Modeling Systemic Risk with Copula-based Methods

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1. Introduction

With the rapid development of the world economic integration and financial innovation, the connection between financial institutions has become closer and their dependence relationship has also developed increasingly complex. Therefore, how to describe dependence structure of financial institutions accurately is the most important issue in financial risk management. An inappropriate model for dependence may lead to mistakes in assessing risk exposures. To improve model's performance, in-depth related research has been carried out. Particularly, copulas offer a powerful tool to model tail and systemic risk.

1.1. Background

Risk modeling has arisen as a major issue in the process of mitigating risks. In order to fairly assess the overall risk exposure, models have to take into account the correlation between individual risks when aggregating them. More importantly, models must capture the dependence between extreme events, thus reflecting the possibility of rare events cumulation.

Traditionally, correlation is used to describe dependence between random variables. However, the time series of financial asset usually have some typical characteristics like higher peak and fatter tail. Therefore, their distribution features need to be fully considered for a better estimate of financial risk. Copulas model offer much more flexibility than the correlation approach.

A copula is a multivariate cumulative distribution function for which the marginal probability distribution of each variable is uniform. It provides an appropriate framework for estimating the asymmetry dependence between two random variables. Lots of scholars utilized copula family methods like Gaussian, Student-t and Clayton to model and minimize tail risk [1].

There are still problems to be solved in classic Copula function application because of "dimension curse". Brechmann and Schepsmeier [2] proved that standard multivariate copula such as multivariate Gaussian or Student-t, lacks flexibility of correctly representing the dependence amongst larger numbers of variables. The recent development of vine copulas [3] (also known as pair copulas) enables the flexible modelling of the dependence structure for portfolios of large dimensions.

1.2. Vine Copula

Vine Copula is a theory of emerging interdependence. It decomposes the multivariate joint into a hierarchical structure by means of a graphical tool, vine, by which researchers are able to disintegrate high dimensional Copulas into a form of a product of several binary Copulas, reducing the computation needed. Regular vines make up the shortcomings of classic copula functions in high-dimensional dependence modeling.

Aas et al. [4] introduced vine copulas into the finance and insurance journal, where they also demonstrated statistical inference techniques for two classes of canonical C- (and D-) vines. Armin P. et al [5] firstly employed partial correlation structure by using vine copulas. Correlation asymmetries and tail dependencies can be taken into account to build more parsimonious models.

In summary, vine copula models combine the flexibility of bivariate copula and the advantages of multivariate copula modelling. We would like to utilize the C-vine and D-vine Copula Models to analyze the complex interdependency structure between different rating groups institutions. The period of the input data will cover the global financial crisis and following European debt crisis, which exhibit the integrity and practicability of our research. Since the whole period of Covid-19 panic is not clear, we will not cover this part.

The remainder of this paper is structured as follows. In Section 2 we utterly describe the data on probability of default (PD) for financial institutions. Section 3 provides a discussion on empirical features of dependence in the joint distribution, which serves as the motivation for our choice of models. Section 4 provides vine copula mathematic definition. Section 5 shows the empirical analysis and potential applications in practice. Section 6 makes an end conclusion.

2. Data Description

We employed PD and corresponding S&P credit rating of global financial institutions, including banks and insurances companies, tracked by Credit Research Initiative every day during the period of Jan 2006 to Dec 2018.

Credit Research Initiative (CRI) offers credit ratings for over 60,000 exchange-listed firms around the world. CRI combines a reduced-form model (based on a forward intensity construction [6]) and a structural model (using Distance-To-Default as one of its input covariates) and employs market-based and accounting-based firm-specific attributes as inputs to predict probability of default of companies on a daily basis. Besides, CRI has developed a system to map PD to the equivalent letter grades of the S&P rating system and therefore enables us to better divide group financial institutions into different credit groups.

We considered 100 traded financial institutions including banks and insurance companies and classify issuers into five credit rating groups. The letter grades and classification method are shown in Table 1. Table 2 reports the descriptive statistics of our data from rating classes 1 through 5 and we can notice that the mean and the standard deviation increase from the first rating class to the fifth rating class, which means changes tend to be higher for low credit issuers. From skewness and kurtosis statistics we found that the PD didn’t strictly follow normal distribution and therefore we considered several other distributions such as Student-t during model construction process to better capture the characteristics of our data.

Rating class	S&P 500 Equivalent Rating	Credit Group
1	AAA, AA+, AA, AA-	High grade
2	A+, A, A-	High grade
3	BBB+, BBB, BBB-	Medium grade
4	BB+, BB, BB-, B+, B, B-	Medium grade
5	CCC+, CCC, CCC-, CC, C	Low grade

Table 1

Rating	Min	Max	Mean	Std	Skewness	Kurtosis
PD in levels						
1	0.003	0.523	0.055	0.074	1.989	4.074
2	0.020	0.695	0.116	0.116	2.050	3.835
3	0.071	1.404	0.241	0.193	3.156	11.887
4	0.113	1.762	0.323	0.233	3.406	13.390
5	0.643	8.534	1.311	0.789	3.642	17.102
PD in Changes						
1	0.000	0.076	0.001	0.004	7.408	80.634
2	0.000	0.218	0.002	0.006	13.947	395.112
3	0.000	0.168	0.004	0.011	7.534	77.076
4	0.000	0.264	0.005	0.012	8.751	125.454
5	0.000	1.016	0.016	0.036	10.241	187.852

Table 2

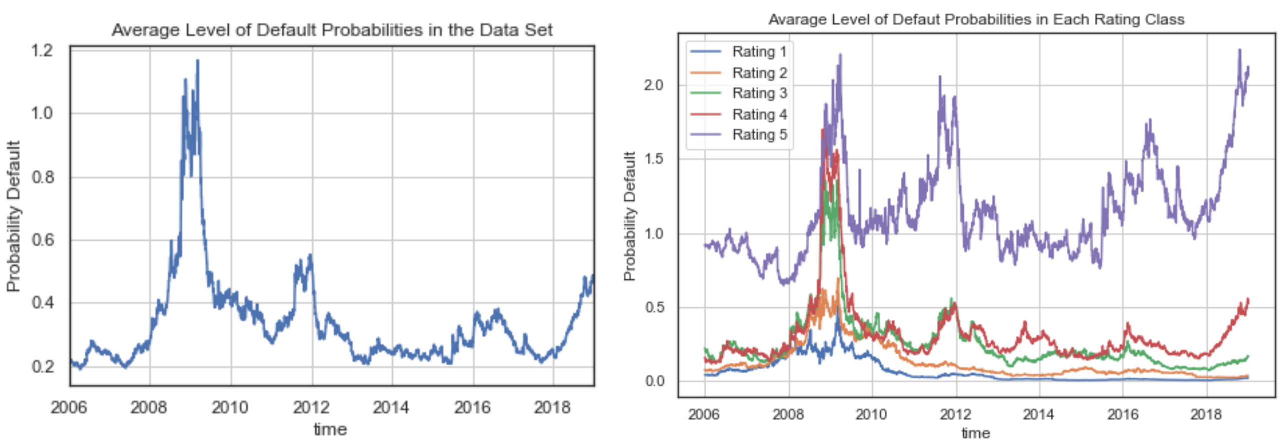


Figure 1 Time series of average PDs

3. Empirical Features of Dependence in The Joint Distribution

In order to compare the statistical properties of joint distributions associated with different copula, we developed a metric, which captures two crucial features of the dependence relationship in the joint distribution: tail-dependence and correlation asymmetry.

3.1. Tail-dependence

Tail-dependence is important in the analysis of the partial correlation structure for the default risk between different issuers. Empirical evidence has shown that financial data in real world often violate the assumption of normality in terms of asymmetry, skewness and heavy tails, and exhibit a very different dependency structure from Gaussian. Especially in highly volatile markets, the use of the normality assumption would underestimate the probability for joint extreme events, which might cause simultaneous losses in a portfolio. Therefore, it's necessary to develop a more robust multivariate model for extreme events.

Vine copula is more flexible compared with standard multivariate copulas, as we can select bivariate copulas from a wide range of parametric families. In the vine copula framework, bivariate copulas are building blocks for higher-dimensional distributions, and the dependency structure is determined by the bivariate copulas and a nested set of trees.

Table 3 reports the tail dependence between the different rating classes. This table presents correlations between the PD level amongst the rating classes based on different tail percentiles. The table indicates that the tail dependence across different groups and levels varies significantly. These findings shed light to our assertion above and the need to leverage vine copula model to address the correlation of tail dependence.

Rating pair		Lower tail dependence			Upper tail dependence		
Rating	Rating	10%	20%	30%	70%	80%	90%
Panel A：tail-dependency in levels							
1	2	0.3875	0.2539	-0.0651	0.7591	0.6492	0.4981
1	3	0.7272	0.4838	0.4281	0.5721	0.4797	0.5435
1	4	-0.1909	-0.1149	-0.0172	0.6079	0.6397	0.5080
1	5	-0.5991	-0.5834	-0.6003	0.3452	0.2444	0.0611
2	3	0.3950	0.2510	0.3451	0.7409	0.7077	0.7303
2	4	-0.0362	-0.0541	0.0935	0.7173	0.7423	0.6954
2	5	-0.4596	-0.4162	-0.4147	0.2570	0.1392	-0.0538
3	4	-0.0577	-0.1664	0.0196	0.9153	0.9216	0.9404
3	5	-0.1231	-0.1161	-0.1169	0.3480	0.1933	-0.0963
4	5	-0.1227	-0.1220	0.0044	0.4687	0.3067	0.0467
Panel B: tail-dependency in change							
1	2	0.5796	0.5736	0.5735	0.3766	0.3609	0.3271
1	3	0.3007	0.3180	0.3267	0.3572	0.3490	0.3679
1	4	0.2289	0.2400	0.2378	0.1877	0.1917	0.1848
1	5	0.2254	0.2032	0.1975	0.1548	0.1654	0.1534
2	3	0.2565	0.2780	0.2894	0.5449	0.5453	0.5411
2	4	0.3349	0.3505	0.3508	0.5536	0.5660	0.5587
2	5	0.0864	0.1065	0.1152	0.0999	0.0994	0.0789
3	4	0.4132	0.4512	0.4654	0.6485	0.6451	0.6358
3	5	0.2174	0.2213	0.2244	0.1484	0.1435	0.1077
4	5	0.1613	0.1578	0.1732	0.1480	0.1393	0.0909

Table 3

3.2. Asymmetric Correlation

The correlation asymmetry is an import phenomenon in financial markets. In the context of credit risk, it indicates that high graded issuers (in many cases large firms) have a larger exposure to systemic risk, whereas low graded issuers (medium and small firms) face more idiosyncratic risk (see Das and Geng, 2004).

We collected the PDs across all financial institutions at each point in time, t, to calculate the correlations for every pair of rating classes. For an exceedance level at quantile q, for pairs of standardized observations $\left(PD_{it}, PD_{jt}\right)$, we selected a subset of observations such that

$$\left\{\left(PD_{it}, PD_{jt}\right)\middle|PD_{it} < q, PD_{jt} < q\right\},$$

and

$$\left\{\left(PD_{it}, PD_{jt}\right)\middle|PD_{it} > 1-q, PD_{jt} > 1-q\right\}.$$

Furthermore, we calculated both correlations when q = 0.5. Therefore, the exceedance correlation ρ^- is lower q quantile, while ρ^+ refers to the joint occurrence of positive changes, above $1-q$. Thus,

$$\rho^+ = corr\left\{\left(PD_{it}, PD_{jt}\right)\middle|PD_{it} > 1-q, PD_{jt} > 1-q\right\}$$
$$\rho^- = corr\left\{\left(PD_{it}, PD_{jt}\right)\middle|PD_{it} < q, PD_{jt} < q\right\}$$

Here i and j denote any two different rating classes.

Figure 1 depicts the correlations for every pair of rating classes in the data. We noted that high graded issuers have greater correlation than lower graded issuers. Additionally, there is clear evidence of correlation asymmetry in all groups.

Further, we can explain the asymmetry tail dependence of credit risk of different issuers from the aspect of systemic risk. From Figure 1, it is noticed that almost all credit ratings show relatively high upper tail dependence to the first two group, which indicates that the system is more vulnerable to higher issuers. Reason may be that due to several policy of capital requirement and asset liquidity, most financial institutions, especially banks, tend to overweight asset issued by higher credit graded issuers, and so if these big institutions fail, the panic will spread much more broadly and quickly within the whole industry. Whereas, the failure of lower credit graded issuers tends to attribute to a much greater variety of firm-specific reasons and combined with the limited liquidity of their bonds and securities, is typically absorbed by the system. This is evidenced by relative lesser tail dependency of all rating groups conditioned to group 5.

The partial correlation structure between the PDs of different credit groups is a measure of systemic risk, since it measures the interdependency between the average credit of the constituents of each group. In this project, we deployed vine-copula to accurately quantify this measure of systemic risk, which, by way of construction, better models the interdependency than the popular pair-copula approach.

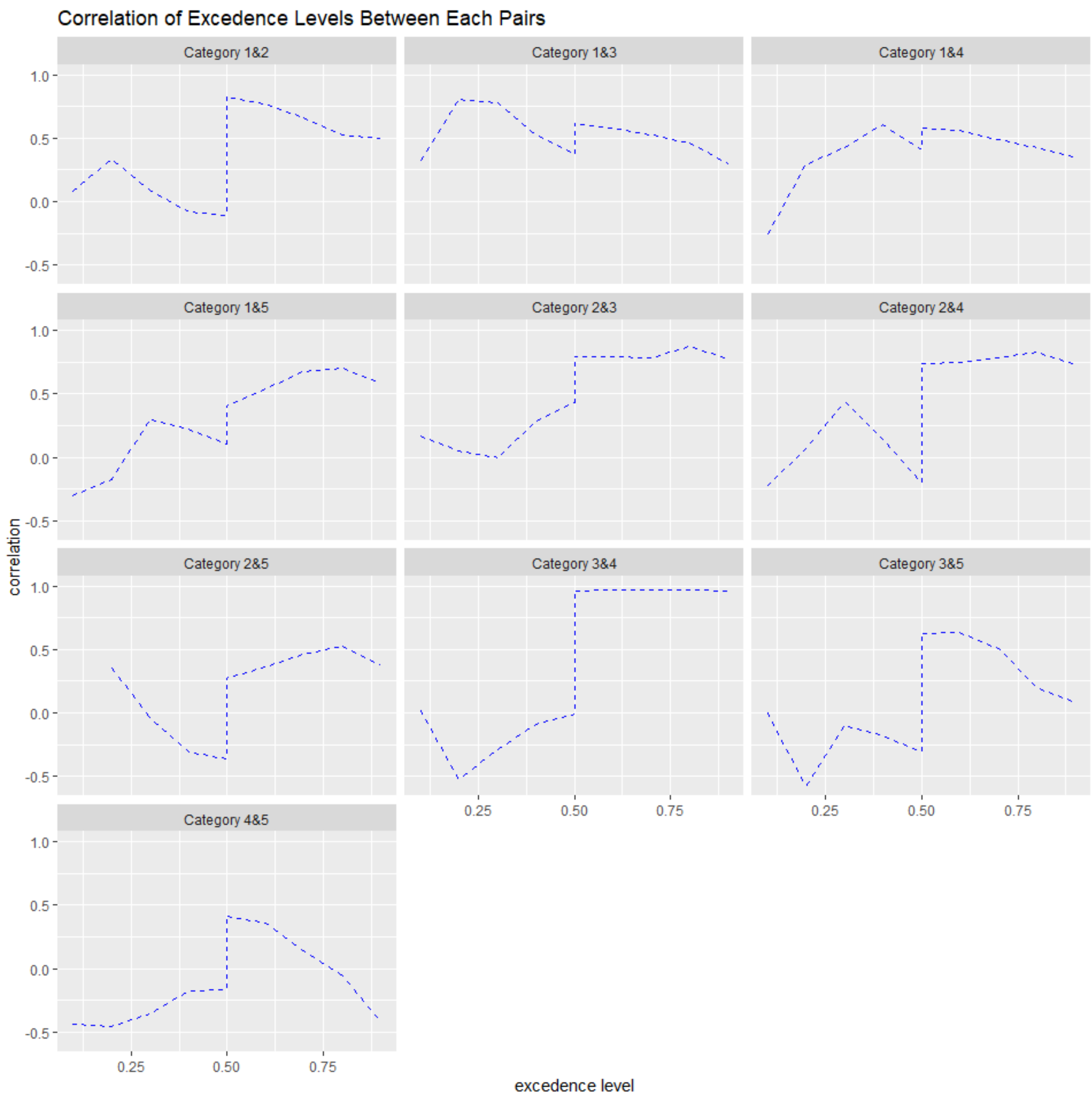


Figure 2

4. Copula functions and vine copula

In statistical terms, a copula is a multivariate function with uniform marginals that represents the dependence structure between 2 random variables. The theory of copulas dates back to Sklar [7], who stated that the joint distribution of 2 continuous random variables X and Y, namely, $F_{XY}(x,y)$ with marginal functions $F_X(x)$ and $F_Y(y)$, is characterized by a copula function C such that:

$$F_{XY}(x,y) = C(F_X(x), F_Y(y))$$

Also, C is uniquely determined on $RanF_X \times RanF_Y$ when the margins are continuous. Thus, copulas can be used to connect margins to a multivariate distribution function which, in turn, can be decomposed into its univariate marginal distributions and a copula that captures the dependence structure. This approach offers greater flexibility in modelling dependence than would be possible with parametric multivariate distributions.

A remarkable feature of the copula is that it allows for tail dependence, which measures the probability that 2 variables experience extreme upward or downward movements. The coefficients of upper (right) and lower (left) tail dependence are obtained (respectively) from the copula as:

$$\lambda_u = lim_{u \to 1} P[X \geq F_X^{-1}(u) | Y \geq F_Y^{-1}(u)] = lim_{u \to 1} \frac{1 - 2u + C(u,u)}{1 - u}$$

$$\lambda_L = lim_{u \to 0} P[X \leq F_X^{-1}(u) | Y \leq F_Y^{-1}(u)] = lim_{u \to 0} \frac{C(u,u)}{u}$$

where $u = F_X^{-1}$ and $v = F_Y^{-1}$ are the marginal quantile functions and $\lambda_u, \lambda_L \in [0,1]$. Two random variables exhibit lower (upper) tail dependence if $\lambda_L > 0 (\lambda_U > 0)$, indicating a non-zero probability of observing an extremely small (large) value for a particular series together with an extremely small (large) value for another series.

4.1. Copula Functions

We considered different copula functions, with different characteristics in terms of tail dependence and time-varying dependence, specified as follows:

The Student-t copula is given by $C_{ST}(u, v; \rho, v) = T(t_v^{-1}(u), t_v^{-1}(v))$, where T is the bivariate Student-t cumulative distribution function, v as the degree of freedom parameter, with correlation ρ, $t_v^{-1}(u)$ and $t_v^{-1}(v)$ are the quantile functions of the univariate Student-t distribution with v as the degree of freedom parameter. It allows for symmetric non-zero dependence in the tails, where tail dependence is given by $\lambda_u = \lambda_L = 2t_{v+1}\left(-\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right) > 0$. This copula converges to the Gaussian when degrees of freedom tend to infinity.

The Clayton copula is given by $C_{CL}(u, v; \alpha) = \max\left\{(u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}, 0\right\}$. It shows asymmetry, as the degree of dependence is higher in the lower tail than in the upper tail, where it equals zero: $\lambda_L = 2^{-1/\alpha}$ $(\lambda_U = 0)$.

The Gumbel copula is given by $C_G(u, v; \delta) = \exp\left(-((-logu)^{\delta} + (-logv)^{\delta})^{1/\delta}\right)$. It is asymmetric with a higher degree of dependence in the upper tail than in the lower tail, where it equals zero; thus, $\lambda_U = 2 - 2^{1/\delta}$ $(\lambda_L = 0)$.

Copula	$\boldsymbol{\lambda_L}$	$\boldsymbol{\lambda_U}$
Gaussian	0	0
Student-t	$2t_{v+1}\left(-\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right)$	$2t_{v+1}\left(-\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right)$
Clayton	$2^{-1/\alpha}$	0
Gumbel	0	$2 - 2^{1/\delta}$

Table 4

4.2. Vine Copula

Vine copulas (Bedford, Cooke, 2002 [8], Joe, 1997 [9], Kurowicka, Joe, 2010 [10]) are multivariate dependence models, constructed using only unconditional and conditional pair copulas. The common case of vine copula is regular vine (R-Vine), and two special and most commonly used implementations are C-Vine and D-Vine, represented by graphical models. C-Vine and D-Vine Copula definitions are displayed below:

C-Vine: If each tree T_i has a unique node of degree $n-i$, then the vine is a canonical vine. The node with maximal degree in T_1 is the root.

D-Vine: If each node in T_i has a degree of at most 2, then the vine is a D-vine.

C-Vine is more effective for ordering by importance while D-Vine is more powerful for temporal ordering of variables because of their different tree structures.

In our research, we examined the rating class '1' and '2' as the driving variable (roots of trees) for the systematic risk. Both of these two groups contain financial institutions with the biggest market capitalization and thus tend to be more organizationally complex than rating class '3', '4' and '5'. This implies that the failures of large financial institutions will bring liquidity stress in the financial system.

For choosing between the rating class '1' and '2', and the order in trees, we dealt with the second criterion proposed by Aas et al. [4]. He placed the strongest bivariate dependencies in the first tree of the vine tree structure. We can get dependencies information from Kendall's τ.

The following graph exhibits the C-Vine and D-Vine trees structure in our research.

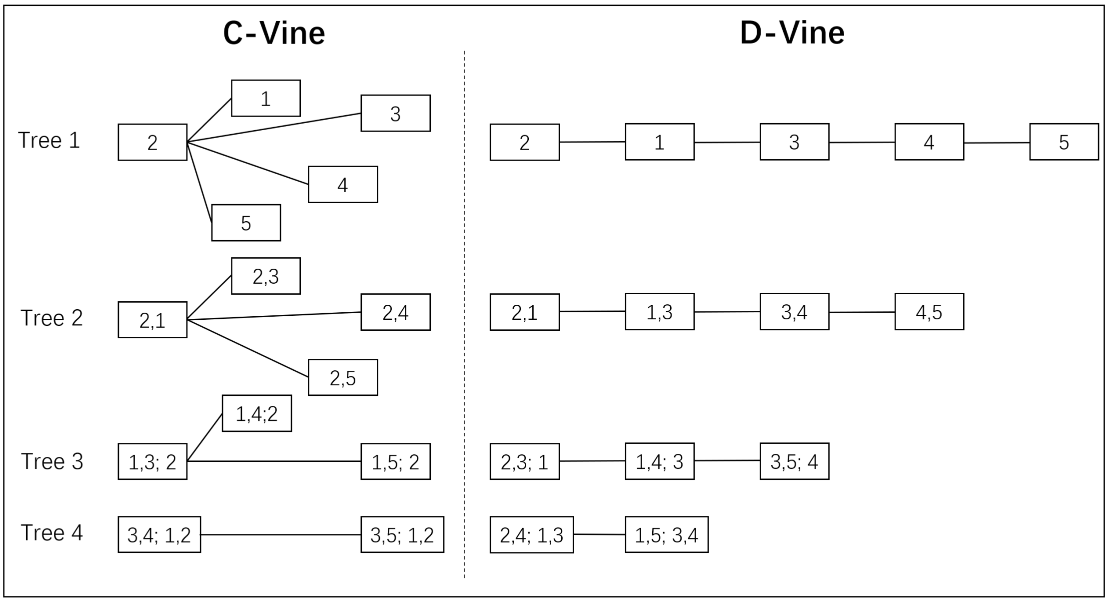


Figure 3

Additionally, we deployed Vuong Test [11] to compart two vine-copula models. Let c_1 and c_2 be two competing vine copulas in terms of their densities and with estimated parameter sets $\widehat{\theta_1}$ and $\widehat{\theta_2}$. We then calculate the standardized sum, υ, of the log differences of their pointwise likelihoods $m_i := \log\left(\frac{c_1(u_i|\widehat{\theta_1})}{c_{2(u_i|\widehat{\theta_2})}}\right)$ for observations $u_i \epsilon [0,1]$.

$$statistic \coloneqq \nu = \frac{\frac{1}{n}\sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N}(m_i - \bar{m})^2}}$$

According to Vuong words, ν is asymptotically standard normal. Under the null-hypothesis,

$$H_0: E[m_i] = 0 \quad \forall\, i = 1,2,\dots,N,$$

We hence conclude that vine model 1 is better than vine model 2 at statistical significance level α if $\nu > \Phi^{-1}(1 - \frac{\alpha}{2})$, where Φ^{-1} is the inverse of the standard normal distribution. If $\nu < \Phi^{-1}(1 - \frac{\alpha}{2})$, we select model 2.

The steps we processed the model are summarized below:

1. Converted the empirical data to uniform distribution, located in [0,1];
2. Constructed the C/D-Vine tree structure and initialized the model;
3. Estimated the parameters of all conditional copula formulae by L-BFGS-B maximum likelihood estimation;
4. Computed the coefficients of conditional copula upper and lower tail dependence based on the parameters estimated in the last step.

5. Empirical Results

In this section, we evaluated the correlation structure of the PDs for 100 financial institutions, which have been classified into 5 rating groups.

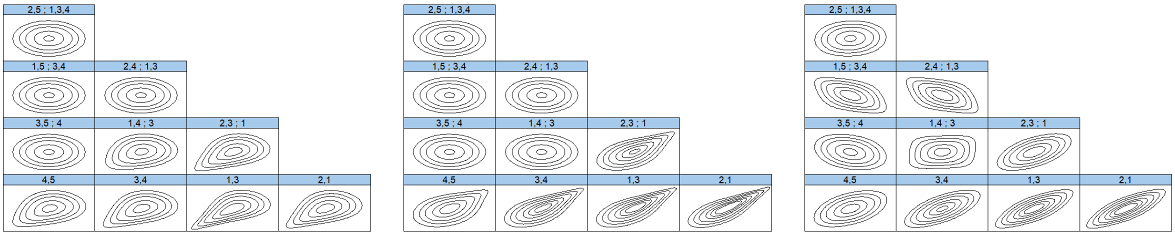
In Table 5, we computed Kendall's τ for default probability of each rating class. Note that the upper right triangle shows the dependence for PD levels, while the lower left triangle expresses the dependence for PD changes. We can see that the average Kendell τ of rating groups 2 is strongest. Therefore, as discussed in section 4.2, we chose the rating group 2 as the driving variable for the systemic risk.

Rating class	1	2	3	4	5
1	1.0000	0.6117	0.5479	0.3820	-0.0663
2	0.2431	1.0000	0.6442	0.2312	-0.0401
3	0.2505	0.2600	1.0000	0.4502	0.0856
4	0.1718	0.2466	0.3471	1.0000	0.3324
5	0.0923	0.0733	0.0992	0.0913	1.0000

Table 5

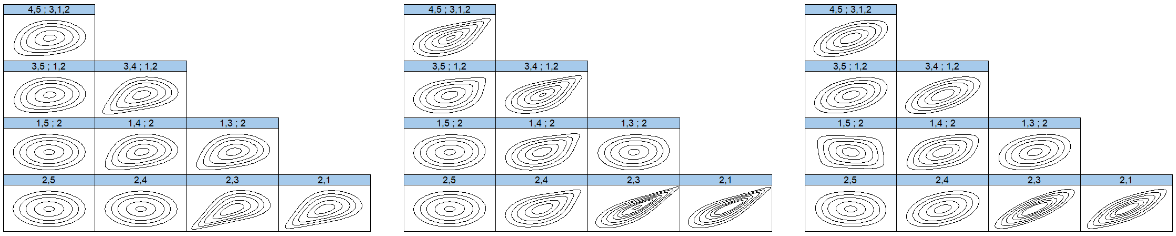
Figure 4 shows empirical contour plots of t-Copula, Clayton and Gumbel for both canonical vine and D-vine. We can see that Clayton copula is lower-tail dependent, and the Gumbel copula is upper-tail dependent. The Student-t copula is both lower- and upper-tail dependent.

D-vine



(a) Clayton (b) Gumbel (c) Student-t

C-vine



(d) Clayton (e) Gumbel (f) Student-t

Figure 4

Table 6 reports the Gaussian Copula partial correlations between the five different rating classes, as well as the tail dependencies using t-Copula, Clayton and Gumbel. The parameters are estimated for both C-vine (top panel) and D-vine (bottom panel). As discussed in Section 2, our data set exhibits a feature of a strong upper-tail dependency and a weaker lower-tail dependency in C-vine structure, but shows a more volatile result in D-vine structure. Similarly, we can see that C-vine structure better captures the tail dependence than D-vine.

D-vine						
Gaussian Copula Partial Correlation						
	$r_{(35\|4)}$	$r_{(14\|3)}$	$r_{(23\|1)}$	$r_{(15\|34)}$	$r_{(24\|13)}$	$r_{(25\|134)}$
	-0.1741	0.1093	0.4139	-0.2278	-0.3047	0.0018
Tail-dependency						
	$\tau_{(35\|4)}$	$\tau_{(14\|3)}$	$\tau_{(23\|1)}$	$\tau_{(15\|34)}$	$\tau_{(24\|13)}$	$\tau_{(25\|134)}$
Student-t	-0.1661	0.0711	0.3747	-0.2910	-0.2912	0.0461
Clayton	0.0000	0.1374	0.2559	0.0000	0.0000	0.0000
Gumbel	0.0001	0.0001	0.4125	0.0001	0.0001	0.0001
C-vine						
Gaussian Copula Partial Correlation						
	$r_{(15\|2)}$	$r_{(14\|2)}$	$r_{(13\|2)}$	$r_{(35\|12)}$	$r_{(34\|12)}$	$r_{(45\|132)}$
	0.0185	0.2970	0.1835	0.2262	0.3906	0.3719
Tail-dependency						
	$\tau_{(15\|2)}$	$\tau_{(14\|2)}$	$\tau_{(13\|2)}$	$\tau_{(35\|12)}$	$\tau_{(34\|12)}$	$\tau_{(45\|132)}$
Student-t	-0.0935	0.2888	0.1946	0.2637	0.3982	0.3621
Clayton	0.0084	0.1802	0.1961	0.0958	0.2592	0.1197
Gumbel	0.0388	0.2756	0.0456	0.2101	0.3844	0.4020

Table 6

Table 6 also reports the partial correlations between different rating groups. The first order partial correlations in both C- and D-vine trees reveal the importance of the rating group 2 in the systemic risk, confirming that the selection of the tree structure is correct.

We performed the Vuong test to compare two vine models with Student-t, Clayton and Gumbel copula models, respectively. The results are presented in Table7. The small p-values and positive statistic values of Clayton and Gumbel indicate that C-vine model is statistical significantly better than D-vine Copula, while the test shows no preference in Student-t model. The result is consistent with that of Table 6.

	Student-t	Clayton	Gumbel
Statistic	-1.268646	11.71612	16.63828
p-value	0.2045	0.0000	0.0000

Table 7

6. Conclusions

In this paper, we firstly reviewed related research in systemic risk modelling. We then put the recently developed C-vine and D-vine copula into practice to estimate the parameters of conditional CDF and tail dependency amongst 100 international financial institutions, which were classified into 5 rating groups. We estimated the partial correlation of 5 rating classes between January 2006 and December 2018, including 2008 GFC and the following European debt crisis. From empirical result, we found that the 2nd class makes the biggest contribution to financial systemic risk among the five credit rating groups and upper-tail dependency is stronger than weaker-tail dependency.

Besides, C-Vine Copula is statistical significantly better than D-Vine Copula in our research. It tells us that the root (Class 2nd) accounts for larger part of the total risk, which is consistent with the empirical result. Our models successfully capture data set features.

Our research may provide some insights into credit products pricing, such as CDS and policy making for regulators. For example, according to our empirical results, C-Vine Copula plays a better role in modeling asymmetry dependence and tail correlation and so we may use this to develop the distribution of n-to-default times by Monte Carlo sampling of many scenarios, which can be further applied to compute the risk neutral expectations in CDS pricing formula.

Nowadays, the Covid-19 outbreak brings serious challenges to global financial institutions. Financial regulators are issuing guidance to protect market participants against from financial crimes. We still need to keep monitoring, controlling and thus minimizing financial institutions' probability of default during this unprecedented time.

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