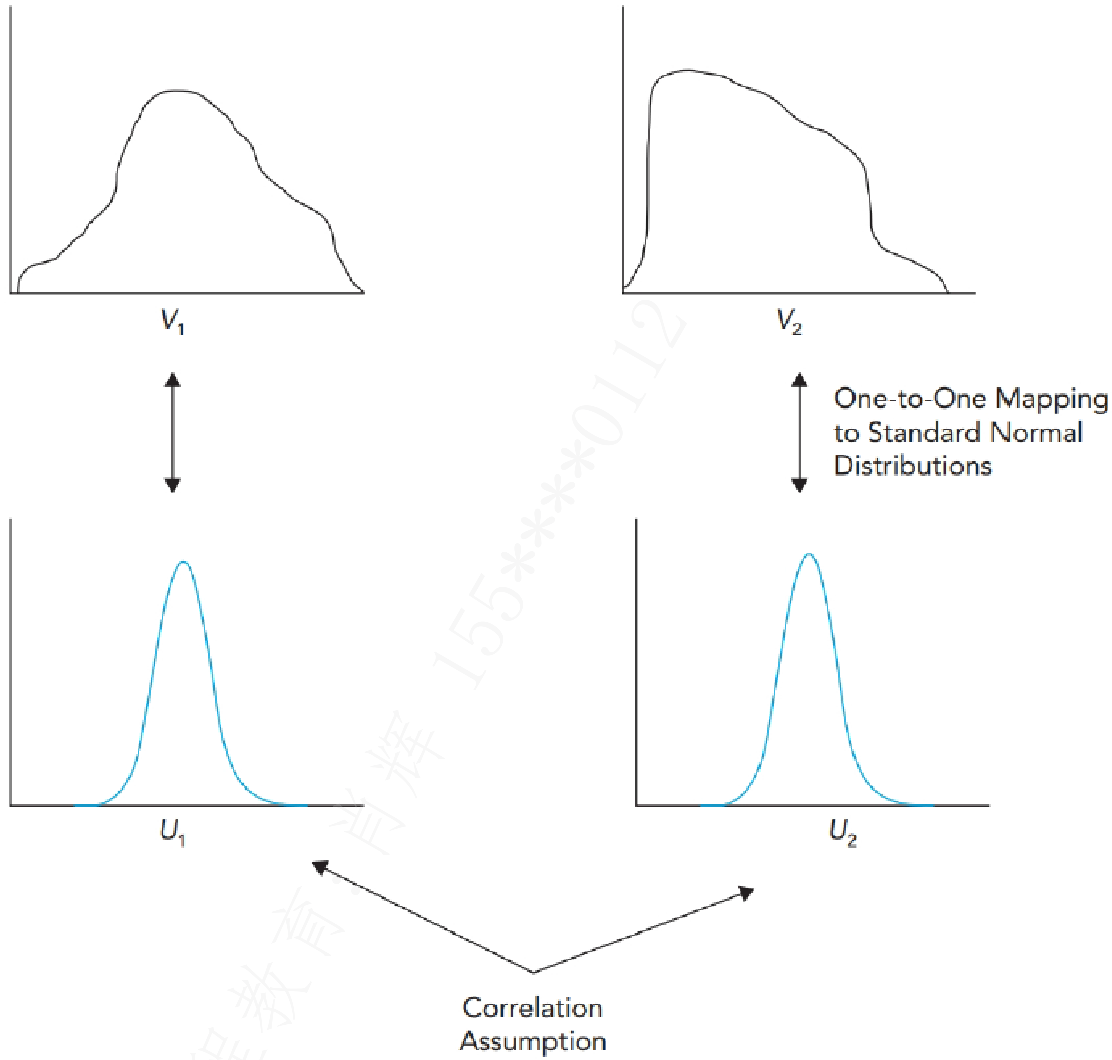
$$\alpha = \frac{\sigma\sqrt{1 + (n-1)\rho}}{L\sqrt{n}} = \frac{0.098\sqrt{1 + 49{,}999 \times 0.1}}{\sqrt{50{,}000} \times 1} = 0.0310$$

Gaussian Copula Model

➢ Gaussian Copula Model

- Suppose we have already known the probability distributions for variables V_1, and V_2, and we also want to define the complete way in which they depend on each other (that is, their joint probability distribution).
- We can assume the joint distribution is bivariate normal distribution as long as both variables are normally distributed.
- If they are not normally distributed, we can transform each distribution to a standard normal distribution by transforming their percentiles to the corresponding percentiles of a standard normal distribution. Then we can define the joint distribution of the transformed variables.

Gaussian Copula Model



One-Factor Correlation Model

- **One-Factor Correlation Model**
  - When we have many variables and each can be mapped to a standard normal distribution U_i. We can use a one-factor model to defining the correlation between the U_i distributions.

$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

  - Where F is a factor common to all the U_i, Z_i is the component that is unrelated to the common factor F, a_i are parameters. The variables F and Z_i have standard normal distributions.

Vasicek Model

➢ **Vasicek Model**

- a_i are assumed to be the same for all i, then the correlation between each pair of U-distributions is a^2.
- For each value of F, the distribution of each U_i has a mean of aF and a standard deviation of $\sqrt{1-a^2}$. For a large portfolio, the default rate is the probability U_i is less than $N^{-1}(PD)$.

$$\text{Default Rate as a function of F} = N\left(\frac{N^{-1}(PD) - aF}{\sqrt{1-a^2}}\right)$$

$$\text{99.9 percentile for default rate} = N\left(\frac{N^{-1}(PD) - aN^{-1}(0.001)}{\sqrt{1-a^2}}\right)$$

Vasicek Model

➢ **Vasicek Model**

- Used by regulators to determine capital for loan portfolios. It uses the Gaussian copula model to define the correlation between defaults.
- The Basel II capital requirement use the IRB approach is:

$$(\mathrm{WCDR} - \mathrm{PD}) \times \mathrm{LGD} \times \mathrm{EAD}$$

- where the WCDR (worst case default rate) is the 99.9 percentile of the default rate distribution.
- Assume the PD is the same for all companies in a large portfolio. The binary probability of the default distribution is mapped to a standard normal distribution U_I. Values in the extreme left tail of this standard normal distribution correspond to a default.

CreditMetrics

- **CreditMetrics**
  - In this model, A one-year transition table is used to define how ratings change.
  - The bank's portfolio of loans is valued at the beginning of a one-year period. A Monte Carlo simulation is then conducted to model how ratings change during the year. In each simulation trial, the ratings of all borrowers at the end of the year are determined and the portfolio is revalued. Credit loss is calculated as the value of the portfolio at the beginning of the year minus the value of the portfolio at the end of the year. The results of many simulation trials are used to produce a complete distribution of credit loss.
  - One of the differences between CreditMetrics and Vasicek model is that it takes into account the impact of **rating changes** as well as **defaults**.

Risk Allocation

Risk Allocation

- Euler's result is concerned with what are termed **homogeneous functions**. These are functions, F, of a set of variables $x_1, x_2, ..., x_n$ having the property:

$$F(\lambda x_1, \lambda x_2, \dots, \lambda x_n) = \lambda F(x_1, x_2, \dots, x_n)$$

✓ for a constant λ. Define

$$Q_i = x_i \frac{\Delta F_i}{\Delta x_i}$$

✓ Q_i is the ratio of ΔF_i to a proportional change, $\Delta x_i / x_i$ in x_i. Euler showed that in the limit, as the size of the Δx_i tends to zero:

$$F = \sum_{i=1}^{n} Q_i$$

- Euler's theorem therefore gives us a way of allocating a risk measure F that is a function of many different trades into its component parts.

Risk Allocation

- **Example**
- Suppose that the losses from loans 1, 2, and 3 have standard deviations of 1.1, 0.9, and 0.9, respectively. The correlations between the losses are as shown:

	Loan 1	Loan 2	Loan 3
Loan 1	1	0	0
Loan 2	0	1	0.7
Loan 3	0	0.7	1

- The standard deviation of the total loss is

$$\sqrt{1.1^2 + 0.9^2 + 0.9^2 + 2 \times 0.7 \times 0.9 \times 0.9} = 1.99$$

Risk Allocation

➢ **Example**

- Suppose that the size of Loan 1 is increased by 1 %. The standard deviation of the loss from Loan 1 increases from 1.1 to $1.1 \times 1.01 = 1.111$.
- The increase in the standard deviation of the loan portfolio is

$$\sqrt{1.111^2 + 0.9^2 + 0.9^2 + 2 \times 0.7 \times 0.9 \times 0.9} - \sqrt{1.1^2 + 0.9^2 + 0.9^2 + 2 \times 0.7 \times 0.9 \times 0.9} = 0.006098$$

- $Q_1 = \frac{0.006098}{0.01} = 0.6098.$
- Similarly, $Q_2 = \frac{0.006924}{0.01} = 0.6924, Q_3 = \frac{0.006924}{0.01} = 0.6924.$
- $Q_1 + Q_2 + Q_3 = 0.6098 + 0.6924 + 0.6924 = 1.99.$

Challenges

- **Challenges in Derivatives Credit Risk Measurement**
  - EAD in derivatives trading is hard to calculate. In the case of derivatives, risk changes every day as the value of the derivative changes.
  - Another challenge is that derivatives are subject to netting agreements. This means that in the event of a counterparty default, all outstanding derivatives related to the counterparty can be treated as a single derivative. This means that cannot be used on a transaction-by-transaction basis. It must be implemented on a counterparty-by counterparty basis.

Challenges

➢ **Challenges in Measuring Credit Risk**

- Banks are faced with the problem of making both through-the-cycle estimates and point-in-time estimates.
- An economic downturn is doubly bad for credit risk because the default rate increases, and the recovery rate decreases.
- A further estimate is exposure at default.
- A consideration might be what is termed wrong-way risk.
- Correlations are hard to estimate.
- Risks are often handled by different departments within a bank, but they are not independent of each other.

Operational Risk

Operational Risk

➢ Definition

- The risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.
- They do not include strategic risks or reputational risks.

➢ Seven Categories of Operational Risk

① Internal Fraud

② External Fraud

③ Employment Practices and Workplace Safety

④ Clients, Products, and Business Practices

⑤ Damage to Physical Assets

⑥ Business Disruption and System Failures

⑦ Execution, Delivery, and Process Management

Operational Risk

- **Three Large Operational Risks**
  - **Cyber Risks**
  - ✓ Cyber Crimes include data destruction, money theft, intellectual property theft, personal and financial data theft, embezzlement, fraud, etc.
  - ✓ Phishing is a common form of hacking. While phishing can take many forms, a common situation involves a hacker targeting a financial institution's customers with an email asking them to confirm account information.

Operational Risk

➢ **Three Large Operational Risks**

- **Compliance Risks**

✓ This is the risk that an organization will incur fines or other penalties because it has intentionally or unintentionally failed to act in accordance with industry laws and regulations, internal policies, or prescribed best practices.

✓ Examples are money laundering, terrorist financing and helping clients evade taxes.

- **Rogue Trader Risk**

✓ This is a risk that employees will take unauthorized action and cause huge losses.

Basel II Regulations

Basel II Regulations

➢ **Basic Indicator Approach (BIA)**

- In the Basic Indicator Approach (BIA), operational risk capital is set equal to 15% of annual gross income over the previous three years. Gross income is defined as net interest income plus noninterest income.

$$\text{Capital}_{\text{BIA}} = \frac{\sum_{\text{i=last three years}} \text{GI}_{\text{i}} \times \alpha}{\text{n}}$$

➢ **Standardized Approach (SA)**

- A bank's activities are divided into eight business lines. The average gross income over the last three years for each business line is multiplied by a "beta factor" for that business line and the result summed to determine the total capital.

$$\text{Capital}_{\text{SA}} = \frac{\sum_{\text{i=last three years}} \max(\sum \text{GI}_{\text{line 1-8}} \times \beta_{\text{line 1-8}}, 0)}{3}$$

Basel II Regulations

➢ **Standardized Approach**

Business Lines	Beta Factor
Corporate Finance	18%
Trading and Sales	18%
Payment and Settlement	18%
Commercial Banking	15%
Agency Services	15%
Retail Banking	12%
Asset Management	12%
Retail Brokerage	12%

Basel II Regulations

- **Advanced Measurement Approach (AMA)**
  - The operational risk regulatory capital requirement is calculated by the bank internally using **qualitative** and **quantitative criteria**.
  - The Basel Committee has listed conditions that a bank must satisfy in order to use the standardized approach or the AMA approach. It expects large internationally active banks to move toward adopting the AMA approach through time.
  - The **capital** charge for **AMA** is calculated as the bank's operational value at risk with a **one-year horizon** and a **99.9%** confidence level.
  - All four elements of the framework must be included in the model: internal loss data, external loss data, scenario analysis, and business environment internal control factors.

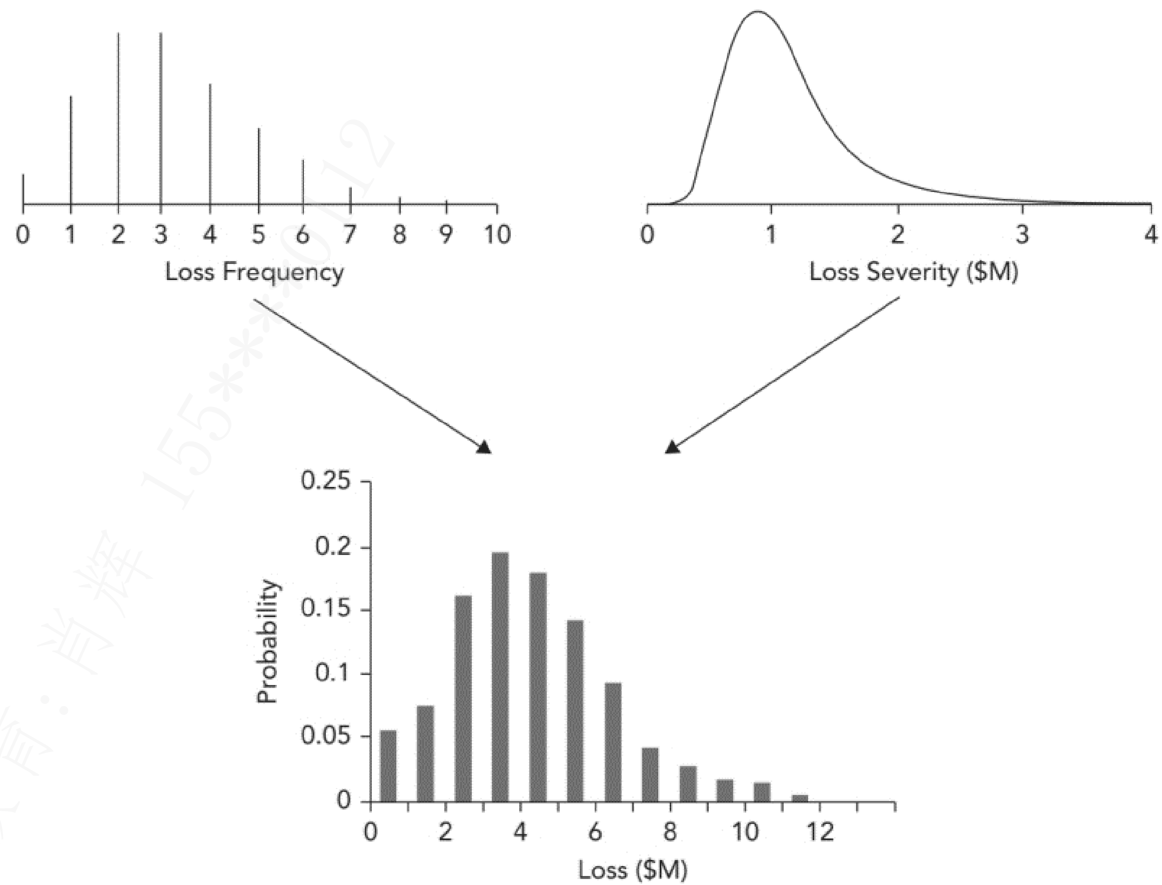
Basel II Regulations

- **Loss Distribution Approach**
  - **Frequency Distribution**
    - ✓ The loss frequency distribution is the distribution of the **number of losses** observed during the time horizon (typically one year). For loss frequency, a common probability distribution is the **Poisson distribution.**
  - **Severity Distribution**
    - ✓ The loss severity distribution is the distribution of the **size of a loss**, given that a loss occurs. For the loss severity distribution, a **lognormal probability** distribution is often uses.

Basel II Regulations

➢ **Monte Carlo Simulation**

- It is typically assumed that **loss severity** and **loss frequency** are **independent**.



Basel II Regulations

➢ **Monte Carlo Simulation**

- The steps are as follows

✓ Step 1: Sample from the Poisson distribution to determine the number of loss events (=n) in a year.

✓ Step 2: Sample n times from the lognormal distribution of the loss size for each of the n loss events.

✓ Step 3: Add up the n loss sizes to determine the total loss.

✓ Step 4: Repeat steps 1 to 3 many times.

Basel II Regulations

- **Data Issues**
  - **Loss frequency** should be estimated either from a bank's **own data** or subjectively by operational risk professionals after careful consideration of the controls in place.
  - When **loss severity** cannot be estimated from a internal data, the losses experienced by **other financial institutions** can sometimes be used.
  - ✓ Data from data providers may be **biased** as only **large losses** are usually reported.
  - ✓ Another potential **bias** has to do with the **size of the losses**.

Basel II Regulations

Data Issues

- Financial institutions also use scenario analysis to estimate loss frequencies and loss severities. It is particularly useful for loss events of low frequency but high severity.
- Scenario analysis considers **losses** that the financial institution has **never experienced**, but that **may occur in the future**.
- Scenario analysis can help firms form strategies for responding to a loss event and/or reducing the probability of it happening in the first place.

Basel II Regulations

➢ **Standardized Measurement**

- The Basel committee has now abandoned the AMA and is **replacing all three Basel methods** with a new standardized approach.
- Bank regulator found AMA method **unsatisfactory** because of the **wide variation** in the calculations of different Banks
- SMA first defines a quantity called **Business Indicator (BI).** It is similar to **gross income**, but it is designed to be a more relevant measure of bank size.
- The Basel committee provides a formula for calculating the required capital from the **loss component** and the **BI component**.

Basel II Regulations

- **Allocation of Economic Capital**
  - Economic capital is allocated to business line for the purpose of calculating the return on capital.
  - If the business line manager can demonstrate that he or she has succeeded in reducing the frequency or severity of losses, less capital will be allocated to the business line.
- **Power Law**
  - If v is the value of the random variable and x is a high value of v, then the power law holds, which is approximately:

$$P_r(v \geq x) \approx Kx^{-\alpha}$$

  - The power law describes how fat the right tail of the probability distribution of v is and can apply to operational risk loss.

Reducing Operational Risk

Reducing Operational Risk

- **Causes of Losses**
  - Sometimes operational risk loss may be related to other **manageable factors**. A cost-benefit analysis should be undertaken, since the costs of reducing operational risk may sometimes outweigh the benefits.
- **Education**
  - It is important to **educate** employees about unacceptable business practices and (more importantly) to create a risk culture that recognizes these practices as unacceptable.
  - Legal departments within financial institutions need to remind employees to be **careful about** what they write in **e-mails** and what they say in **phone calls.**

Reducing Operational Risk

- **Risk Control and Self Assessment**
  - Risk control and self assessment (RCSA) is a way for financial institutions to try to understand operational risks and establish operational risk awareness among employees.
  - ✓ Line managers and their staff are required to identify risk exposures.

    ① Interviewing line managers and their staff;

    ② Asking line managers to complete risk questionnaires;

    ③ Reviewing risk incident history with line managers.

    ④ ……
  - ✓ The evaluation process should be repeated regularly. (e.g., every year).
  - ✓ May lead to improvements reducing the frequency of losses, the severity of losses, or both.

Reducing Operational Risk

➢ **Key Risk Indicators**

- These are data points which may indicate an increased likelihood of operational risk losses in certain areas. In some cases, remedial action can be taken before it is too late.
  - ✓ Staff turnover,
  - ✓ Failed transactions,
  - ✓ Positions filled by temps, and
  - ✓ Unfilled positions.
- To use these indicators effectively, it is important to track how they change through time so that unusual behavior can be identified.

Reducing Operational Risk

- **Insurance**
  - Many operational risks can be insured against, insuring against a loss can not only reduce the severity of losses, but also reduce capital requirements.
  - **Moral Hazard**
  - ✓ Moral hazard is the risk that the existence of an insurance contract will cause the insured entity to behave in a way that makes a loss more likely.
  - ✓ Most policies include deductibles, coinsurance provisions, and policy limits.
  - **Adverse Selection**
  - ✓ Adverse selection is the problem an insurance company faces in distinguishing low-risk situations from high-risk situations. If it charges the same premium for a certain type of risk to all financial institutions, it will inevitably attract clients with the highest risk.
  - ✓ Insurance companies deal with adverse selection by finding out researching potential customers before providing a quote.

Stress Testing

Stress Testing versus VaR and ES

➢ **Stress Testing versus VaR and ES**

	Stress Testing	**VaR and ES**
Analysis	forward-looking	backward-looking
Scenarios	few scenarios (**all negative** for the organization)	a wide range of scenarios (**both good and bad** for the organization)
Horizon	longer period	short time

Stress Testing versus VaR and ES

- **Stressed VaR and Stressed ES**
  - In stressed VaR and stressed ES, data is collected from particularly stressful times.

1. Stressed VaR and Stressed ES & Stress testing

✓ **Stress testing** usually has a **longer time horizon**.

2. Stressed VaR and Stressed ES & Traditional VaR measures

✓ **Traditional VaR** measures are designed to quantify all possible outcomes and can therefore be **back-tested**.

✓ It is not possible to back-test stressed VaR or the output from stress testing in this way, because these measurements focus on extreme results that we do not expect to observe with any particular frequency.

Choosing Scenarios

➢ **Choosing Scenarios**

- **First step is to select a time horizon.**

✓ The time horizon should be long enough for the full impact of the scenarios to be evaluated and very long scenarios can be necessary in some situations.

- **Historical Scenarios**

✓ Assume that all relevant variables behave as they did in the past.

✓ Sometimes, a moderately adverse situation from the past is more extreme for all risk factors to exercise multiplied by a certain amount.

- **Stress Key Variables**

✓ One approach to scenario building is to assume that a large change takes place in one or more key variables.

Choosing Scenarios

➢ **Choosing Scenarios**

- **Ad Hoc Stress Tests**

✓ History does not exactly repeat, and management judgments are necessary to generate new scenarios or modify existing ones based on past data.

- **Using the Results**

✓ Senior management and boards should carefully assess the results of the stress tests and decide whether some form of risk mitigation is necessary.

Model Building and Reverse Stress Testing

- **Model Building**
  - Scenarios (and AD hoc scenarios) built by emphasizing key variables usually specify only a few key risk factors or the movement of economic variables. Variables specified in a scenario definition are sometimes referred to as core variables, while other variables are referred to as peripheral variables.
  - One approach is to carry out an analysis (e.g., linear regression between peripheral variables and core variables).
  - **Knock-on Effects**
- **Reverse Stress Testing**
  - Reverse stress testing takes the opposite approach: It asks what combination of circumstances could lead to the failure of the financial institution?

Regulatory Stress Testing

> **Regulatory Stress Testing**

- In the United States, for example, the federal reserve conducted stress tests on all Banks with combined assets of more than $50 billion, known as the Comprehensive Capital Analysis Review (CCAR).
- Banks are required to consider four scenarios:

✓ Baseline

✓ Adverse

✓ Severely adverse

✓ An internal scenario

Regulatory Stress Testing

- **Regulatory Stress Testing**
  - Banks must **submit a capital plan**, documentation to justify the models they use, and the results of their stress tests.
  - If they fail the stress test because their capital is insufficient, they are likely to be required to **raise more capital** and **restrict the dividends** they can pay until they have done so.
  - Banks with consolidated assets between USD 10 billion and USD 50 billion are subject to the Dodd-Frank Act Stress Test (DFAST).
  - The scenarios in DFAST are like those in CCAR. However banks are **not required to submit a capital plan**.

Governance of Stress Testing

Governance

- **Governance over Stress Testing**
  - The Board and Senior Management
  - Policies and Procedures
  - Validation and Independent Review
  - Internal audit
- **The Board**
  - ✓ Has a responsibility to **oversee** key strategies.
  - ✓ Responsible for the company's risk appetite and risk culture.
  - ✓ **Dictate** how the stress tests are conducted.
  - ✓ **Determine** the processes used to create the scenarios and the ways in which assumptions and models are used to evaluate them.

Governance

- **The Senior Management**
  - ✓ **Ensuring** that the stress test activities mandated by the board are conducted by competent staff and are regularly reported to the board.
  - ✓ Responsible for **ensuring** that the organization complies with appropriate policies and procedures.
  - ✓ **Ensure** that the situation changes as the economic environment changes and new risks emerge.
  - It is important for **the board and senior management** to ensure that the stress tests **cover all lines** of business and exposures.
  - The same scenario should be used across the financial institution, and the results should be aggregated to provide an **enterprise-wide view** of risk.
  - They should carefully consider whether the results of stress tests indicate that more **capital** should be held or that **liquidity** should be improved.

Governance

➢ **Policies and Procedures**

- Financial institution should have written policies and procedures for stress testing and ensure that they are adhered to.
- The policies and procedures should:

✓ Describe **why** stress testing is carried out.

✓ Explain stress-testing **procedures** to be followed throughout the company .

✓ Define the **roles** and responsibilities for those involved in stress testing.

✓ Define the **frequency** at which stress testing is to be performed.

✓ Explain the procedures to be used in building and **selecting scenarios**.

Governance

- **Policies and Procedures**
  - The policies and procedures should:
  - ✓ Explain how **independent reviews** of the stress-testing function will be carried out.
  - ✓ Provide clear **documentation** on stress testing to third parties (such as regulators, external auditors, and rating agencies) as appropriate.
  - ✓ Indicate **how** the results of stress testing are to be used and by **whom**.
  - ✓ Be **updated** as appropriate as stress-testing practices will change as market conditions change.
  - ✓ Allow management to **track** how the results of stress tests **change** through time.
  - ✓ **Document** the operation of models and other software acquired from **vendors** or other third parties.

Governance

➢ **Validation and Independent Review**

- It is important that the reviewer of the stress test process be **independent** of the employee performing the stress test.
- The review should:

✓ Covers **qualitative** or **judgmental** aspects of stress tests.

✓ Ensure that tests are based on **sound** theory.

✓ Be sure to **recognize limitations** and **uncertainties**.

✓ **Continuously monitor** the results.

- It is also important to ensure that **models** obtained from suppliers are subject to the same **rigorous scrutiny** as internal models.

Governance

➢ **Internal Audit**

- The internal audit function assesses practices used throughout the financial institution to ensure they are **consistent.**
- Sometimes it will be able to find ways to improve governance, control, and accountability.
- It could then **advise** senior management and the board on changes it considered desirable