Valuation and Risk Models

FRM一级培训讲义-强化班

讲师：Cindy

101% Contribution Breeds Professionalism

Topic Weightings in FRM Part Ⅰ

Session NO.	Contents	Weightings
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Study Session 2	Quantitative Analysis	20
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Framework

- Bond Market
  - Bond Valuation
  - Bond Market Risk
- Options Valuation
  - Binomial Trees
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Bond Market

Topic 1: Bond Valuation

Bond Valuation

- **Bond Valuation**
  - Method 1:$P = \sum PV(CF_t)$
  - Method 2:Replication Approach
- **Bond Return**
  - YTM
- **P&L Components**
  - Price appreciation
  - Cash-carry

Bond Valuation method 1

- **How to determine the price of a bond?**
  - Spot Rate

$$P = \frac{CF_1}{1+S_1} + \frac{CF_2}{(1+S_2)^2} + \cdots + \frac{CF_T}{(1+S_T)^T}$$

  - Forward Rate

$$P = \frac{CF_1}{1+f(1)} + \frac{CF_2}{\big(1+f(1)\big)\big(1+f(2)\big)} + \cdots + \frac{CF_T}{\big(1+f(1)\big)\big(1+f(2)\big)\dots\big(1+f(T)\big)}$$

  - Discount Factor

$$P = \sum CF_t \times d_t$$

Exercise

- There are two U.S. Treasury bond. The first has a price of 99.98, matures in six months, and pays a semi-annual coupon at a rate of 3%. The second has a price of 101.11, matures in one year, and pays a semi-annual coupon at a rate of 4%. What are, respectively, the six-month and one-year discount factor?
  A. d(0.5) = 0.9790, d(1.0) = 0.9830
  B. d(0.5) = 0.9850, d(1.0) = 0.9720
  C. d(0.5) = 1.0020, d(1.0) = 0.9830
  D. d(0.5) = 0.9650, d(1.0) = 1.0340

- Answer: B

Bond Valuation method 2

- **Replication Approach**
  - **Law of One Price:** Absent confounding factors (e.g., liquidity, financing, taxes, credit risk), **identical sets of cash flows** should sell for **same price**.
  - It turns out that a **deviation** from the law of one price implies the existence of an **arbitrage opportunity.**
  - Example: three bond yields and prices are shown below.

	Maturity	YTM	Coupon	Price (% of par)
1	1 year	5%	0%	95.238
2	2 years	6%	0%	89.00
3	2 years	6%	6%	100

  - The 2-year spot rate is 6.03%. Is there an arbitrage opportunity using these three bonds? If so, describe the trades necessary to exploit the arbitrage opportunity?

Bond Valuation method 2

- **Example**

$$0.06\mathrm{B}_1 + 1.06\mathrm{B}_2 = \mathrm{B}_3$$

$$0.06 \times 95.238 + 1.06 \times 89.00 = 100.05$$

Bonds 3 is undervalued, so we buy bond 3. Bond 1 and Bond 2 are overvalued, then we sell them together.

Exercise

- Three different U.S. Treasury notes pay semi-annual coupons and mature in exactly one year; i.e., each pays the next coupon in six months and matures six months subsequently. The price of Bond A with a coupon rate of 2.0% per annum is \$99.02 and the price of Bond C with a coupon rate of 7.0% per annum is \$103.91. If Bond B has a coupon rate of 4.0% per annum, what is the price of Bond B?
  A. \$99.12
  B. \$100.56
  C. \$100.98
  D. \$101.12

- Answer: C

Bond Return

- **Gross Realized Returns**

$$R_{t,t+1} = \frac{P_{t+1} + c - P_t}{P_t}$$

- **Net Realized Returns**
  - <u>Incorporates funding cost</u>

$$R_{t,t+1} = \frac{P_{t+1} + c - B_{\text{funded price}}}{P_t}$$

- **Spread**
  - Important measures of **relative** value.
  - Assume we define the term structure as a set of forward rates, then we can find a spread, s, that **equates the discounted cash flows to the price of the bond.**

Bond Return

- **YTM(Yield to maturity)**
  - **Single discount rate**, when used to discount a bond's cash flow, produces the bond's market price.

$$P=\frac{CF_1}{1+YTM}+\frac{CF_2}{(1+YTM)^2}+\cdots+\frac{CF_n}{(1+YTM)^n}$$

  - It can be viewed as the **realized return** on the bond assuming all **cash flows are reinvested at the YTM** and the **bond is held to maturity.**

$$P=\frac{CF_1}{1+R_1}+\frac{CF_2}{(1+R_2)^2}+\cdots+\frac{CF_n}{(1+R_n)^n}$$

  - YTM is a kind of **average** of all the spot rates.
- **Relationship between spot rates and forward rates**
  - Spot rates are an **average** of forward rates.

Exercise

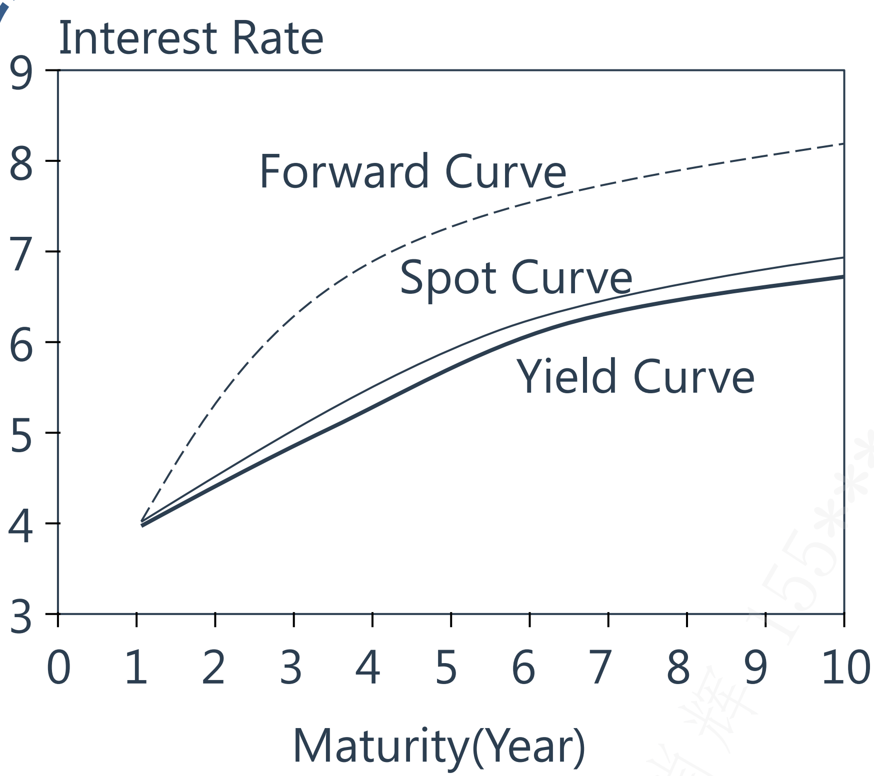
- A 3-year bond with a current price of $105.91 pays a semi-annual coupon with a coupon rate of 5%. What is the bond's YTM?
  A. 1.97%
  B. 2.25%
  C. 2.93%
  D. 3.56%

- Answer: C

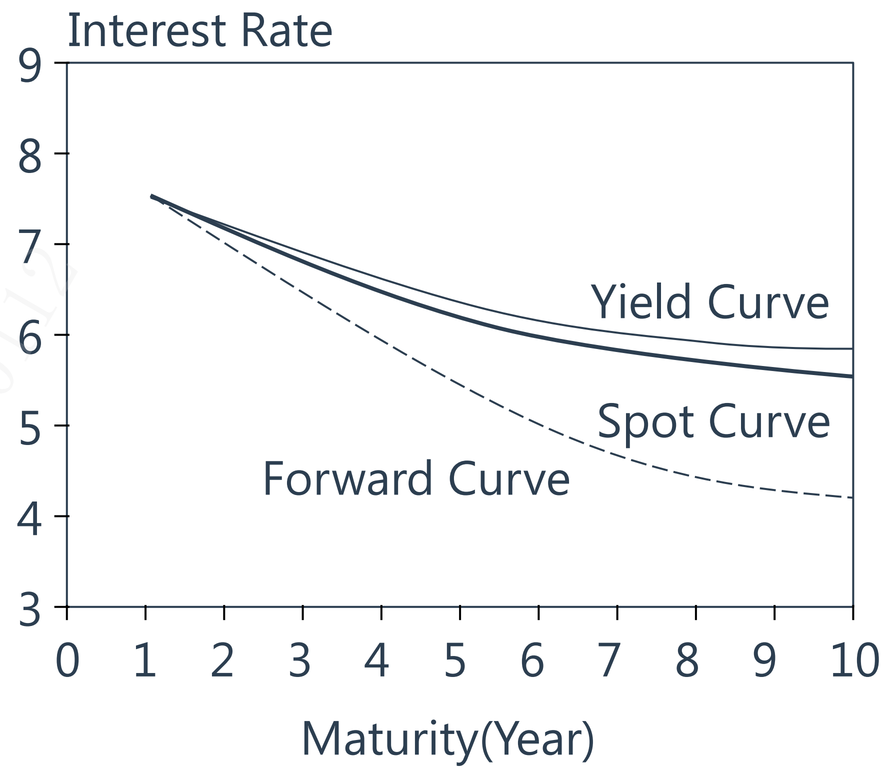
Bond Return

- **Coupon Effect**
  - The fact that correctly priced bonds with the **same maturity** but **different coupons** have **different yields to maturity** is called the **coupon effect**.
  - ✓ **Upward-sloping trend**, the **yield to maturity falls as the coupon rises**.
  - ✓ **Downward-sloping trend**, the **yield to maturity rises as the coupon rises.**

Bond Return



Upward-Sloping Term Structure



Downward-Sloping Term Structure

P&L Components

- **Decomposition of P&L**
  - The bond's profit and loss consists of both:
    - ✓ **Price appreciation (or depreciation)**; a.k.a., capital gain or loss.
    - ✓ **Cash-carry**: cash flows such as **coupon** payments.
  - **Price appreciation** can be decomposed into three components:
    - ① **Carry-roll-down**: the price change due to the **passage of time** where rates move as expected but with no change in the spread.
      - ◆ The most common assumption when the carry roll-down is calculated is that forward rates are realized (i.e., the forward rate for a future period remains unchanged as we move through time).

P&L Components

- **Decomposition of P&L**

  ② **Rate change**: the price effect of **rates changing** from the intermediate term structure to the term structure that actually prevails at time t+1.

  ③ The price appreciation due to a **spread change** is the price effect due to the bond's individual spread changing from s(t) to s(t+1).

Bond Market

Topic 2: Bond Market Risk

Bond Market Risk

- **Parallel Term Structure Shifts**
  - **Plain bond**:
    ① Macaulay Duration & Modified Duration & Dollar Duration & DV01
    ② Convexity
  - **Embedded option**:
    ① Effective duration & DVDZ & DVDF
    ② Effective convexity
- **Non-Parallel Term Structure Shifts**
  ① PCA
  ② Key-Rate Exposure
  ③ Forward Bucket Shift

Parallel Term Structure Shifts

➤ **Macaulay Duration**

- Average period of cash flow returning weighted by discounted cash flow.

$$\mathrm{Mac.D} = \sum_{t=1}^{T}\left(\frac{PV(CF_t)}{P} \times t\right) = \sum_{t=1}^{T}(w_t \times t)$$

- For a **plain bond**, the Macaulay duration is **less than** or equal to its maturity.
- For a **zero coupon bond**, the Macaulay duration **equals** to its maturity.

Parallel Term Structure Shifts

- **Modified Duration**

$$\text{Modified Duration(MD)} = -\frac{\Delta P/P}{\Delta y} = \frac{\text{Macaulay Duration}}{1 + y/m}$$

  - If the yield is measured with **continuous compounding**

$$\text{Macaulay Duration} = \text{Modified Duration}$$

  - The approximate duration relationship of bond is:

$$\Delta P = -MD \times P \times \Delta y$$

- **Dollar Duration**

$$DD = MD \times P = \Delta P/\Delta y$$

Parallel Term Structure Shifts

➢ **DV01**

$$DV01 = MD \times P \times 0.0001 = DD \times 0.0001$$

- **DV01 Hedge**

✓ If DV01 is expressed in terms of a fixed face amount, hedging a position of F^A face amount of security A requires a position of F^B of security B where:

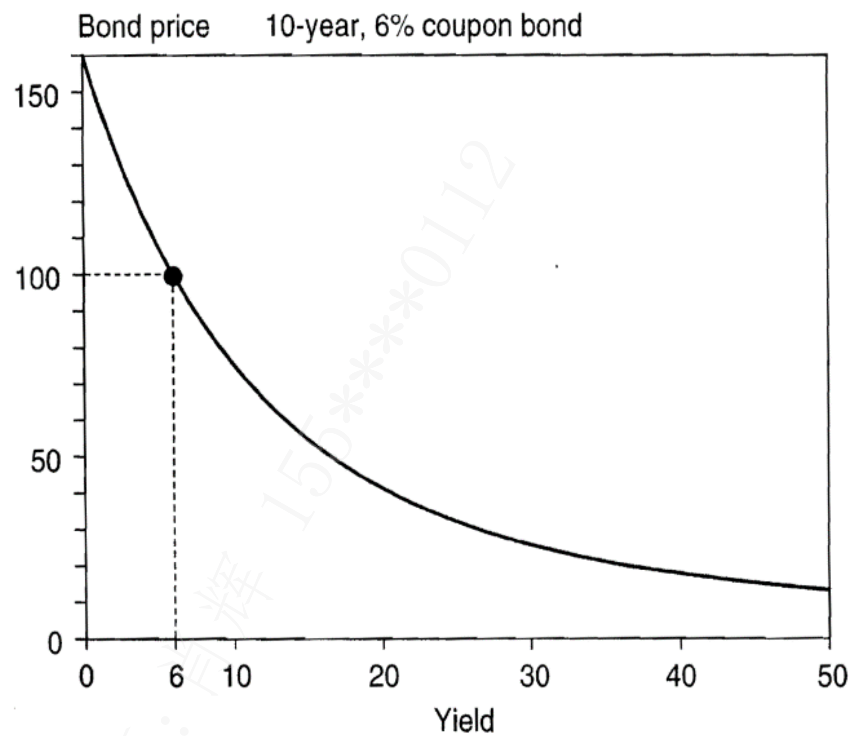
$$F^B = \frac{F^A \times DV01^A}{DV01^B}$$

- **Duration Hedge**

$$N^* = \frac{P_P \times D_p}{P_F \times D_F}$$

Parallel Term Structure Shifts

- **Convexity**
  - Non-linear relationship



$$\Delta P = -DP\Delta y + \frac{1}{2}CP(\Delta y)^2$$

Parallel Term Structure Shifts

- **Convexity**
  - Example
  - Estimate the effect of a 100 bps increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach with a duration of 7 and a convexity of 90.

$$\text{Percentage bond price change} \approx \text{Duration effect} + \text{Convexity effect}$$

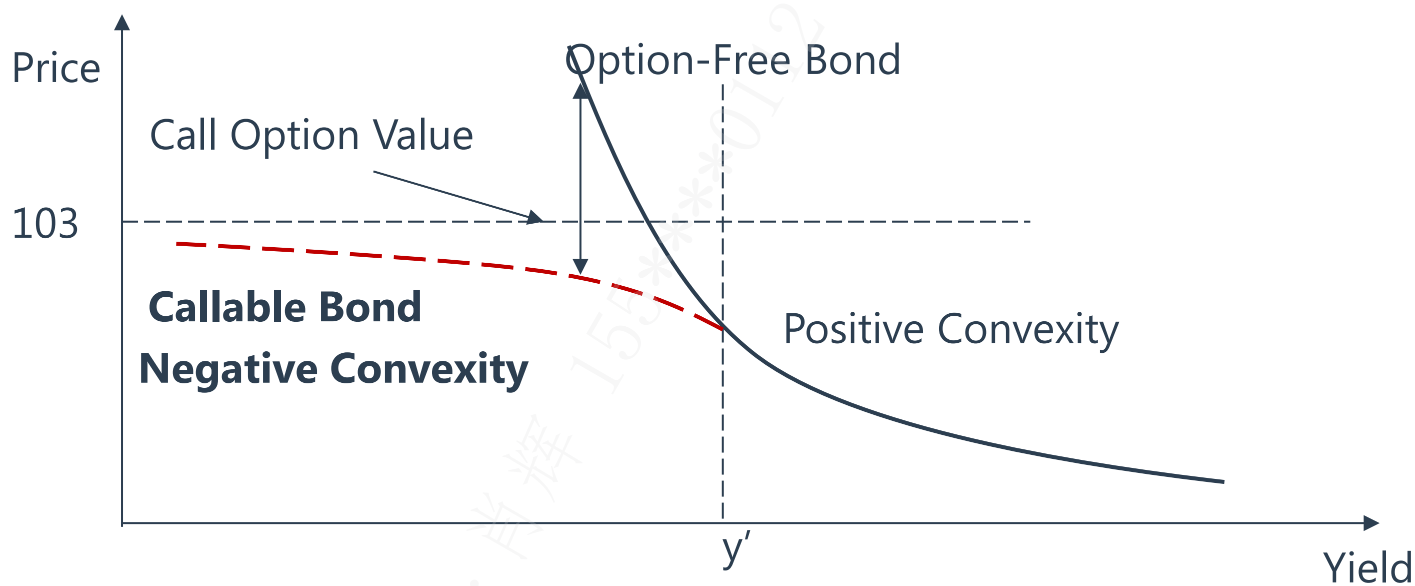
✓ $\Delta B_{+\Delta y} \approx [-7 \times 100 \times 0.01] + [(1/2) \times 90 \times 100 \times 0.01^2] = -6.55$

✓ $\Delta B_{-\Delta y} \approx [-7 \times 100 \times (-0.01)] + [(1/2) \times 90 \times 100 \times (-0.01)^2] = 7.45$

Parallel Term Structure Shifts

Negative Convexity

- Most mortgage bonds are negatively convex, and callable bonds usually exhibit negative convexity at lower yields.



Exercise

- MTGE4. MTGE7. MTGE10 are mortgage-backed securities (MBS) that pay 4%. 7%. and 10% coupons, respectively prevailing mortgage rates are 10% Assuming these securities have the same maturity and coupon frequency, which of the following is correct?
  - A. In most cases, convexity is sufficient to approximate MBS price changes resulting from yield changes for purpose of estimating VaR.
  - B. In most cases, duration is sufficient to approximate MBS price changes resulting from yield changes for purpose of estimating VaR.
  - C. The Optionality embedded in a MBS makes the implementation of the duration-convexity method less appropriate the purpose of estimating VaR.
  - D. As rates fall, MTGE10 price change approximations using the duration-convexity method are likely to be better than MTGE4 price change approximations.
- Answer: C

Parallel Term Structure Shifts

➢ **Effective Duration and Effective Convexity**

- When the bond contains **embedded options**, we prefer **effective duration**:

$$\boxed{D^E = -\frac{\Delta P/P}{\Delta y} = \frac{P_- - P_+}{2P_0 \Delta y}}$$

- **Effective convexity**: an approximate measure of convexity

$$\boxed{C^E = \frac{D_- - D_+}{\Delta y} = \frac{P_- + P_+ - 2P_0}{P_0 \Delta y^2}}$$

- **DVDZ or DPDZ**: The change in price from a **one-basis-point** increase in **all spot (i.e., zero) rates**.
- **DVDF or DPDF**: The change in price from a **one-basis-point** increase in **forward rates**.

Exercise

- Suppose there is a 10-year option-free noncallable bond with an annual coupon of 7% trading at par. If interest rates rise by 30 bps, estimated price is 97.922. If interest rates fall by 30 bps, estimated price is 102.137. Calculate the convexity of this bond.

$$\text{Convexity} = \frac{102.137 + 97.922 - 2 \times 100}{100 \times (0.003)^2} = 65.56$$

Parallel Term Structure Shifts

➢ **Portfolio Duration and Convexity**

- In regard to both modified (effective) duration and convexity, portfolio duration and convexity equal the **weighted sum** of individual, respectively, durations and convexities where each component's **weight** is **its value as a percentage of portfolio value**.
- Bullet versus Barbell Portfolio

✓ Consider three bonds:

1. A 5-year bond with a 2% coupon.
2. A 10-year bond with a 4% coupon.
3. A 20-year bond with a 6% coupon.

✓ 2---bullet portfolio, 1+3---barbell portfolio.

Non-Parallel Term Structure Shifts

- **Principal Components Analysis**
  - This technique looks at the daily changes in interest rates corresponding to various maturities and identifies factors that have the following characteristics:

① These factors are **uncorrelated**;

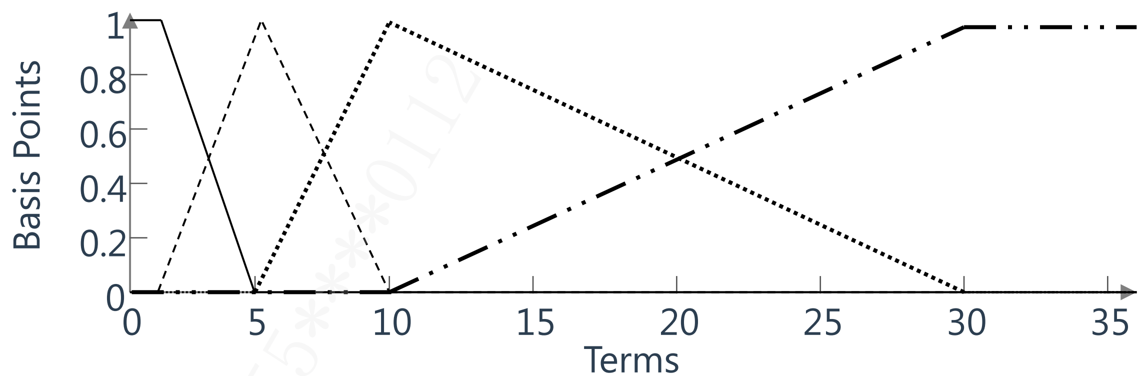
② Daily changes in term structure are **linear combinations** of the factors;

③ The **first two or three** factors account for the **majority** of the observed daily movements.

Non-Parallel Term Structure Shifts

➢ Key Rate Exposure

- Shifts in the key-rates are **decline linearly.**
- The rate of a given maturity is affected solely by its **closest key-rate**.



- Key Rate Duration

✓ Which is the key rate equivalent of durations.

- Key Rate 01s

✓ Which is the key rate equivalent of DV01.

$$D_{key} = -\frac{1}{P} \cdot \frac{\Delta P}{\Delta y} \qquad DV01_{key} = -0.0001 \times \frac{\Delta P}{\Delta y}$$

Exercise

- The following table provides the initial price of a C-STRIP and its present value after application of a one basis shift in three key rates.

	Value
Initial value	24.122
2-year shift	24.126
5-year shift	24.127
10-year shift	24.073

① What is the key rate '01 for a 10-year shift?

A. 0.014 B. 0.016 C. 0.303 D. 0.049

② What is the key-rate duration for a 10-year shift?

A. 20.31 B. 23.450 C. 19.60 D. 36.14

- Correct Answer

① D ② A

Exercise

- An underlying exposure has a 5-year key-rate '01 of +$23,970. If this key rate exposure can be hedged by trading five-year bond that itself has a 5-year key rate '01 of $0.048 per 100 face amount, what is the hedge trade?
  - A. Buy $499.375 in face amount
  - B. Buy $49.94 million in face amount
  - C. Sell $499.375 in face amount
  - D. Sell $49.94 million in face amount

- Answer: D

Option Valuation

Option Valuation

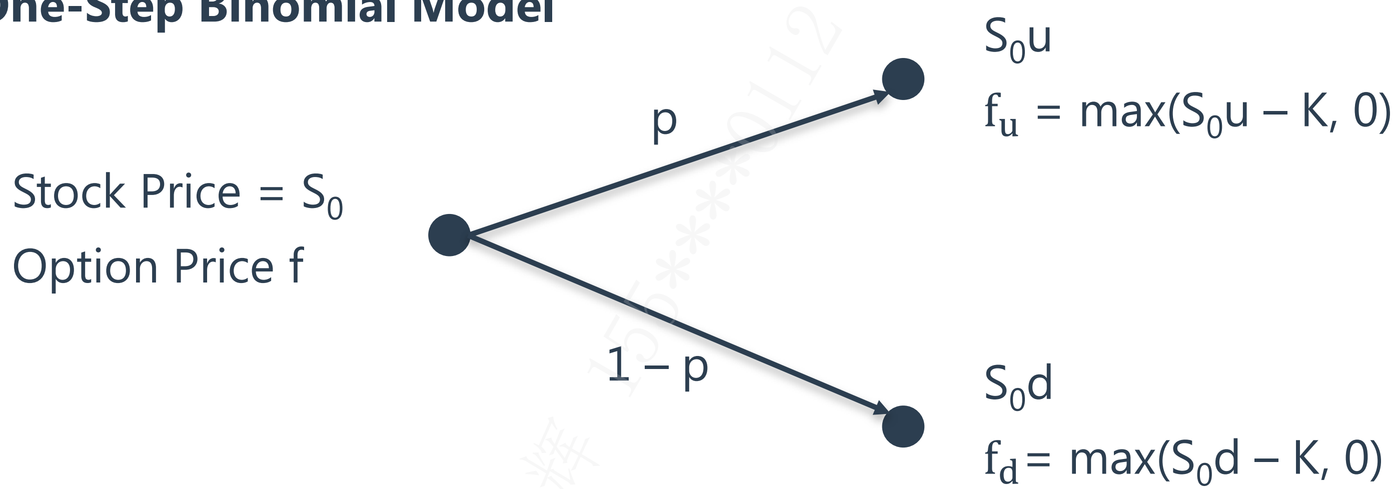
- **Binomial Trees**
  - European Options
  - American Options
- **The Black-Scholes-Merton Model**
  - Assumptions
  - Formulas
  - Warrants
- **Option Sensitivity Measures**
  - Delta
  - Gamma
  - Vega
  - Theta
  - Rho

Binomial Trees

➢ **Risk-Neutral Valuation**

- investors do not adjust their expected return based on risk, so the expected return on all assets is risk-free interest rate.

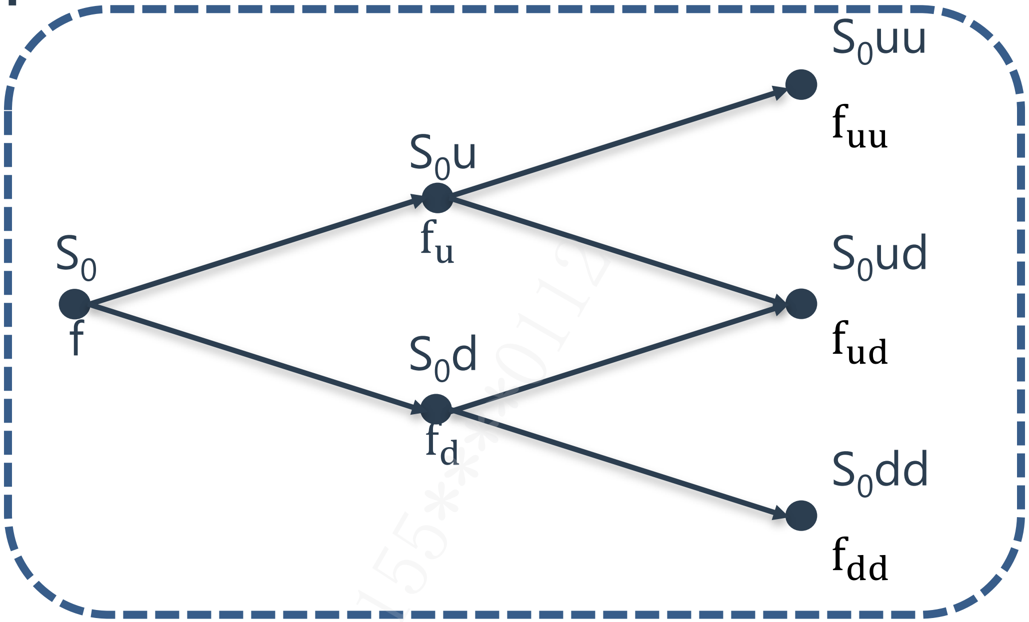
➢ **One-Step Binomial Model**



$$\boxed{p = \frac{e^{r\Delta t} - d}{u - d} \qquad f = [pf_u + (1-p)f_d]e^{-r\Delta t} \qquad u = e^{\sigma\sqrt{\Delta t}}; \qquad d = e^{-\sigma\sqrt{\Delta t}}}$$

Multi-Step Trees

- **European Options**

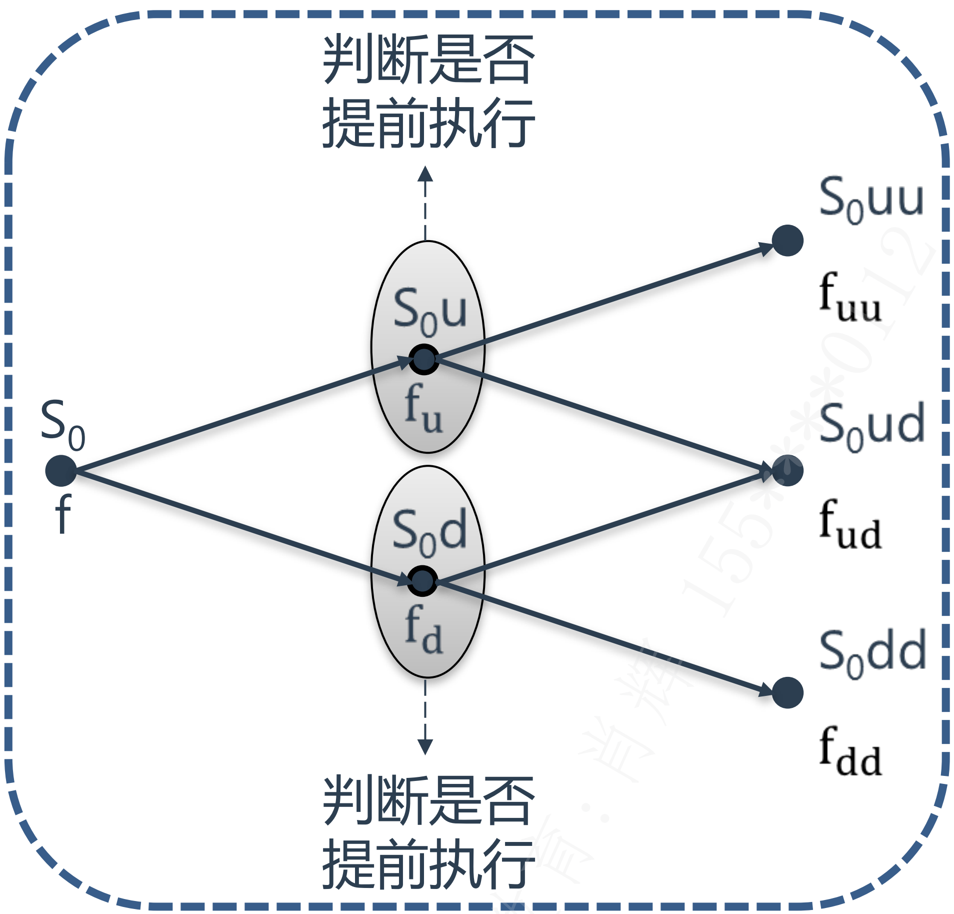


$$p = \frac{e^{r\Delta t} - d}{u - d}$$

$$f = e^{-2r\Delta t}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$

Multi-Step Trees

➢ **American Options**



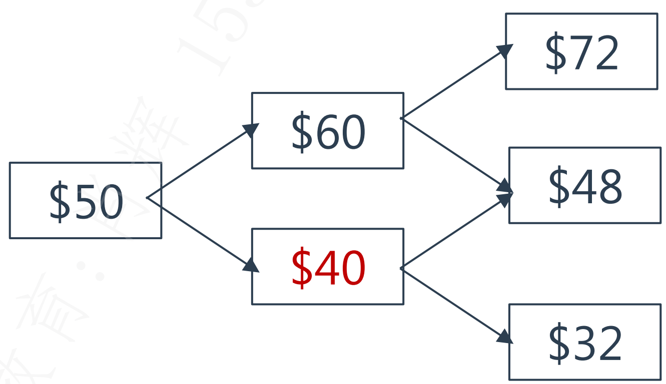
- Make sure that the option value at each node is **no less than the intrinsic value.**

Multi-Step Trees

- **Example: Put Option with price jump of +/- 20%**

Asset	Strike	Time	Riskless	Div. Yield
$50	$52	2	5%	0%

u	d	p
1.2	0.8	0.6282



Multi-Step Trees

➢ **Example: Put Option with price jump of +/- 20%**

➢ **European put**

$$\left(2 \times 4 \times 0.6282 \times (1 - 0.6282) + 20 \times (1 - 0.6282)^2\right)e^{-0.05\times 2} = 4.1923$$

➢ **American put**

$$\left(0 \times 0.6282 + 4 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 1.4147$$

$$\left(4 \times 0.6282 + 20 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 9.4634$$

$$\left(1.4147 \times 0.6282 + 12 \times (1 - 0.6282)\right)e^{-0.05\times 1} = 5.09$$

Other Assets

- **Options on Stocks with Dividends**

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

  - Everything else about the tree is the same as before.

- **Options on Stock Indices**
  - Usually, index provides a dividend yield. Therefore, the valuation should involve the modification as above.

- **Options on Currencies**
  - Currency can be considered as an asset providing a yield.

- **Options on Futures**
  - It costs noting to enter into a futures contract and we can treat a futures contract like a stock paying a dividend yield of r. Therefore, we get:

$$p = \frac{1 - d}{u - d}$$

Black-Scholes-Merton Model

- **Assumptions**
  - The stock price follows the process with μ and σ constant.
  - There are no transaction costs or taxes. All securities are perfectly divisible.
  - There are no dividends during the life of the derivative.
  - There are no riskless arbitrage opportunities.
  - Security trading is continuous.
  - The risk-free rate of interest, r, is constant and the same for all maturities.
  - The options being considered cannot be exercised early.
- **Valuation**

$$\mathrm{call} = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$\mathrm{put} = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_{1,2} = \frac{\ln(S_0/K) + \left(r \pm \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$

Black-Scholes-Merton Model

- A stock price is USD 50 with a volatility of 22%. The risk free rate is 3%. Use the Black- Scholes-Merton formula to value
  - A European call option and
  - A European put option

  when the strike price is USD 50, and the time to maturity is nine months

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = 0.2134$$

$$d_2 = \frac{\ln(S_0/K) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} 0.0228$$

$$\text{call} = S_0N(d_1) - Ke^{-rT}N(d_2) = 50 \times 0.5832 - 50e^{-3\% \times 0.75} \times 0.508 = 4.3$$

$$\text{put} = Ke^{-rT}N(-d_2) - S_0N(-d_1) = 50e^{-3\% \times 0.75} \times 0.492 - 50 \times 0.4168 = 3.2$$

Black-Scholes-Merton Model

- **The early exercise of American options**

$$AC: D_n \geq K\left(1 - e^{-r(T-t_n)}\right)$$

$$AP: D_n \leq K(1 - e^{-r(T-t_n)})$$

- **Options on Stocks with Dividends**

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$d_{1,2} = \frac{\ln(S_0/K) + (r - q \pm \sigma^2/2)T}{\sigma\sqrt{T}}$$

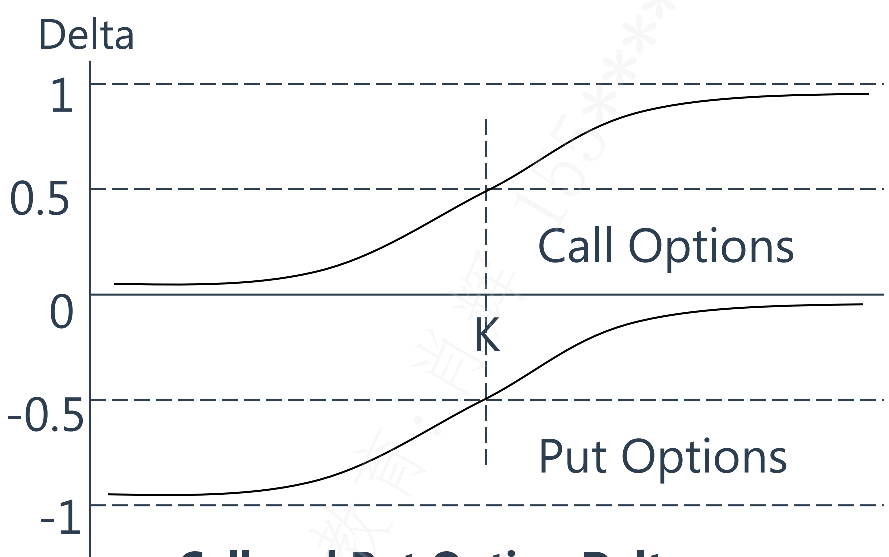
- **Warrants**
  - The payoff to an option holder if the option is exercised is

$$\frac{NS_T + MK}{N + M} - K = \frac{N}{N + M}(S_T - K) = \frac{N}{N + M} \cdot c$$

Greek Letters

➢ **Impact of Underlying Asset Price – Delta**

- The delta of an option, Δ, is defined as the ratio of change in option price to change in underlying asset price.
- **Call** option Δ range from **0** to **1**
- **Put** option Δ range from **-1** to **0**

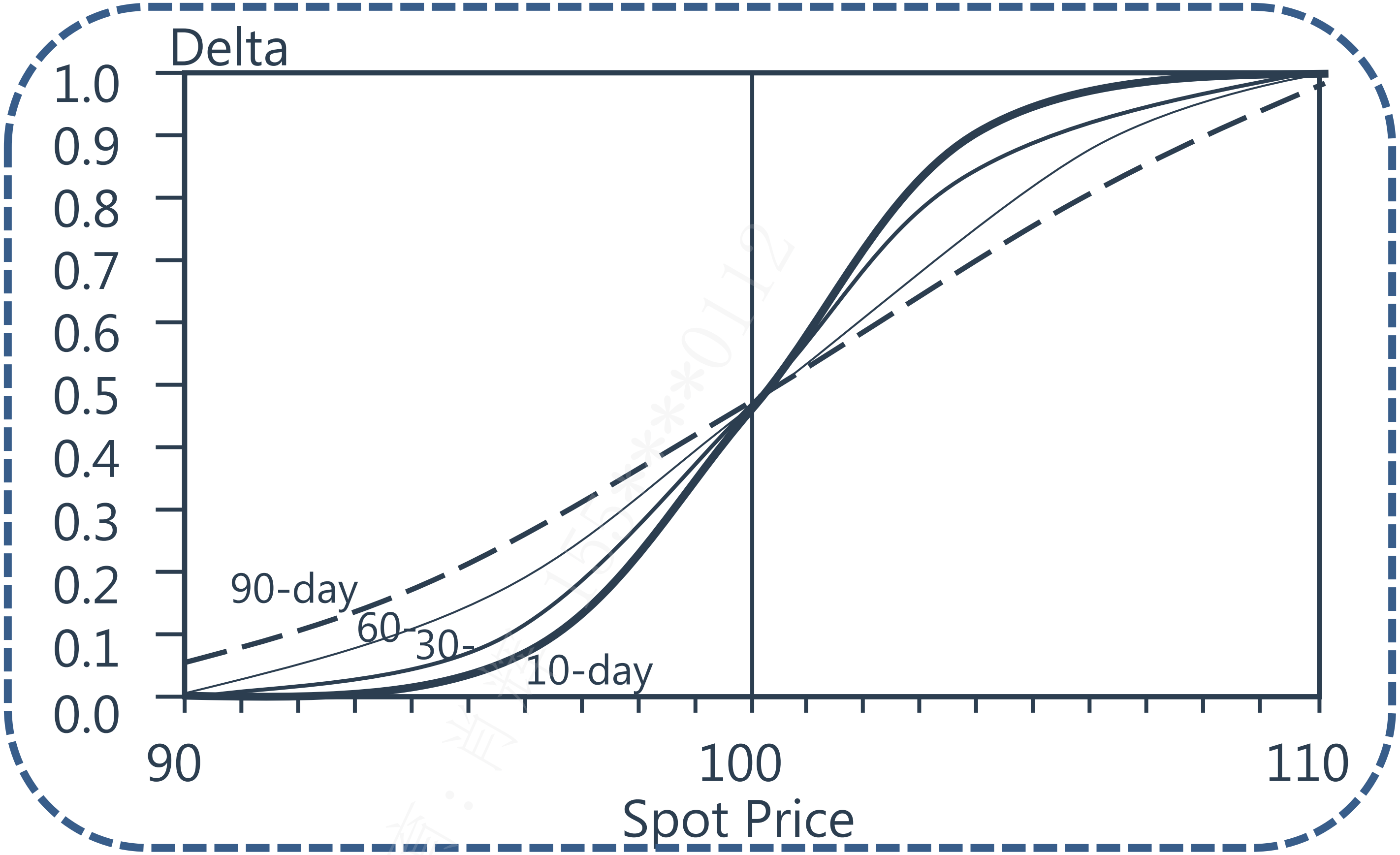


Call and Put Option Delta

Long Call	Δ > 0
Short Call	Δ < 0
Long Put	Δ < 0
Short Put	Δ > 0

Greek Letters

➢ **Delta (cont'd)**



- When t → T, Delta is unstable.

Greek Letters

- **Impact of Underlying Asset Price – Delta (cont'd)**
  - **According to the BSM Model, option delta is as follow**

$$\Delta = \frac{\partial c}{\partial S} = e^{-qT}N(d_1)$$

$$\Delta = \frac{\partial p}{\partial S} = e^{-qT}[N(d_1) - 1]$$

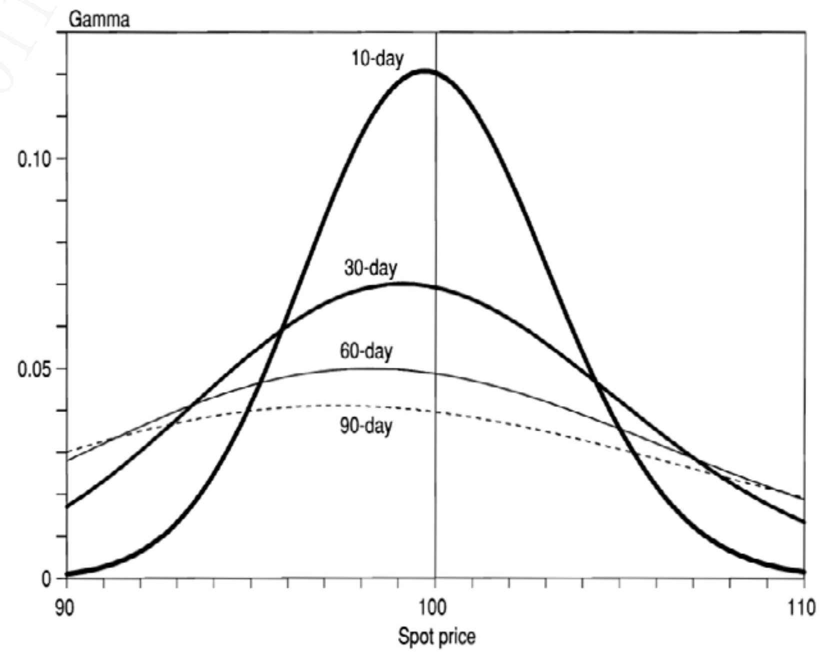
  - **Portfolio Delta:** Summation of product of each position and its delta.
  - ✓ **Forward Delta:** 1 or e^{-qT}
  - ✓ **Futures Delta:** e^{rT} or $e^{(r-q)T}$
- **Delta Hedge**
  - A position with a delta of zero is called a delta neutral position.
  - Hedge against small changes in asset price.

Greek Letters

- **Impact of Underlying Asset Price – Gamma**
  - Rate of **delta change** with respect to **price** change of **underlying asset**.
  - If gamma is large, delta is very sensitive to the price change of the underlying asset.

① Largest when is at-the-money.
② Same for call and put options.
③ t→T，Gamma↑

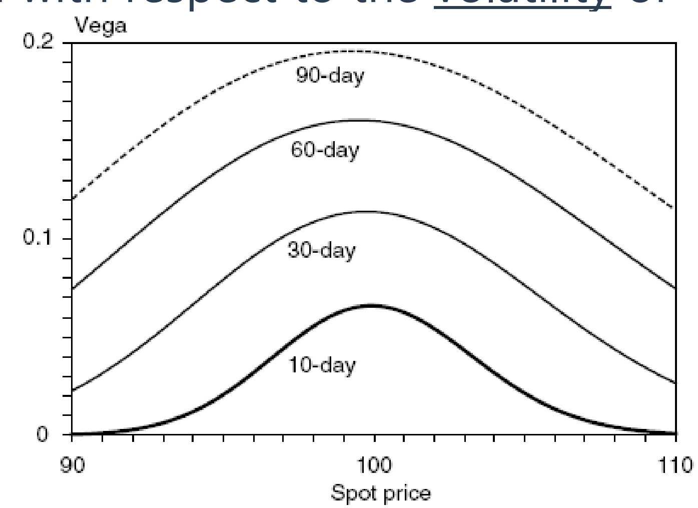
Long Call	Short Call	Long Put	Short Put
$\Gamma > 0$	$\Gamma < 0$	$\Gamma > 0$	$\Gamma < 0$



- **Gamma Hedge**
  - Hedge against larger changes

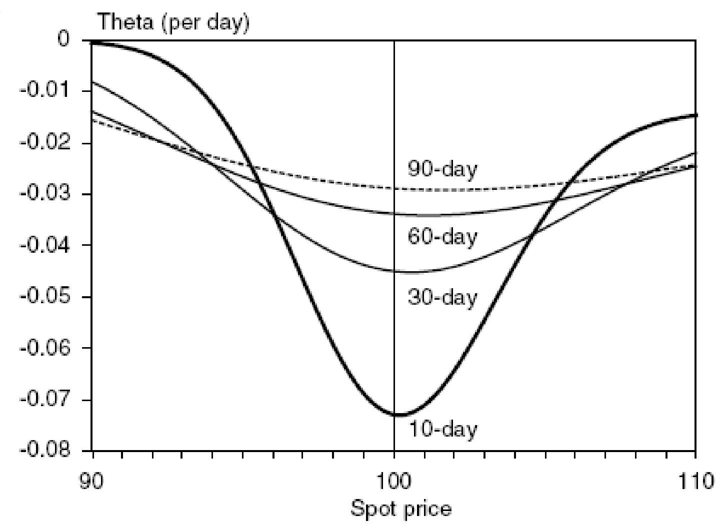
Greek Letters

- **Delta and Gamma Hedge**
  - **Example:** Suppose that a portfolio is delta neutral and has a gamma of -3,000. The delta and gamma of a particular traded call option are 0.62 and 1.50, respectively. Create a gamma-neutral position.
  - ✓ Buy 3,000/1.5 = 2,000 options
  - ✓ Sold 2,000 ×0.62 = 1,240 shares of the underlying position
- **Impact of Volatility – Vega**
  - Rate of change of the value of the option with respect to the volatility of the underlying asset.
  - ① Largest when is at-the-money.
  - ② Same for call and put options.
  - ③ t→T，Vega↓



Greek Letters

- **Impact of Maturity – Theta**
  - Rate of **change of the value of option** with respect to the **passage of time.**
  - ① **Time decay**. As time to maturity decreases, option tends to become less valuable, so theta is usually **negative for an long position**, means option lose value as time goes by.
  - ② Largest when is at-the-money.
  - ③ t→T，Theta↑
  - ④ Deep ITM Eruopean put, $\theta > 0$
  - ⑤ θ is not a risk factor.



- **Impact of Interest Rate – Rho**
  - Sensitivity to the interest rate
  - **In the money** calls and puts are **more sensitive** to changes in rates than out-of-the-money options.

Market Risk Models

Market Risk Models

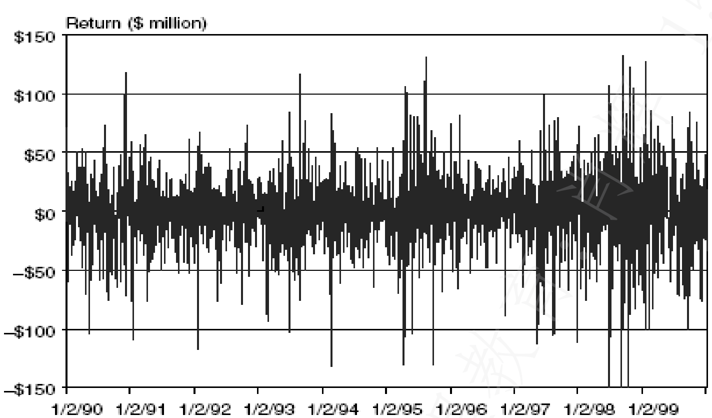
- **Measures of Financial Risk**
  - Coherent Risk Measures
  - VaR
  - ES
- **Calculating and Applying VaR**
  - Delta-normal method
  - Delta-gamma method
- **Measuring and Monitoring Volatility**
  - EWMA
  - GARCH

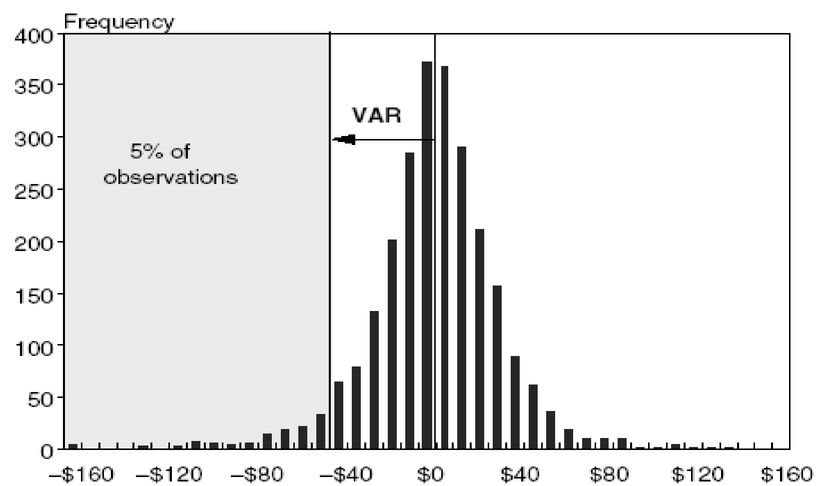
Coherent Risk Measure

- **Features of Coherent Risk Measures**
  - **Monotonicity**
    - ✓ If one portfolio consistently produces **worse results** than another, it should have a **higher risk** metric.
  - **Subadditivity**
    - ✓ For any two portfolios A and B, the **risk measure for the portfolio** formed by combining A and B should **not be greater than** the **sum of the risk** measures for portfolio A and B.
  - **Homogeneity**
    - ✓ Changing the **size** of the portfolio by **multiplying** the total amount of assets by λ results in a measure of **risk multiplied by** λ.
  - **Translation Invariance**
    - ✓ If an amount of **cash K** is added to a portfolio, its **risk** measure should **decrease by K.**

Measures of Financial Risk

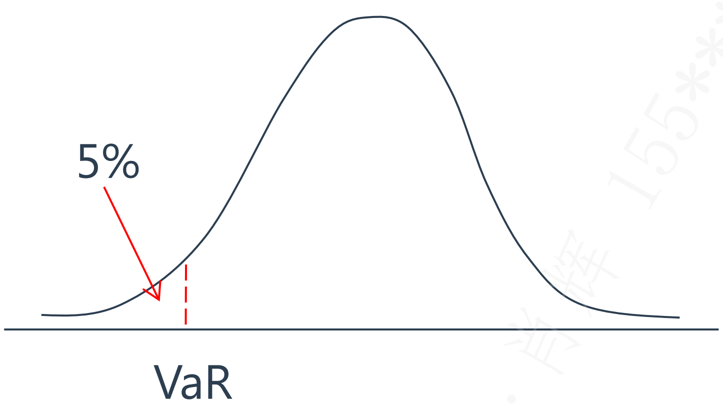
- **Mean-Variance Framework**
  - We model financial **risk** in terms of the **mean and variance** (or **standard deviation**) of **P/L (or returns).**
  - We assume the **daily P/L** or returns obey a **normal distribution.**
- **Value at Risk**
  - VaR is the **maximum loss** over a **target horizon** and for a **given confidence level.**

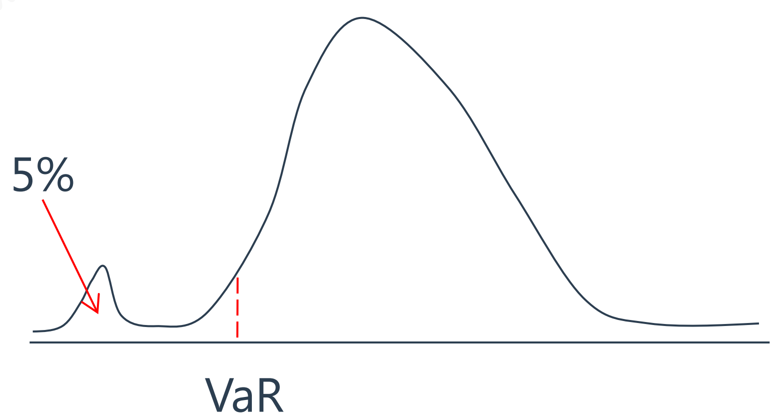




Measures of Financial Risk

- **Disadvantages of VaR**
  - ① Did not contain worst conditions, did not describe tail loss.
  - ② Not sub-additive.
- **Expected Shortfall/Conditional VaR/Tail loss**
  - **Average** of the worst 100×(1-α)% of losses.





Measures of Financial Risk

- A one-year project has a 3% chance of losing USD10 million, a 7% chance of losing USD 3 million, and a 90% chance of losing USD 1 million. What are
  - VaR
  - Expected shortfall

  when the confidence level is 95% and the time horizon is one year?

- Answer:
  - VaR is USD 3 million
  - Expected shortfall (USD) is $10\times0.6+3\times0.4=7.2$.

1.Delta-Normal Model

Basic Measurement of VaR

- Given an expected return other than zero, VaR can be measured as:

$$\mathrm{VaR}(X\%) = |E(R) - Z_{X\%} \times \sigma|$$

$$\mathrm{VaR}(\$) = |E(R) - Z_{X\%} \times \sigma| \times V$$

Square Root Rule

- If fluctuations in a stochastic process from one period to the next are independent (i.e., there are **no serial correlations or other dependencies**).
- In the longer term, the assumption is that

$$\mathrm{VaR}(T, X) = \sqrt{T} \times \mathrm{VaR}(1, X)$$

$$\mathrm{ES}(T, X) = \sqrt{T} \times \mathrm{ES}(1, X)$$

1.Delta-Normal Model

- **Delta-Normal Approximation**
  - The linear approximation is assumed and the underlying factor is assumed to follow a **normal distribution**. It is not good for derivatives with extreme nonlinearities (MBS, Fixed-income securities with embedded option).

$$\mathrm{VaR}(dP) = |-D \times P| \times \mathrm{VaR}(dy)$$

$$\mathrm{VaR}(df) = |\Delta| \times \mathrm{VaR}(dS)$$

- **Delta-Gamma Approximation**

$$\mathrm{VaR}(dP) = |-D \times P| \times \mathrm{VaR}(dy) - \frac{1}{2} \times C \times P \times \mathrm{VaR}(dy)^2$$

$$\mathrm{VaR}(df) = |\Delta| \times \mathrm{VaR}(dS) - \frac{1}{2} \times \Gamma \times \mathrm{VaR}(dS)^2$$

1.Delta-Normal Model

- A non-linear portfolio depends on a stock price. The delta is 30 and the gamma is 5. Estimate the impact of a USD 2 change in the stock price on the value at risk of the portfolio with all else remaining the same.

- Answer:

$$\mathrm{VaR(df)} = |\Delta| \times \mathrm{VaR(dS)} - \frac{1}{2} \times \Gamma \times \mathrm{VaR(dS)}^2 = 30 \times 2 - \frac{1}{2} \times 5 \times 2^2 = 50$$

2.Historical Simulation

➢ **Historical Simulation**

- **Percentage change**: Stock prices and exchange rates
- **Actual change**: Interest rates and credit spreads

✓ Generate portfolio values for each of the scenarios and then calculate losses.

✓ We sort the losses from the largest to the smallest.

✓ Suppose we are interested in the VaR and expected shortfall with a one-day horizon and a 99% confidence level. The VaR can be calculated as the fifth worst loss.

◆ The expected shortfall is usually calculated as the average of the four losses that are worse than the VaR level.

3.Monte Carlo Simulation

➢ **Monte Carlo Simulation**

- Monte Carlo simulations generate scenarios by taking random samples from a hypothetical distribution of the risk factors (rather than using historical data).
- Suppose there is a total of 1000 simulation trials. The 99% VaR for the period considered will be the tenth worst loss.(1000*1%=10)

Quantifying Volatility in VaR Models

- **Unconditional and Conditional Normality**

① Unconditional Normality

✓ The probability distribution of the return each day has the **same normal distribution** and the **same standard deviation.**

✓ This results in a return distribution with **fatter tail** than the normal distribution.

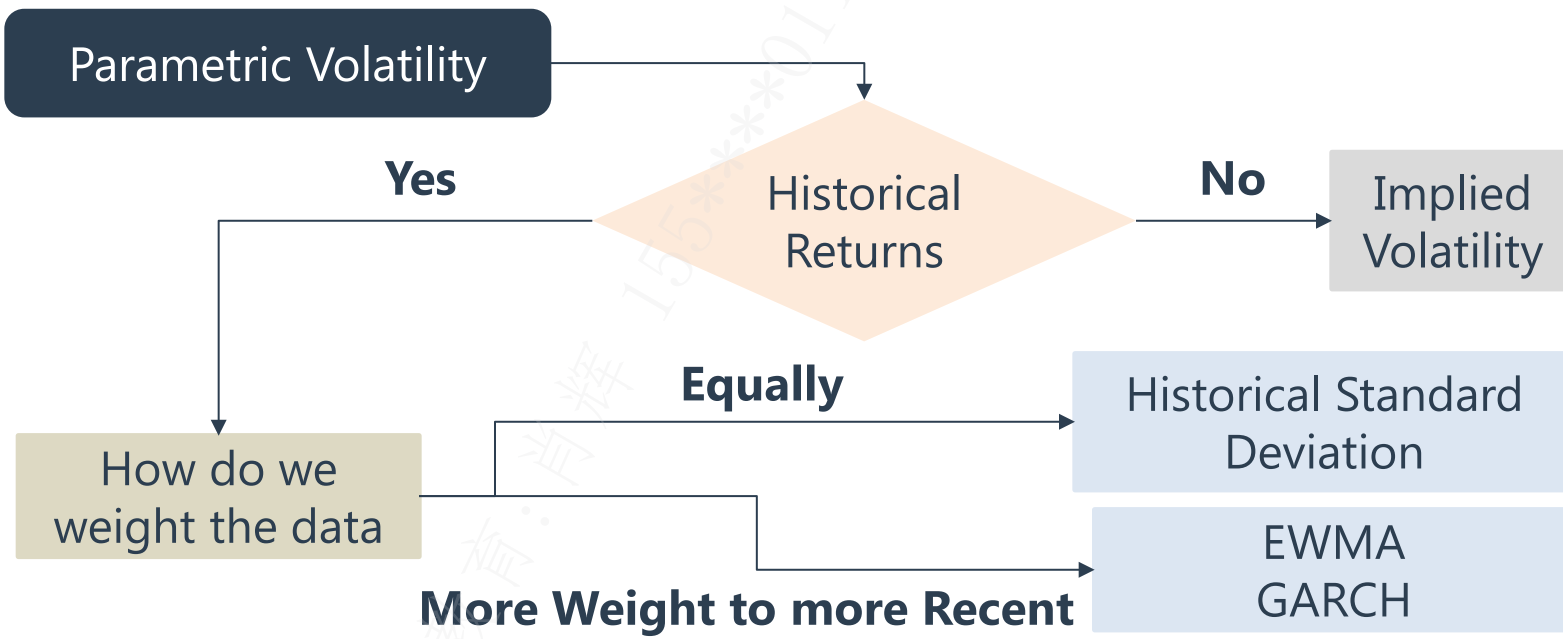
② Conditional Normality

✓ Asset returns are constantly normal is to assume they are **normal conditioned on the volatility.**

✓ **Regime switching model.**

Quantifying Volatility in VaR Models

- **Parametric Approach**
  - Historical Standard Deviation
  - EWMA(Exponentially Weighted Moving Average)
  - GARCH(Generalized AutoRegressive Conditional Heteroskedasticity)



Quantifying Volatility in VaR Models

- **Parametric Approach (cont'd)**
  - **Historical Standard Deviation Approach**
    - ✓ Assuming equally weighted
    - ✓ Raw returns are used instead of returns around the mean (i.e., the **expected mean is assumed zero**).

$$\sigma_n^2 = \left(\frac{1}{M}\right)\sum_{i=1}^{M} u_{n-i}^2$$

  - **Exponential Smoothing Method**
    - ✓ Exponential smoothing places **exponentially declining weights** on historical data, placing more weight on more recent information and less weight on past information.
    - ✓ Two models, **EWMA** and **GARCH** employ exponential smoothing.

Quantifying Volatility in VaR Models

- **Parametric Approach (cont'd)**
  - **EWMA**

$$\boxed{\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2}$$

$$\text{cov}_n = \lambda \text{cov}_{n-1} + (1-\lambda)x_{n-1}y_{n-1}$$

  - **GARCH**

$$\boxed{\begin{gathered}\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2 \\ V_L = \frac{\omega}{1-\alpha-\beta}\end{gathered}}$$

✓ In GARCH (1,1), the sum of the alpha (α) and beta (β) parameters is called persistence.

✓ GARCH (1, 1) is unstable if the persistence > 1. A persistence of 1.0 implies no mean reversion. A persistence of less than 1.0 implies "reversion to the mean," where a lower persistence implies greater reversion to the mean.

✓ EWMA is a **special case** of GARCH.

Quantifying Volatility in VaR Models

- Suppose that the price of an asset at the close of trading yesterday was USD 20 and its volatility was estimated as 1.4% per day. The price at the close of trading today is USD 19. What is the new volatility estimate using the EWMA with a λ of 0.9?
  - The new return is -1/20=-0.05. The new variance rate estimate is
    $$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$
    $$= 0.9 \times 0.014^2 + (1-0.9) \times (-0.05)^2 = 0.000426$$
  - The new volatility is the square root of this or 2.06%

Quantifying Volatility in VaR Models

➢ If $\omega = 0.000002, \alpha = 0.04$, and $\beta = 0.94$ in a GARCH model, what is the long-run average variance rate? What volatility does this correspond to?

- The long-run average variance rate is

$$V_L = \frac{\omega}{1-\alpha-\beta} = \frac{0.000002}{1-0.04-0.94} = 0.0001.$$

- This corresponds to a volatility of 1% per day
- The new variance rate is:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$= 0.000002 + 0.04 \times (-0.05)^2 + 0.94 \times 0.014^2 = 0.000286$$

- The new volatility is the square root of this which is 1.69%

Credit Risk Models

Credit Risk Models

- **Ratings**
  - External Ratings
  - Transition Matrices
  - Internal Ratings
  - Hazard Rates
- **Country Risk**
  - Credit Spread
- **Capital Model**
  - EL/UL
  - Mean and Standard Deviation of Credit Loss
  - Measuring Credit Risk

Rating Scales

➢ Long-Term Ratings

- Ratings for bonds are termed long-term ratings.

Explanation	S&P/Fitch	Moody's Services
Investment grade:		
Highest grade	AAA	Aaa
High grade	AA	Aa
Upper medium grade	A	A
Medium grade	BBB	Baa
Speculative grade:		
Lower medium grade	BB	Ba
Speculative	B	B
Poor standing	CCC	Caa
Highly speculative	CC	Ca
Lowest quality, no interest	C	C
In default	D	
Modifiers: A+, A, A-, and A1, A2, A3		

Rating Scales

➢ Short-Term Ratings

- Ratings for money market instruments are termed short-term ratings.

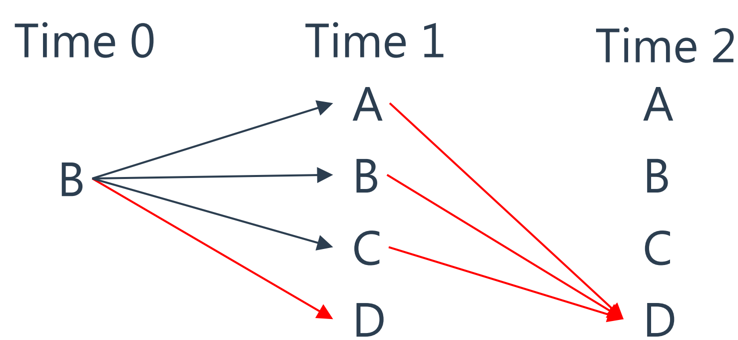
Moody's	S&P	Fitch
Investment Grade		
P-1	A-1+	F1+
	A-1	F1
P-2	A-2	F2
P-3	A-3	F3
Non-Investment Grade		
NP	B	B
	C	C
	D	D

External and Internal Ratings

Rating and Default

- Agencies publish cumulative default rates categorized by rating (i.e., the cumulative default rate per rating category) and transition matrices. Transition matrices plot the frequency of rating migrations over time;

Rating From	Rating To			
	A	B	C	D
A	97%	3%	0%	0%
B	2%	93%	2%	3%
C	1%	12%	64%	23%
D	0%	0%	0%	100%



External and Internal Ratings

- **Point-in-Time and Through-the-Cycle**
  - **Point-in-Time**: assesses credit quality over the **near term**; i.e., a few months or one year.
  - **Through-the-Cycle**: rating agencies try to incorporate business cycles.
  - Through-the-cycle ratings try to **"filter out" cycle fluctuations**. Because they **incorporate an average**.
  - ✓ when economic conditions vary from the average, through-the-cycle may **over- and under-estimate credit quality**.

External and Internal Ratings

- **Alternative to Ratings**
  - The **default** happens if the value of the **assets falls below the face value of the debt** repayment that is required at that time.
  - If V is the value of the assets and D is the face value of the debt, the firm defaults when V<D. The value of the **equity** at the future time is:

    $$\max(V-D,0)$$

  - Value of Equity=Call on Firm= $VN(d_1) - De^{-rT}N(d_2)$
  - The firm defaults if the option is not exercised.

External and Internal Ratings

- **Internal Ratings**
  - Their ratings are usually based on several factors. In general, each factor is scored, and a **weighted average score** is calculated to determine the overall final rating.
  - Banks must **back-test** their procedures for calculating internal ratings.
- **Altman's Z-score**
  - This is used to come up with a rule for distinguishing between those firms that default from those that do not.
  - ✓ A Z-score **above 3** indicated that the firm was **unlikely to default**.
  - ✓ A Z-score **below 1.8** had a **very high probability of defaulting.**

Hazard Rates

- **Hazard Rate** is the rate at which default are happening.
  - Suppose that $\bar{\lambda}$ is the average hazard rate between time zero and time t, the default probability between time zero and time t is:

$$\boxed{1 - e^{-\bar{\lambda}t}}$$

- **Example**
  - If the hazard rate is 1.5% per year for the first two years. What is the probability of default during the first two years?
  - The probability of default during the first two years is

$$1 - e^{-\bar{\lambda}t} = 1 - e^{-0.015\times 2} = 0.02955$$

Country Risk

- **Sources of Country Risk**
  - GDP Growth Rates
  - Political Risk
  - Legal Risk
  - The Economy
- **Composite Measures of Country Risk**
  - Political Risk Services (PRS)
  - Euromoney
  - The Economist
  - The World Bank
- **Sovereign Default**
  - Foreign Currency Defaults
  - Local Currency Defaults

Country Risk

- **Sovereign Default**
  - **Sovereign Default Spread**
  - ✓ When a government issues bonds, denominated in a foreign currency, the interest rate on the bond can be compared to a rate on a riskless investment in that currency.
  - **Advantage**
  - ① Market differentiation for risk is more granular than the ratings agencies
  - ② Market-based spreads are more dynamic than ratings
  - **Disadvantages**
  - ① Tend to be more volatile than ratings
  - ② Can be affected by variables that have nothing to do with default.
  - ✓ Liquidity and investor demand can cause shifts in spreads that have little or nothing to do with default risk.

Credit Risk

- **Credit Risk**
  - ① **Default**-related events
  - ② **Credit** migrations
- **Capital Model**
  - **Expected losses** refer to the losses that banks **take into account** when setting lending rates.
  - The expected loss of a loan over a certain period of time is

$$\text{Probability of Default} \times \text{Loss Given Default}$$

$$\text{RR} = \frac{\text{recovery}}{\text{exposure}} = 1 - \frac{\text{LGD}}{\text{exposure}}$$

$$\text{Probability of Default} \times (1 - \text{Recovery Rate})$$

Mean and Standard Deviation of Credit Losses

- **Mean and Standard Deviation**
  - Assume: L_i is the amount borrowed in the ith loan; p_i is the probability of default for the ith loan; R_i is the recovery rate.

$$\boxed{\begin{gathered}\mu_i = p_i \times L_i(1-R_i) + (1-p_i) \times 0 = p_i L_i (1-R_i) \\ \sigma_i = \sqrt{p_i - p_i^2}\,(L_i(1-R_i))\end{gathered}}$$

  - Assume all loans have the same principal L, all recovery rates are the same and equal to R, all default probabilities are the same and equal to p. The standard deviation of the loss is then the same for all i.

$$\boxed{\sigma_P^2 = n\sigma^2 + n(n-1)\rho\sigma^2}$$

Mean and Standard Deviation of Credit Losses

- A USD 1 million loan has a probability of 0.5% of defaulting in a year. The recovery rate is estimated to be 40%. What is the expected credit loss and the standard deviation of the credit loss?
- Answer:
  - The expected loss in USD is $0.005 \times 1 \times (1 - 0.4) = 0.003$
  - ✓ This is USD 3,000.
  - The variance of the loss is

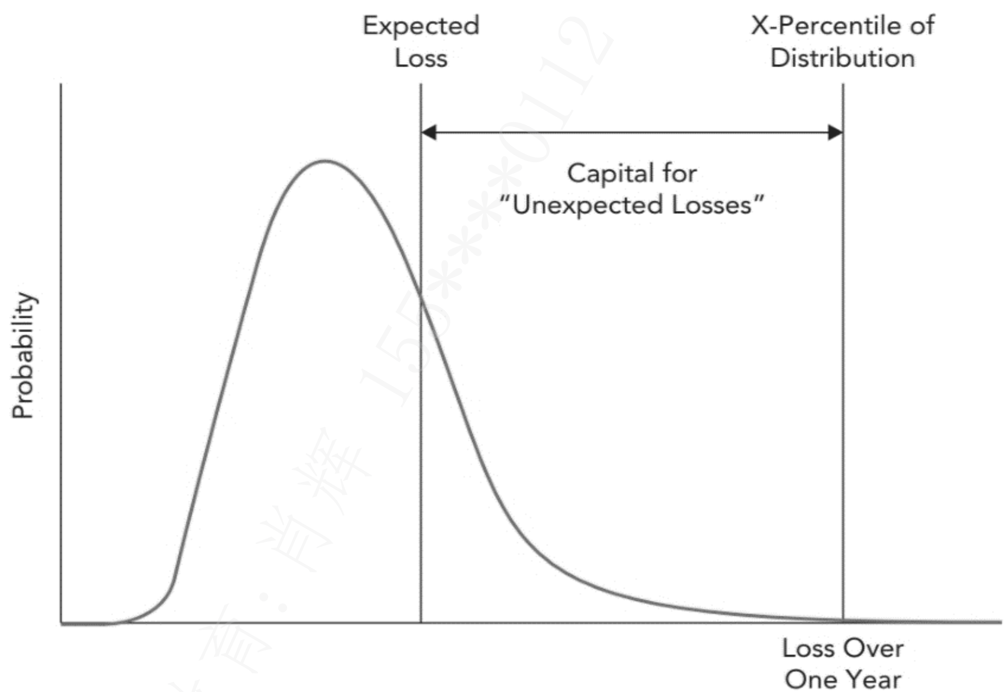
$$0.005 \times 0.6^2 - (0.005 \times 0.6)^2 = 0.001791.$$

  - The standard deviation is the square root of this, or USD 0.04232 million.
  - ✓ This is USD42,320

Model for Determining Capital

Capital Model

- The bank's **capital** is a buffer against **unexpected loss**
  - ✓ The actual loss in a given year above the expected loss.



Vasicek Model and CreditMetrics

- **Vasicek Model(-Regulatory capital)**
  - Used by regulators to determine capital for loan portfolios. It uses the **Gaussian copula** model to define the correlation between defaults.
  - The **Basel II** capital requirement use the **IRB approach** is:

$$(\mathrm{WCDR} - \mathrm{PD}) \times \mathrm{LGD} \times \mathrm{EAD}$$

  - where the WCDR (worst case default rate) is the **99.9 percentile** of the default rate distribution.
  - Values in the extreme left tail of this standard normal distribution correspond to a default.
- **CreditMetrics(-Economic capital)**
  - One of the differences between CreditMetrics and Vasicek model is that it takes into account the impact of **rating changes** as well as **defaults.**

Operational Risk Models

Operational Risk Models

- Definition
- Categories
- Regulatory Capital Requirement
- Reducing Operational Risk

Operational Risk

- **Operational Risk**
  - The risk of loss resulting from
  ① Inadequate or Failed Internal Processes
  ② People
  ③ Systems
  ④ From External Events.
  - Three Large Operational Risks
  ① Cyber Risks
  ② Compliance Risks
  ③ Rogue Trader Risk

Operational Risk

- **Operational Risk**
  - Basel’s Seven Categories of Operational Risk

① Internal Fraud

② External Fraud

③ Employment Practices and Workplace Safety

④ Clients, Products, and Business Practices

⑤ Damage to Physical Assets

⑥ Business Disruption and System Failures

⑦ Execution, Delivery, and Process Management

Regulatory Capital Requirement

- **Basic Indicator Approach:** through a simple calculation of the **average gross revenue** for the **past 3 years**, multiplied by **15%**.

$$\boxed{\mathrm{ORC}^{\mathrm{BIA}} = [(\mathrm{GI}_{1,\ldots,n} \times \alpha)]/n, \ \ \alpha = 15\%}$$

- **Standardized Approach:** The total capital charge is calculated as the **three-year average** of the simple summation of the regulatory capital charges across each of the business lines in each year.

$$\boxed{\mathrm{ORC}^{\mathrm{TSA}} = \frac{\Sigma_{\text{year } 1-3} \max(\Sigma(\mathrm{GI}_{1-8} \times \beta_{1-8}), 0)}{3}}$$

Corporate Finance	18%	Angency and custody services	15%
Trading and sales	18%	Asset management	12%
Settlement and payment activities	18%	Retail brokerage	12%
Commercial banking	15%	Retail banking	12%

Regulatory Capital Requirement

➢ **Advanced Measurement Approach (AMA)**

- Allows a bank to design its **own model** for calculating operational risk capital.

✓ Must hold capital for a **1-year horizon** at **99.9%** confidence level.

$$\boxed{ORC^{AMA} = UL(1-\text{year},\ 99.9\%\ \text{confidence})}$$

✓ All **four elements** of the framework must be included in the model:

① Internal loss data,

② External loss data,

③ Scenario analysis,

④ Business environment internal control factors.

Loss Distribution Approach

- **Loss Frequency Distribution**
  - Models number of losses.
  - Common probability distribution: **Poisson Distribution**
- **Loss Severity Distribution**
  - Models size of a loss.
  - Common probability distribution: **Lognormal Distribution**
- **Loss Distribution**
  - Assume that loss severity and loss frequency are **independent**
  - The frequency and severity distributions must be combined; **Monte Carlo simulation** can be used for this purpose.

Regulatory Capital Requirement

- **Standardized Measurement Approach**
  - However, bank **regulators** have found the approach **unsatisfactory** due to the **high degree of variation** in the calculations carried out by different banks.
  - **Two banks** presented with the **same data** were liable to come up with quite **different capital** requirements **under AMA**.
  - SMA first defines a quantity called **Business Indicator (BI).** It is similar to **gross income,** but it is designed to be a more relevant measure of bank size.
  - The Basel committee provides a formula for calculating the required capital from the **loss component** and the **BI component.**

Reducing Operational Risk

- **Causes of Losses**
  - Sometimes operational risk loss may be related to other **manageable factors.**
- **Education**
  - It is important to **educate** employees about unacceptable business practices
- **Risk Control and Self Assessment**
  - May lead to improvements reducing the frequency of losses, the severity of losses, or both.
- **Key Risk Indicators**

  ① Staff turnover,

  ② Failed transactions,

  ③ Positions filled by temps, and

  ④ Unfilled positions.

Reducing Operational Risk

- **Insurance**
  - Moral Hazard
    - ✓ Deductibles,
    - ✓ Coinsurance provisions,
    - ✓ Policy limits.
  - Adverse Selection
    - ✓ Finding out researching potential customers before providing a quote.

Stress Testing

Stress Testing

- Stress Testing versus VaR and ES
- Governance over Stress Testing
- Principles for Sound Stress Testing

Stress Testing and Other Risk Management Tools

- **Stress Testing**
  - A key question for financial institutions is whether they have enough **capital** and **liquid assets** to cope with various situations.
- **Stress Testing versus VaR and ES**

	Stress Testing	VaR and ES
Analysis	forward-looking	backward-looking
Scenarios	few scenarios （**all negative** for the organization）	a wide range of scenarios （**both good and bad** for the organization）
Horizon	longer period	short time

Governance over Stress Testing

- **Key Elements of Effective Governance over Stress Testing**
  - The Board and Senior Management
  - Policies and Procedures
  - Validation and Independent Review
  - Internal Audit
- **The Board and Senior Management**
  - **Board of Director:** Has the responsibility to oversee the key strategies. It is also responsible for the firm's risk appetite and risk culture.
  - **Senior Management:** Is responsible for ensuring that stress testing activities authorized by the board are implemented correctly. Senior Management is also responsible for ensuring the organization is adhering to the appropriate policies and procedures.
  - It is important for the board and senior management to ensure stress testing **covers all business lines and exposures**.

Governance over Stress Testing

- **Policies and Procedures**
  - Describe the overall **purpose** of stress-testing activities.
  - Indicate stress-testing **roles** and responsibilities.
  - Define the **frequency** at which stress testing is to be performed
  - Outline the **process** for choosing stressful conditions for tests.
  - Be reviewed and **updated** as necessary to ensure that stress testing practices remain appropriate and keep up to date with changes.
  - **Document** the operation of **models** and other **software** acquired from **vendors** or other third parties.
  - ✓ Documentation is important so far as it ensures continuity if key employees leave and satisfies the needs of senior management, regulators, and other external parties.

Governance over Stress Testing

- **Validation and Independent Review**
  - The reviews themselves should be **unbiased** and provide assurance to the board that stress testing is being carried out in accordance with the firm's policies and procedures.
  - The reviewers of stress-testing procedures be **independent** of the employees conducting the stress test. The review should:
  - ✓ Cover the **qualitative** or **judgemental** aspects of a stress test
  - ✓ Ensure that tests are based on **sound theory**
  - ✓ Ensure that **limitations** and **uncertainties** are acknowledged
  - ✓ **Monitor** results on an **ongoing** basis.

Governance over Stress Testing

- **Internal Audit**
  - It should ensure that stress tests are carried out by **employees** with appropriate **qualifications**, that **documentation** is **satisfactory**, and that the **models** and **procedures** are **independently validated**.
  - It assesses the practices used across the whole financial institution to ensure they are **consistent**.
  - It can then **provide advice** to senior management and the board on changes it considers to be desirable .

Choosing Scenarios

- **Historical Scenarios**
  - It is assumed that **all relevant variables** will behave as they did in the past.
  - ✓ The 2007- 2008 U.S. housing-related recession.
- **Stress Key Variables**
  - Assume that a large change takes place in one or more **key variables**.
  - ✓ A 25% decline in equity prices
- **Ad Hoc Stress Tests**
  - It is important for firms to develop other scenarios **reflecting current economic conditions**, the particular exposures of the financial institution, and an **up-to-date** assessment of possible future adverse events.
- **Using the Results**
  - They should carefully consider whether the results of stress tests indicate that **more capital** should be held or that **liquidity** should be improved.

Principles for Sound Stress Testing

- **Stress Testing Principles for Banks**
  - Providing **forward-looking** assessments of risk
  - Overcoming the **limitations** of models and historical data
  - Supporting internal and external **communications**
  - Feeding into **capital and liquidity** planning procedures
  - Informing and setting of **risk tolerance**, and
  - Facilitating the development of **risk mitigation** or **contingency plans** across a range of stressed conditions.

It’s not the end but just beginning.

Thought is already is late, exactly is the earliest time.

感到晚了的时候其实是最快的时候。

问题反馈

- 如果您认为金程**课程讲义/题库/视频**或其他资料中**存在错误，欢迎您告诉我们**，所有提交的内容我们会在最快时间内核查并给与答复。
- **如何告诉我们?**
  - 将您发现的问题通过电子邮件告知我们，具体的内容包含：
  - ✓ 您的姓名或网校账号
  - ✓ 所在班级（eg.2205FRM一级长线无忧班）
  - ✓ 问题所在科目（若未知科目，请提供章节、知识点）和页码
  - ✓ 您对问题的详细描述和您的见解
  - 请发送电子邮件至：academic.support@gfedu.net
- **非常感谢您对金程教育的支持，您的每一次反馈都是我们成长的动力**。后续我们也将开通其他问题反馈渠道（如微信等）。