

1.

2. To get ~~largest~~ largest  $O(\sqrt{n})$  number  
 $k = O(\sqrt{n})$

Total time =  $k$ th smallest + sorting

$$= O(n) + O(\sqrt{n}) * \log(\sqrt{n}) = O(n)$$

3. a) Optimal value for each and every node has  $j$  contain  $j > 1$  where the node contain  $i$ .

$$OPT(i) = \max(OPT(j) + 1)$$

$OVER[i] =$  longest path between  $V_1$  to  $V_i$

$$OVER[i] = \max(OVER[j] + 1) \text{ while } j < i$$

int longestPath()

for (int  $i=1$ ;  $i \leq V$ ;  $i++$ ) {

$OVER[i] = -1$ ;

$OVER[i] = 0$ ;

    for (int  $j=1$ ;  $j < i$ ;  $j++$ ) {

        if ( $OVER[j] \geq 0$ ) {

            for ( $i$  in ' $V$ ' to  $V$ ) {

$OVER[V] = \max(OVER[V] + 1, OVER[j]);$

            return  $OVER[V]$ ;

b) - Two nodes can be connected such that  $V_1$  is connected to  $(V_2, V_3, \dots, V_n)$

-  $V_j$  is connected with  $(V_{j+1}, V_{j+2}, \dots, V_n)$

- number of vertices as  $V=n$

$$\text{edges} = E = n \times (n+1) / 2 = O(n^2)$$

$$OPT[i] = \max(1 \leq k < i) \{ E(OPT[k] + 1) \}$$

-  $P[1] = 0$

- for  $i=2$  to  $n$

- Algorithm see for all edges going into  $i$  store all max longest path has edge  $+1$

- It set  $P[i]$  to this value

- if the loop complete  $P[n]$

$$\text{Run time} = O(n^2)$$



4. One bowl = 40 = 1 h and 4 lbs  
 one mug = 50 = 2 h and 3 lbs  
 Total = 15

	Bowls	Mugs
No#	24	8
Profit	40	50
labor	1	2
clay	4	3

~~Maximize Profit~~ Maximum Profit is 136.

5. To find longest palindromic subsequence, we have the string that given and reverse the given string.

```
int dp[100][100];
```

```
int lcs(string s, string t, int n, int m) {
```

```
    if (n == 0 || m == 0) return 0;
```

```
    if (dp[n][m] != -1) return dp[n][m];
```

```
    if (s[n-1] == t[m-1]) return dp[n][m] = 1 + lcs(
```

```
        s, t, n-1, m-1);
```

```
    else return dp[n][m] = max(lcs(s, t, n-1, m),
```

```
        lcs(s, t, n, m-1));
```

```
int main() {
```

```
    int n; cin >> n;
```

```
    string s; cin >> s;
```

```
    string t = s;
```

```
    reverse(t.begin(), t.end());
```

```
    memset(dp, -1, sizeof(dp));
```

```
    cout << lcs(s, t, n, n);
```

```
}
```

Q. Part 1) - Every Problem in NP can be reduced to the same problem in polynomial time.

- It is in NP.

Part 2) The maximum independent set problem of a graph can be reduced to the largest complete subgraph.

1. Find all the independent vertices in the graph.
  2. Take set  $C$  be the complement graph of  $M_1$ . It has the vertices that are not in  $M_1$  and has edges that are not ~~not~~ ~~incide~~ incident of any of the vertices in  $M_1$ .
  3. If number of vertices in  $C$  is  $k$ , answer is yes, else no.
- Complete subgraph of size  $= n - |M_1|$ .