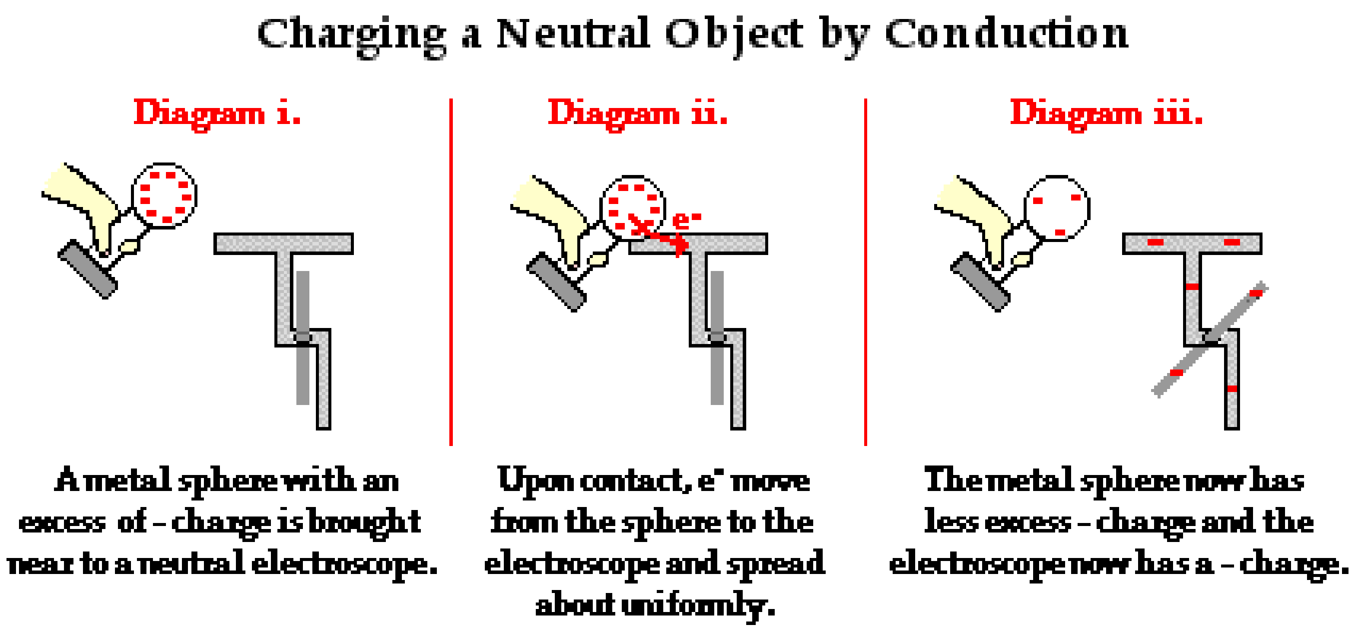
1.1 - Electric Charge & Coulomb's Law

Tuesday, February 28, 2017 9:25 AM

Conductors and Insulators

- Resistivity is a material property, measured in (Ω·m)

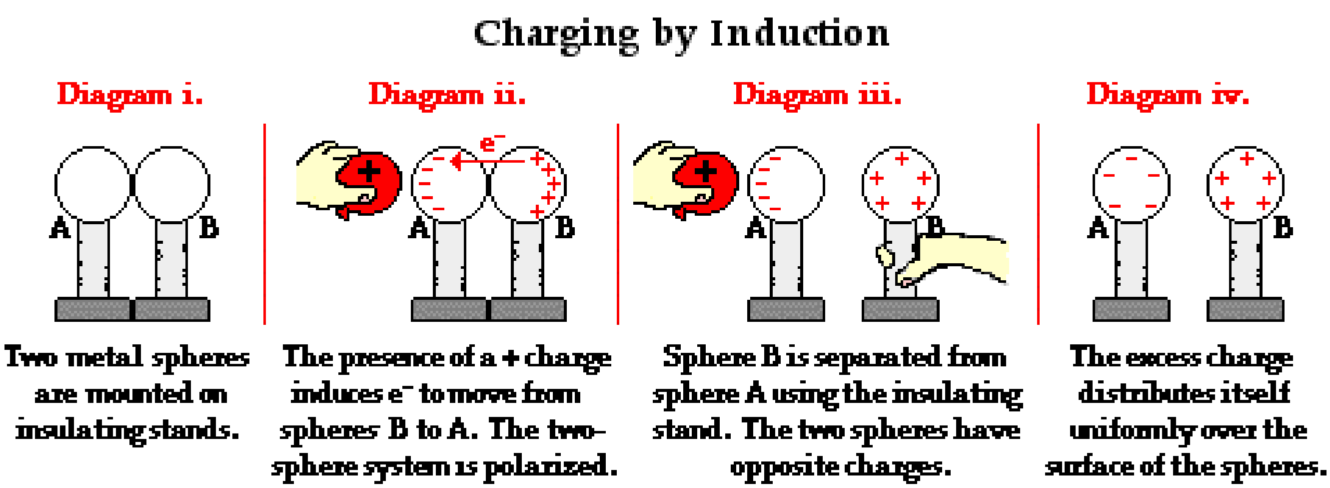
Charging by Conduction

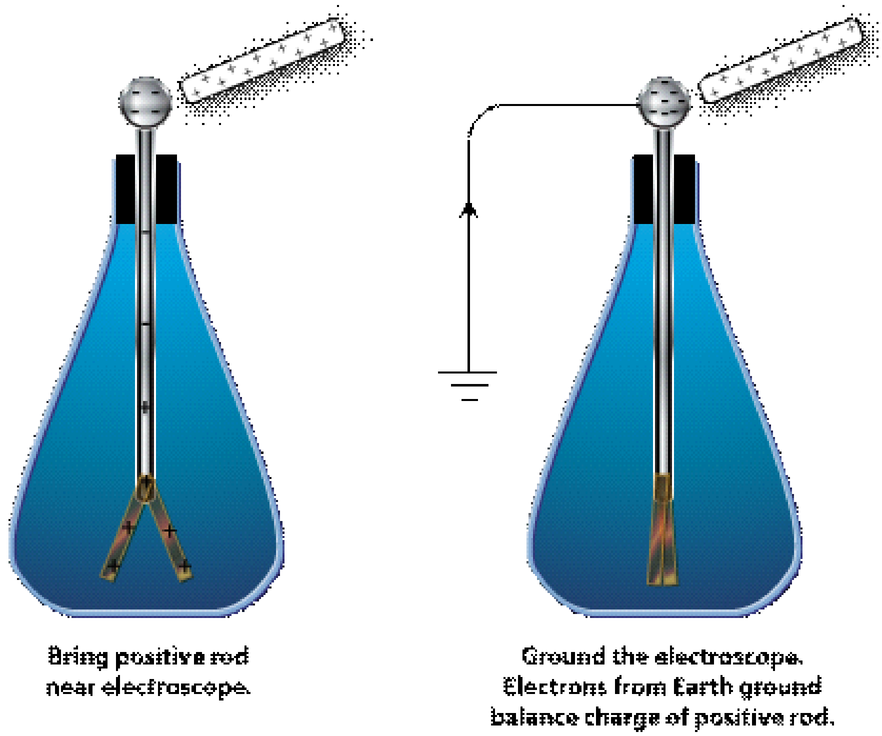


The Electroscope

- An electroscope is used to detect small electric charges based on conduction

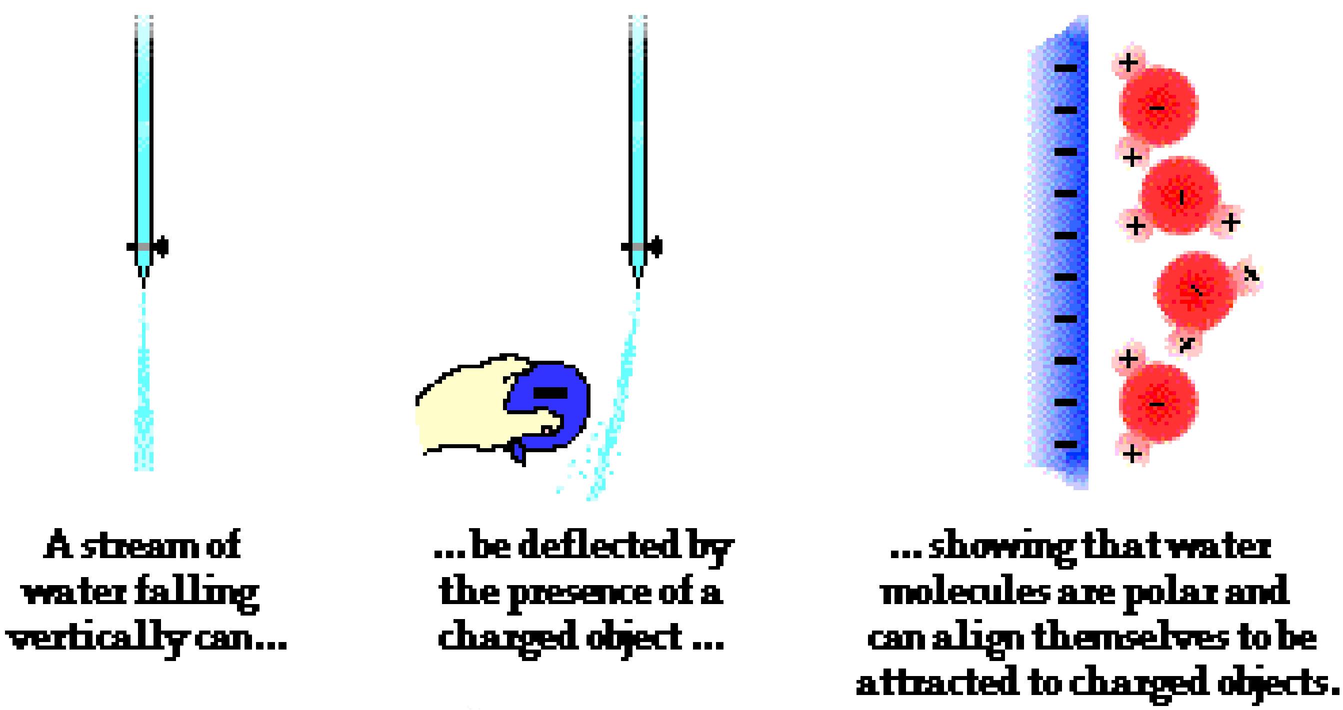
Charging by Induction

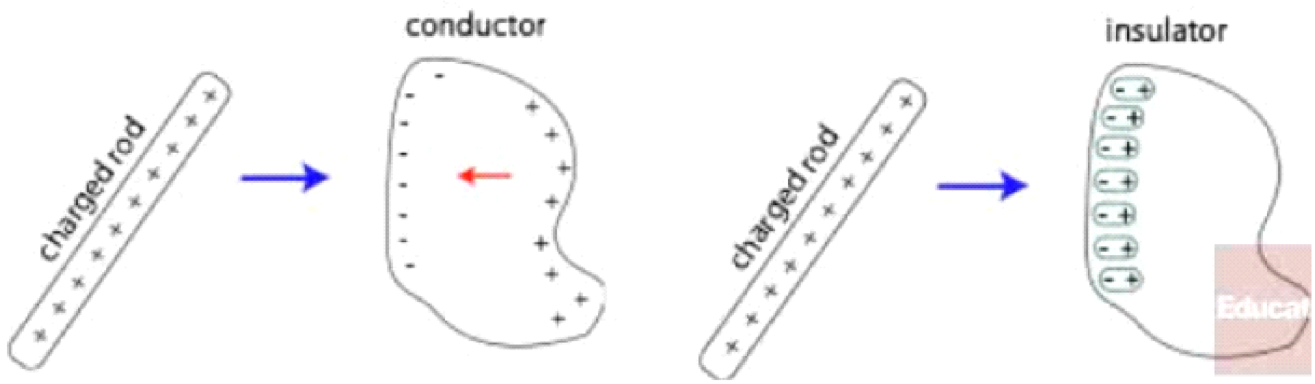




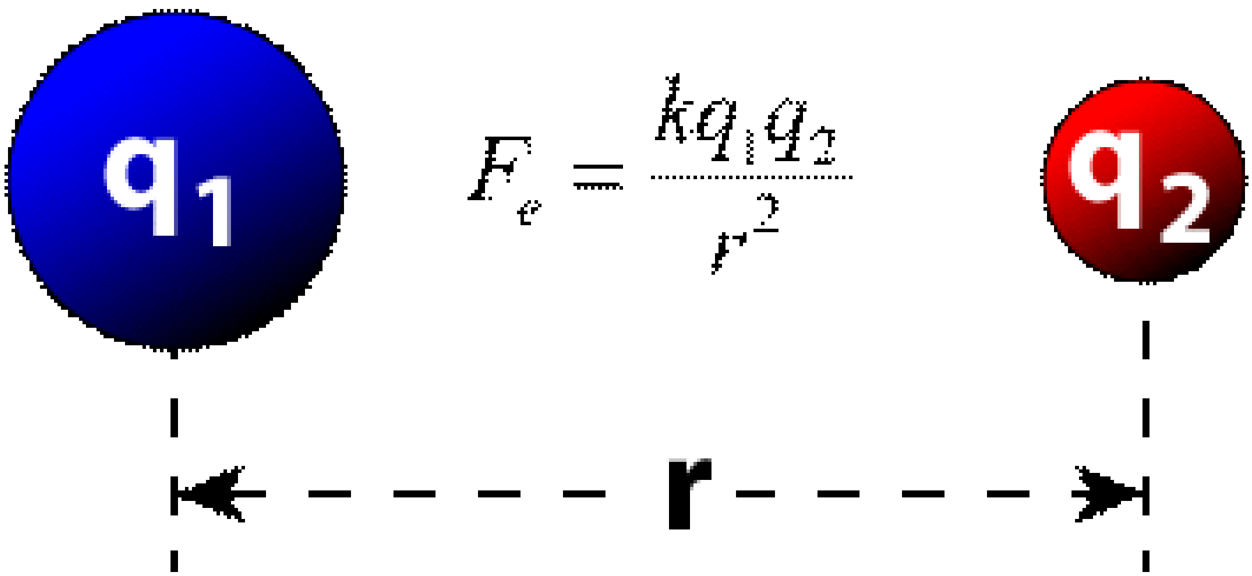
Polarization and Electric Dipole Moment

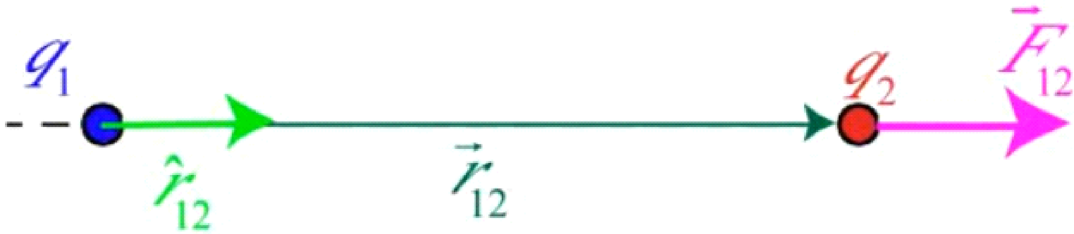
- When a charged object is brought near a conductor, the electrons in the conductor are free to move
- When a charged object is brought near an insulator, the electrons are not free to move, bu they may spend a little more time on the side of their orbit than another, creating a net separation of charge in a process known as **polarization**
- The distance between the shifted positive and negative charges, multiplied by the charge, is known as the **electric dipole moment**





Coulomb's law





$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$\varepsilon_0 = 8.85\times10^{-12}\ {C^2}/{N\bullet m^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

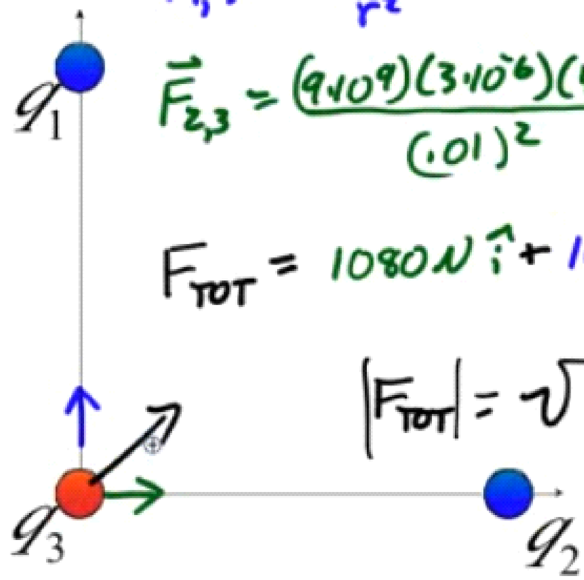
Practice Question 1

Three point charges are located at the corners of a right triangle as shown, where q1 = q2 = 3μC and q3 = -4μC. If q1 and q2 are each 1 cm from q3, find the net force on q3.

$$\vec{F}_{1,3} = \frac{k q_1 q_3}{r^2} = \frac{(9\cdot10^9)(3\cdot10^{-6})(4\cdot10^{-6})}{(.01)^2} = 1080\,N \uparrow = 1080\,N\,\hat{j}$$

$$\vec{F}_{2,3} = \frac{(9\cdot10^9)(3\cdot10^{-6})(4\cdot10^{-6})}{(.01)^2} = 1080\,N\,\hat{i}$$

$$F_{TOT} = 1080N\,\hat{i} + 1080N\,\hat{j} = \langle 1080N, 1080N \rangle$$

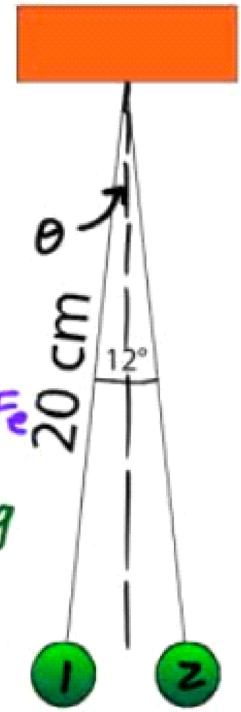


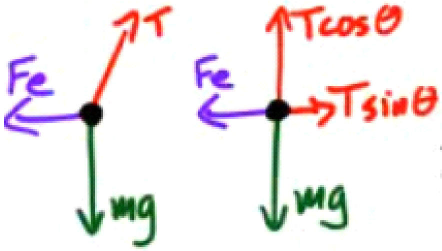
$$|F_{TOT}| = \sqrt{1080^2 + 1080^2} = 1527N$$

@ 45° N of E

Practice Question 2

Two identical charged balls of mass 5 mg are hung from the ceiling by a light string of length 20 cm. The total angle between them is 12 degrees. Find the magnitude of the charge on each ball.





$$F_{NET_x} = T\sin\theta - F_e = 0 \Rightarrow T\sin\theta = F_e$$

$$F_{NET_y} = T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{mg} \Rightarrow \tan\theta = \frac{F_e}{mg} \Rightarrow F_e = mg\tan\theta = \frac{k q_1 q_2}{r^2}$$

$$\xrightarrow{r = 2L\sin\theta} \frac{kq^2}{4L^2\sin^2\theta} = mg\tan\theta \Rightarrow q = \sqrt{\frac{4mgL^2\sin^2\theta\tan\theta}{k}} \Rightarrow$$

$$q = \sqrt{\frac{4(.005)(9.8)(.2)^2\sin^2(6^\circ)\tan 6^\circ}{9\cdot 10^9}} = \boxed{3.16\cdot 10^{-8}\,C}$$

1.2 - Electric Fields

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Electric Fields

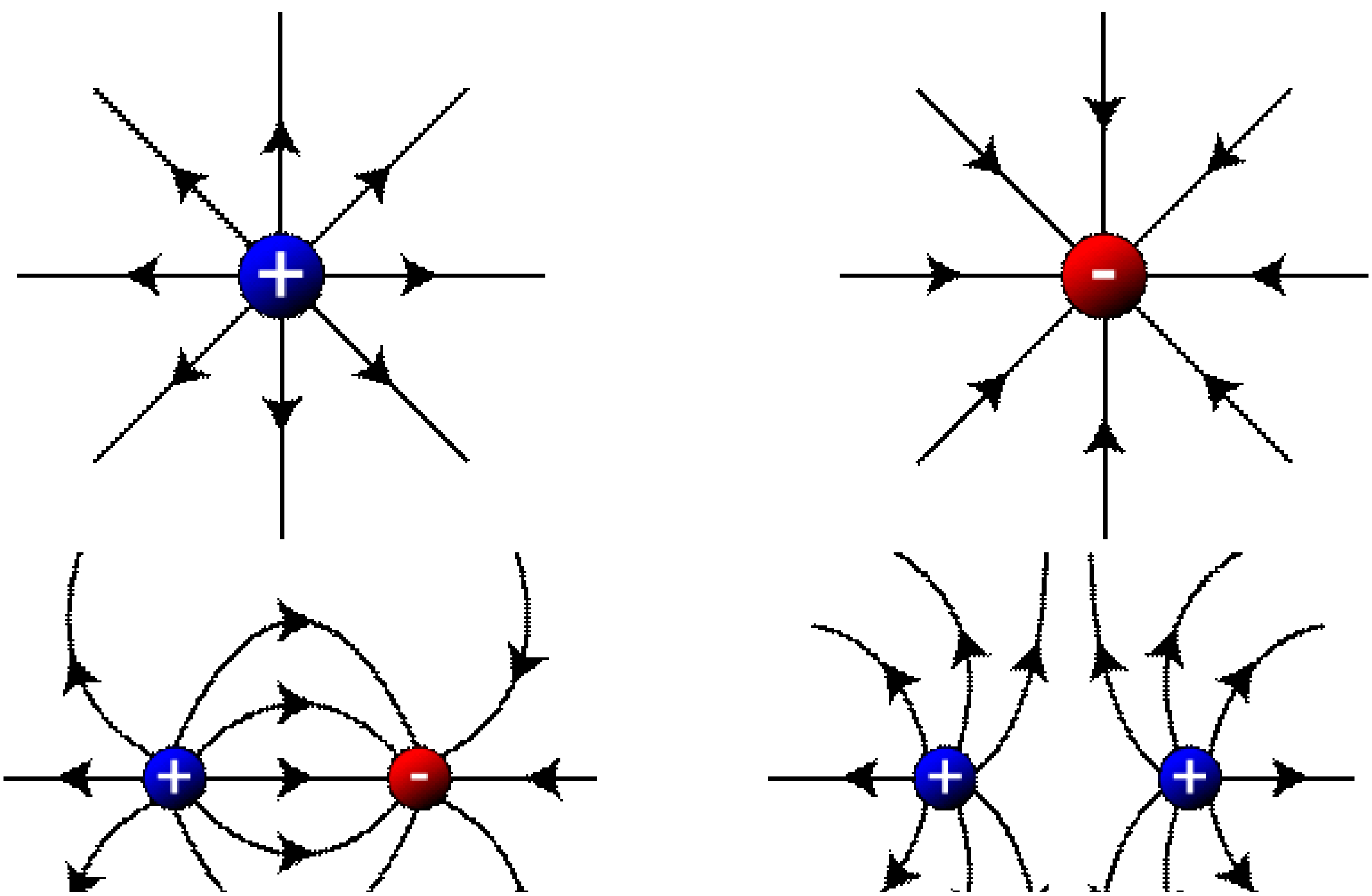
- The electric field strength vector E is the amount of electrostatic force observed by a charge per unit of charge.

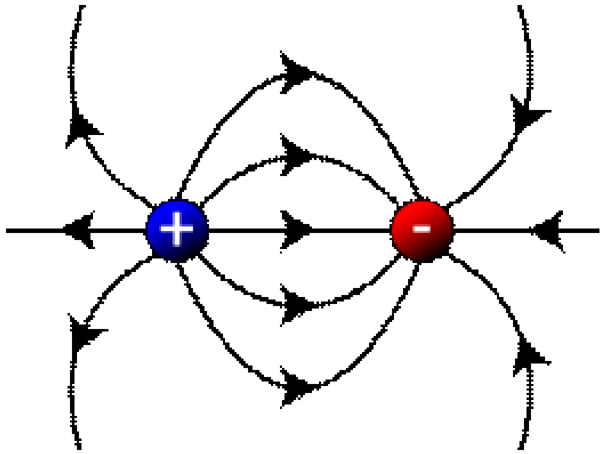
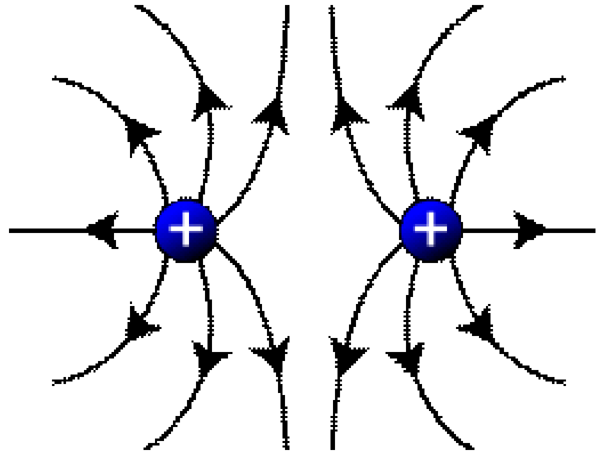
$$E = \frac{F}{q} = \frac{k \cdot \cancel{q} \cdot Q / d^2}{\cancel{q}} = \frac{k \cdot Q}{d^2}$$

$$\boxed{E = \frac{k \cdot Q}{d^2}}$$

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

Visualizing the Electric Fields



Comparing Electrostatics & Gravity

- Gravity only attracts
- Electrostatics attract and repel

Electrostatics	**Gravity**
Force: $F_e = \frac{kq_1q_2}{r^2}$	Force: $F_g = \frac{Gm_1m_2}{r^2}$
Field Strength: $E = \frac{F_e}{q}$	Field Strength: $g = \frac{F_g}{m}$
Field Strength: $E = \frac{kq}{r^2}$	Field Strength: $g = \frac{Gm}{r^2}$
Electrostatic Constant: $k = 8.99 * 10^9 \frac{N \cdot m^2}{C^2}$	Gravitational Constant: $G = 6.67 * 10^{-11} \frac{N \cdot m^2}{kg^2}$
Charge Units: Coulombs	Mass Units: kilograms

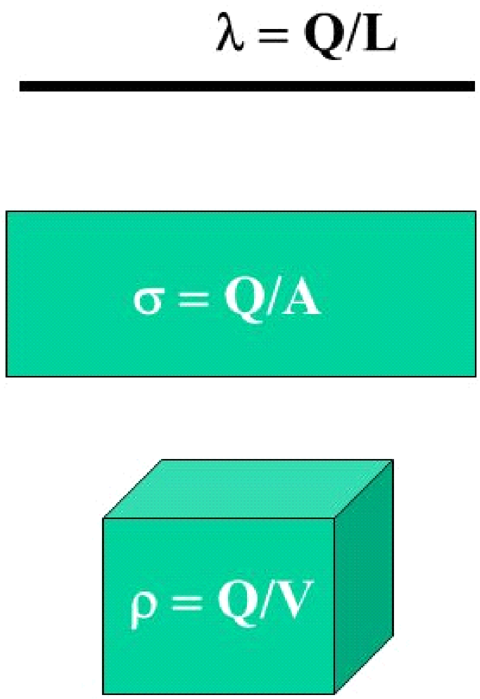
Charge Densities

Useful idea: charge density

Line of charge:
charge per unit length $= \lambda$

Sheet of charge:
charge per unit area $= \sigma$

Volume of charge:
charge per unit volume $= \rho$



Practice Question 1

Find the electric field at the origin due to the three charges shown in the diagram.

$$\mathcal{E} = \frac{kq}{r^2} = \frac{(9\cdot10^9)(2)}{(8)^2} = 2.81\cdot10^8\,\frac{N}{C} = \left\langle 0, -2.81\cdot10^8\,\frac{N}{C}\right\rangle$$

$$\mathcal{E} = \frac{kq}{r^2} = \frac{(9\cdot10^9)(2)}{(8)^2} = 2.81\cdot10^8\,\frac{N}{C} = \left\langle 2.81\cdot10^8\,\frac{N}{C}, 0\right\rangle$$

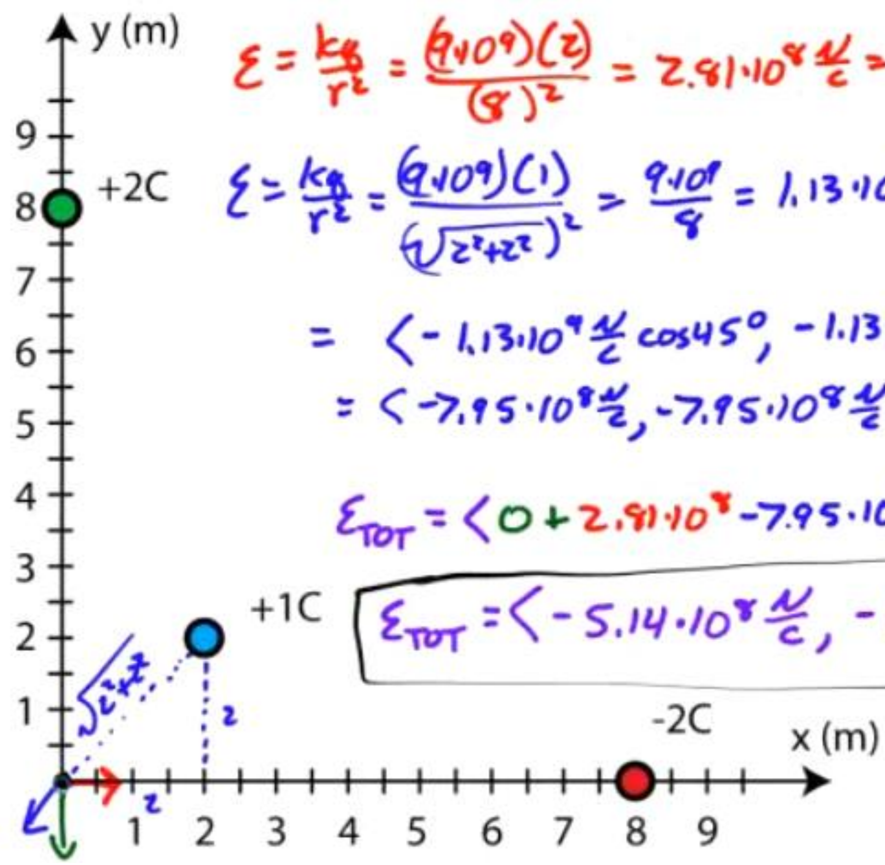
$$\mathcal{E} = \frac{kq}{r^2} = \frac{(9\cdot10^9)(1)}{(\sqrt{2^2+2^2})^2} = \frac{9\cdot10^9}{8} = 1.13\cdot10^9\,\frac{N}{C}\ \swarrow$$

$$= \left\langle -1.13\cdot10^9\,\frac{N}{C}\cos45^\circ, -1.13\cdot10^9\,\frac{N}{C}\sin45^\circ\right\rangle$$

$$= \left\langle -7.95\cdot10^8\,\frac{N}{C}, -7.95\cdot10^8\,\frac{N}{C}\right\rangle$$

$$\mathcal{E}_{TOT} = \left\langle 0 + 2.81\cdot10^8 - 7.95\cdot10^8, -2.81\cdot10^8 + 0 - 7.95\cdot10^8\right\rangle$$

$$\boxed{\mathcal{E}_{TOT} = \left\langle -5.14\cdot10^8\,\frac{N}{C}, -1.18\cdot10^9\,\frac{N}{C}\right\rangle}$$



Practice Question 2

Determine the x-coordinate where the electric field is zero using the diagram below.

$\mathcal{E}_1 = \frac{kq_1}{r^2}$ $\quad\quad$ $\mathcal{E} = \frac{kq_2}{(11-r)^2}$

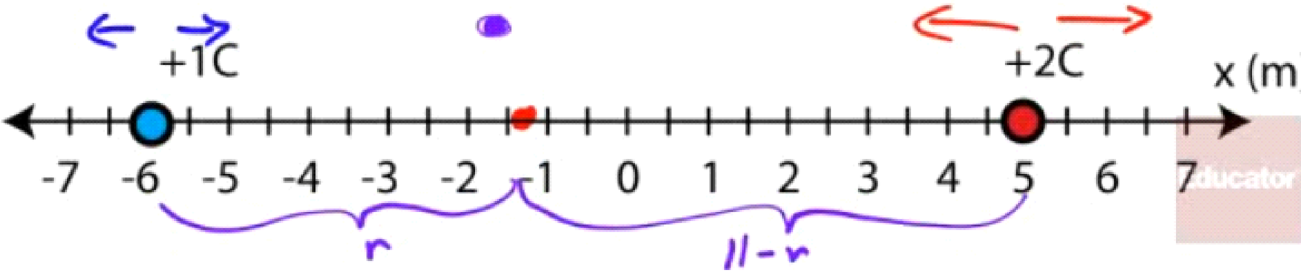
$\mathcal{E}_{TOT} = \frac{k1}{r^2} - \frac{k2}{(11-r)^2} = 0 \Rightarrow \frac{1}{r^2} = \frac{2}{121-22r+r^2} \Rightarrow$

$2r^2 = r^2 - 22r + 121 \Rightarrow r^2 + 22r - 121 = 0$ Quad Formula

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-22 \pm \sqrt{22^2 - 4(-121)}}{2} \Rightarrow r = 4.56$, $\cancel{r = -26.6}$

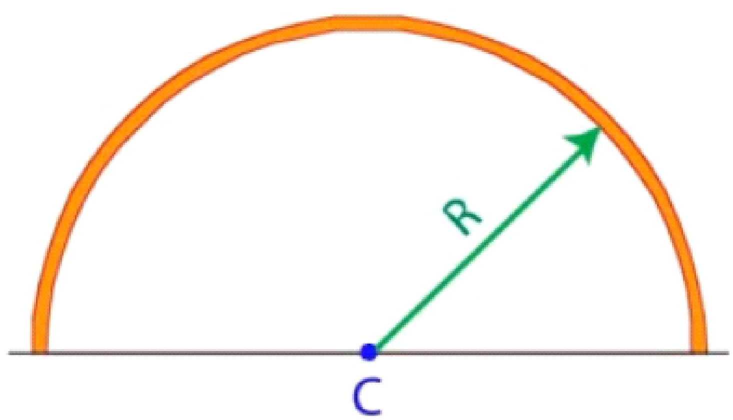
$r = 4.56$

$x = -6 + 4.56 = \boxed{-1.44\,m}$

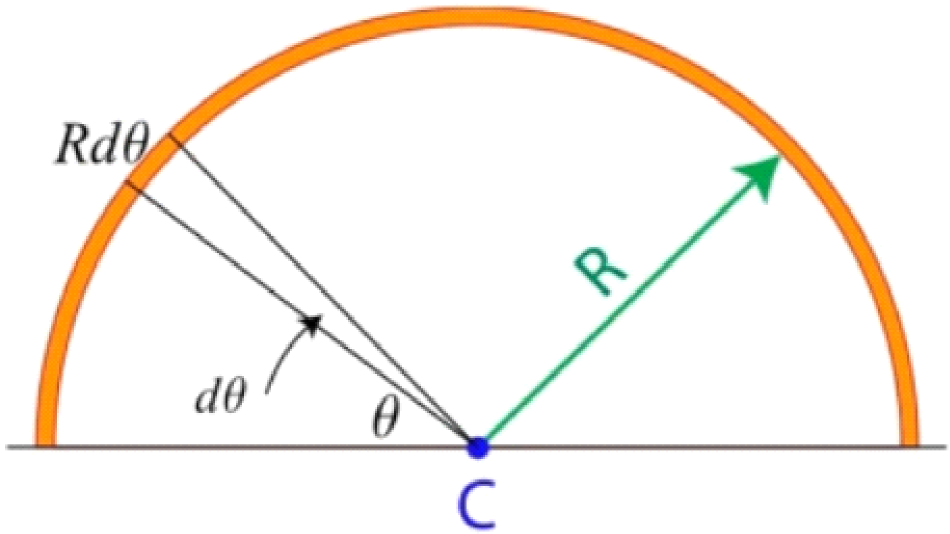


Practice Question 3

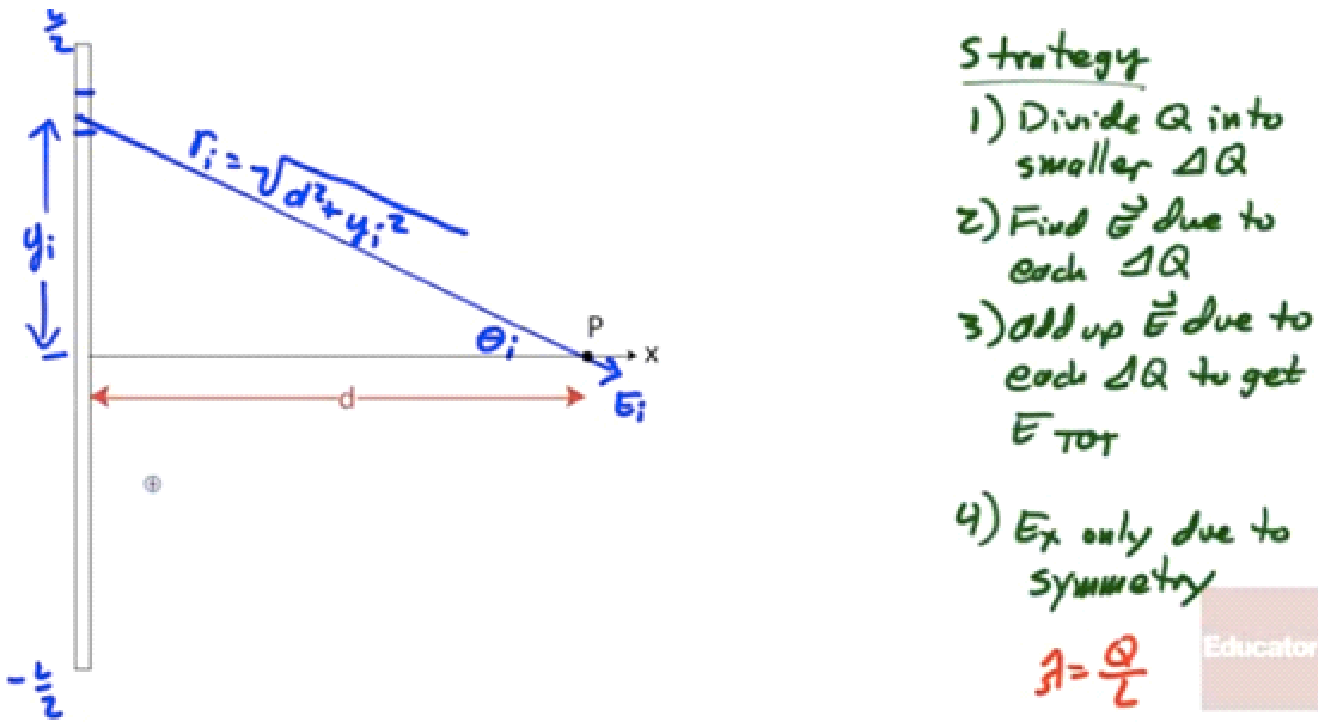
A thin insulating semicircle of charge Q with radius R is centered around point C. Determine the electric field at point C due to the semicircle of charge.



- $\lambda = \frac{\Delta Q}{\Delta L} = \frac{Q}{\pi R}$
- $dQ = \lambda R d\theta$
- $dE_y = \frac{1}{4\pi\epsilon_0}\frac{dQ}{R^2} sin\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda R d\theta}{R^2} sin\theta = \frac{\lambda}{4\pi\epsilon_0 R} sin\theta \, d\theta$
- $E_y = \frac{\lambda}{4\pi\epsilon_0 R}\int_0^{\pi} sin\theta \, d\theta = \frac{\lambda}{2\pi\epsilon_0 R}$
- $E_y = \frac{Q/\pi R}{2\pi\epsilon_0 R} = \frac{Q}{2\pi^2\epsilon_0 R^2}$



Practice Question 4



- Find the electric field a distance d from a long straight insulating rod of length L at point P perpendicular to the wire and eduidistant from each end of the wire if the wire is uniformly charged
  - $E_{i_x} = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r_i^2} \, cos\theta_i$

- $\because r_i = \sqrt{y_i^2 + d^2}, \qquad cos\,\theta_i = \frac{d}{r_1}, \qquad dQ = \lambda dy = \frac{Q}{L}dy$

- $\therefore E_{i_x} = \frac{1}{4\pi\epsilon_0}\frac{\frac{Q}{L}dy}{y_i^2 + d^2}\frac{d}{\sqrt{y_i^2 + d^2}} = \frac{1}{4\pi\epsilon_0}\ \frac{Qd}{L\left(y_i^2 + d^2\right)^{\frac{3}{2}}}dy$

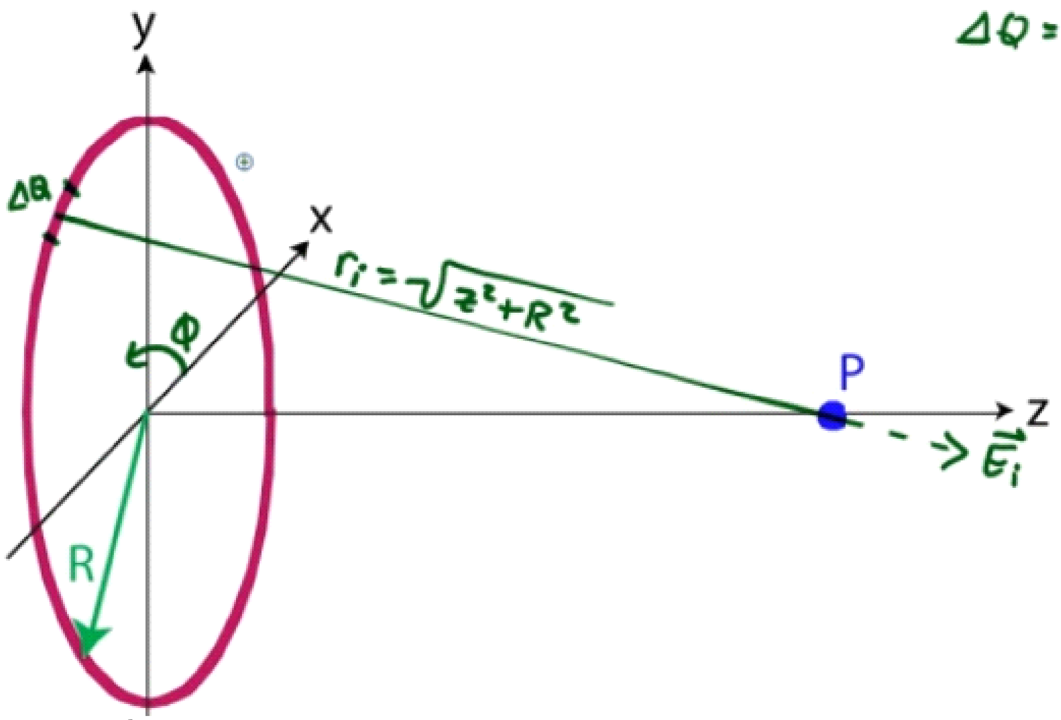
- $\therefore E_x = \frac{1}{4\pi\epsilon_0}\frac{Qd}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dy}{\left(y_i^2 + d^2\right)^{\frac{3}{2}}} = \frac{Qd}{4\pi\epsilon_0 L}\left(\frac{y}{d^2\sqrt{y^2 + d^2}}\right)\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{Q}{4\pi\epsilon_0 d\sqrt{\left(\frac{L}{2}\right)^2 + d^2}}$

- What if the line is infinitely long
    - $E_x = \lim_{L\to\infty}\left(\frac{Q}{4\pi\epsilon_0 d\sqrt{\left(\frac{L}{2}\right)^2 + d^2}}\right)$
    - $\because Q = \lambda L$
    - $\therefore E_x = \frac{\lambda L}{4\pi\epsilon_0 dL/2} = \frac{\lambda}{2\pi\epsilon_0 d}$
- What is the E field if the distance d is infinite?
    - $E_x = \lim_{d\to\infty}\left(\frac{Q}{4\pi\epsilon_0 d\sqrt{\left(\frac{L}{2}\right)^2 + d^2}}\right) = \frac{Q}{4\pi\epsilon_0 d^2}$

Practice Question 4

Find the electric field at a point on the axis (perpendicular to the ring) of a thin insulating ring of radius R that is uniformly charged. $\lambda = \frac{Q}{L} = \frac{Q}{2\pi R}$

$\Delta Q = \lambda R\, d\phi$

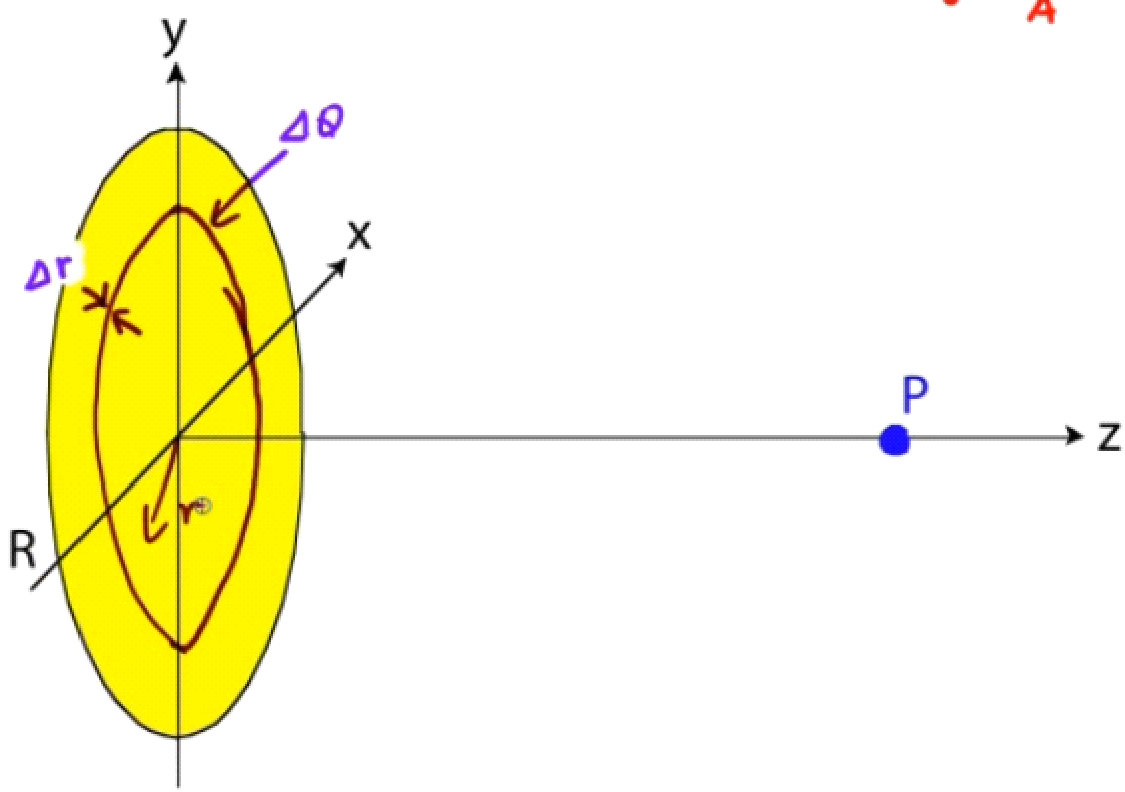


- $E_{i_z} = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r_i^2}\cos\theta_i$
- $\because dQ = \lambda R\, d\phi, \qquad r_i = \sqrt{(z^2+R^2)}$
- $\therefore E_{i_z} = \frac{1}{4\pi\epsilon_0}\frac{\lambda R\, d\phi}{z^2+R^2}\frac{z}{\sqrt{(z^2+R^2)}} = \frac{1}{4\pi\epsilon_0}\frac{z\lambda R}{(z^2+R^2)^{\frac{3}{2}}}d\phi$
- $\therefore E_z = \frac{1}{4\pi\epsilon_0}\frac{z\lambda R}{(z^2+R^2)^{\frac{3}{2}}}\int_0^{2\pi} d\phi = \frac{1}{4\pi\epsilon_0}\frac{z\lambda R}{(z^2+R^2)^{\frac{3}{2}}} \times 2\pi$
- $\because \lambda = \frac{Q}{L} = \frac{Q}{2\pi R}$
- $\therefore E_z = \frac{1}{4\pi\epsilon_0}\frac{z \times \frac{Q}{2\pi R} \times R}{(z^2+R^2)^{\frac{3}{2}}} \times 2\pi = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2+R^2)^{\frac{3}{2}}}$

Practice Question 5

Find the electric field due to a uniformly charged insulating disk of radius R at a point P perpendicular to the disk as shown in the diagram.

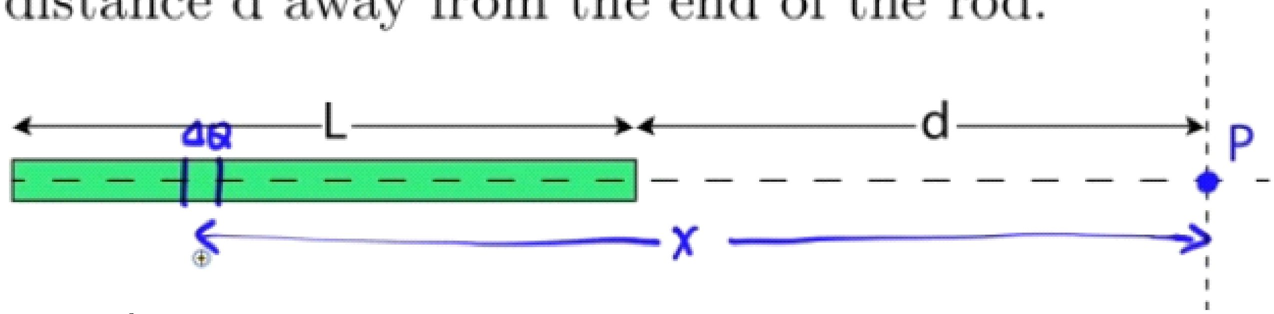
$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$



- $\because dQ = \sigma\, dA, \qquad dA = 2\pi r_i\, dr$
- $\therefore dQ = \sigma 2\pi r_i\, dr$
- $E_{i_z} = \dfrac{1}{4\pi\epsilon_0}\dfrac{z\, dQ_i}{(z^2+R^2)^{\frac{3}{2}}}$
- $\because dQ = \sigma 2\pi r_i\, dr$
- $\therefore E_{i_z} = \dfrac{1}{4\pi\epsilon_0}\dfrac{z\sigma 2\pi r_i}{(z^2+R^2)^{\frac{3}{2}}}\, dr = \dfrac{z\sigma}{2\epsilon_0}\dfrac{r_i}{(z^2+R^2)^{\frac{3}{2}}}\, dr$
- $\therefore E_z = \dfrac{z\sigma}{2\epsilon_0}\displaystyle\int_{r=0}^{r=R}\dfrac{r}{(z^2+R^2)^{\frac{3}{2}}}\, dr = \dfrac{z\sigma}{4\epsilon_0}\displaystyle\int_{r=0}^{r=R}\dfrac{2r\, dr}{(z^2+R^2)^{\frac{3}{2}}}$
- $Let\ u = z^2 + r^2, then\ du = 2r\, dr$
- $\therefore E_z = \dfrac{z\sigma}{4\epsilon_0}\displaystyle\int_{u=z^2}^{u=z^2+R^2} u^{-\frac{3}{2}}\, du = \dfrac{z\sigma}{4\epsilon_0}\left(-\dfrac{2}{\sqrt{u}}\right)\Bigg|_{z^2}^{z^2+R^2} = \dfrac{z\sigma}{2\epsilon_0}\left(\dfrac{1}{\sqrt{u}}\right)\Bigg|_{z^2+R^2}^{z^2} = \dfrac{z\sigma}{2\epsilon_0}\left(\dfrac{1}{z} - \dfrac{1}{\sqrt{z^2+R^2}}\right) = \dfrac{\sigma}{2\epsilon_0}\left(1 - \dfrac{z}{\sqrt{z^2+R^2}}\right)$
- $\because \sigma = \dfrac{Q}{A} = \dfrac{Q}{\pi R^2}$
- $\therefore E_z = \dfrac{Q}{2\pi R^2\epsilon_0}\left(1 - \dfrac{z}{\sqrt{z^2+R^2}}\right)$

Practice Question 6

Find the electric field due to a finite uniformly charged rod of length L lying on its side at some distance d away from the end of the rod.



- $E_{i_x} = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{x^2}$
- $\because dQ = \lambda\, dx$
- $\therefore E_{i_x} = \dfrac{\lambda}{4\pi\epsilon_0}\dfrac{1}{x^2}dx$
- $\therefore E_x = \dfrac{\lambda}{4\pi\epsilon_0}\displaystyle\int_{x=d}^{x=d+l}\dfrac{1}{x^2}dx = \dfrac{\lambda}{4\pi\epsilon_0}\left(-\dfrac{1}{x}\right)\Bigg|_{d}^{d+L} = \dfrac{\lambda L}{4\pi\epsilon_0 d(d+L)}$
- $\because \lambda = \dfrac{Q}{L}$
- $\therefore E_x = \dfrac{Q}{4\pi\epsilon_0 d(d+L)}$

1.3 - Gauss's Law

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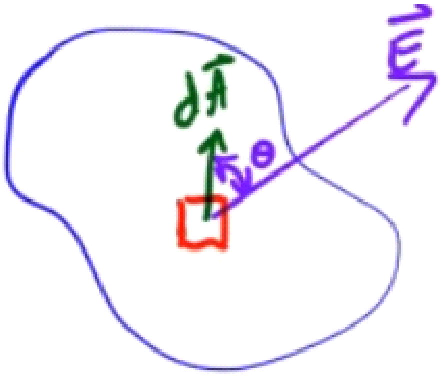
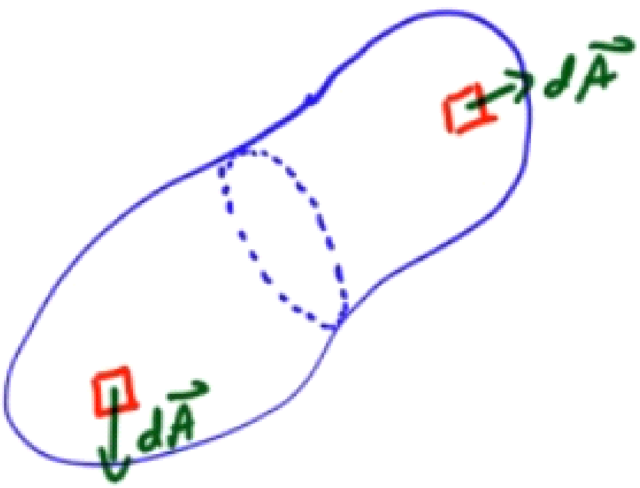
Electric Flux

- Electric Flux is the amount of electric field penetrating a surface



$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta \Rightarrow$$

$$\Phi = \int d\Phi = \int_A \vec{E}\cdot d\vec{A}$$



$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

Point Charge Inside a Hollow Sphere

- Place a point charge inside a hollow sphere of radius R.
- Determine the flux through the sphere

$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta \xrightarrow[\cos\theta=1]{\theta=0} d\Phi = E\,dA$$

$$\Phi = \int d\Phi = \oint E\,dA = E\oint dA = EA \xrightarrow{A=4\pi R^2}$$

$$\Phi = E\,4\pi R^2 \xrightarrow{\vec{E}=\frac{Q}{4\pi\epsilon_0 R^2}\hat{r}} \Phi = \frac{\cancel{4\pi}\cancel{R^2}\,Q}{\cancel{4\pi}\epsilon_0\cancel{R^2}} = \frac{Q}{\epsilon_0} \Rightarrow$$

$$\boxed{\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}}$$

Gauss's Law

Gauss's Law

- Useful for finding the electric field due to charge distributions for cases of:
  - Spherical Symmetry
  - Cylindrical Symmetry
  - Planar Symmetry

Gauss's Law for Electric Fields

from Fleisch

$$\Phi_E = \oint_S \vec{E} \bullet \hat{n}\, da = \frac{q_{enc}}{\varepsilon_0}$$

Electric Flux

Reminder that this integral is over a closed surface

Reminder that the electric field is a vector

Dot product tells you to find the part of $\vec{E}$ parallel to $\hat{n}$ (perpendicular to the surface)

The unit vector normal to the surface

The amount of net charge in coulombs

Reminder that only the enclosed charge contributes

The electric field in N/C

An increment of surface area in m^2

The electric permittivity of the free space

Tells you to sum up the contributions from each portion of the surface

Reminder that this is a surface integral (not a volume or a line integral)

The flux of an electric field passing through any closed surface is proportional to the total charge contained within that surface.

Practice Question 1

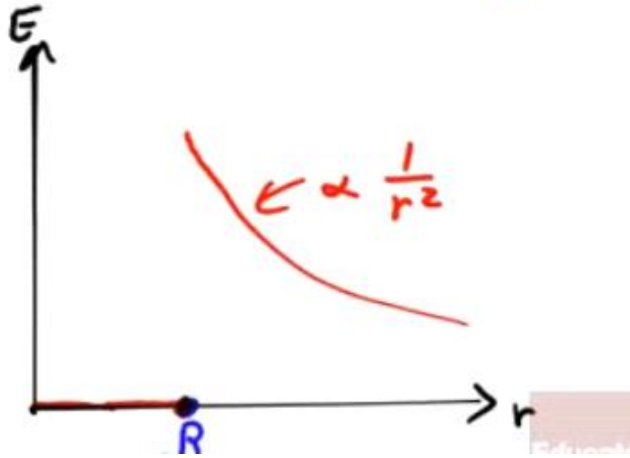
- Find the electric field inside and outside a thin hollow shell of uniformly distributed charge Q

$r_i < R$: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \xRightarrow{A=4\pi r_i^2} EA = \frac{Q_{enc}}{\epsilon_0} \Rightarrow$

$E(4\pi r_i^2) = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E = \frac{Q_{enc}}{4\pi\epsilon_0 r_i^2} \xRightarrow{Q_{enc}=0} \boxed{E=\emptyset}$

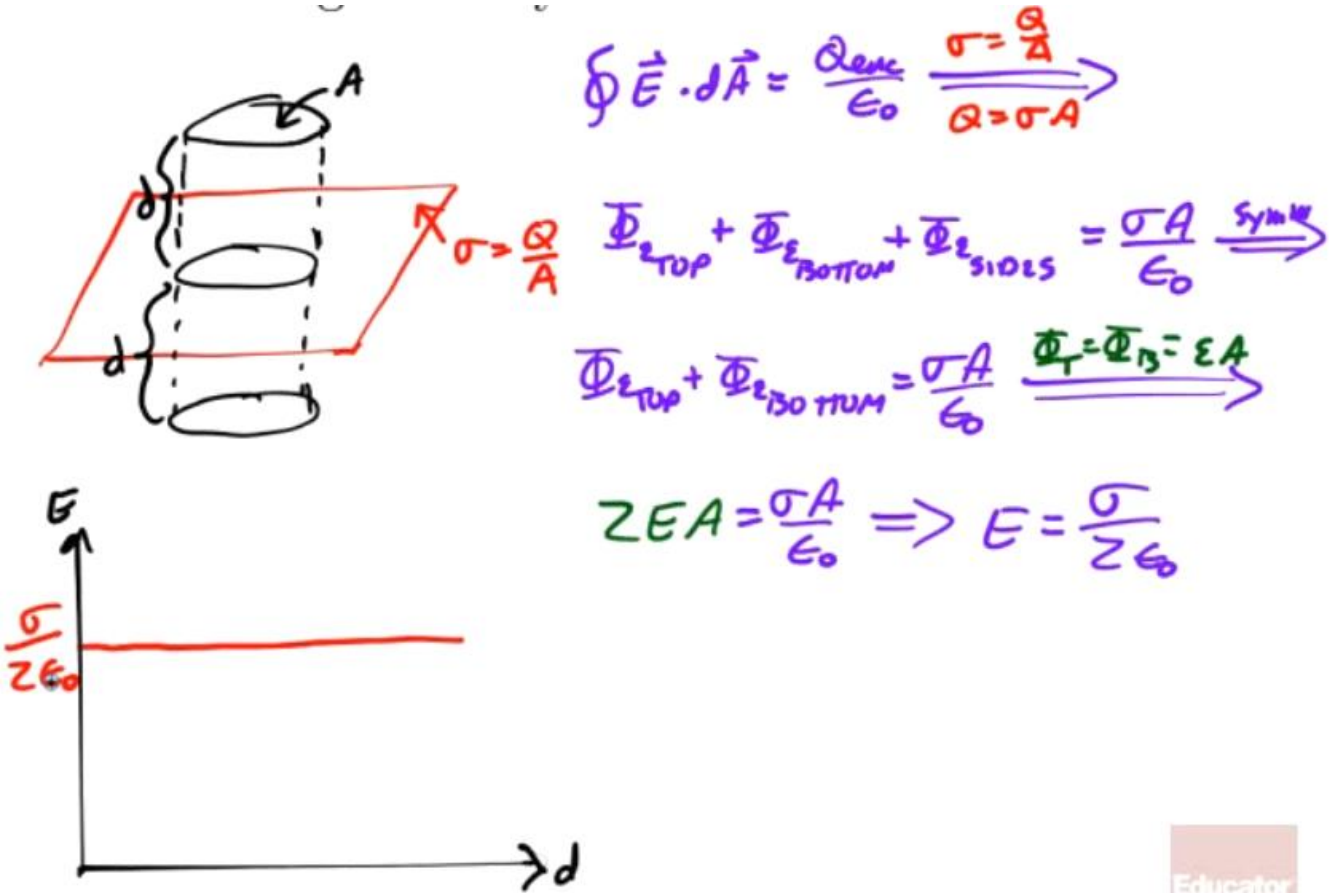
$r_o > R$: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r_o^2) = \frac{Q_{enc}}{\epsilon_0} \Rightarrow$

$E = \frac{Q_{enc}}{4\pi\epsilon_0 r_o^2} = \frac{Q}{4\pi\epsilon_0 r_o^2}$



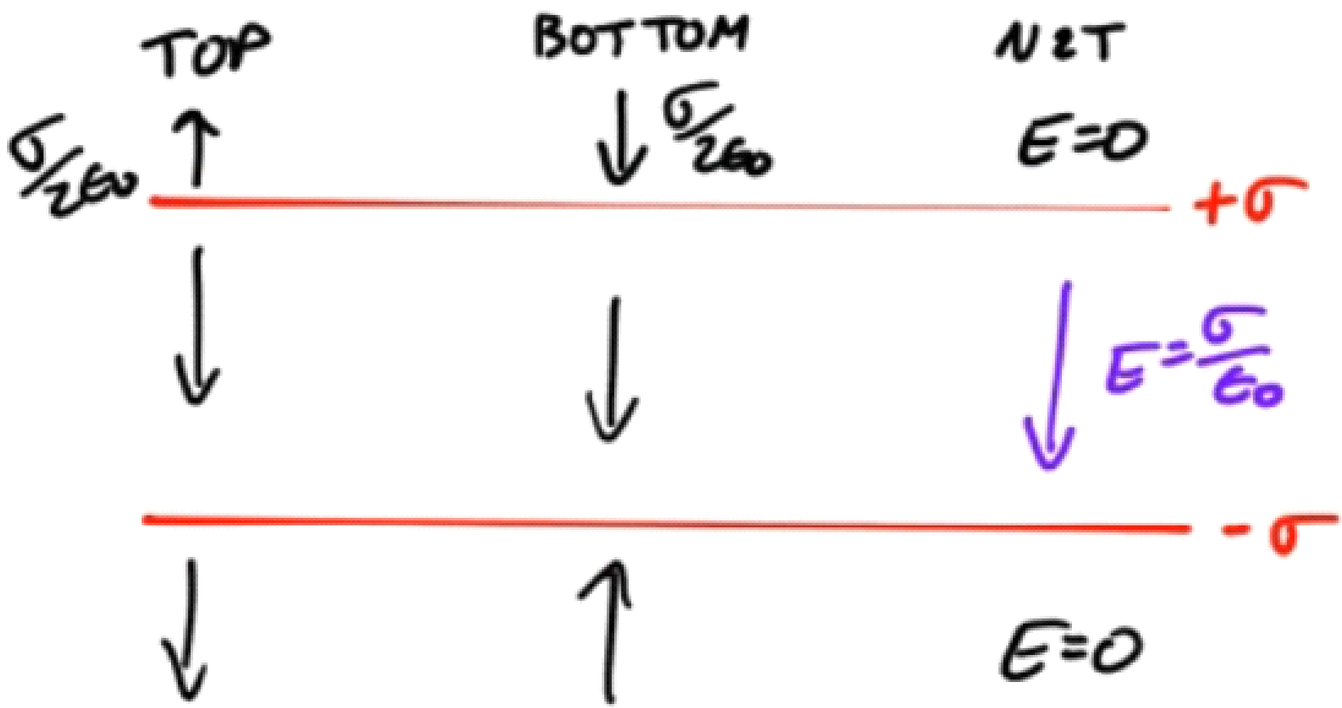
Practice Question 2

- Find the electric field E to an infinite plane of uniform charge density σ.



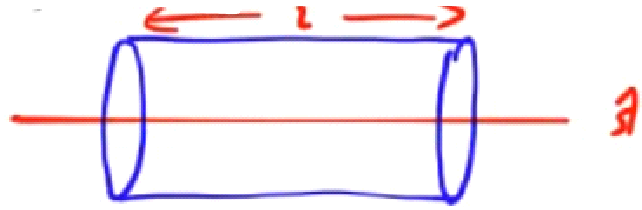
Practice Question 3

- Find the electric field outside and between two oppositely-charged parallel planes or plates



Practice Question 4

- Find the electric field strength at a distance R from an infinitely long uniformly charged wire of linear charge density λ

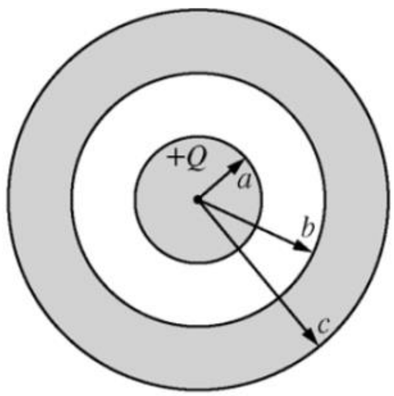


$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow$$

$$\Phi_L+\Phi_R+\Phi_{cyl} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow[\Phi_L+\Phi_R=0]{\text{Symmetry}} \Phi_{cyl}=\frac{Q_{enc}}{\epsilon_0} \xrightarrow{\Phi_{cyl}=2\pi r L E}$$

$$2\pi r L E = \frac{Q_{enc}}{\epsilon_0} \xrightarrow[Q_{enc}=\lambda L]{\lambda=\frac{Q}{L}} 2\pi r L E = \frac{\lambda L}{\epsilon_0} \Rightarrow \boxed{E=\frac{\lambda}{2\pi\epsilon_0 r}}$$

2008 Free Response Question 1



E&M. 1.

A metal sphere of radius a contains a charge $+Q$ and is surrounded by an uncharged, concentric, metallic shell of inner radius b and outer radius c, as shown above. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the induced charge on each of the following and explain your reasoning in each case.

i. The inner surface of the metallic shell

ii. The outer surface of the metallic shell

(b) Determine expressions for the magnitude of the electric field E as a function of r, the distance from the center of the inner sphere, in each of the following regions.

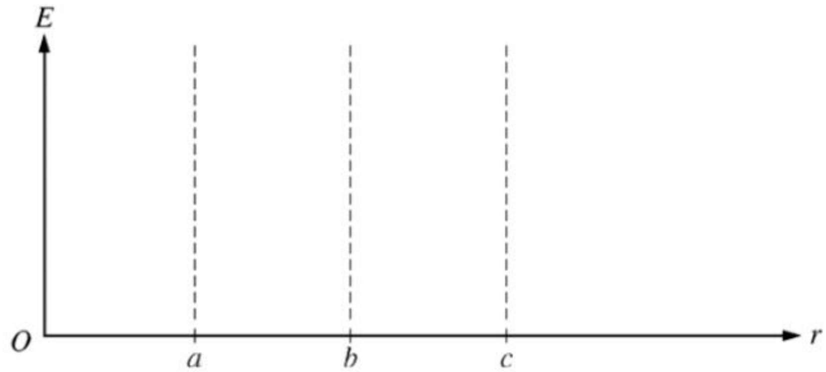
i. $r < a$

ii. $a < r < b$

iii. $b < r < c$

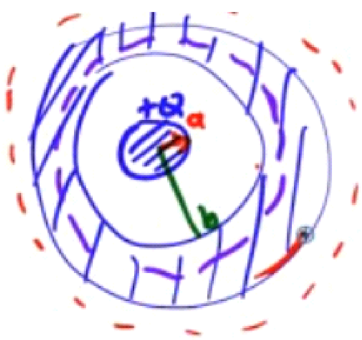
iv. $c < r$

(c) On the axes below, sketch a graph of E as a function of r.



(d) An electron of mass m_e carrying a charge $-e$ is released from rest at a very large distance from the spheres. Derive an expression for the speed of the particle at a distance $10r$ from the center of the spheres.

a)



$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow[\text{is zero}]{E \text{ in conductor}} 0 = \frac{Q_{enc}}{\epsilon_0} \Rightarrow$

$Q_{enc} = 0 = +Q + Q_b = 0 \Rightarrow \boxed{Q_b = -Q}$

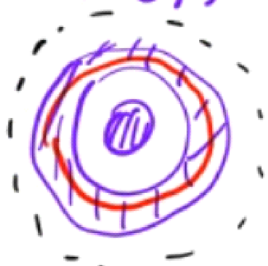
ii) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = +Q \Rightarrow Q_{enc} = Q_a + Q_b + Q_c \Rightarrow$

$+Q = +Q + -Q + Q_c \Rightarrow \boxed{Q_c = +Q}$

i) $r<a$: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = 0 \Rightarrow \boxed{E=0}$

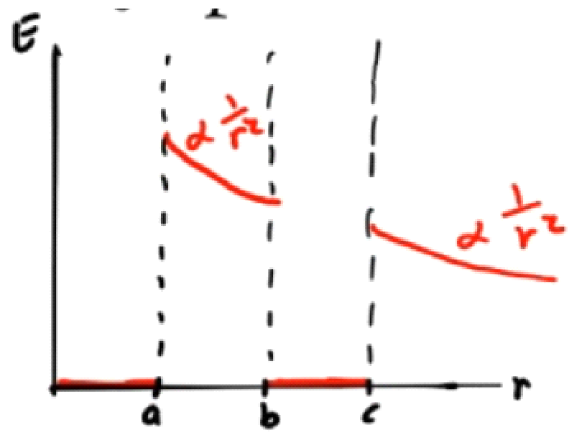
ii) $r\in(a,b)$: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

iii) $r\in(b,c)$: $E=0$



iv) $r>c$: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

c)



d) $U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$

$K+U=0 \Rightarrow \frac{1}{2}m_e v^2 = \frac{Qe}{4\pi\epsilon_0(10c)} \Rightarrow$

$v^2 = \frac{Qe}{2\pi\epsilon_0 m_e(10c)} \Rightarrow \boxed{v = \sqrt{\frac{Qe}{20\pi\epsilon_0 m_e c}}}$

2011 Free Response Question 1

A nonconducting, thin, spherical shell has a uniform surface charge density σ on its outside surface and no charge anywhere else inside.

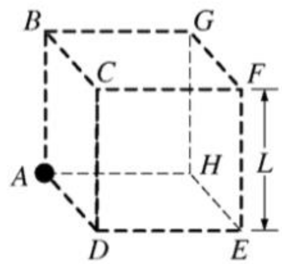
(a) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Describe the Gaussian surface that you use.

(b) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell?

____ Yes ____ No ____ It cannot be determined from the information given.

Justify your answer.

Now consider a small conducting sphere with charge $+Q$ whose center is at corner A of a cubical surface, as shown below.



(c) For which faces of the surface, if any, is the electric flux through that face equal to zero?

____ *ABCD* ____ *CDEF* ____ *EFGH* ____ *ABGH* ____ *BCFG* ____ *ADEH*

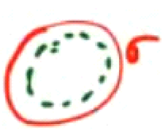
Explain your reasoning.

(d) At which corner(s) of the surface does the electric field have the least magnitude?

(e) Determine the electric field strength at the position(s) you have indicated in part (d) in terms of Q, L, and fundamental constants, as appropriate.

(f) Given that one-eighth of the sphere at point A is inside the surface, calculate the electric flux through face *CDEF*.

a) Use a spherical shell inside the charged shell as our Gaussian surf



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow{Q_{enc}=0} E(4\pi r^2)=0 \Rightarrow \boxed{E=0}$$

b) NO. Without the symmetry of charges to cancel each other out, you may have an internal E field.

c) ADEH, ABCD, ABGH   Field lines run parallel to these surfaces, therefore NO flux through these surfaces

d) A is inside the conducting sphere & $\vec{E}$ is 0 inside a conductor @ equilibrium

e) $E_A = 0$

f) $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{8\epsilon_0}$

$$\frac{1}{3} \frac{Q}{8\epsilon_0} = \boxed{\frac{Q}{24\epsilon_0}}$$

1.4 - Electric Potential & Electric Potential Energy

Tuesday, February 28, 2017 4:53 PM

Electric Potential Energy

- When an object was lifted against gravity by applying a force for some distance, work was done to give that object gravitational potential energy
- When a charged object is moved against an electric field by applying a force for some distance, work is done to give that object electric potential energy.
- The work done per unit charge in moving a charge between two points in an electric field is a scalar known as the electric potential V (or voltage)
  - Units: volts (1V = 1 J/C)
  - The work done is equal to the change in the object's electric potential energy (U_E)
  - $U_E = qV$

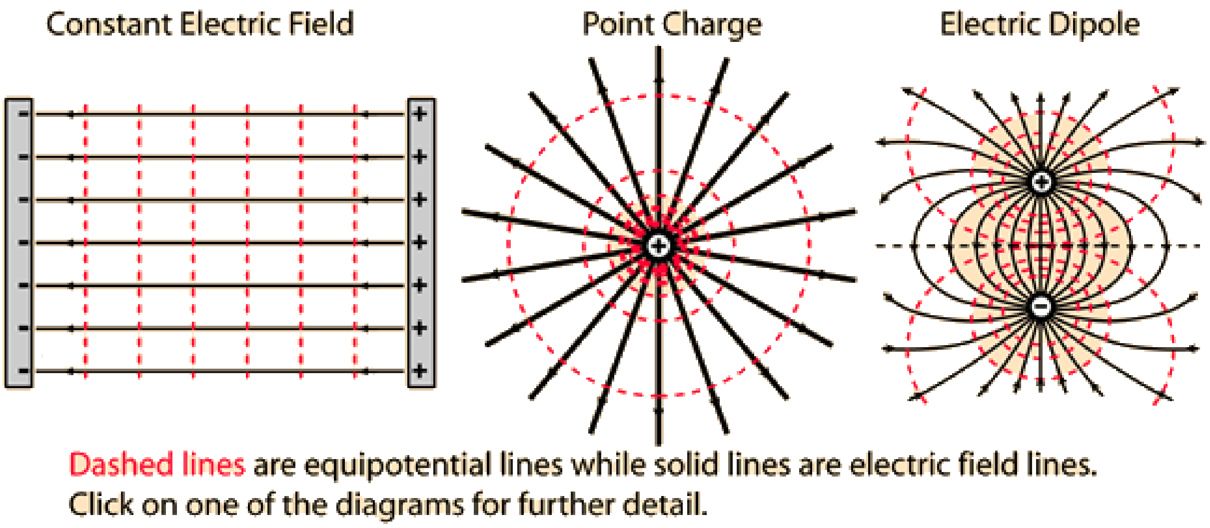
$$\overbrace{W}^{\text{Work}} = \underbrace{F}_{\text{Force}}d = q(\underbrace{E}_{\text{Electric Field}}d) = \overbrace{q\underbrace{V}_{\text{Voltage}}}^{\text{Electric Potential Energy}} = \underbrace{mgh}_{\text{Gravitational Potential Energy}} = \overbrace{\frac{1}{2}m{\underbrace{v}_{\text{velocity}}}^2}^{\text{Kinetic Energy}}$$

The Electron-Volt

- Oftentimes the electrical energy and/or work done is a very small portion of a joule
- A smaller, alternative, non-standard unit of energy is often used for convenience, known as the electron volt (eV)
  - 1 eV is the amount of work done in moving an elementary charge through a potential difference of 1 volt
  - $1\ eV = 1.6 \times 10^{-19}\ joules$

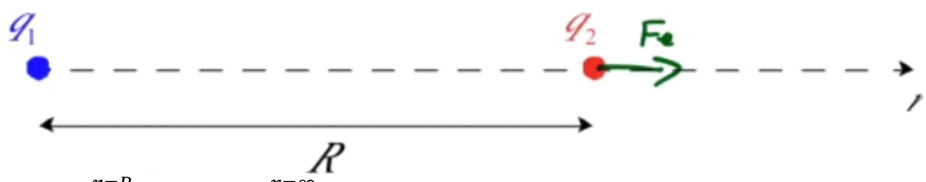
Equipotential Lines

- Topographic maps show you lines of equal altitude, or equal gravitational potential
- Lines connecting points of equal electrical potential are known as equipotential lines.
  - Equipotential lines always cross electrical field lines at right angles
  - If you move a charged particle in space, and stay on an equipotential line, no work will be done.
  - As equipotential lines get closer together, the gradients of the potential increases (stepper "slope" of potentials)
  - Electric field points from high to low potential



Electric Potential Energy Due to a Point Charge

- Find the work required to take a point charge q_2 from infinity ($U_e = 0$) to some point a distance R away from point charge q_1



- $W = \int_{r=\infty}^{r=R} -F_e \, dl = \int_{r=R}^{r=\infty} F_e \, dr$
- $\because F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$
- $\therefore W = \int_{r=R}^{r=\infty} \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\, dr = \frac{q_1 q_2}{4\pi\epsilon_0}\int_{r=R}^{r=\infty} r^{-2}\, dr = \frac{q_1 q_2}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_R^\infty = \frac{q_1 q_2}{4\pi\epsilon_0}\left(\frac{1}{R}\right) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{R}$

Electric Force from Electric Potential Energy

- $F = -\frac{dU}{dl} = -\frac{d}{dr}\left(\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}\right) = -\frac{1}{4\pi\epsilon_0} \times q_1 q_2 \times \frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

Electric Potential Due to a Point Charge

- Electric potential (voltage) is the work per unit charge required to bring a charge from infinity to some point R in an electric field
- $U = \frac{W}{q} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{R} = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$
- $U = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

Example 1: Electric Potential Due to a Point Charge

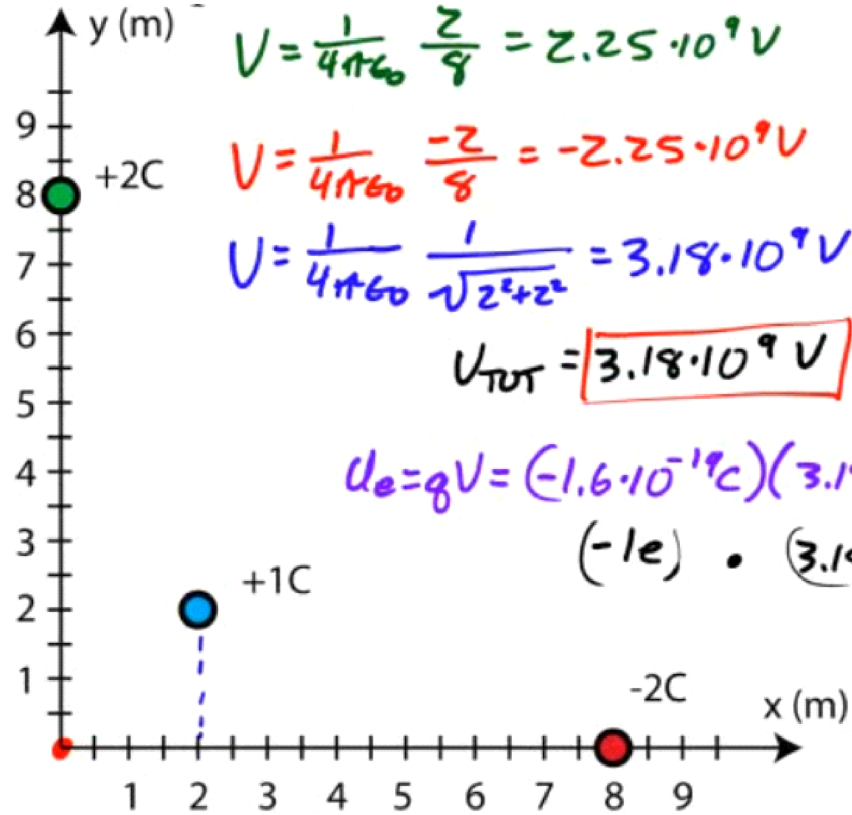
- Find the electric potential at point P, located 3 meters away from a -2 C charge.
- What is the electric potential energy of a 0.5 C charge situated at point P?

$$V_P = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{-2}{3} = \boxed{-6\cdot 10^{-9}\ V}$$

$$U_e = qV = (0.5C)(-6\cdot 10^{-9}\ V) = \boxed{-3\cdot 10^{-9}\ J}$$

Example 2: Electric Potential Due to a Point Charge

- Find the electric potential at the origin due to the three charges shown in the diagram. If an electron is placed at the origin, what potential energy does it possess?



$$V = \frac{1}{4\pi\epsilon_0}\frac{2}{8} = 2.25\cdot 10^{9}\ V$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{-2}{8} = -2.25\cdot 10^{9}\ V$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{2^2+2^2}} = 3.18\cdot 10^{9}\ V$$

$$V_{TOT} = \boxed{3.18\cdot 10^{9}\ V}$$

$$U_e = qV = (-1.6\cdot 10^{-19}\ C)(3.18\cdot 10^{9}\ V) = \boxed{-5.1\cdot 10^{-10}\ J}$$

$$(-1e)\cdot(3.18\cdot 10^{9}\ V) = -3.18\cdot 10^{9}\ eV$$

Finding Electric Field from Electric Potential

- $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
- $\frac{dV}{dr} = \frac{q}{4\pi\epsilon_0} \times \frac{d}{dr}\left(\frac{1}{r}\right) = \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r^2}\right) = \frac{-q}{4\pi\epsilon_0 r^2}$
- $\because \vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r} = -\frac{dV}{dr}\hat{r}$

Finding Electric Potential from Electric Field

- $U = \frac{W}{q} = \int_r^{\infty} \frac{\vec{F_e}}{q} \cdot d\hat{r}$
- $\because \vec{E} = \frac{\vec{F_e}}{q}$

- $\therefore U = \int_r^\infty \vec{E} \cdot d\hat{r}$

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} = \frac{\Delta U_e}{q}$$

Example 3: Electric Potential Due to a Point Charges

Find the electric potential at the origin due to the following charges: 2µC at (3,0); -5µC at (0,5); and 1µC at (4,4).

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\left(\frac{2\cdot 10^{-6}}{3} + \frac{-5\cdot 10^{-6}}{5} + \frac{1\cdot 10^{-6}}{\sqrt{4^2+4^2}}\right) \Rightarrow$$

$$\boxed{V = -1410\ V}$$

Example 4: Electric Field from Potential

Given an electric potential of $V(x)=5x^2-7x$, find the magnitude and direction of the electric field at x=3m.

$$\vec{E} = -\frac{dV}{dr}\hat{r} = -\frac{d}{dx}(5x^2-7x)\hat{i} = -(10x-7)\hat{i} = (7-10x)\hat{i}$$

$$\xrightarrow{x=3m} \vec{E} = [7-10(3)]\hat{i} = \boxed{-23\ \frac{V}{m}\ \hat{i}}$$

Example 5: Speed of an Electron Released in an Electric Field

An electron is released from rest in a uniform electric field of 500 N/C. What is its velocity after it has traveled one meter?

$$\Delta K = -\Delta U \xrightarrow{\Delta U = -q\int_A^B \vec{E}\cdot d\vec{r}} \Delta K = q\int_A^B \vec{E}\cdot d\vec{r} \Rightarrow$$

$$\Delta K = qE\int_A^B dr = qE\Delta r \Rightarrow \frac{1}{2}mv^2 = qE\Delta r \Rightarrow$$

$$v = \sqrt{\frac{2qE\Delta r}{m}} = \sqrt{\frac{2(1.6\cdot 10^{-19})(500)(1)}{9.11\cdot 10^{-31}\,kg}} = \boxed{1.33\cdot 10^{7}\ \frac{m}{s}}$$

Example 6: Work Required to Establish a Charge System

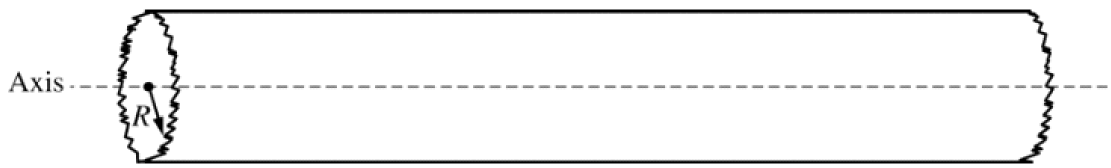
Two point charges (5 μC and 2 μC) are placed 0.5 meters apart. How much work was required to establish the charge system?

$$U_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(5\cdot 10^{-6})(2\cdot 10^{-6})}{.5} = \boxed{0.18\,J}$$

What is the electric potential halfway between the two charges?

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{5\cdot 10^{-6}}{.25} + \frac{2\cdot 10^{-6}}{.25}\right) = \boxed{252\ kV}$$

2013 Free Response Question 1



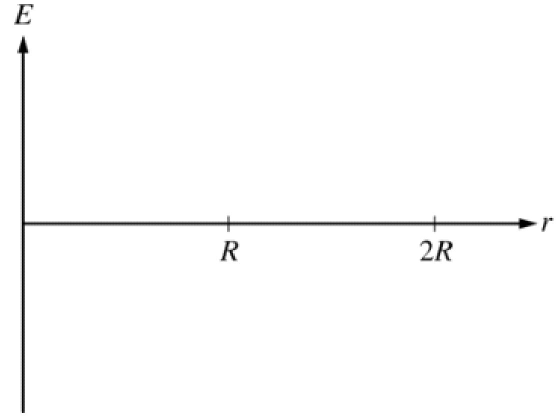
E&M 1.

A very long, solid, nonconducting cylinder of radius R has a positive charge of uniform volume density ρ. A section of the cylinder far from its ends is shown in the diagram above. Let r represent the radial distance from the axis of the cylinder. Express all answers in terms of r, R, ρ, and fundamental constants, as appropriate.

(a) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r < R$. Draw an appropriate Gaussian surface on the diagram.

(b) Using Gauss's law, derive an expression for the magnitude of the electric field at a radius $r > R$.

(c) On the axes below, sketch the graph of electric field E as a function of radial distance r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



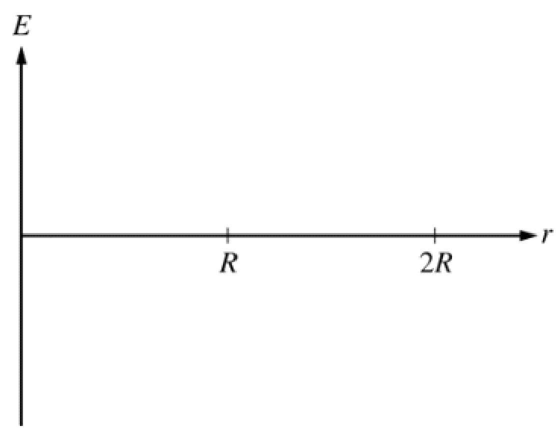
(d)

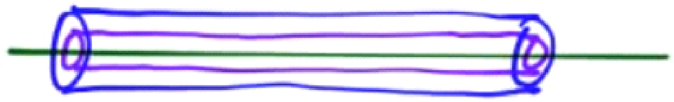
i. Derive an expression for the magnitude of the potential difference between $r = 0$ and $r = R$.

ii. Is the potential higher at $r = 0$ or $r = R$?

____ $r = 0$ ____ $r = R$

(e) The nonconducting cylinder is replaced with a conducting cylinder of the same shape and same <u>linear</u> charge density. On the axes below, sketch the electric field E as a function of r for $r = 0$ to $r = 2R$. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.





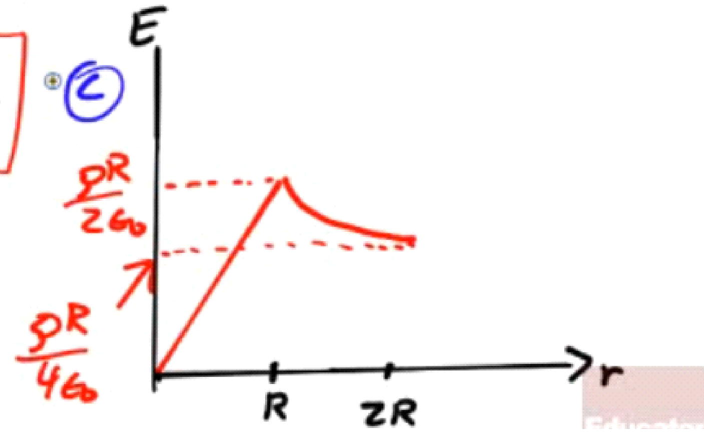
a) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\rho V}{\epsilon_0} \Rightarrow E(2\pi r \ell) = \frac{\rho(\pi r^2 \ell)}{\epsilon_0} \Rightarrow$

$E = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{\ell}}{2\cancel{\pi}\cancel{r}\cancel{\ell}\,\epsilon_0} = \boxed{\frac{\rho r}{2\epsilon_0}}$

b) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\rho V}{\epsilon_0} \Rightarrow E(2\pi r \ell) = \frac{\rho(\pi R^2 \ell)}{\epsilon_0} \Rightarrow$

$E = \frac{\rho \cancel{\pi} R^2 \cancel{\ell}}{2\cancel{\pi} r \cancel{\ell}\,\epsilon_0} = \boxed{\frac{\rho R^2}{2\epsilon_0 r}}$

c)



d) $\vec{E} = -\frac{dV}{dr}\hat{r} \Rightarrow \frac{dV}{dr} = -E \Rightarrow V = -\int \vec{E}\cdot d\vec{r} = -\int_{r=0}^{R} \frac{\rho r}{2\epsilon_0} dr \Rightarrow$

$V = \frac{-\rho}{2\epsilon_0}\int_0^R r\,dr = \frac{-\rho}{2\epsilon_0}\left(\frac{r^2}{2}\right)\Big|_0^R = \frac{-\rho}{2\epsilon_0}\left(\frac{R^2}{2} - 0\right) \Rightarrow$

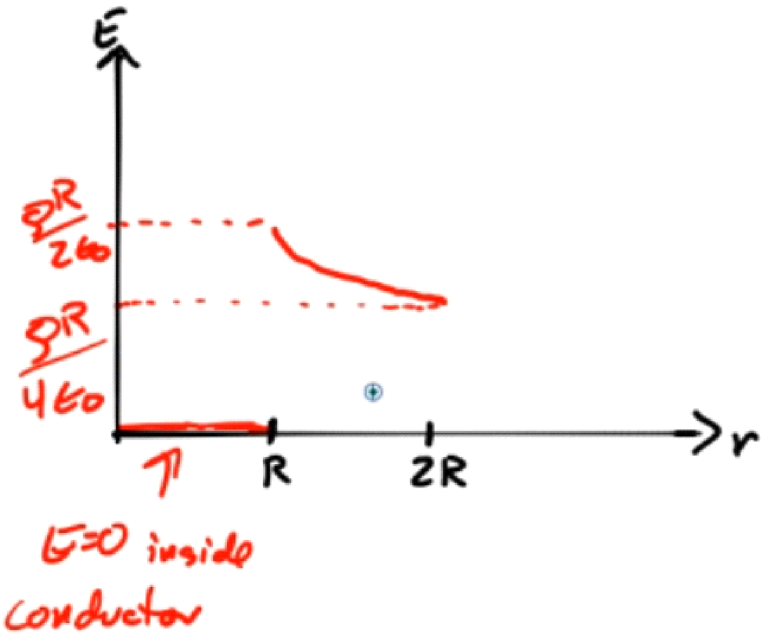
$V = \frac{-\rho R^2}{4\epsilon_0} \Rightarrow |V| = \boxed{\frac{\rho R^2}{4\epsilon_0}}$

ii) $V_B - V_A = -\frac{\rho R^2}{4\epsilon_0}$

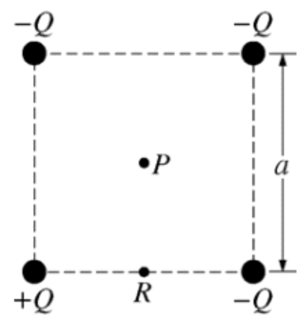
$\boxed{\text{higher @ } r = \phi}$

$V_A - V_B = \frac{\rho R^2}{4\epsilon_0} \Leftarrow$ A must be higher

$\uparrow r=0 \quad \uparrow r=R$



2006 Free Response Question 1



E&M 1.

The square of side a above contains a positive point charge $+Q$ fixed at the lower left corner and negative point charges $-Q$ fixed at the other three corners of the square. Point P is located at the center of the square.

(a) On the diagram, indicate with an arrow the direction of the net electric field at point P.

(b) Derive expressions for each of the following in terms of the given quantities and fundamental constants.

i. The magnitude of the electric field at point P

ii. The electric potential at point P

(c) A positive charge is placed at point P. It is then moved from point P to point R, which is at the midpoint of the bottom side of the square. As the charge is moved, is the work done on it by the electric field positive, negative, or zero?

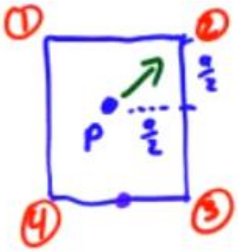
____ Positive ____ Negative ____ Zero

Explain your reasoning.

(d)

i. Describe one way to replace a single charge in this configuration that would make the electric field at the center of the square equal to zero. Justify your answer.

ii. Describe one way to replace a single charge in this configuration such that the electric potential at the center of the square is zero but the electric field is not zero. Justify your answer.



b) $E_{P_2} = \frac{Q}{4\pi\epsilon_0 r^2}$ $\quad E_{P_4} = \frac{Q}{4\pi\epsilon_0 r^2}$

$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = 2\left(\frac{a}{2}\right)^2 \Rightarrow r^2 = \frac{2a^2}{4} = \frac{a^2}{2} \Rightarrow r = \frac{a}{\sqrt{2}}$

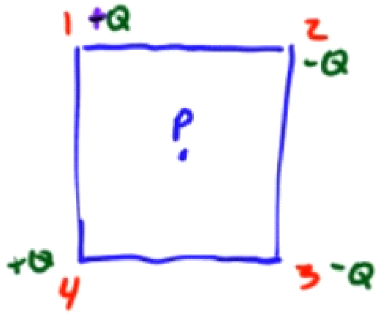
$E_p = E_{P_2} + E_{P_4} = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{2Q\,2}{4\pi\epsilon_0 a^2} = \boxed{\frac{Q}{\pi\epsilon_0 a^2}}$

ii) $V_p = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0\left(\frac{a}{\sqrt{2}}\right)}(Q-Q-Q-Q) = \frac{-2Q\sqrt{2}}{4a\pi\epsilon_0} = \boxed{\frac{-Q}{\sqrt{2}\pi\epsilon_0 a}}$

c) Negative. Field ↗

Charge is moved against field so $W<0$

d)

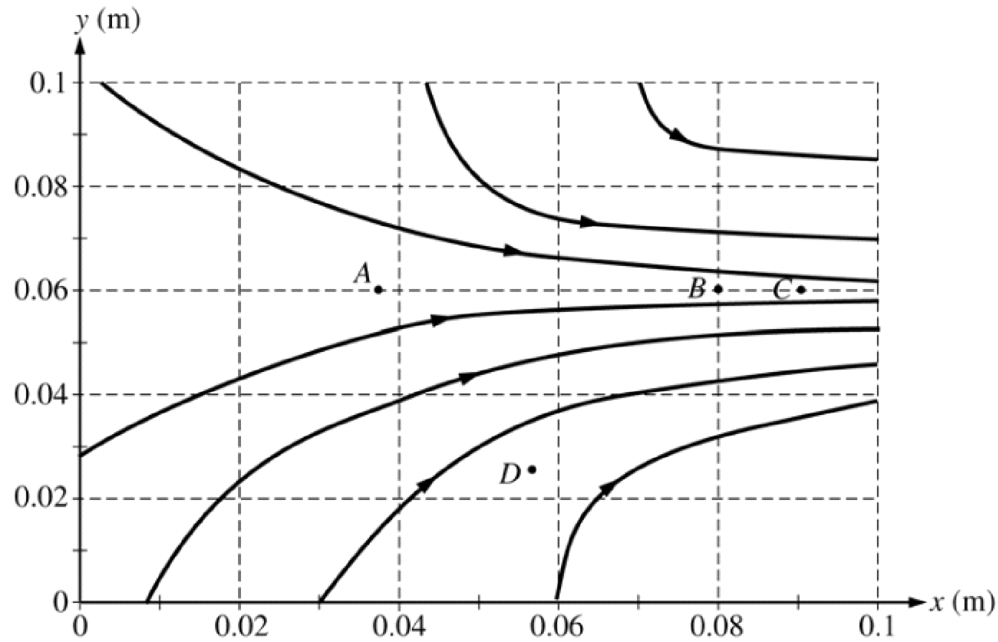


i) Replace Q_2 with $+Q$ so fields cancel @ Point P

ii) replace Q_1 with $+Q$ so $V = \sum \frac{q}{4\pi\epsilon_0 r} = 0$
but field no longer cancels @ Point P

$E_p \rightarrow$

2005 Free Response Question 1



E&M. 1.

Consider the electric field diagram above.

(a) Points A, B, and C are all located at $y = 0.06$ m .

i. At which of these three points is the magnitude of the electric field the greatest? Justify your answer.

ii. At which of these three points is the electric potential the greatest? Justify your answer.

(b) An electron is released from rest at point B.

i. Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.

ii. Calculate the electron's speed after it has moved through a potential difference of 10 V.

(c) Points B and C are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.

(d) On the diagram, draw an equipotential line that passes through point D and intersects at least three electric field lines.

a i) Greatest @ C since field lines are most dense

ii) A => Field points from high to low potential (L->R)
A is farthest left ∴ A at highest potential

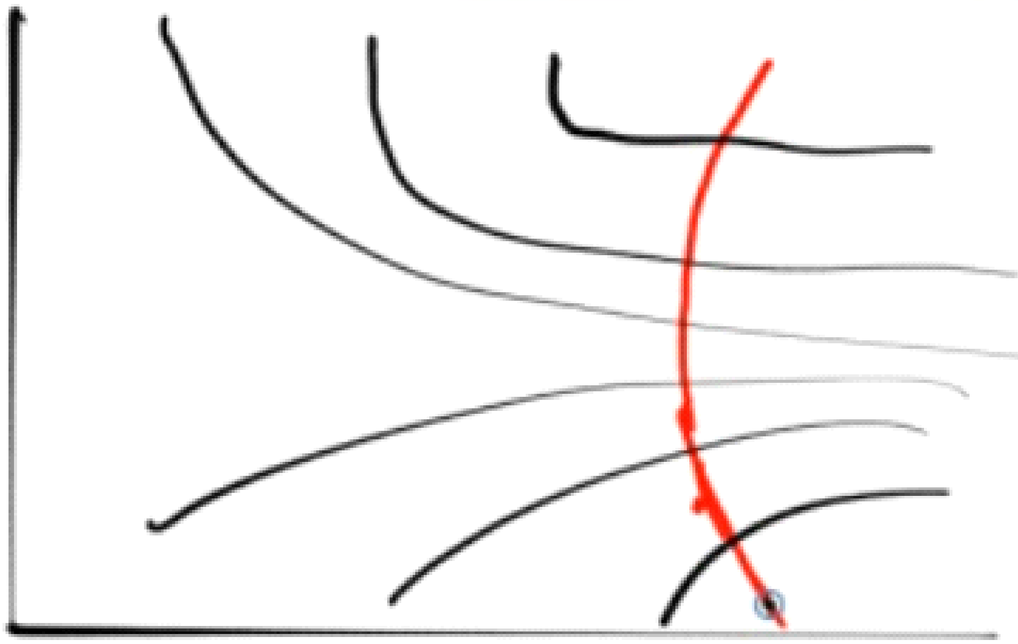
b) Electron accelerates toward the left with increasing speed but speed increases by smaller & smaller amounts per unit time as it moves because acceleration is decreasing.

ii) $W = qV \Rightarrow \frac{1}{2}mv^2 = qV \Rightarrow v^2 = \frac{2qV}{m} \Rightarrow v = \sqrt{\frac{2qV}{m}} \Rightarrow$

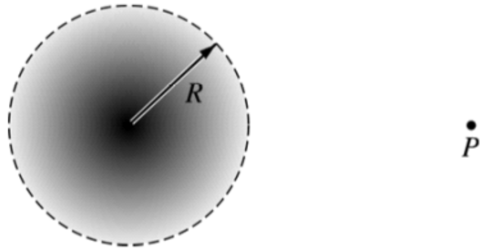
$$\Rightarrow v = \sqrt{\frac{2qV}{m}} \Rightarrow$$

$$v = \sqrt{\left(\frac{2(1.6\cdot 10^{-19})(40)}{9.11\cdot 10^{-31}}\right)} = 1.87\cdot 10^{6}\ \frac{m}{s}$$

c) $E = \frac{V}{d} = \frac{20V}{.01m} = \boxed{2000\ \frac{V}{m}}$ assuming roughly uniform $\vec{E}$ field



2003 Free Response Question 1



E&M. 1.

A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right) \text{ for } r \le R \text{ and}$$
$$\rho = 0 \text{ for } r > R,$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants.

(a) Determine the following as a function of r for $r > R$.

i. The magnitude E of the electric field

ii. The electric potential V

(b) A proton is placed at point P shown above and released. Describe its motion for a long time after its release.

(c) An electron of charge magnitude e is now placed at point P, which is a distance r from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of r as it strikes the cloud.

(d) Derive an expression for ρ_0.

(e) Determine the magnitude E of the electric field as a function of r for $r \le R$.

a) i) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

ii) $V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r} = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{-Q}{4\pi\epsilon_0}\int_{\infty}^{r} r^{-2}\,dr = \frac{-Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{\infty}^{r} = \frac{Q}{4\pi\epsilon_0 r}$

b) Proton will move away from sphere (to right) at increasing velocity w/a decreasing acceleration

c) Cons E: $U_{e_r} = U_{e_R} + K_R \Rightarrow K_R = U_{e_r} - U_{e_R} \xrightarrow{U = \frac{q_1 q_2}{4\pi\epsilon_0 r}}$

$K_R = \frac{-eQ}{4\pi\epsilon_0 r} - \frac{-eQ}{4\pi\epsilon_0 R} \Rightarrow \boxed{K = \frac{eQ}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)}$

$$Q = \int_{r=0}^{R} \rho\, dV \xrightarrow[\rho = \rho_0 (1 - \frac{r}{R})]{dV = 4\pi r^2 dr} Q = \int_0^R \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr \Rightarrow$$

$$Q = 4\pi\rho_0 \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)\Bigg|_0^R =$$

$$4\pi\rho_0 \left(\frac{R^3}{3} - \frac{R^{\cancel{4}\,3}}{4\cancel{R}}\right) = 4\pi\rho_0 \left(\frac{4R^3}{12} - \frac{3R^3}{12}\right) \Rightarrow$$

$$Q = 4\pi\rho_0 \frac{R^3}{12} \Rightarrow \rho_0 = \frac{12Q}{4\pi R^3} = \boxed{\frac{3Q}{\pi R^3}}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow[Q_{enc} = \int_0^r \rho\, dV]{A = 4\pi r^2} 4\pi r^2 E = \frac{1}{\epsilon_0} \int_{r=0}^{r} \rho\, dV \xrightarrow{dV = 4\pi r^2 dr}$$

$$4\pi r^2 E = \frac{1}{\epsilon_0} \int_0^r \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr \Rightarrow \cancel{4\pi} r^2 E = \frac{\cancel{4\pi}\rho_0}{\epsilon_0} \int_0^r \left(r^2 - \frac{r^3}{R}\right) dr \Rightarrow$$

$$r^2 E = \frac{\rho_0}{\epsilon_0} \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)\Bigg|_0^r \Rightarrow r^2 E = \frac{\rho_0}{\epsilon_0} \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \xrightarrow{\rho_0 = \frac{3Q}{\pi R^3}}$$

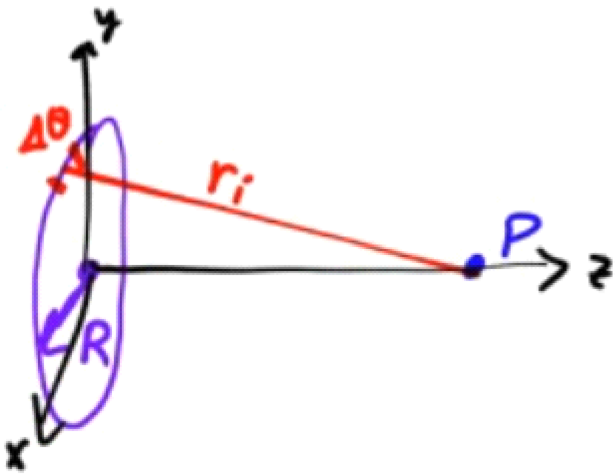
$$E = \frac{3Q}{\pi R^3 r^2 \epsilon_0} \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = \frac{3Q}{\pi \epsilon_0 R^3} \left(\frac{r}{3} - \frac{r^2}{4R}\right) = \frac{Q}{\pi \epsilon_0 R^3} \left(r - \frac{3r^2}{4R}\right)$$

$$\Rightarrow \boxed{E = \frac{Qr}{\pi \epsilon_0 R^3} \left(1 - \frac{3r}{4R}\right)}$$

1.5 - Electric Potential Due to Continuous Charge Distributions

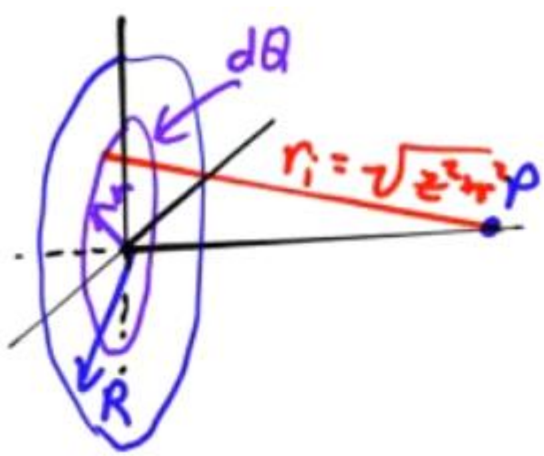
Wednesday, March 1, 2017 2:50 PM

Potential Due to a Charged Ring



- Find the electric potential on the axis of a uniformly charged ring of radius R and total charge Q at point P located a distance z from the center of the ring
- $V_p = \sum_i V_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{Q_i}{r_i} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r_i} \int dq$
- $\because r_i = \sqrt{z^2 + r^2}, \qquad \int dq = Q$
- $\therefore V_p = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{z^2 + r^2}}$

Potential Due to a Charged Disk



- Find the electric potential on the axis of a uniformly charged disk of radius R and total charge Q at point P located a distance z from the center of the ring

$\because dQ = 2\pi r \times dr \times \sigma, \qquad \sigma = \frac{Q}{\pi r^2}$

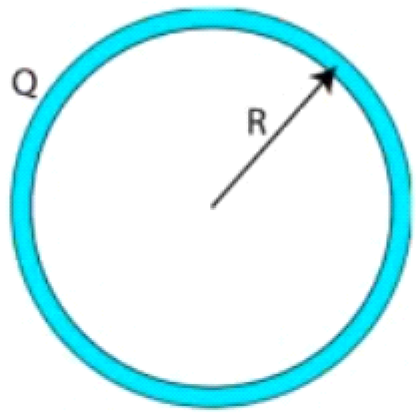
$$\therefore dQ = \frac{2Qr}{R^2}\, dr$$

$$V_p = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\int_{r=0}^{R} \frac{2Qr}{r_i R^2}\, dr = \frac{Q}{2\pi\epsilon_0 R^2}\int_{r=0}^{R} \frac{1}{r_i}\, dr$$

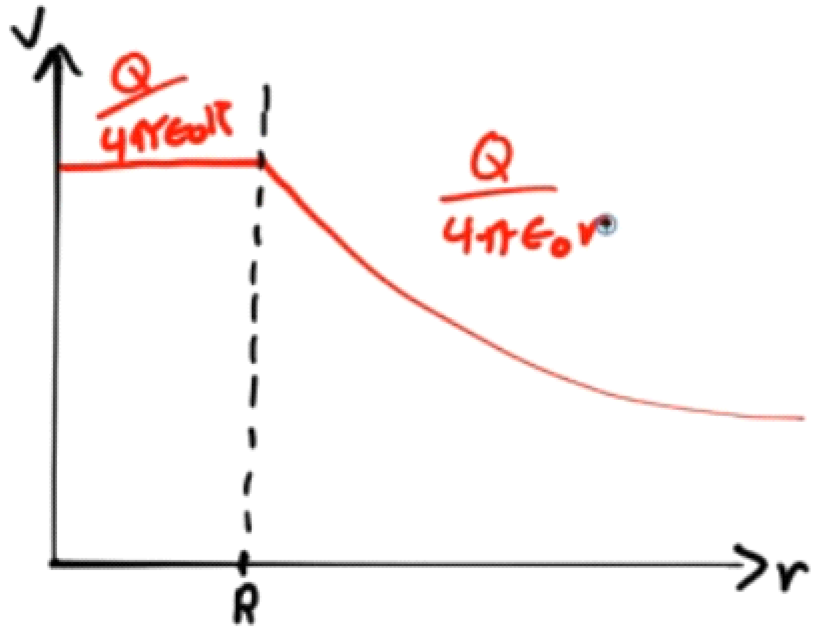
$$\because r_i = \sqrt{z^2 + r^2}$$

$$\therefore V_p = \frac{Q}{2\pi\epsilon_0 R^2}\int_{r=0}^{R} (z^2 + r^2)^{-\frac{1}{2}}\, dr = \frac{Q}{2\pi\epsilon_0 R^2}\left[z(z^2 + r^2)^{\frac{1}{2}}\right]\Big|_0^R = \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{z^2 + R^2} - z\right)$$

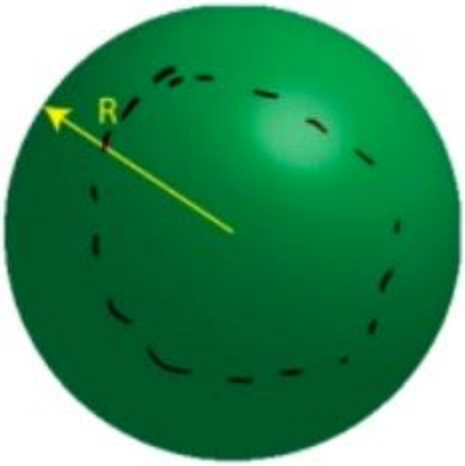
Potential Due to a Spherical Shell of Charge



- Find the electric potential both inside and outside a uniformly charged shell of radius R and total charge Q
- $V_{out} = -\int \vec{E} \cdot d\vec{l} = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} dr = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r} r^{-2} dr = \frac{Q}{4\pi\epsilon_0 r}$
- $V_{inside} = -\int \vec{E} \cdot d\vec{l} = -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} dr - \int_{R}^{r} 0\, dr = \frac{Q}{4\pi\epsilon_0 R}$

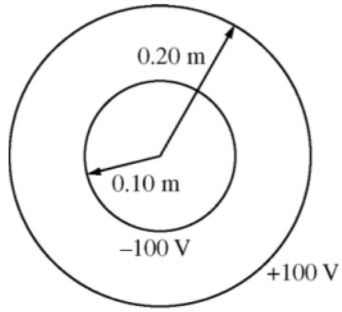


Potential Due to a Uniform Solid Sphere



- Find the electric field and electric potential inside a uniformly charged solid insulating sphere of radius R and total charge Q
- $\because \oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\epsilon_0}$
- $\therefore E \times 4\pi r^2 = \dfrac{\rho V}{\epsilon_0}$
- $\because \rho = \dfrac{Q}{\frac{4}{3}\pi R^3}, \qquad V = \dfrac{4}{3}\pi r^3$
- $\therefore E \times 4\pi r^2 = \dfrac{Q r^3}{R^3 \epsilon_0}$
- $\therefore E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3}$
- $V = -\int \vec{E} \cdot d\vec{l} = -\int_\infty^R \vec{E} \cdot d\vec{r} - \int_R^r \vec{E} \cdot d\vec{r} = -\int_\infty^R \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2} dr - \int_R^r \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3}\, dr = \dfrac{Q}{8\pi\epsilon_0 R}\left(3 - \dfrac{r^2}{R^2}\right)$

2012 Free Response Question 1



E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of -100 V, and the outer shell is set at an electric potential of $+100$ V, with each potential defined relative to the conventional reference point. Let Q_i and Q_o represent the net charge on the inner and outer shells, respectively, and let r be the radial distance from the center of the shells. Express all algebraic answers in terms of Q_i, Q_o, r, and fundamental constants, as appropriate.

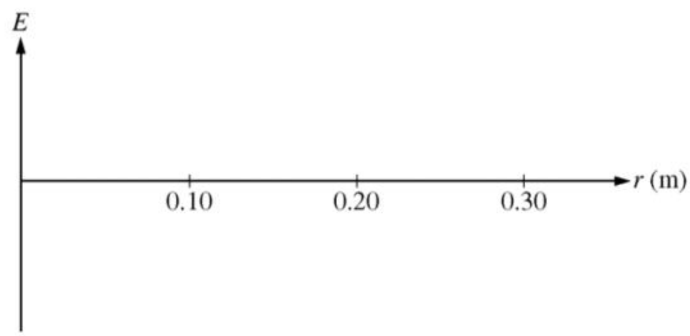
(a) Using Gauss's Law, derive an algebraic expression for the electric field $E(r)$ for $0.10 \text{ m} < r < 0.20 \text{ m}$.

(b) Determine an algebraic expression for the electric field $E(r)$ for $r > 0.20 \text{ m}$.

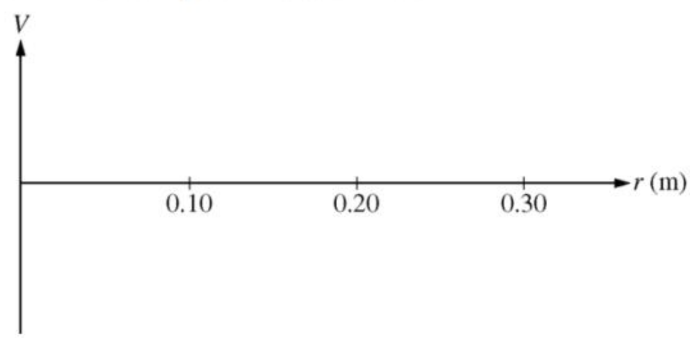
(c) Determine an algebraic expression for the electric potential $V(r)$ for $r > 0.20 \text{ m}$.

(d) Using the numerical information given, calculate the value of the total charge Q_T on the two spherical shells $(Q_T = Q_i + Q_o)$.

(e) On the axes below, sketch the electric field E as a function of r. Let the positive direction be radially outward.



(f) On the axes below, sketch the electric potential V as a function of r.



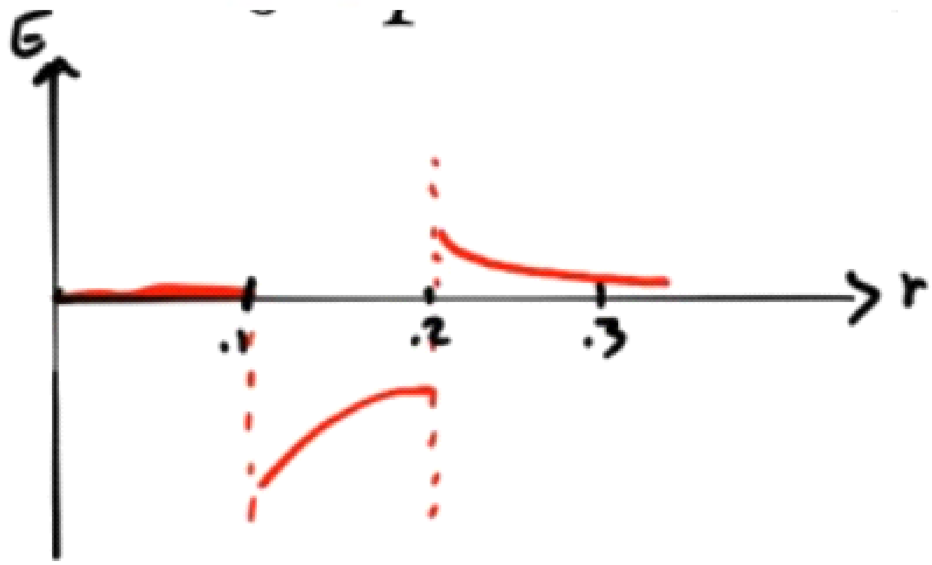
a) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_i}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q_i}{4\pi\epsilon_0 r^2}}$

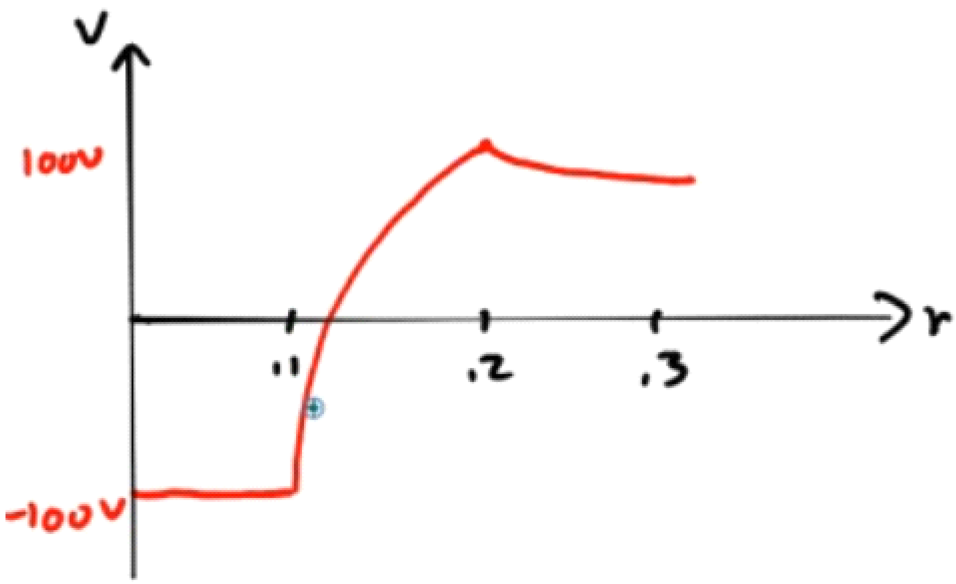
b) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_i+Q_0}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q_i+Q_0}{4\pi\epsilon_0 r^2}}$

c) $V = -\int \vec{E}\cdot d\vec{l} = -\int_{\infty}^{r} \frac{Q_i+Q_0}{4\pi\epsilon_0 r^2}\,dr = \frac{-(Q_i+Q_0)}{4\pi\epsilon_0}\int_{\infty}^{r} r^{-2}\,dr \Rightarrow$

$V = \frac{-(Q_i+Q_0)}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{\infty}^{r} = \frac{-(Q_i+Q_0)}{4\pi\epsilon_0}\left(-\frac{1}{r}\right) = \boxed{\frac{Q_i+Q_0}{4\pi\epsilon_0 r}}$

$V = \frac{Q_T}{4\pi\epsilon_0 r} \xrightarrow[V=100V]{r=.2m} Q_T = 4\pi\epsilon_0 r V = 4\pi\epsilon_0(0.2)(100) = \boxed{2.23\ nC}$





2010 Free Response Question 1

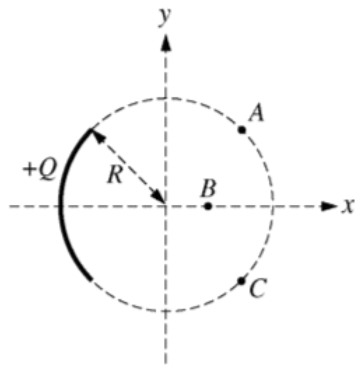


Figure I

E&M. 1.

A charge $+Q$ is uniformly distributed over a quarter circle of radius R, as shown above. Points A, B, and C are located as shown, with A and C located symmetrically relative to the x-axis. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Rank the magnitude of the electric potential at points A, B, and C from greatest to least, with number 1 being greatest. If two points have the same potential, give them the same ranking.

____V_A ____V_B ____V_C

Justify your rankings.

Point P is at the origin, as shown below, and is the center of curvature of the charge distribution.

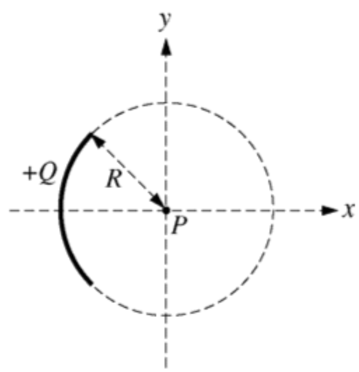
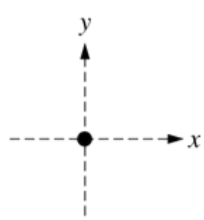


Figure II

(b) Determine an expression for the electric potential at point P due to the charge Q.

(c) A positive point charge q with mass m is placed at point P and released from rest. Derive an expression for the speed of the point charge when it is very far from the origin.

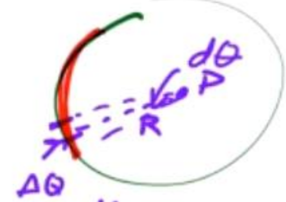
(d) On the dot representing point P below, indicate the direction of the electric field at point P due to the charge Q.



(e) Derive an expression for the magnitude of the electric field at point P.

a) $V_B > V_A = V_C$

b) $V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\int_{\theta=\frac{3\pi}{4}}^{\theta=\frac{5\pi}{4}} \frac{\lambda R\,d\theta}{R} \Rightarrow$



$\lambda R\,d\theta$

$\lambda = \frac{Q}{l} = \frac{Q}{\frac{\pi R}{2}} = \frac{2Q}{\pi R}$

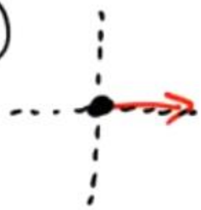
$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} d\theta = \frac{\lambda}{4\pi\epsilon_0}\frac{\pi}{2} \xrightarrow{\lambda = \frac{2Q}{\pi R}}$$

$$V = \frac{2Q}{\pi R}\frac{\pi}{4\pi\epsilon_0 2} = \boxed{\frac{Q}{4\pi\epsilon_0 R}}$$

c) $U_{e_i} = K_f \Rightarrow qV_P = \frac{1}{2}mv^2 \xrightarrow{V_P = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}} \frac{qQ}{4\pi\epsilon_0 R} = \frac{1}{2}mv^2 \Rightarrow$

$$v^2 = \frac{\cancel{2}qQ}{2\cancel{4}\pi\epsilon_0 mR} \Rightarrow \boxed{v = \sqrt{\frac{qQ}{2\pi\epsilon_0 mR}}}$$

d)



e) $E = E_x = \int dE_x = \int dE\cos\theta = \int \frac{dQ}{4\pi\epsilon_0 R^2}\cos\theta$

$$\xrightarrow[dQ = \lambda R\,d\theta = \frac{2Q}{\pi R}R\,d\theta]{\lambda = \frac{2Q}{\pi R}} E = \int \frac{2Q}{\pi\, 4\pi\epsilon_0 R^2}\cos\theta\, d\theta$$

$dQ = \frac{2Q}{\pi}d\theta$

$$E = \frac{Q}{2\epsilon_0\pi^2 R^2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos\theta\, d\theta = \frac{Q}{2\epsilon_0\pi^2 R^2}\left(\sin\frac{\pi}{4} - \sin -\frac{\pi}{4}\right) \Rightarrow$$

$$E = \frac{Q}{2\epsilon_0\pi^2 R^2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{Q\sqrt{2}}{2\epsilon_0\pi^2 R^2} = \boxed{\frac{Q}{\sqrt{2}\epsilon_0\pi^2 R^2}}$$

2009 Free Response Question 1

A spherically symmetric charge distribution has net positive charge Q_0 distributed within a radius of R. Its electric potential V as a function of the distance r from the center of the sphere is given by the following.

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \quad \text{for } r < R$$

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \quad \text{for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

____ Radially inward ____ Radially outward

ii. $r > R$

____ Radially inward ____ Radially outward

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i. $r < R$

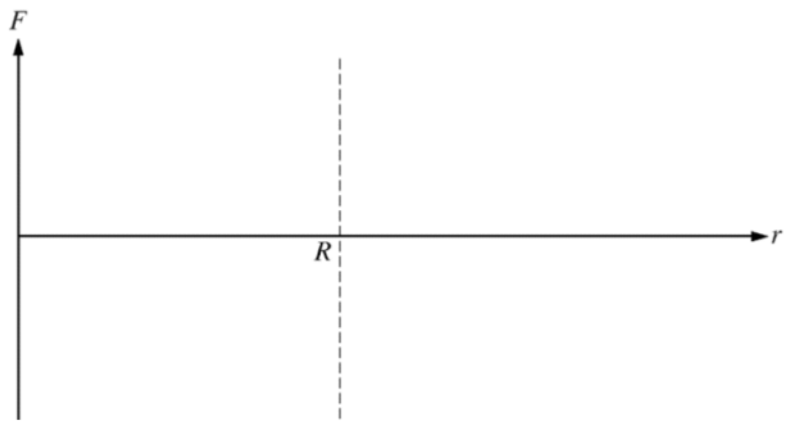
ii. $r > R$

(c) Is there any charge on the surface of the sphere ($r = R$) ?

____ Yes ____ No

If there is, determine the charge. In either case, explain your reasoning.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



ai) $E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q_0}{4\pi\epsilon_0 R}\left[-2+3\left(\frac{r}{R}\right)^2\right]\right) = \frac{-Q_0}{4\pi\epsilon_0 R}\frac{d}{dr}\left(-2+3\left(\frac{r}{R}\right)^2\right) \Rightarrow$

$E = \frac{-Q_0}{4\pi\epsilon_0 R}\left(\frac{6r}{R^2}\right) = \frac{-3Q_0 r}{2\pi\epsilon_0 R^3} \Rightarrow |E| = \frac{3Q_0 r}{2\pi\epsilon_0 R^3}$ inward $E<0$

ii) $E = -\frac{dV}{dr} = \frac{-d}{dr}\left(\frac{Q_0}{4\pi\epsilon_0 r}\right) = \frac{-Q}{4\pi\epsilon_0}\frac{d}{dr}\left(r^{-1}\right) = \frac{Q}{4\pi\epsilon_0 r^2}$ Outward $E>0$

bi) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0} \Rightarrow Q_{enc} = \epsilon_0 E(4\pi r^2) \xrightarrow{E = \frac{-3Q_0 r}{2\pi\epsilon_0 R^3}}$

$Q_{enc} = \frac{\epsilon_0(-3)Q_0 r}{2\pi\epsilon_0 R^3}(4\pi r^2) \Rightarrow \boxed{Q_{enc} = \frac{-6Q_0 r^3}{R^3}}$

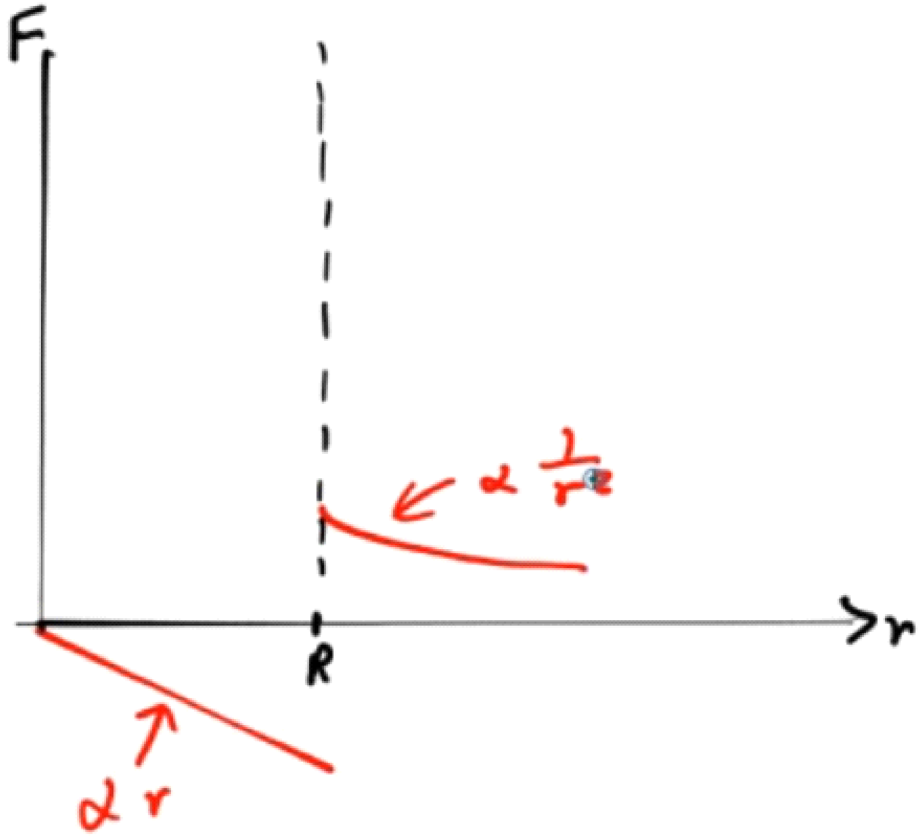
$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow 4\pi r^2 \epsilon_0 E = Q_{enc} \xrightarrow{E = \frac{Q_0}{4\pi\epsilon_0 r^2}}$

$\frac{4\pi r^2 \epsilon_0 Q_0}{4\pi\epsilon_0 r^2} = Q_{enc} \Rightarrow \boxed{Q_{enc} = Q_0}$

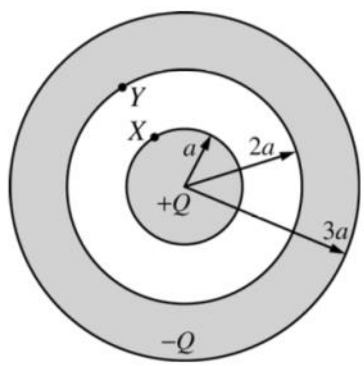
c) yes, there is charge on the surface

$r>R$ $\quad Q_{enc_{r>R}} = Q_{surface} + Q_{enc_{r=R}} \Rightarrow Q_{sur} = Q_{enc} - Q_{enc_{r=R}} \Rightarrow$

$Q_{sur} = Q_{enc_{r>R}} - Q_{enc_{r=R}} = Q_0 - \left(\frac{-6Q_0 R^3}{R^3}\right) = \boxed{7Q_0}$



2007 Free Response Question 2



E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

ai) $\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{3Q}{4\pi a^3}$

a i) $\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{3Q}{4\pi a^3}$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0} \xrightarrow{\rho = \frac{3Q}{4\pi a^3}}$$

$$E = \frac{\cancel{3}Q\,\cancel{\frac{4}{3}}\cancel{\pi} r^{\cancel{3}}}{4\pi a^3 \epsilon_0\,\cancel{4}\cancel{\pi} \cancel{r^2}} \Rightarrow \boxed{E = \frac{Qr}{4\pi\epsilon_0 a^3}}$$

a ii) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

a iii) $Q_{enc} = Q + \rho_b V_b$ $\qquad \rho_b = \frac{-Q}{V_b} = \frac{-Q}{\frac{4}{3}\pi(3a)^3 - \frac{4}{3}\pi(2a)^3} = \frac{-Q}{\frac{4}{3}\pi(27a^3-8}$

$$\rho_b = \frac{-Q}{\frac{4}{3}\pi(19a^3)}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2)\epsilon_0 = Q + \left(\frac{-Q}{\frac{4}{3}\pi(19a^3)}\right)\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi(2a)^3\right) \Rightarrow$$

$$E(4\pi\epsilon_0 r^2) = Q\left(1 - \frac{r^3}{19a^3} + \frac{8a^3}{19a^3}\right) = Q\left(1 - \frac{r^3}{19a^3} + \frac{8}{19}\right) \Rightarrow$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}\left(1 - \frac{r^3}{19a^3} + \frac{8a^3}{19a^3}\right) = \frac{Q}{4(19)\pi\epsilon_0 r^2}\left(19 - \frac{r^3}{a^3} + 8\right) \Rightarrow$$

$$E = \frac{Q}{76\pi\epsilon_0 r^2}\left(27 - \frac{r^3}{a^3}\right)$$

a iv) $Q_{enc} = 0 \Rightarrow E = 0$

b) $E = -\frac{dV}{dr} \Rightarrow V = -\int E\,dr = -\int_{\infty}^{3a} E\cdot dr = -\int 0 = 0$

c) $V_x - V_Y = \Delta V = \int_a^{2a} \vec{E}\cdot d\vec{r} = \int_a^{2a} \frac{Q}{4\pi\epsilon_0 r^2}dr = \frac{Q}{4\pi\epsilon_0}\int_a^{2a} r^{-2}dr =$

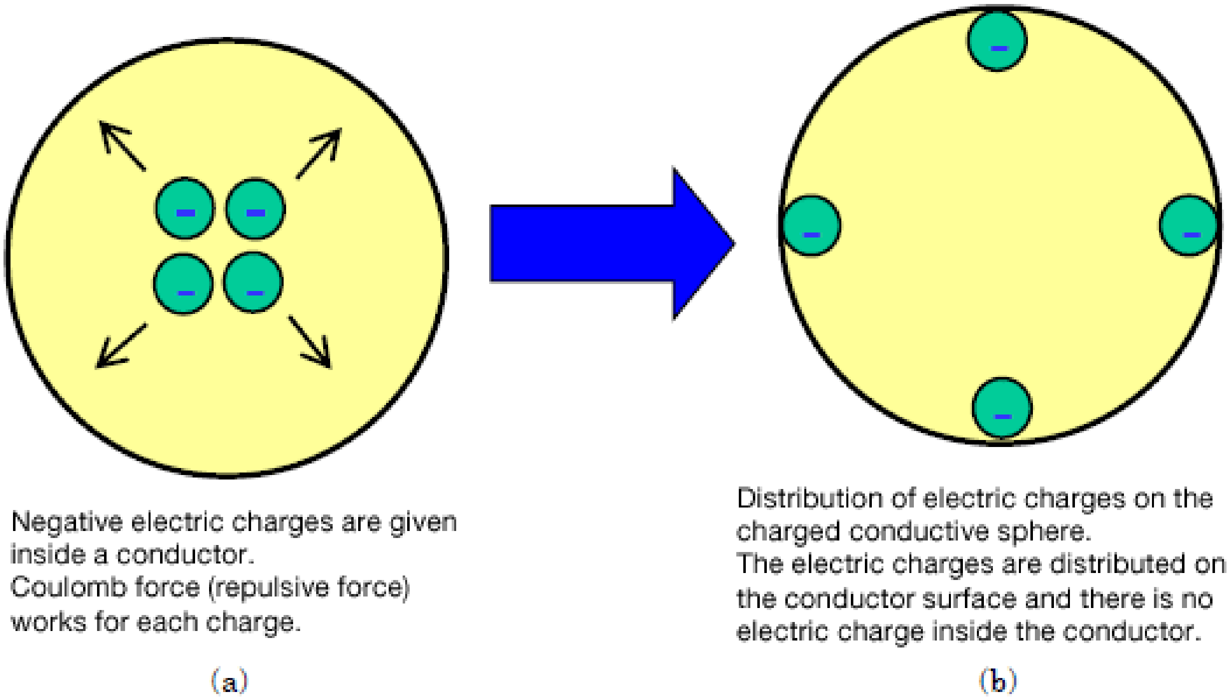
$$\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]\Big|_a^{2a} = \frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{2a} - \frac{-1}{a}\right) = \frac{Q}{8\pi a\epsilon_0}$$

1.6 - Conductors

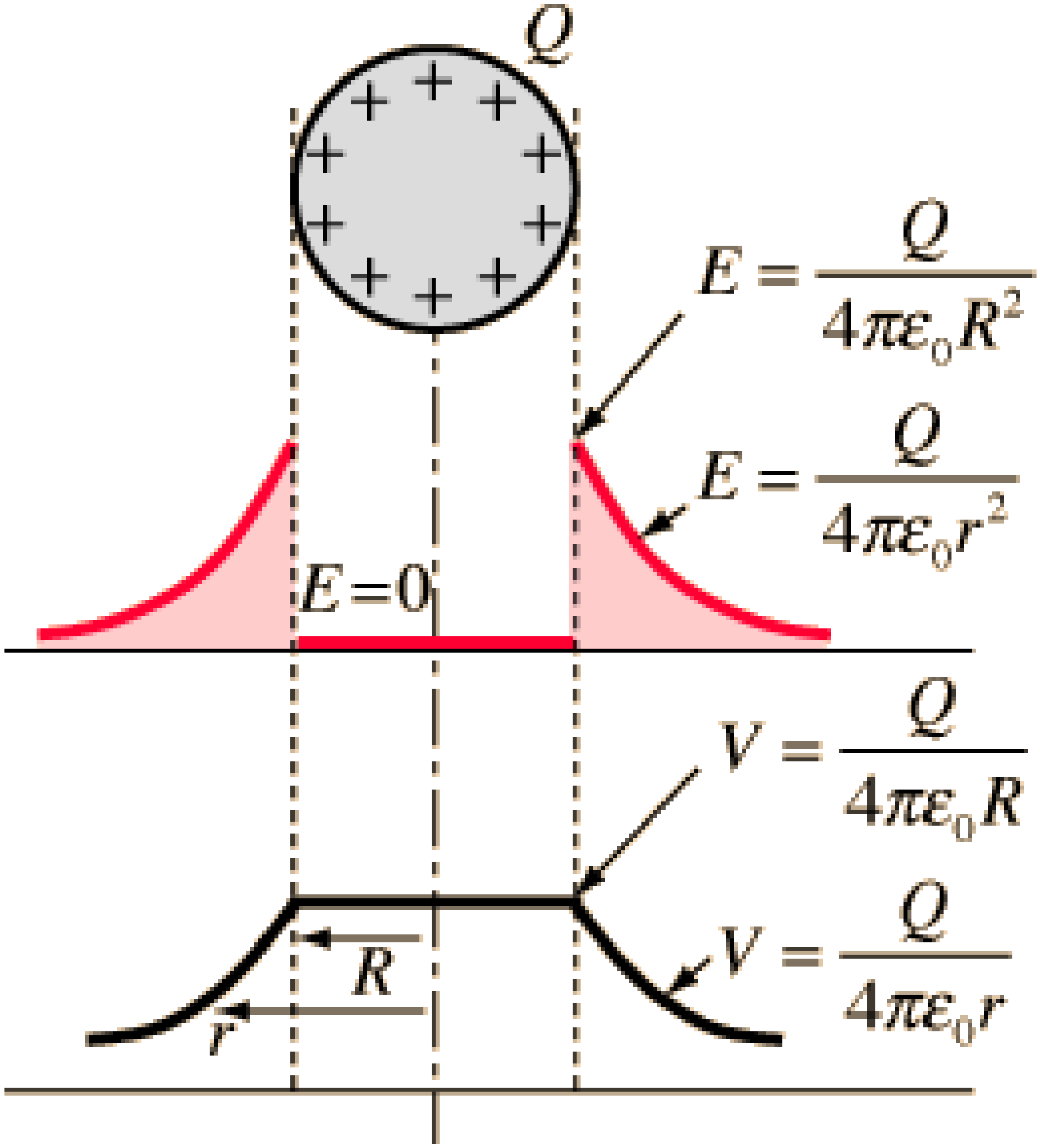
Wednesday, March 1, 2017 4:11 PM

Charges in a Conductor

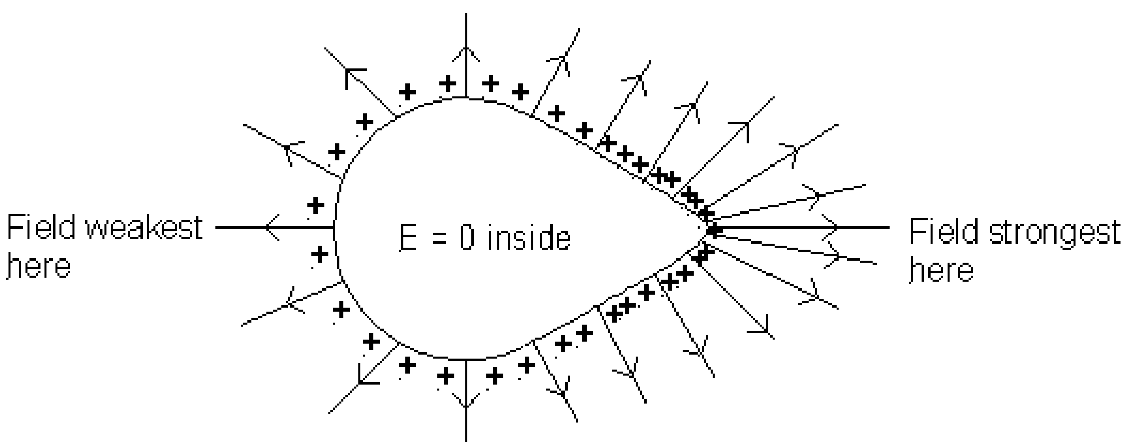
- Charge is free to move until the E=0



- All charge resides at surface



- Field lines are perpendicular to the surface



(d) Electric field & charge distribution around a pear-shaped conductor

Electric Field at the Surface of a Conductor

- $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$
- $\because Q_{enc} = \sigma A$
- $\therefore EA = \frac{\sigma A}{\epsilon_0}$
- $\therefore E = \frac{\sigma}{\epsilon_0}$
- E increases as r σ increases.

Hollow Conductors

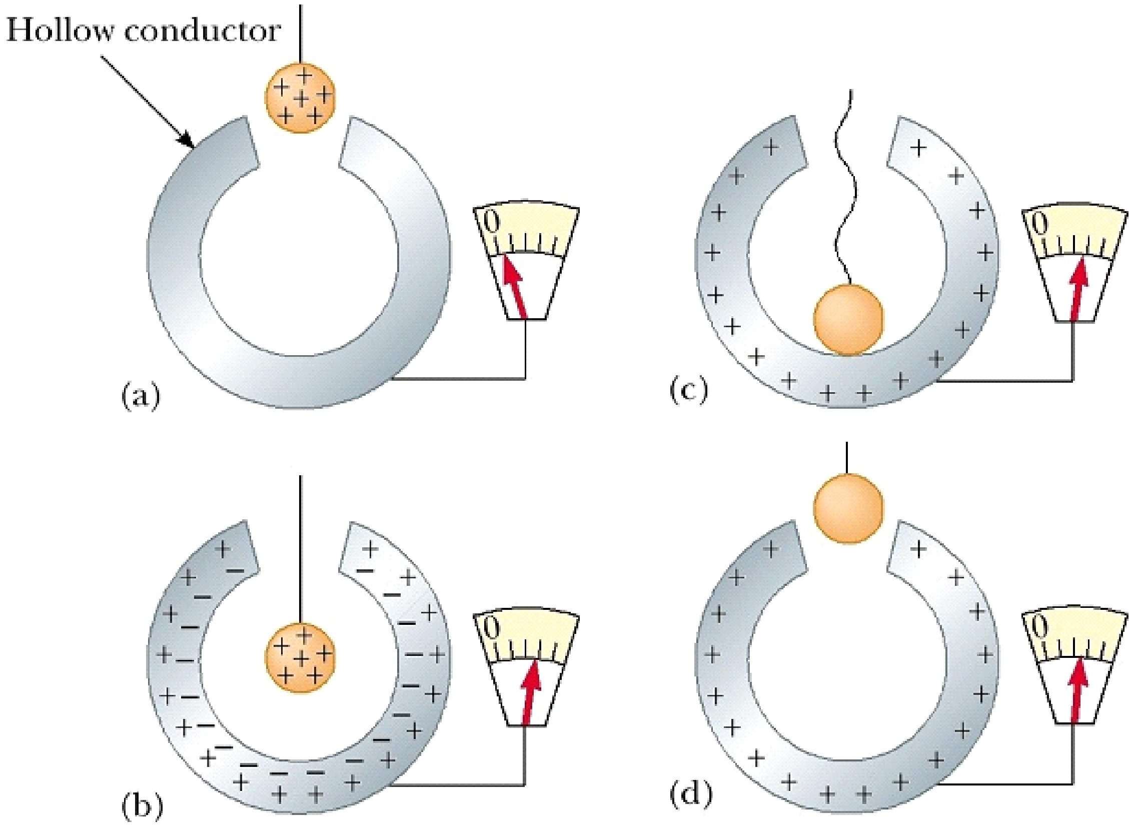
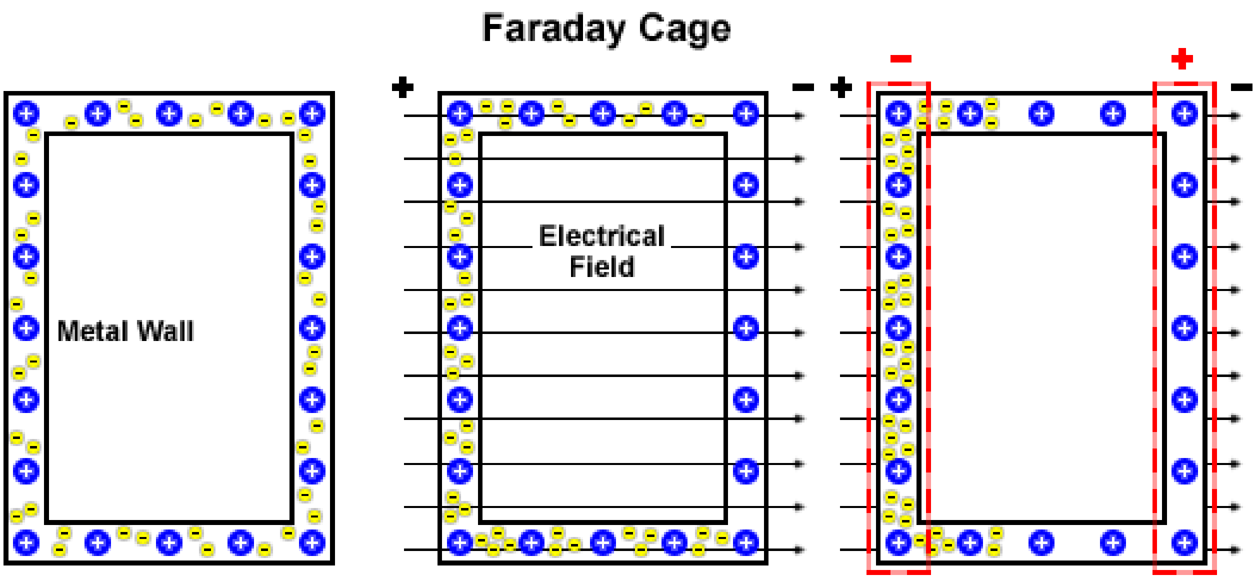


Figure 24.20 An experiment showing that any charge trans-ferred to a conductor resides on its surface in electrostatic equilibrium. The hollow conductor is insulated from ground, and the small metal ball is supported by an insulating thread.

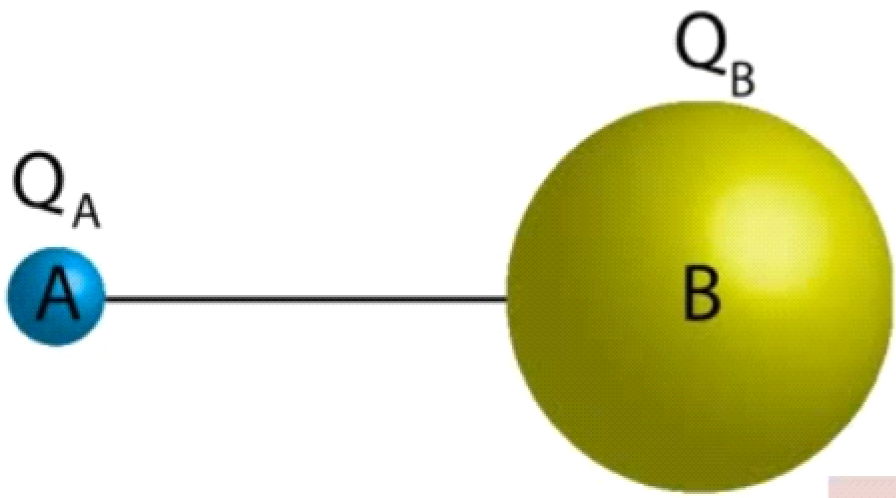


Faraday Cage in the absence of an electrical field.

The charged particles in the wall of the Faraday cage respond to an applied electrical field.

Electrical fields generated inside the wall cancel out the applied field, neutralizing the interior of the cage.

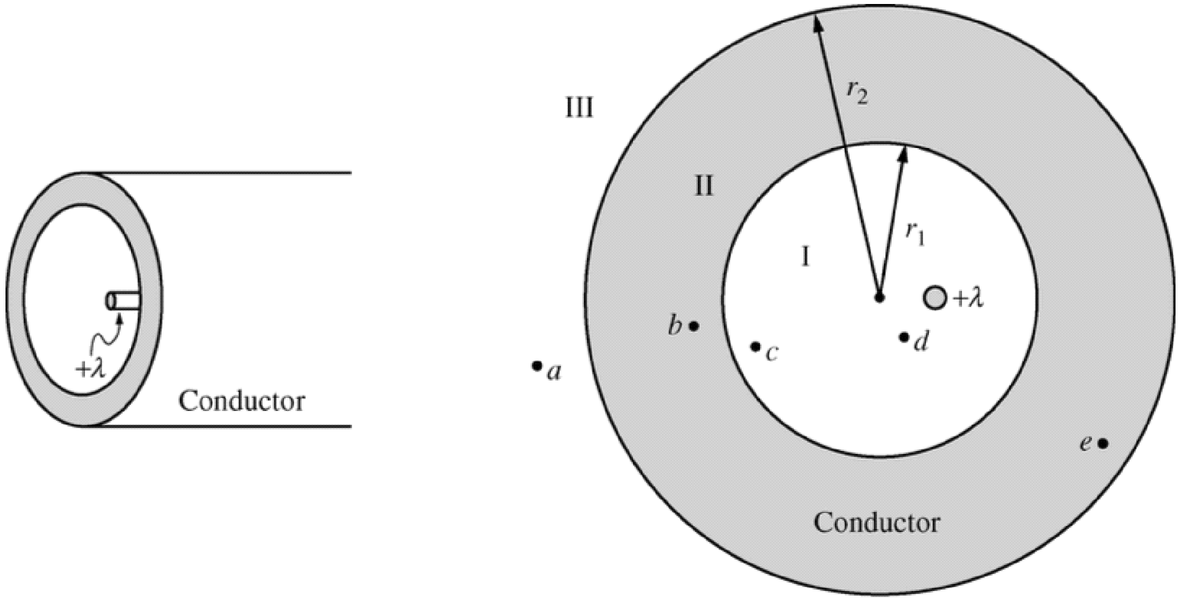
Example 1: Conducting Spheres Connected by a Wire



- Two conducting spheres, A and B, are placed a large distance from each other. The radius of Sphere A is 5 cm, and the radius of Sphere B is 20 cm. A charge Q of 200 nC is placed on Sphere A, while Sphere B is uncharged. The spheres are then connected by a wire. Calculate the charge on each sphere after the wire is connected
- $V_A = \dfrac{Q_A}{4\pi\epsilon_0 R_A}, \qquad V_B = \dfrac{Q_B}{4\pi\epsilon_0 R_B}$
- $\because V_A = V_B$
- $\therefore \dfrac{Q_A}{R_A} = \dfrac{Q_B}{R_B}$
- $\because Q = Q_A + Q_B$
- $\therefore \dfrac{Q_A}{R_A} = \dfrac{Q - Q_A}{R_B}$

- $\therefore Q_A = \frac{QR_A}{R_A + R_B} = 40nC, \qquad Q_B = 160nC$

2004 Free Response Question



Cross Section

E&M. 1.

The figure above left shows a hollow, infinite, cylindrical, uncharged conducting shell of inner radius r_1 and outer radius r_2. An infinite line charge of linear charge density $+\lambda$ is parallel to its axis but off center. An enlarged cross section of the cylindrical shell is shown above right.

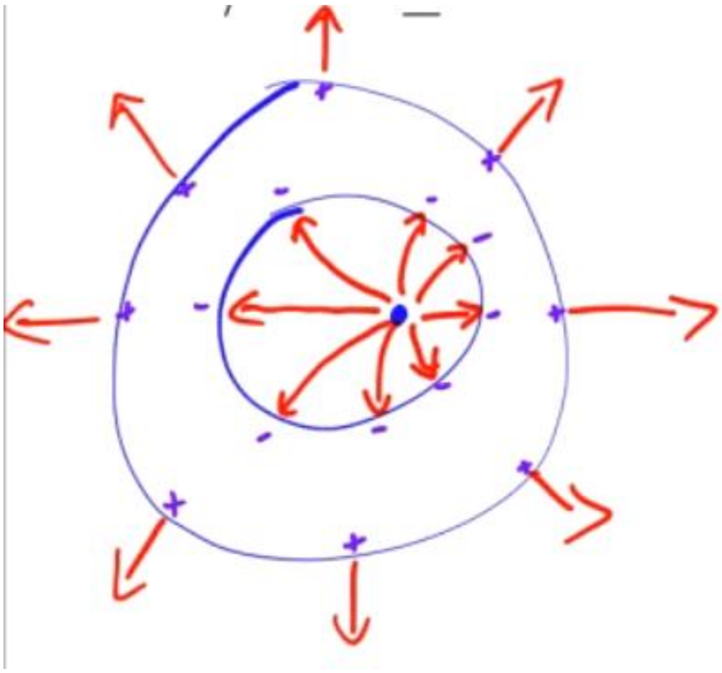
(a) On the cross section above right,

i. sketch the electric field lines, if any, in each of regions I, II, and III and

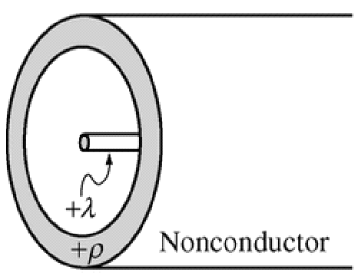
ii. use + and – signs to indicate any charge induced on the conductor.

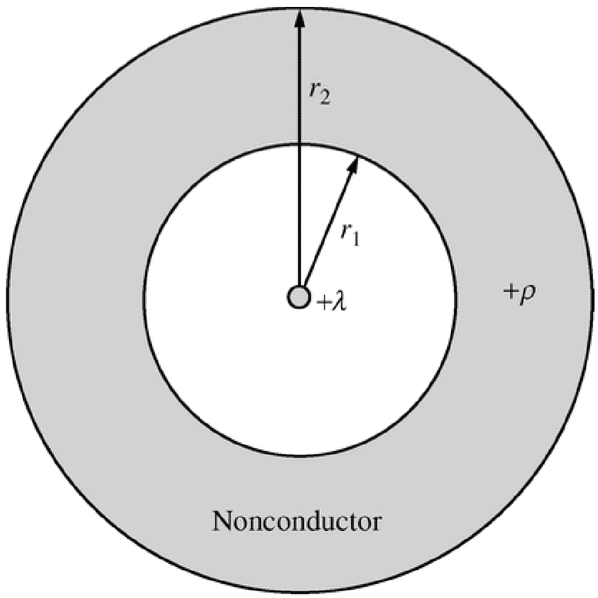
(b) In the spaces below, rank the electric potentials at points a, b, c, d, and e from highest to lowest (1 = highest potential). If two points are at the same potential, give them the same number.

____ V_a ____ V_b ____ V_c ____ V_d ____ V_e



b) $V_D > V_C > V_B = V_2 > V_A$





Cross Section

(c) The shell is replaced by another cylindrical shell that has the same dimensions but is nonconducting and carries a uniform volume charge density $+\rho$. The infinite line charge, still of charge density $+\lambda$, is located at the center of the shell as shown above. Using Gauss's law, calculate the magnitude of the electric field as a function of the distance r from the center of the shell for each of the following regions. Express your answers in terms of the given quantities and fundamental constants.

i. $r < r_1$

ii. $r_1 \le r \le r_2$

iii. $r > r_2$

c)

i) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(2\pi r L) = \frac{\lambda L}{\epsilon_0} \Rightarrow E = \frac{\lambda L}{2\pi r L \epsilon_0} = \boxed{\frac{\lambda}{2\pi \epsilon_0 r}}$

ii) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(2\pi r L) = \frac{\lambda L + \pi r^2 L \rho - \pi r_i^2 L \rho}{\epsilon_0} \Rightarrow$

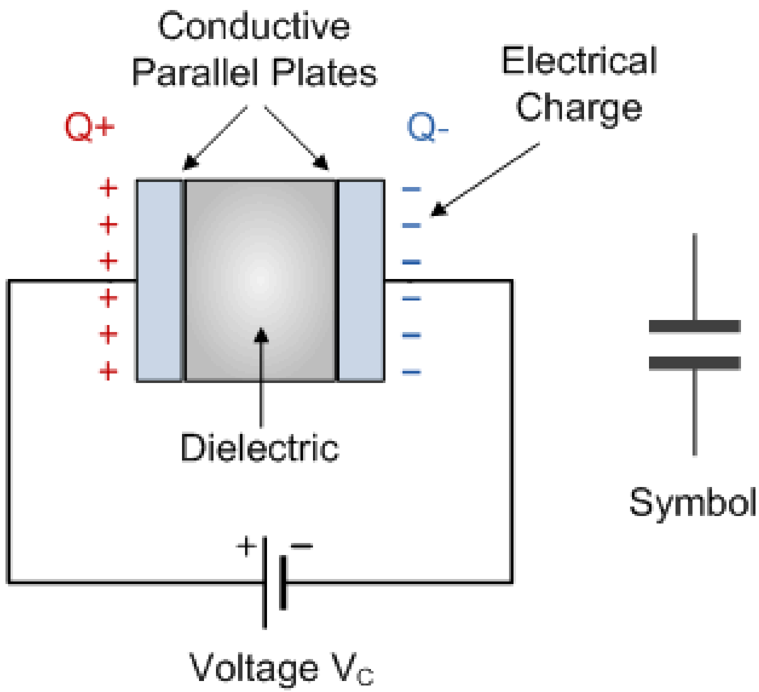
$E = \frac{\lambda + \rho\pi r^2 - \rho \pi r_i^2}{2\pi \epsilon_0 r} = \boxed{\frac{\lambda}{2\pi\epsilon_0 r} + \frac{\rho}{2\epsilon_0 r}(r^2 - r_i^2)}$

1.7 - Capacitors

Wednesday, March 1, 2017 4:57 PM

What is a Capacitor?

- A capacitor is an electric device used to store electrical energy
  - Two conducting plates
  - Insulating material between (dielectric)
- Place opposite charges on each plate
- Develop a potential difference across the plates
- Energy is stored in the electric field between the plates

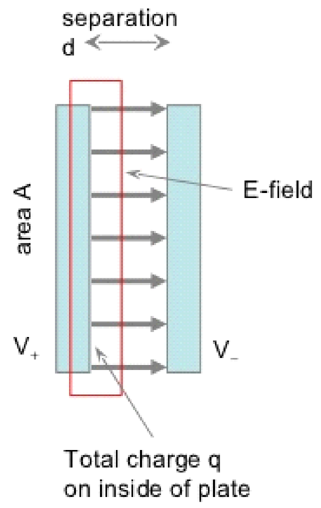


Capacitance

- Capacitance (C) is the ratio of the charge separated on the plates of a capacitor to the potential difference between the plates
- Units of capacitance are coulombs/volt, or farads (F)
  - A farad is a very large amount of capacitance

Capacitance for Parallel Plates

To calculate capacitance, need to determine the E field between the plates. We use Gauss' Law, with one end of our gaussian surface closed inside one plate, and the other closed in the region between the plates



Gauss: $\Phi_E = E \cdot Area = E \cdot A = \frac{q}{\varepsilon_0}$

So $q = \varepsilon_0 EA$ or $E = \frac{q}{\varepsilon_0 A}$

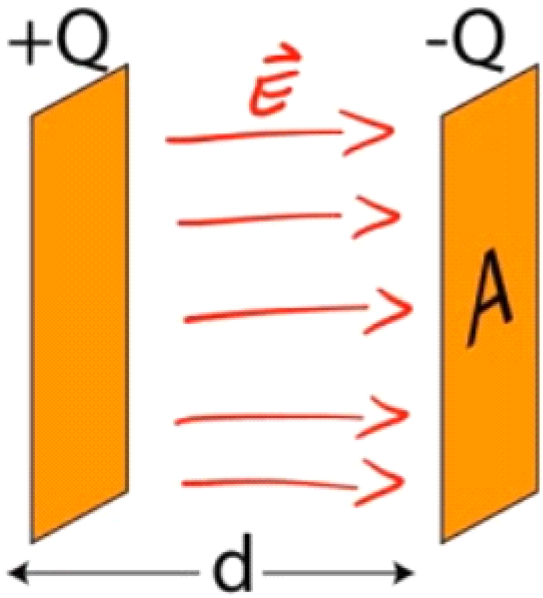
Potential = force/q x distance: $V = Ed$

$$\Rightarrow C = q/V = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$

Calculating Capacitance

1. Assume a charge of +Q and -Q on each conductor
2. Find the electric field between the conductors (Gauss's Law)
3. Calculate V by intergrating the electric field ($V = -\int \vec{E} \cdot d\vec{l}$)
4. Utilize $C = \frac{Q}{V}$ to solve for capacitance.

Example 1: Parallel Plates



- Determine the capacitance between identical parallel plates of area A separated by a distance d

1. Assume +Q and -Q
2. $E = \dfrac{\sigma}{\epsilon_0}$
3. $V = -\int \vec{E} \cdot d\vec{l} = -Ed = -\dfrac{\sigma d}{\epsilon_0}$

   $\because \sigma = \dfrac{Q}{A}$

   $\therefore V = \dfrac{Qd}{\epsilon_0 A}$
4. $C = \dfrac{Q}{V} = \dfrac{Q}{Qd/\epsilon_0 A} = \dfrac{\epsilon_0 A}{d}$

Example 2: Cylindrical Capacitor

- Determine the capacitance of a long, thin hollow conducting cylinder of a radius R_B surrounding a long solid conducting cylinder of radius R_A

1. Assume +Q and -Q
2. $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{enc}}{\epsilon_0}$

   $\therefore E(2\pi r L) = \dfrac{\lambda L}{\epsilon_0}$

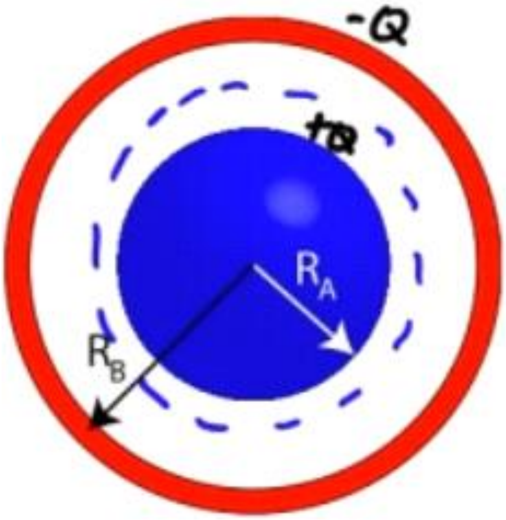
   $\therefore E = \dfrac{\lambda}{2\pi\epsilon_0 r}$
3. $V = -\int \vec{E} \cdot d\vec{l} = -\int_{r=R_A}^{R_B} \dfrac{\lambda}{2\pi\epsilon_0 r} dr = -\dfrac{\lambda}{2\pi\epsilon_0}\int_{r=R_A}^{R_B} \dfrac{1}{r}\, dr = -\dfrac{\lambda}{2\pi\epsilon_0}(\ln r)\Big|_{R_A}^{R_B} = \dfrac{\lambda}{2\pi\epsilon_0} \ln\dfrac{R_A}{R_B}$

   $\because \lambda = \dfrac{Q}{L}$

   $\therefore V = \dfrac{Q}{2\pi\epsilon_0 L} \ln\dfrac{R_A}{R_B}$

4. $C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{ln\frac{R_A}{R_B}}$

Example 3: Spherical Capacitor



- Determine the capacitance of a thin hollow conducting shell of radius R_B concentric aroung a solid conducting sphere of radius R_A

1. Assume +Q and -Q
2. $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

   $\therefore E(4\pi r^2) = \frac{Q}{\epsilon_0}$

   $\therefore E = \frac{Q}{4\pi\epsilon_0 r^2}$

3. $V = -\int \vec{E} \cdot d\vec{l} = -\int_{r=R_A}^{R_B} \frac{Q}{4\pi\epsilon_0 r^2} dr = -\frac{Q}{4\pi\epsilon_0}\int_{r=R_A}^{R_B} r^{-2} dr = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{R_A}^{R_B} = \frac{Q}{4\pi\epsilon_0}\frac{R_A - R_B}{R_A R_B}$

   $\because R_A < R_B$

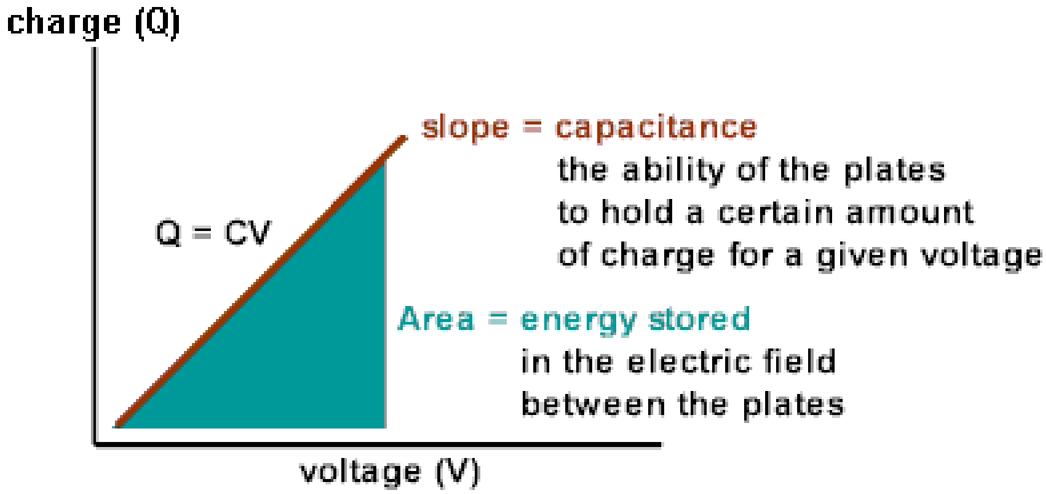
   $\therefore V = \frac{Q}{4\pi\epsilon_0}\frac{R_B - R_A}{R_A R_B}$

4. $C = \frac{Q}{V} = \frac{4\pi\epsilon_0 R_A R_B}{R_B - R_A}$

Energy Stored in a Capacitor

- Work is done charging a capacitor, allowing the capacitor to store energy.
- $U_{cap} = \int_{q=0}^{Q} \frac{q}{C} dq = \frac{Q^2}{2C}$
- $\because C = \frac{Q}{V}$
- $U_{cap} = \frac{1}{2}QV$
- $\because Q = CV$

- $U_{cap} = \frac{1}{2}CV^2$



Field Energy Density

- The amount of energy stored as electric field per unit volume between the plates of capacitor is known as the field energy density u_e
- $u_e = \frac{1}{2}\epsilon_0 E^2$

$$Energy = \frac{1}{2}QV$$

$$Energy = \frac{1}{2}CV^2$$

$$Energy = \frac{1}{2}\left(\frac{\varepsilon_o A}{d}\right)(Ed)^2$$

$$Energy = \frac{1}{2}\varepsilon_o AE^2 d$$

$$\frac{Energy}{Ad} = \frac{1}{2}\varepsilon_o E^2$$

$$\frac{Energy}{Volume} = \frac{1}{2}\varepsilon_o E^2$$

Dielectrics

- Insulating Materials

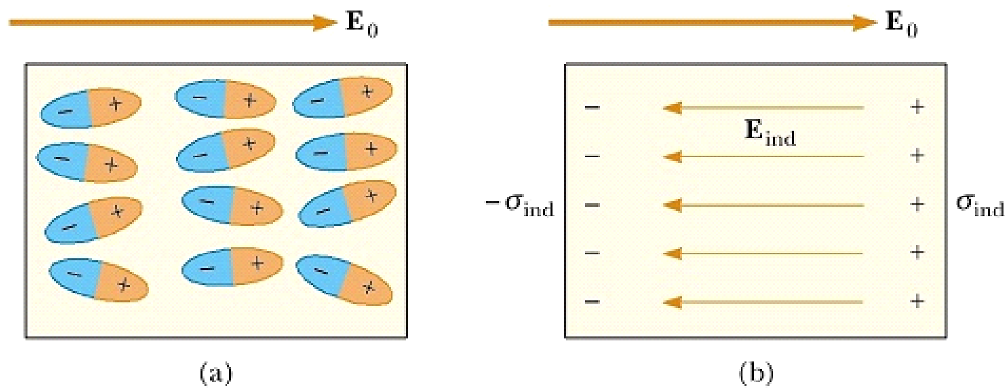
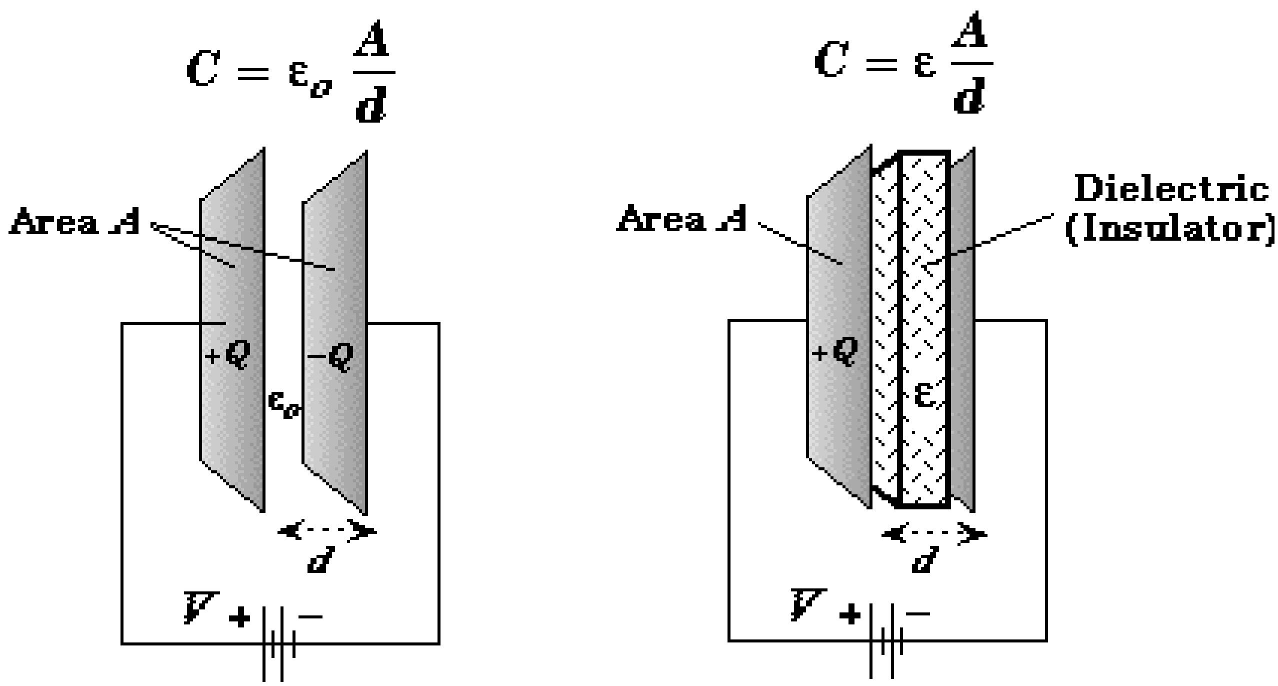
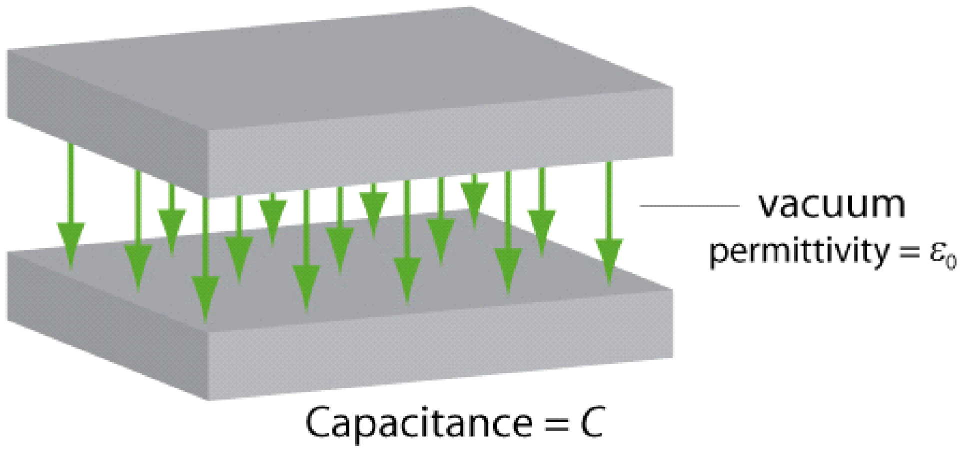
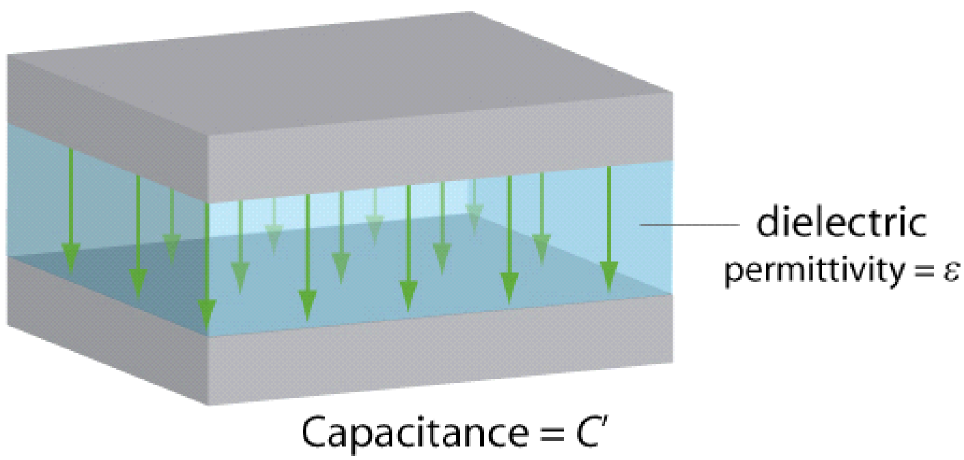


Figure 26.24 (a) When a dielectric is polarized, the dipole moments of the molecules in the dielectric are partially aligned with the external field $\mathbf{E}_0$. (b) This polarization causes an induced negative surface charge on one side of the dielectric and an equal induced positive surface charge on the opposite side. This separation of charge results in a reduction in the net electric field within the dielectric.

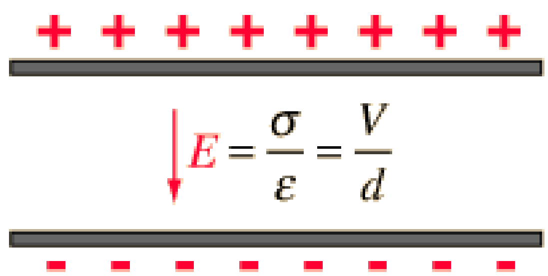


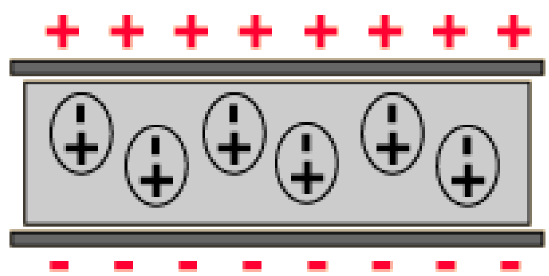
Dielectric Constant (κ)





$$\boxed{\kappa = C'/C} \qquad \boxed{\kappa = \varepsilon/\varepsilon_0}$$





For air, $\varepsilon \approx \varepsilon_0$

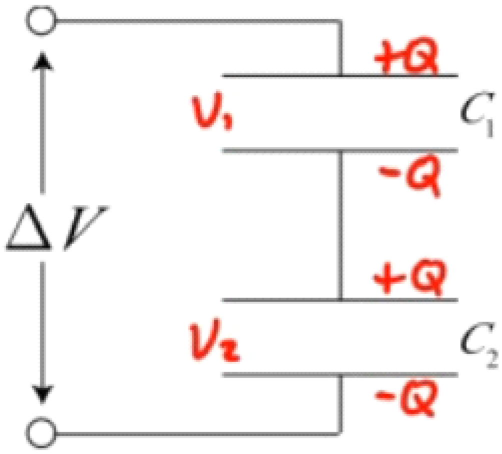
$$E_{effective} = E - E_{polarization} = \frac{\sigma}{k\varepsilon_0}$$

$$C = \frac{\varepsilon_0 A}{d}$$

The capacitance is increased by the factor k.

$$C = \frac{k\varepsilon_0 A}{d}$$

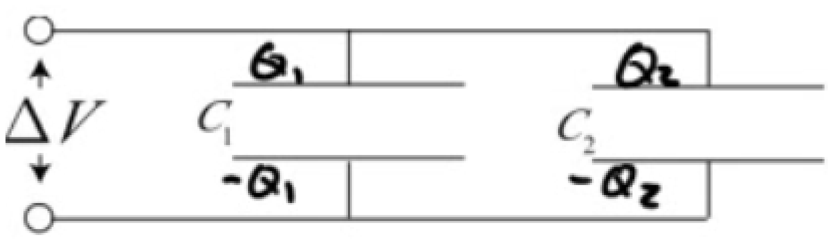
Example 4: Capacitors in Series



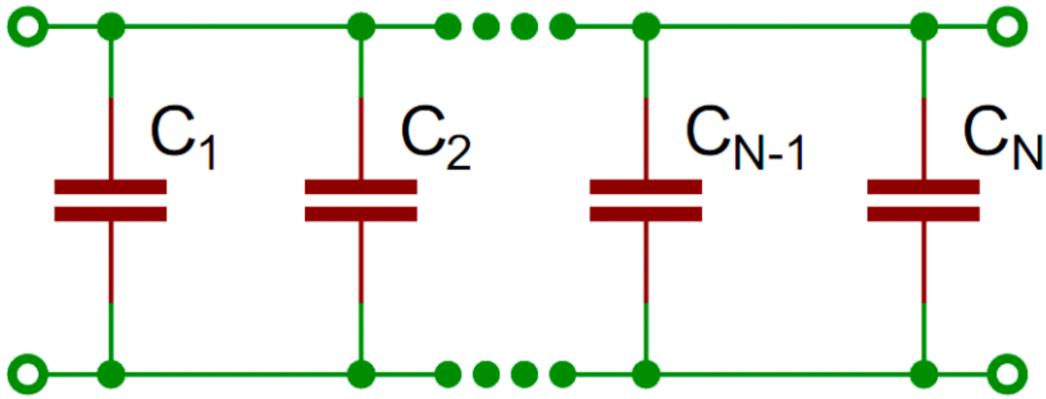
- Determine the equivalent capacitance of two capacitors in series
- $C_{eq} = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_{Tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_{N-1}} + \frac{1}{C_N}$$

Example 5: Capacitors in Parallel



- Determine the equivalent capacitance of two capacitors in parallel
- $C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$



$$C_{Tot} = C_1 + C_2 + \ldots + C_{N-1} + C_N$$

Example 6: Capacitance

- A capacitor stores 3 microcoulombs of charge with a potential difference of 1.5 volts across the plates. What is its capacitance? How much energy is stored in the capacitor?
- $C = \frac{Q}{V} = \frac{3 \times 10^{-6}C}{1.5V} = 2 \times 10^{-6}F$
- $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6}F \times 1.5V = 2.25 \times 10^{-6}J$

Example 7: Charge on a Capacitor

- How much charge sits on the top plate of a 200 nF capacitor when charged to a potential difference of 6 volts?
  - How much energy is stored in the capacitor when it is fully charged?
  - How much energy is stored in the capacitor when the voltage across its plates is 3 volts?

$C = 200 \cdot 10^{-9}F$
$V = 6V$

$C = \frac{Q}{V} \Rightarrow Q = CV = (200 \cdot 10^{-9}F)(6V) = \boxed{1.2 \cdot 10^{-6}C}$

$U = \frac{1}{2}CV^2 = \frac{1}{2}(200 \cdot 10^{-9}F)(6V)^2 = \boxed{3.6 \cdot 10^{-6}J}$

$U = \frac{1}{2}CV^2 = \frac{1}{2}(200 \cdot 10^{-9}F)(3V)^2 = \boxed{9 \cdot 10^{-7}J}$

Example 8: Designing a Capacitor

How far apart should the plates of an air-gap capacitor be if the area of the top plate is 5×10^{-4} m^2 and the capacitor must store 50 mJ of charge at an operating potential difference of 100 volts?

$A = 5 \cdot 10^{-4}\ m^2$

$U = .05\ J$

$V = 100\ V$

$$U = \frac{1}{2}CV^2 \xrightarrow{C = \frac{\epsilon_0 A}{d}} U = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2 \Rightarrow$$

$$d = \frac{\epsilon_0 A V^2}{2U} = \frac{(8.85 \cdot 10^{-12})(5 \cdot 10^{-4})(100)^2}{2(.05)} \Rightarrow$$

$$\boxed{d = 4.43 \cdot 10^{-10}\ m}$$

Example 9: Calculating Capacitance

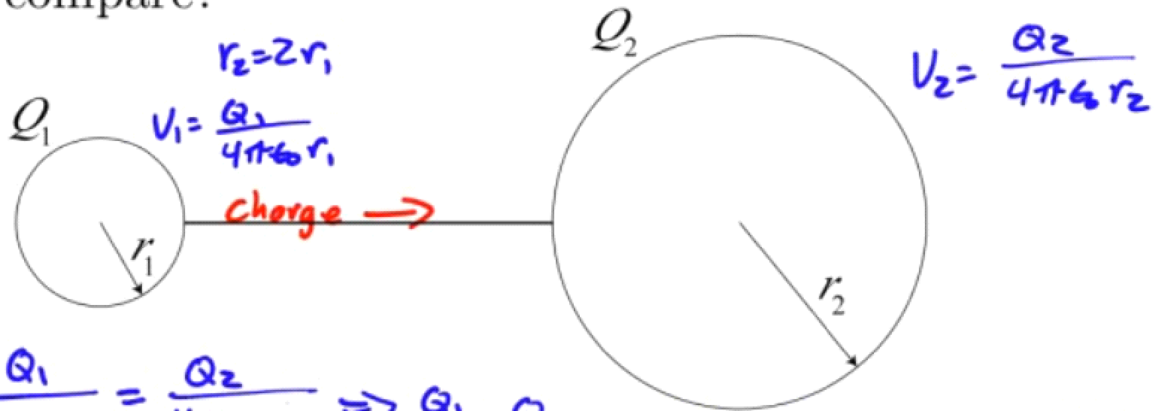
- Find the capacitance of two parallel plates of length 1 mm and width 2 mm if they are separated by 3 micrometers of air.
- What would the device's capacitance be if the 3 micrometers of air were replaced by 3 micrometers of SiO_2 (glass) which has a dielectric constant (relative permittivity) of 3.9?

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 (.002)(.001)}{3 \cdot 10^{-6}} = 5.9 \cdot 10^{-12}\ F = \boxed{5.9\ pF}$$

$$C = \frac{\epsilon A}{d} = \frac{3.9(8.854 \cdot 10^{-12})(.002)(.001)}{3 \cdot 10^{-6}} = 2.3 \cdot 10^{-11}\ F = \boxed{23\ pF}$$

Example 10: Two Conducting Spheres

Two conducting spheres, each with charge Q, are connected by a wire as shown. Do any charges flow between the spheres? How do their potentials compare?



$$\frac{Q_1}{4\pi\epsilon_0 r_1} = \frac{Q_2}{4\pi\epsilon_0 r_2} \Rightarrow \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{r_1} = \frac{Q_2}{2r_1} \Rightarrow Q_2 = \frac{2Q_1 r_1}{r_1} \Rightarrow$$

$$Q_2 = 2Q_1$$

potentials must be <u>equal</u>

Example 11: Inserting a Dielectric

An air-gap parallel plate capacitor is connected to a source of constant potential difference. Inserting a dielectric between the plates of the capacitor increases which of the following quantities? (Choose all that apply)

✓A) Charge on the capacitor

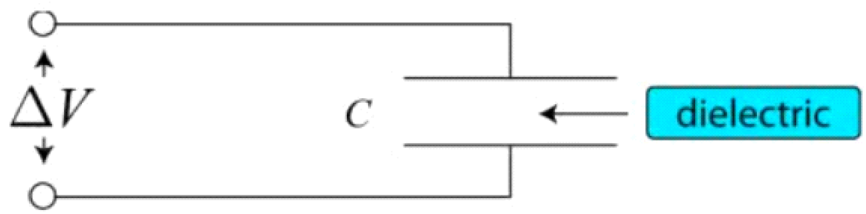
✗B) Voltage across the capacitor

✓C) Capacitance of the capacitor

✓D) Energy stored in the capacitor

✗E) Electric field strength between the plates

- V constant
- $C = \frac{\epsilon A}{d} \Rightarrow C \uparrow$
- $C = \frac{Q}{V} \Rightarrow Q = CV \Rightarrow Q \uparrow$
- $U = \frac{1}{2}CV^2 \Rightarrow U \uparrow$
- $V = \frac{E}{d} \Rightarrow E = Vd \Rightarrow E_{\text{constant}}$



2.1 - Current & Resistance

Thursday, March 2, 2017 8:32 AM

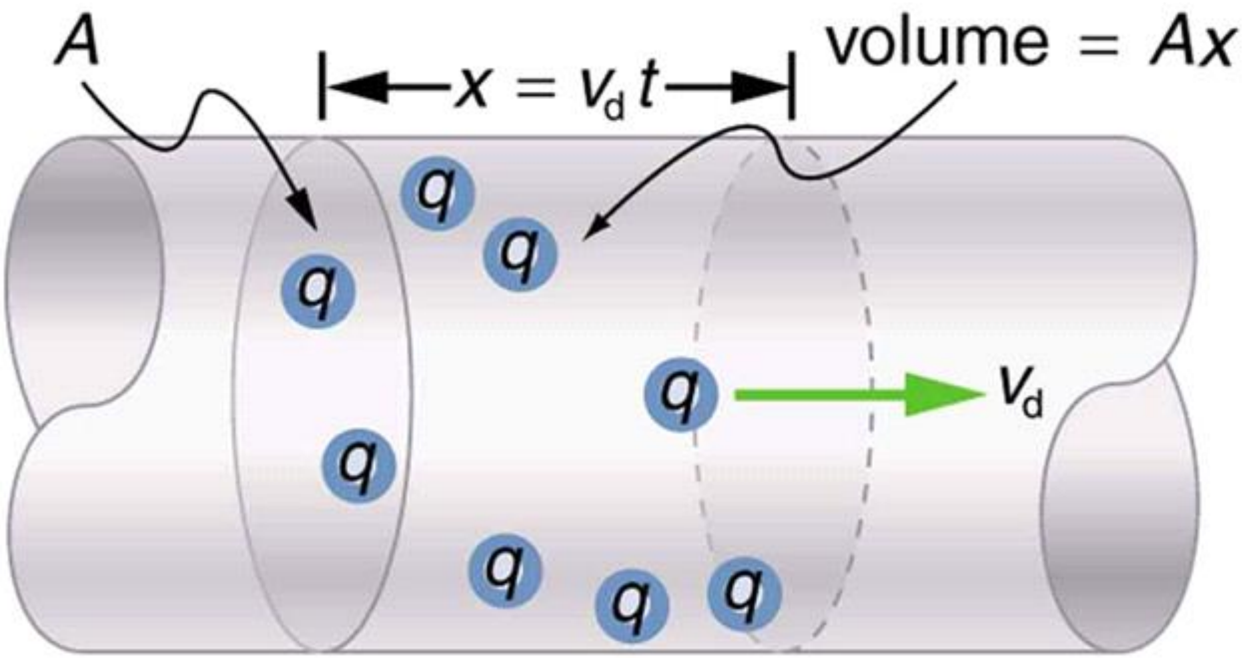
Electric Current

- Electric current is the flow rate of electric charge
  - Units are C/s, or amperes (A)
  - Positive current flow is the direction of the flow of positive charges, which is opposite the direction of electron flow
- $I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

Drift Velocity

- In a conductor, electrons are inconstant thermal motion.
- Net electron flow, however, is zero because the motion is random
- When an electric field is applied, a small net flow in a direction opposite the electric field is observed
- The average velocity of these electrons due to the electric field is known as the electron drift velocity v_d

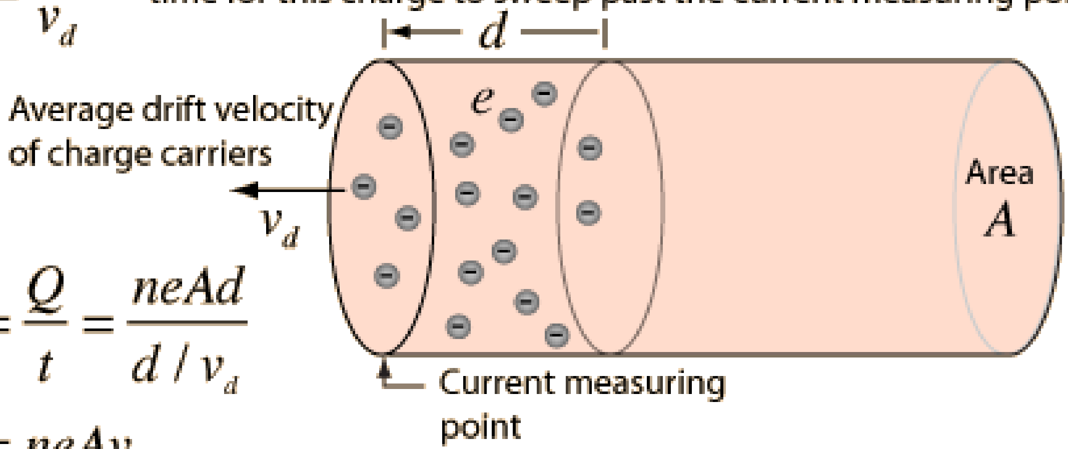
Derivation of Current Flow



n = number of charges e per unit volume

$Q = neAd$ = total mobile charge in length d of the conductor

$t = \dfrac{d}{v_d}$ = time for this charge to sweep past the current measuring point.

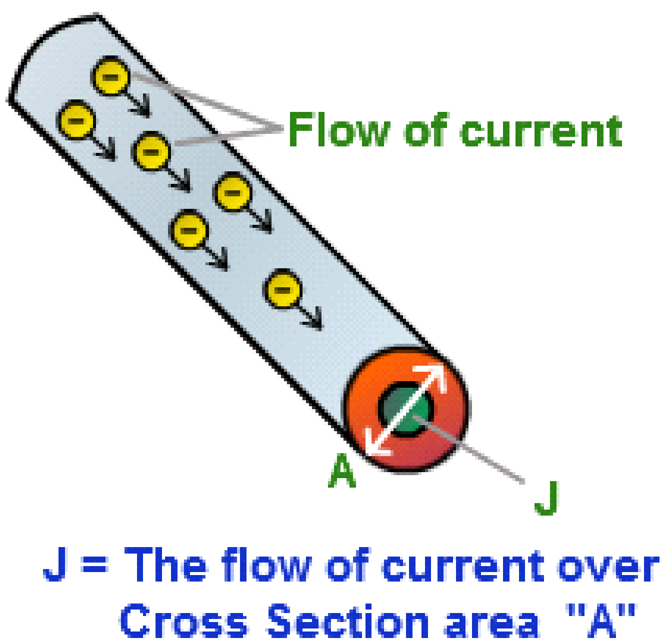


$$I = \frac{Q}{t} = \frac{neAd}{d / v_d}$$

$$I = neAv_d$$

Current Density

- Current Density through a surface is the current per area, and is a vector quantity $\vec{J}$

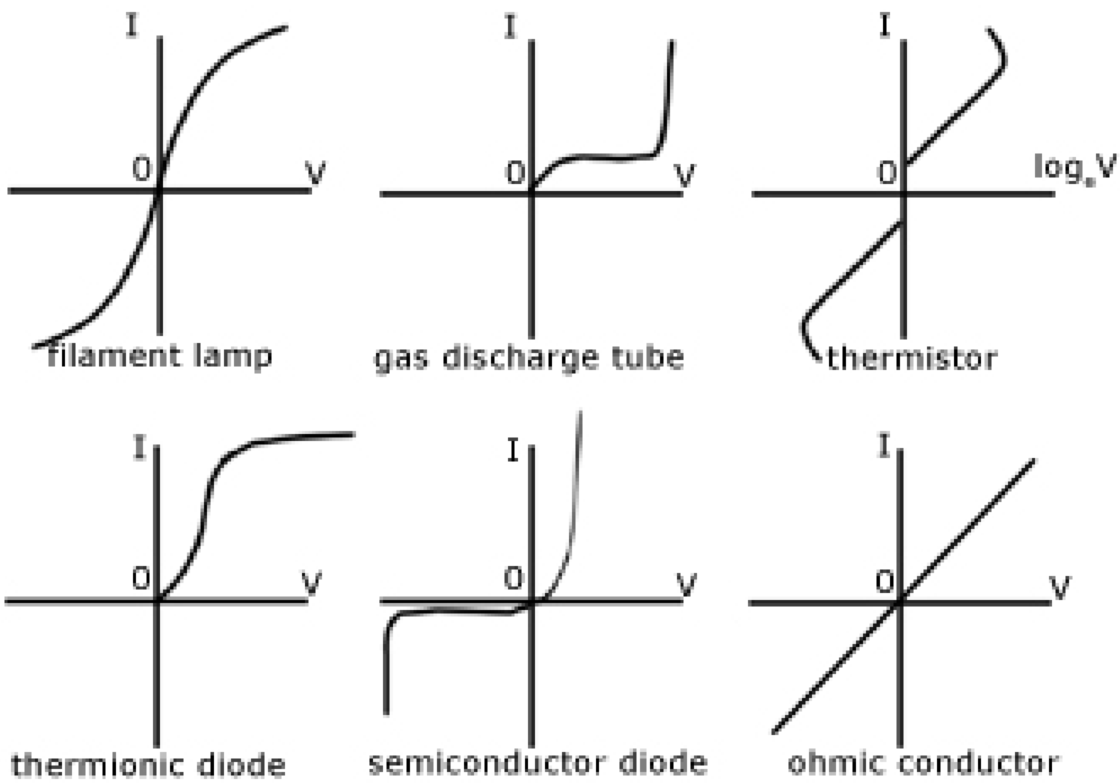


$$\mathbf{I} = \int_{current\ flow\ region} \mathbf{J} \cdot d\mathbf{S}$$

[Total Current] = Sum of Current Density over the region where the current flows

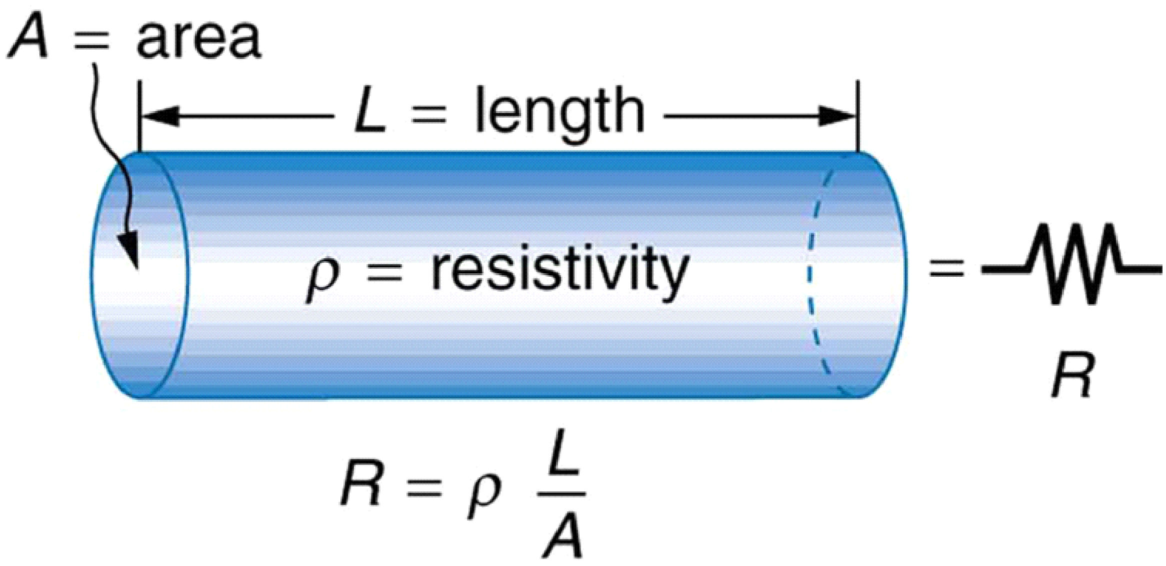
Resistance

- Resistance is the ratio of the potential drop across an object to the current flowing through the object.
- Object which have a fixed resistance (not a function of current of potential drop) are known as Ohmic materials and are said to follow Ohm's Law (an empirical law)



Resistance of a Wire

- The resistance of a wire depends on the geometry of the wire as well as a material property known as resistivity ρ (Unit: Ω·m)
- Resistivity relates to the ability of a material to resist the flow of electrons



Refining Ohm's Law

- $V = IR = \frac{I\rho L}{A}$
- $E = \frac{V}{L} = \frac{\rho I}{A}$
- $\because J = \frac{I}{A}, \qquad I = JA$
- $\therefore \vec{E} = \rho\vec{J}$

Conversion of Electric Energy to Thermal Energy

- $W = qV$
- $\therefore P = \frac{dW}{dt} = \frac{d}{dt}(qV) = V\frac{dq}{dt} = VI$
- $P = IV = I^2R = \frac{V^2}{R}$

Example 1: Silver Wire

- A silver wire with a 0.5-mm-radius cross-section is connected to the terminal of a 1V battery. The wire is 0.1 m long. The resistivity of silver is $1.59 \times 10^{-8}\ \Omega \cdot \mathrm{m}$, its molar mass is 107.9 $g/mole$, and its mass density is 10.5 g/cm^3

1. Determine the resistance of the wire
   - $R = \frac{\rho L}{A} = \frac{(1.59 \times 10^{-8})(0.1)}{\pi(0.0005)^2} = 0.00202\ \Omega$
2. Determine the current flowing through the wire
   - $I = \frac{V}{R} = \frac{1}{0.00202} = 494\ A$
3. Determine the drift velocity of the free electrons in the wire (assume one free electron per atom)
   - $N = \frac{N_A}{V} = \frac{N_A}{M/\rho_{silver}} = \frac{N_A\rho_{silver}}{M}$
   - $\because I = Nev_dA$

- $\therefore v_d = \frac{I}{NeA} = \frac{IM}{N_A \rho_{silver} eA} = \frac{(494)(0.1079)}{(6.02 \times 10^{23})(10500)(1.6 \times 10^{-19})(\pi \times 0.0005^2)} = 0.067\ m/s$

4. Determine the average time required for electrons to pass from the negative terminal of the battery to the positive terminal
   - $t = \frac{L}{v_d} = \frac{0.1}{0.067} = 1.49\ s$

Example 2: Aluminum Wire

- 12-gauge aluminum wire with a cross-sectional area of $3.31 \times 10^{-6}\ m^2$ carries a 4A current. The density of alumnium is 2.7 g/cm^3. Find the drift velocity of the electrons in the wire, assuming each aluminum atom supplies one conduction electron
- $v_d = \frac{I}{NeA} = \frac{IM}{N_A \rho_{Al} eA} = \frac{(4)(0.027)}{(6.02 \times 10^{23})(2700)(1.6 \times 10^{-19})(3.31 \times 10^{-6})} = 1.25 \times 10^{-4}\ m/s$

2.2 - Circuits I: Series Circuits

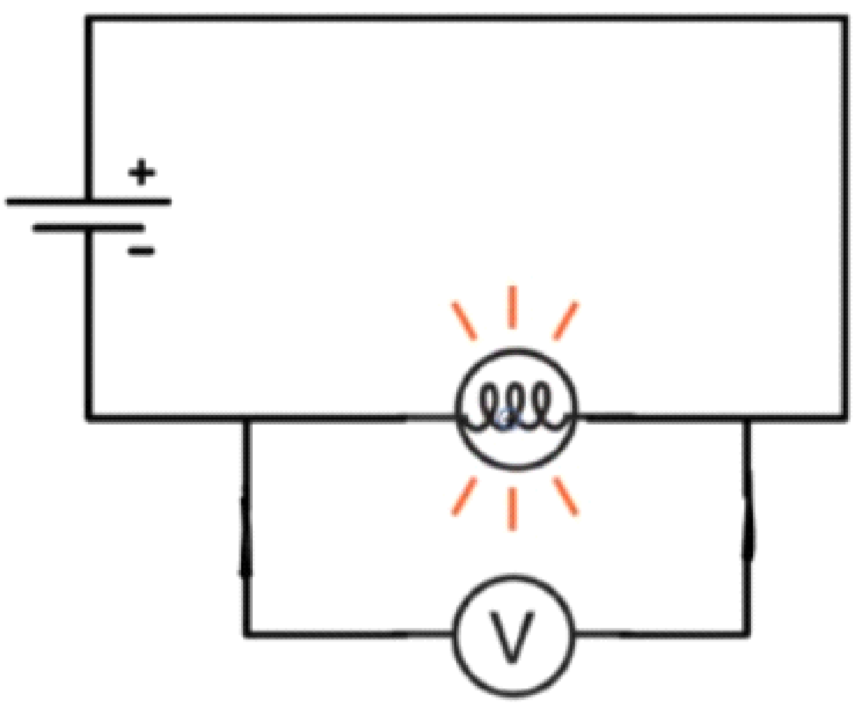
Thursday, March 2, 2017 8:32 AM

Electrical Circuits

- An electrical circuit is a closed-loop path through which current can flow
- An electrical circuit can be made up of most any materials but practically speaking, circuits are typically comprised of electrical devices
  - Wires
  - Batteries
  - Resistors
  - Switches
- Conventional current flows from high potential to low potential

Voltmeters

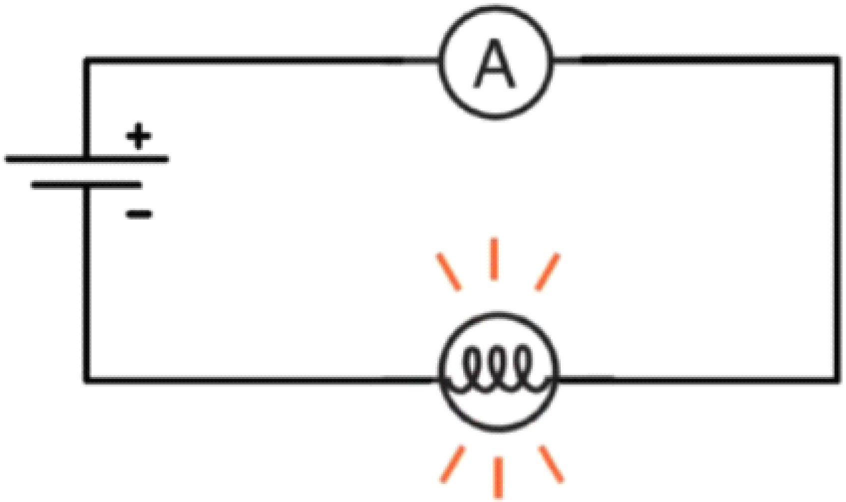
- Voltmeters measure the potential difference between two points in a circuit
- Voltmeters are connected in parallel with the element to be measured
- If a voltmeter is connected correctly, you can remove it from the circuit without breaking the circuit
- Voltmeters have very high resistance



Ammeters

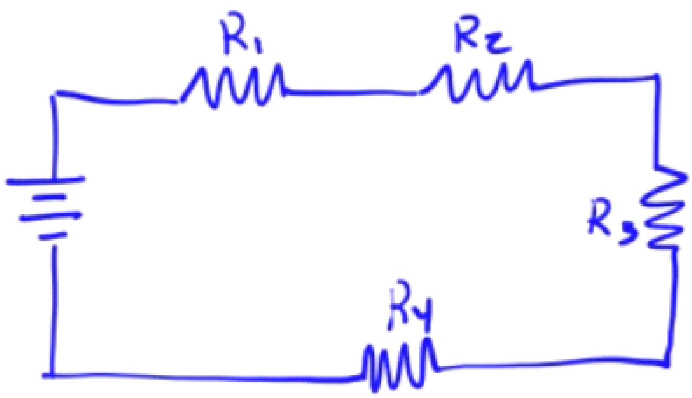
- Ammeters measure the current flowing through an element of a circuit
- Ammeters are connected in series with the circuit, so that the current to be measured flows through the ammeter
- The circuit must be broken to correctly insert an ammeter

- Ammeters have very low resistance to minimize the potential drop through the ammeter



Series Circuits

- Series circuits have only a single current path
- Removal of any circuit element causes an open circuit



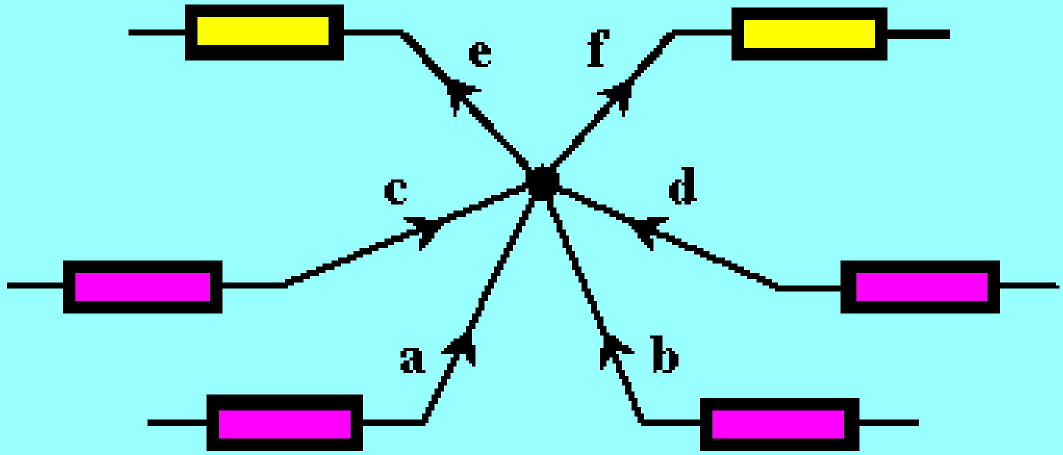
$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

$$I_{TOT} = I_1 = I_2 = I_3 = \dots$$

$$V_{TOT} = V_1 + V_2 + V_3 + \dots$$

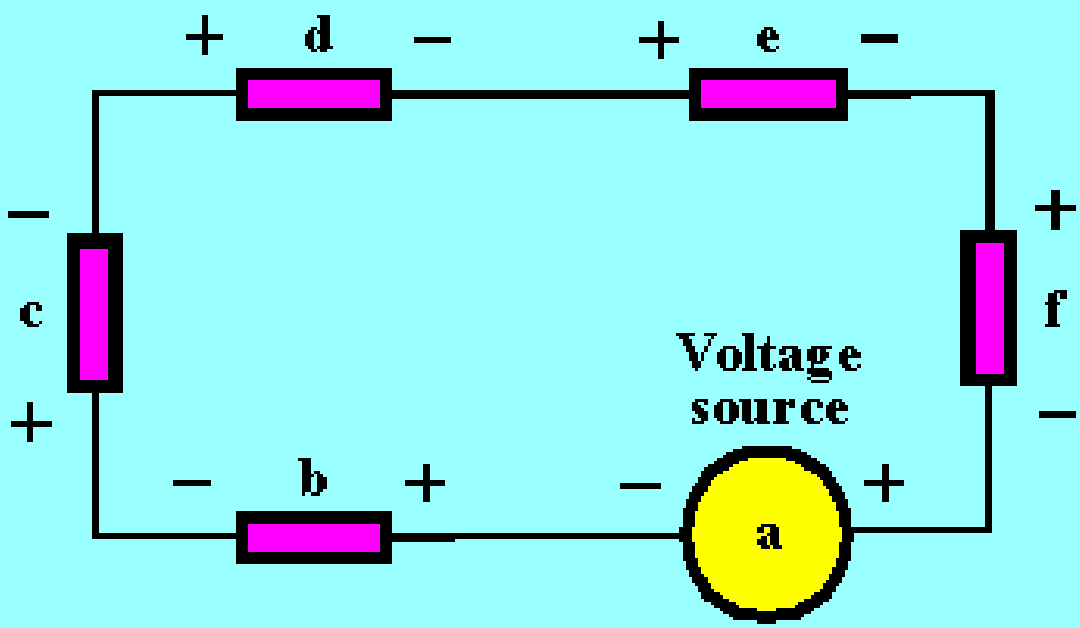
Kirchhoff's Laws

- Kirchhoff's Laws are tools utilized in analyzing circuits
- Kirchhoff's Current Law (KCL) states that the sum of all current entering any point in a circuit equals the sum of all current leaving any point in a circuit
    - Restatement of conservation of charge
    - aka "Junction Rule"
- Kirchhoff's Voltage Law (KVL) states that the sum of all the potential drops in any closed loop of a circuit has to equal zero
    - Restatement of conservation of energy
    - aka "Loop Rule"



First Law

a + b + c + d = e + f



Second Law

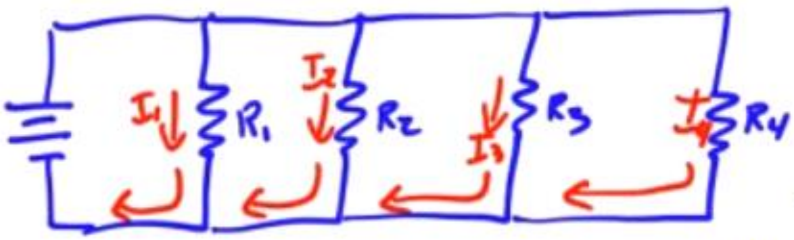
a + b + c + d + e + f = 0

2.3 - Circuits II: Parallel Circuits

Thursday, March 2, 2017 8:32 AM

Parallel Circuits

- Parallel Circuits have multiple current paths
- Removal of a circuit element may allow other branches of the circuit to continue operating



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

$$I_{TOT} = I_1 + I_2 + \cdots$$

$$V_{TOT} = V_1 = V_2 = V_3 = \cdots$$

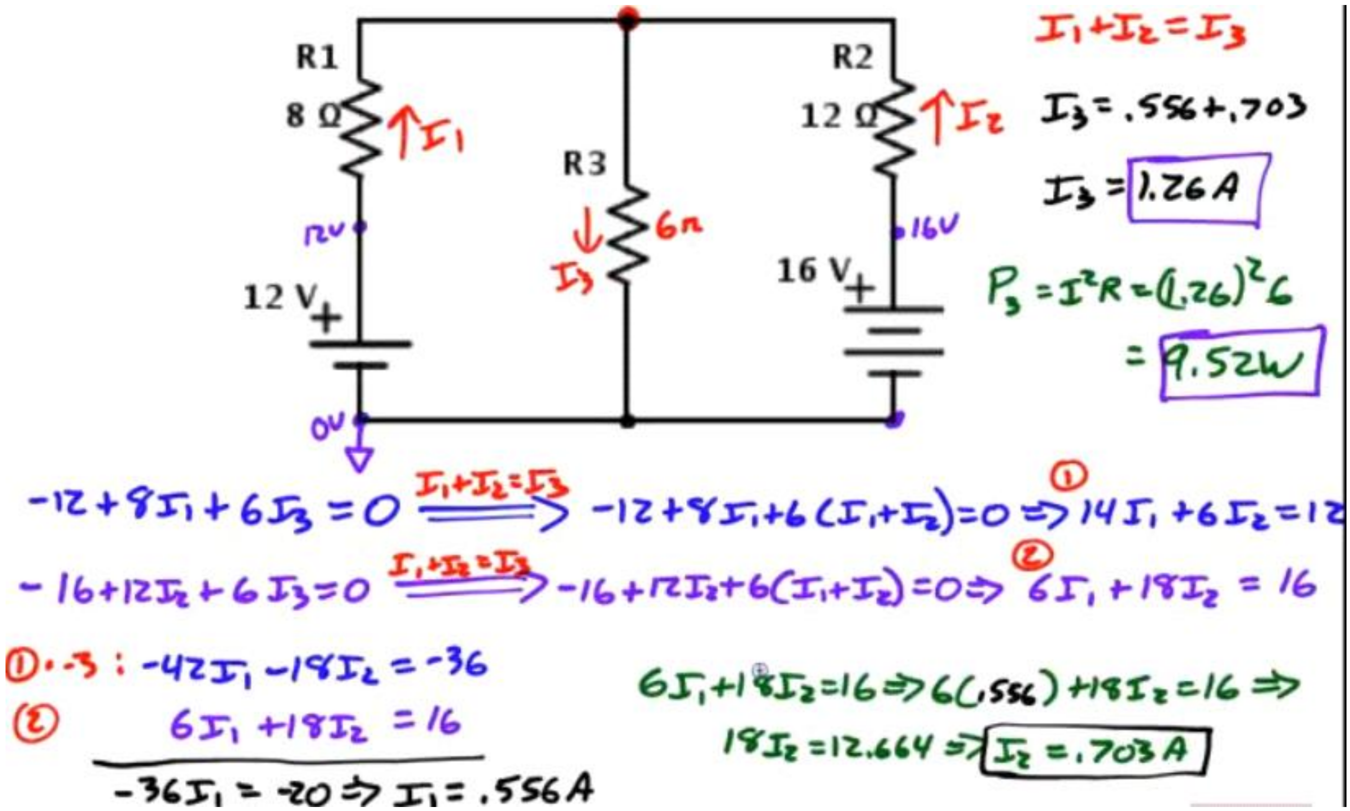
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Combination Series-Parallel Circuits

- A circuit doesn't have to be completely serial or parallel
- First, look for portions of the circuit with parallel elements.
- Replace parallel resistors with an equivalent single resistor
- Work back to original circuit using KCL and KVL until you know the current, voltage, and resistance of each individual circuit element
- Often times will lead to systems of equations to solve

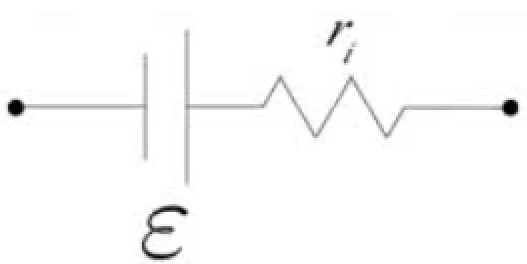
Example 1: Two Voltage Sources

- Find the current flowing through R3 if R3 has a value of 6 ohms. What is the power dissipated in R3?

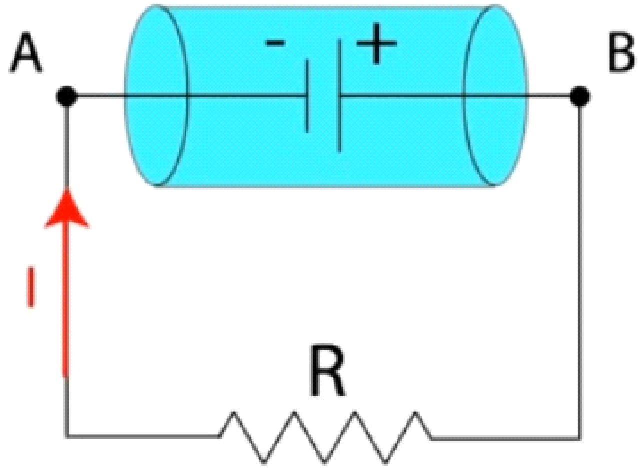


Batteries

- A cell or battery (combination of cells) provides a potential difference, oftentimes referred to as an electromotive force or emf.
- A battery can be thought of as a pump for charge, raising it from a lower potential to a higher potential
- Ideal batteries have no resistance
- Real batteries have some amount of resistance to the flow of charge within the battery itself, known as the internal resistance r_i
- In real batteries, the terminal voltage is slightly lower than the battery's emf

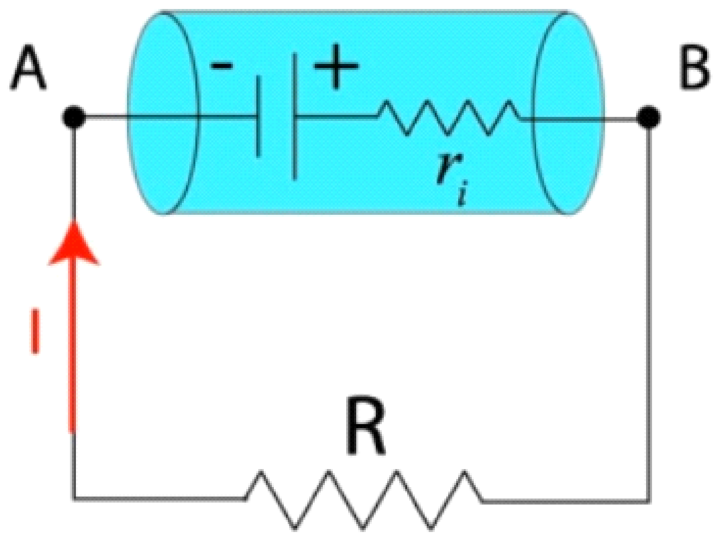


Ideal Battery



$V_{battery} = \Delta V = V_B - V_A \Rightarrow V_{Battery} = \mathcal{E} = V_T$

Real Battery



$V_{battery} = \Delta V = V_B - V_A$

$V_{battery} = IR = \boxed{\mathcal{E} - Ir_i = V_T}$

2.4 - RC Circuits: Steady State

Thursday, March 2, 2017 9:55 AM

Capacitors in Parallel

- Capacitors store charge on their plates
- Capacitors in parallel can be replaced with an equivalent capacitor
- $C_{eq} = C_1 + C_2 + C_3 + \cdots$

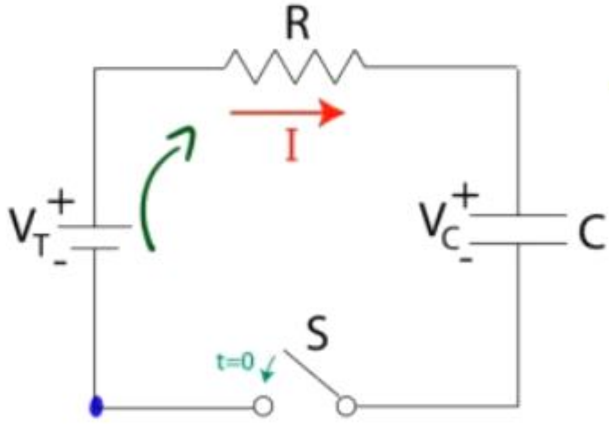
Capacitors in Series

- Charge on capacitors must be the same
- Capacitors in series replaced with an equivalent capacitor
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

RC Circuits

- RC Circuits are circuits comprised of a source of potential difference, a resistor network, and one or more capacitors
- We will look at RC circuits from the steady-state perspective
  - What happens when first turned on
  - What happens after a "long" time has elapsed
- Key to understanding RC Circuit Performance
  - Uncharged capacitors act like wires
  - Charged capacitors act like opens

Charging an RC Circuit



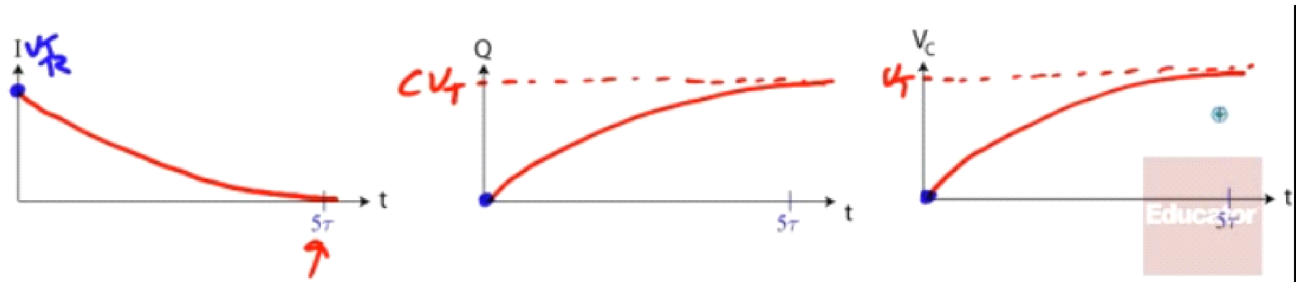
$-V_T + IR + V_c = 0 \xrightarrow[V_c = Q/c]{C = \frac{Q}{V_c}}$

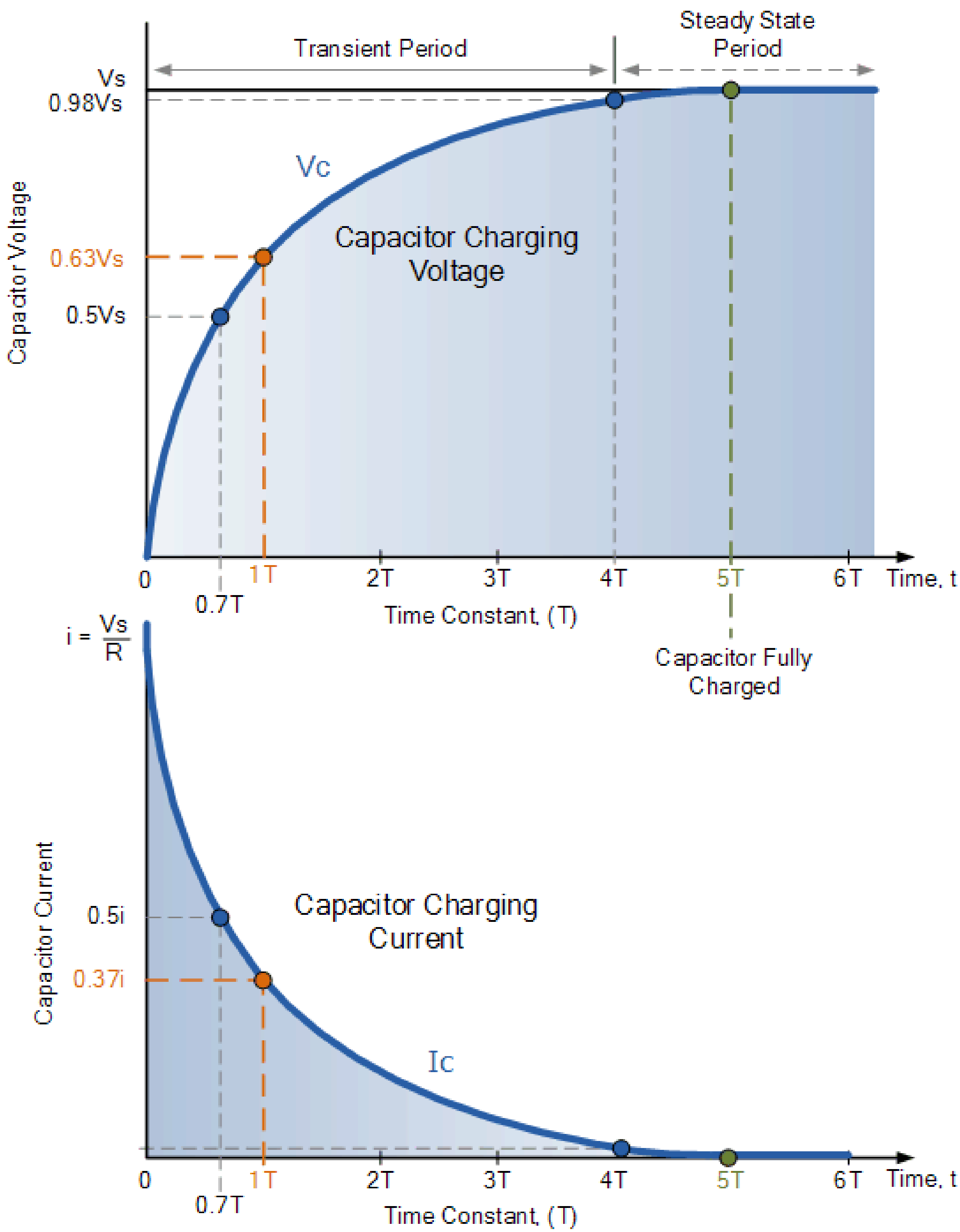
$-V_T + IR + \frac{Q}{C} = 0 \xrightarrow[Q(t=0)=0]{}$

$-V_T + IR = 0 \Rightarrow V_T = IR$

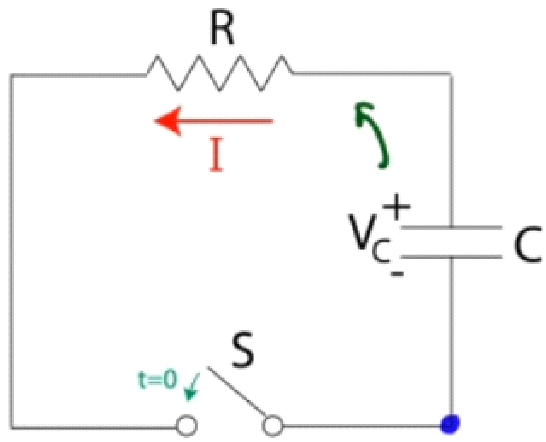
Cap acts like an open

$t \to \infty: -V_T + IR + V_c = 0 \xRightarrow{I=0} V_T = V_c$


I
V_T/R
t
5τ
Q
CV_T
t
5τ
V_C
V_T
t
5τ



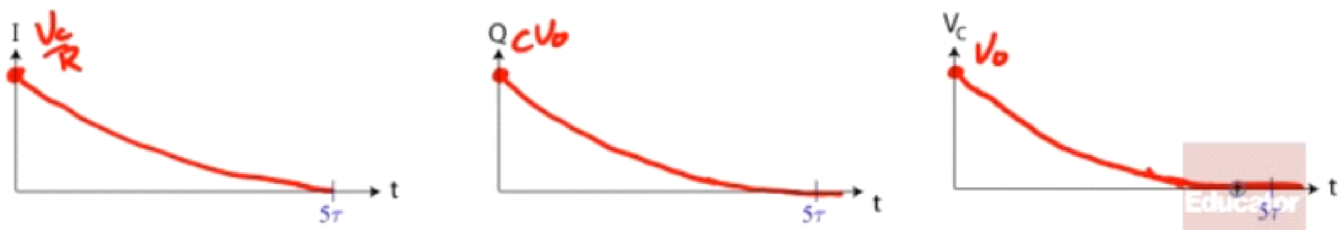
Discharging an RC Circuit

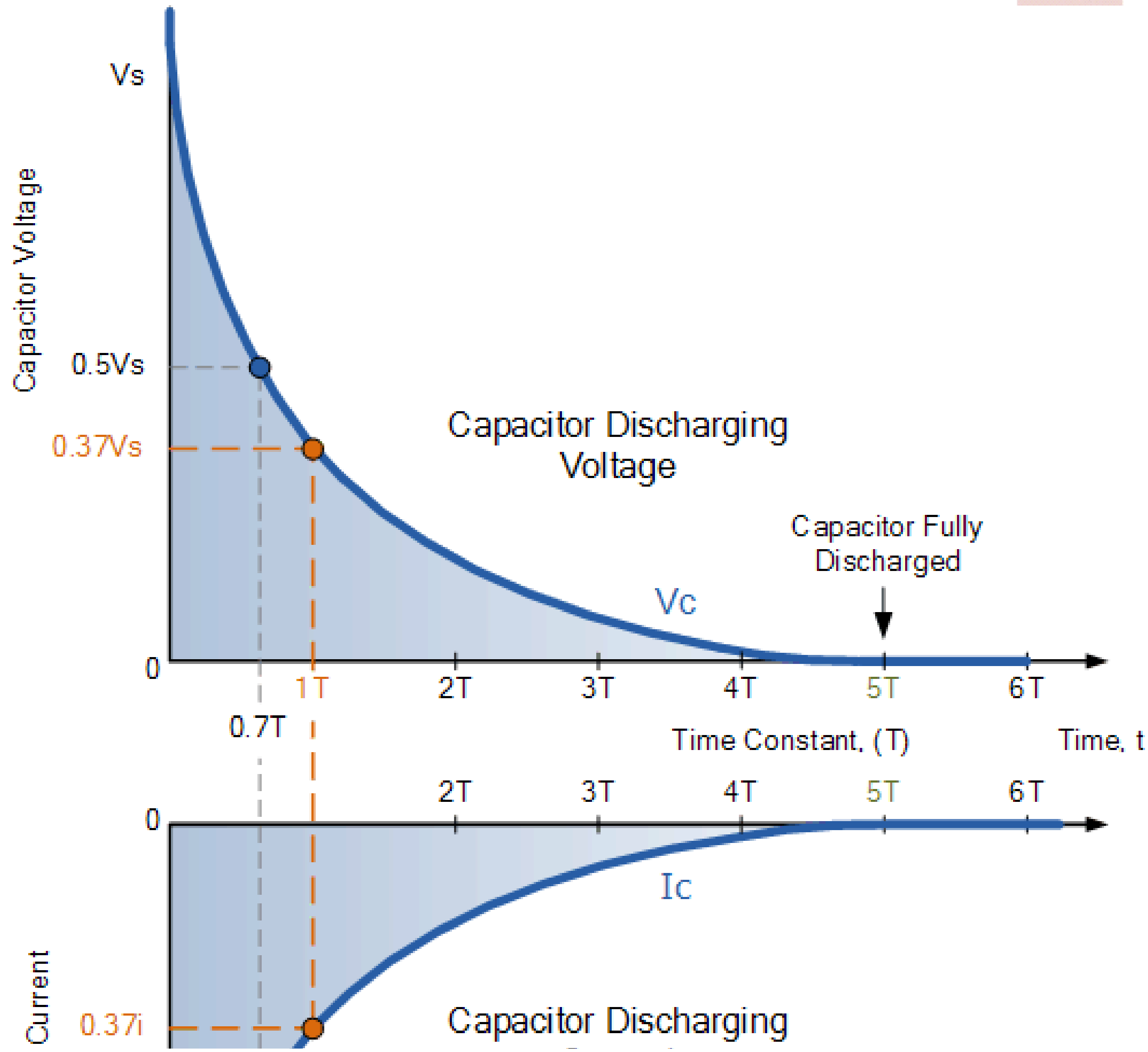


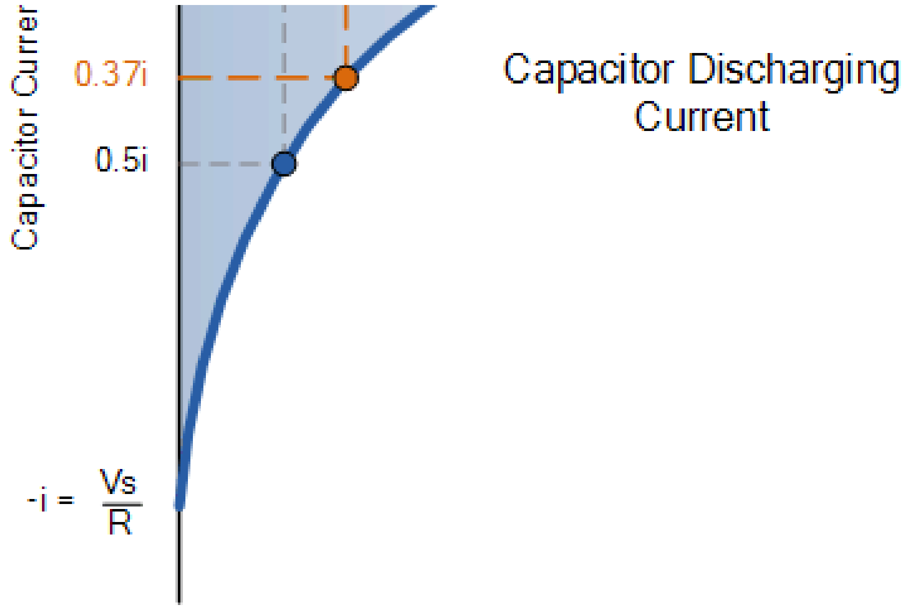
$-V_c + IR = 0 \Longrightarrow I = \frac{V_c}{R}$ $\quad Q = CV$

Long time -> Capacitor acts like wire

$-V_c + IR = 0 \xRightarrow{V_c=0} I = 0 \quad Q = 0$





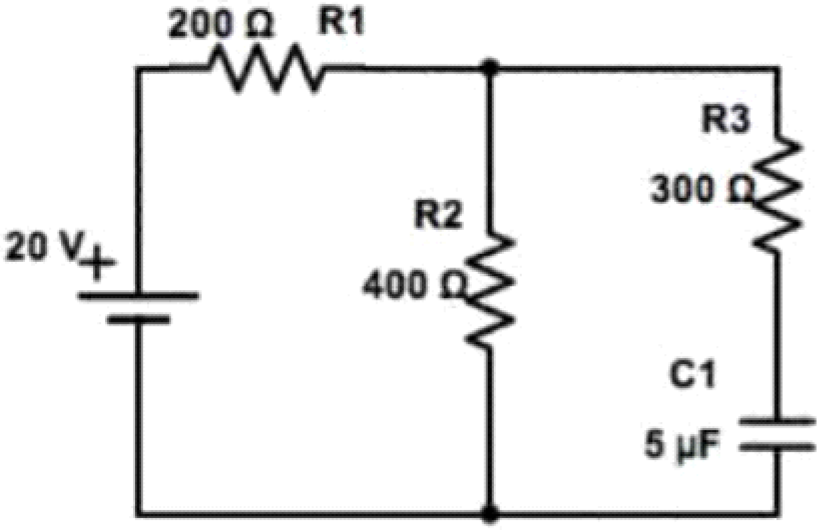


The Time Constant

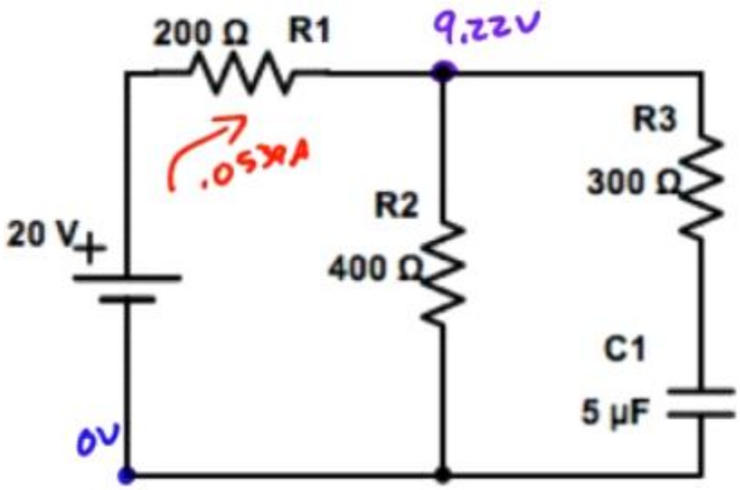
- The time constant in an RC circuit ($\tau = RC$) indicates the time at which the quantity under observation has achieved $1 - \frac{1}{e} \approx 63\%$ of its final value
- By 5 time constants ($t = 5\tau = 5RC$), the quantity under observation is within 1 percent of its final value

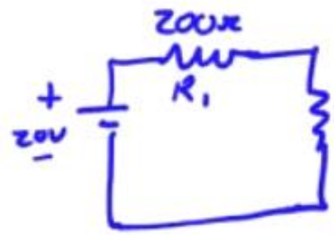
Time Constant	RC Value	Percentage of Maximum	
		Voltage	Current
0.5 time constant	0.5T = 0.5RC	39.3%	60.7%
0.7 time constant	0.7T = 0.7RC	50.3%	49.7%
1.0 time constant	1T = 1RC	63.2%	36.8%
2.0 time constants	2T = 2RC	86.5%	13.5%
3.0 time constants	3T = 3RC	95.0%	5.0%
4.0 time constants	4T = 4RC	98.2%	1.8%
5.0 time constants	5T = 5RC	99.3%	0.7%

Example 1: RC Analysis



- What is the current through R2 when the circuit is first connected?





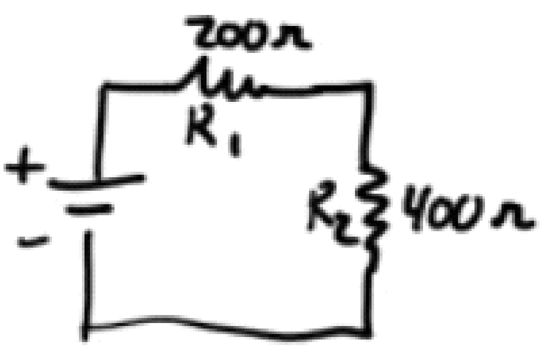
$R_{23} = \frac{(400)(300)}{700} = 171\,\Omega$

$I = \frac{V}{R} = \frac{20V}{371} = .0539\,A$

$I_2 = \frac{V_2}{R_2} = \frac{9.22V}{400\Omega} = \boxed{.0231A}$

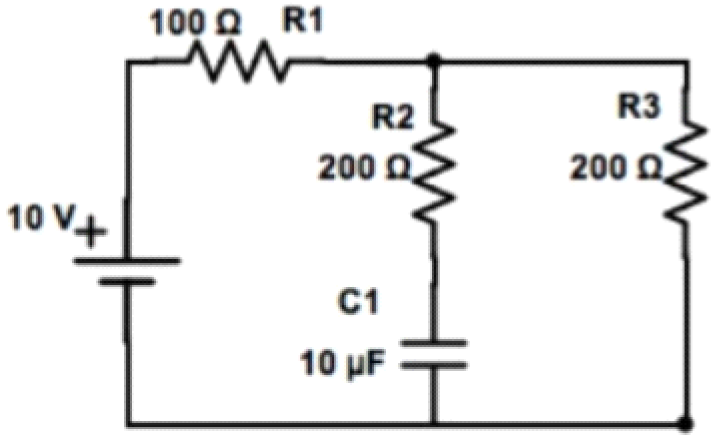
- What is the current through R2 a long time after the circuit has been connected?

Long Time ⇒ Cap acts like an open

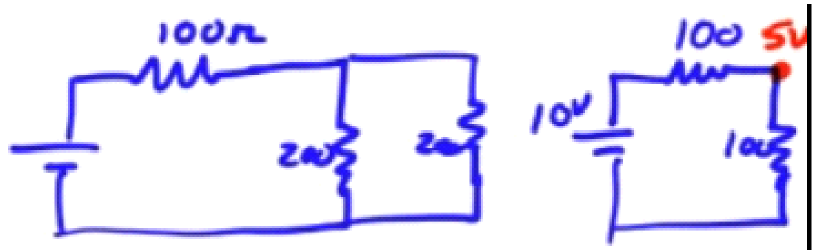


$I = \frac{V}{R} = \frac{20V}{600\Omega} = \boxed{.0333A}$

Example 2: More RC Analysis

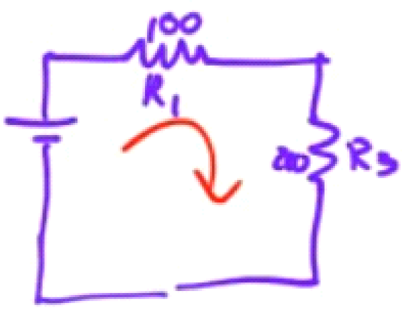


- What is the current through R3 when the circuit is first connected?



$$I_3 = \frac{V}{R} = \frac{5V}{200\Omega} = \boxed{.025\,A}$$

- What is the current through R2 a long time after the circuit has been connected

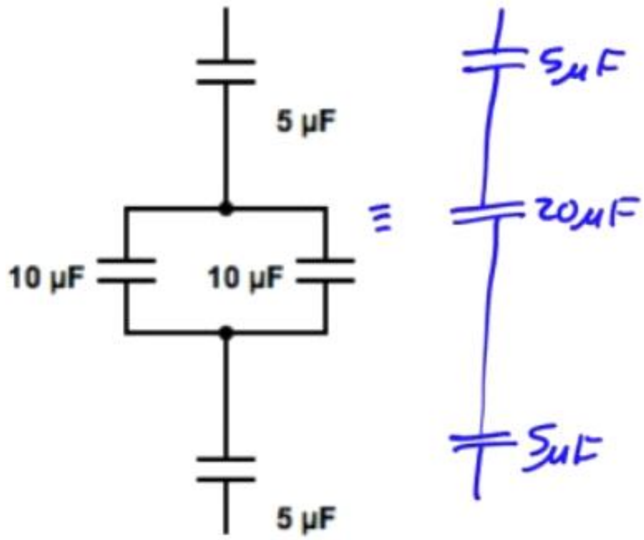


$$I = \frac{V}{R} = \frac{10V}{300\Omega} = .0333\,A$$

$$I_2 = 0$$

Example 3: Equivalent Capacitance

- What is the equivalent capacitance of the capacitor network shown below?



$$\frac{1}{C_{eq}} = \frac{1}{5\mu F} + \frac{1}{20\mu F} + \frac{1}{5\mu F}$$

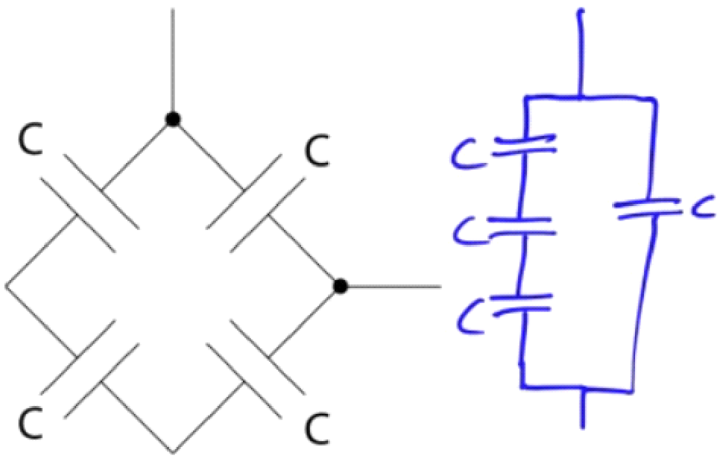
$$\frac{1}{C_{eq}} = 450{,}000 \Rightarrow$$

$$C_{eq} = \frac{1}{450{,}000} = 2.22 \cdot 10^{-6} F$$

$$\boxed{2.22\mu F}$$

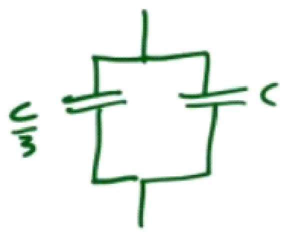
Example 4: More Equivalent Capacitance

- What is the equivalent capacitance of the capacitor network shown below?



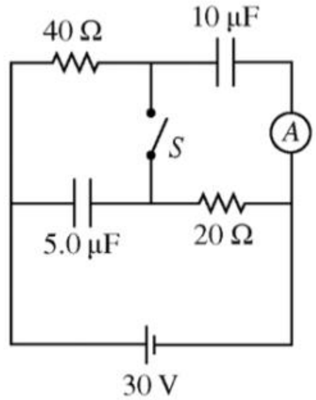
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C_{eq} = \frac{C}{3}$$



$$\boxed{C_{TOT} = \frac{4C}{3}}$$

2010 Free Response Question 2



E&M. 2.

In the circuit illustrated above, switch *S* is initially open and the battery has been connected for a long time.

(a) What is the steady-state current through the ammeter?

(b) Calculate the charge on the 10 μF capacitor.

(c) Calculate the energy stored in the 5.0 μF capacitor.

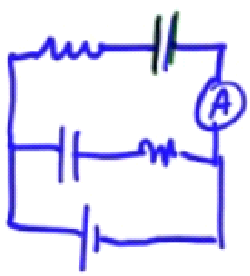
The switch is now closed, and the circuit comes to a new steady state.

(d) Calculate the steady-state current through the battery.

(e) Calculate the final charge on the 5.0 μF capacitor.

(f) Calculate the energy dissipated as heat in the 40 Ω resistor in one minute once the circuit has reached steady state.

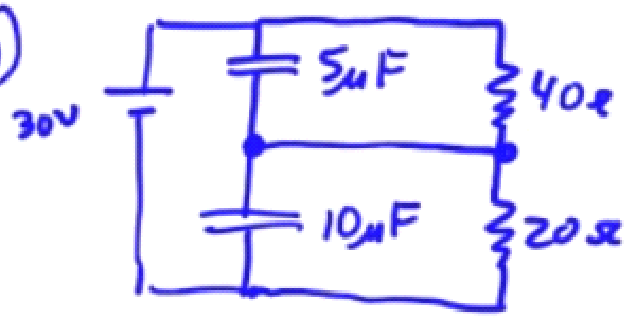
a) after a long time, $I_{cap}=0 \Rightarrow$ $\boxed{\phi \text{ current}}$ through ammeter



b) $C=\frac{Q}{V} \Rightarrow Q=CV=(10\cdot10^{-6}F)(30V)=\boxed{300\mu C}$

c) $U=\frac{1}{2}CV^2=\frac{1}{2}(5\cdot10^{-6}F)(30V)^2=2250\mu J=\boxed{2.25\, mJ}$

d)



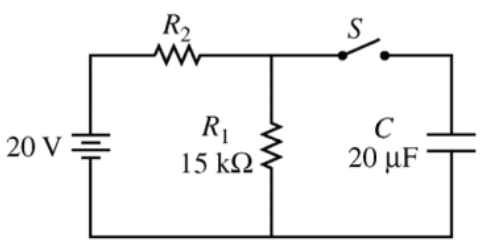
$I = \frac{V}{R} = \frac{30V}{60\Omega} = \boxed{0.5A}$

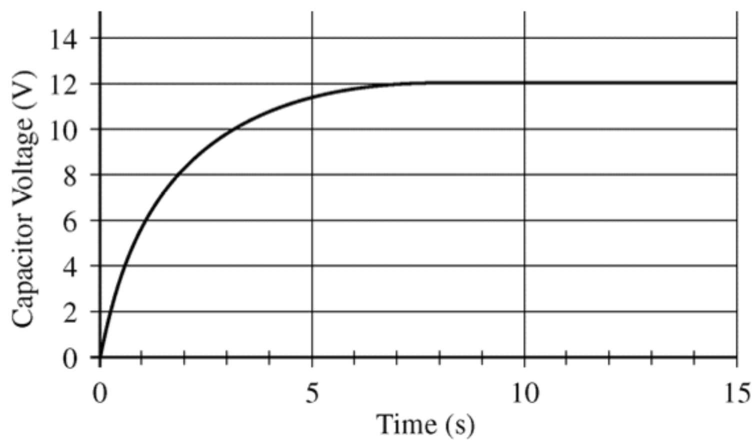
e) $Q = CV = (5 \cdot 10^{-6} F)(20V) = \boxed{100\mu C}$

f) $P = I^2 R = (0.5A)^2 (40\Omega) = \boxed{10W}$

$E = Pt = (10W)(60s) = \boxed{600J}$

2004 Free Response Question 2





E&M. 2.

In the circuit shown above left, the switch S is initially in the open position and the capacitor C is initially uncharged. A voltage probe and a computer (not shown) are used to measure the potential difference across the capacitor as a function of time after the switch is closed. The graph produced by the computer is shown above right. The battery has an emf of 20 V and negligible internal resistance. Resistor R_1 has a resistance of 15 kΩ and the capacitor C has a capacitance of 20 μF.

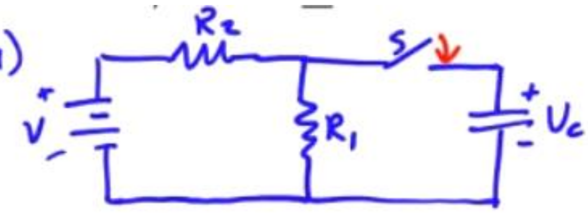
(a) Determine the voltage across resistor R_2 immediately after the switch is closed.

(b) Determine the voltage across resistor R_2 a long time after the switch is closed.

(c) Calculate the value of the resistor R_2.

(d) Calculate the energy stored in the capacitor a long time after the switch is closed.

a)



$@ t=0, V_c \Rightarrow V_{R_1}=0 \Rightarrow V_{R_2} = \boxed{20V}$

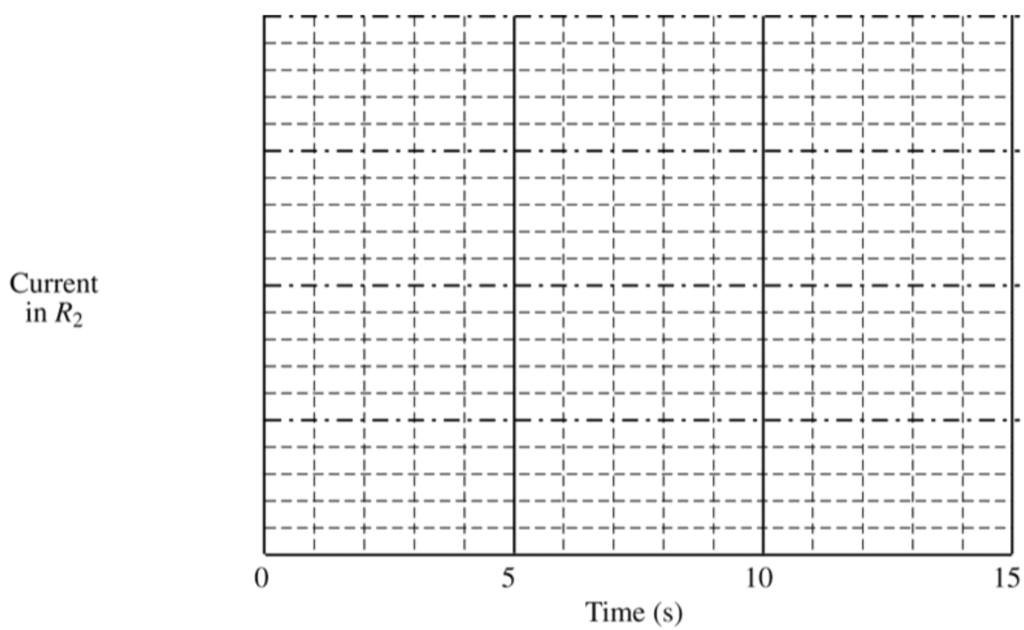
b) $t \to \infty, V_c = 12V \Rightarrow V_{R_1} = 12V \Rightarrow V_{R_2} = \boxed{8V}$

c) $t \to \infty, V_{R_1} = 12V \Rightarrow I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{12V}{15000} \Rightarrow .0008\,A$

$V_{R_2} = 8V = I_{R_2} R_2 \Rightarrow R_2 = \frac{V_{R_2}}{I_{R_1}} = \frac{8V}{.0008A} = 10{,}000\Omega = \boxed{10k\Omega}$

d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(20 \cdot 10^{-6}F)(12V)^2 = .00144J = \boxed{1.44\,mJ}$

(e) On the axes below, graph the current in R_2 as a function of time from 0 to 15 s. Label the vertical axis with appropriate values.



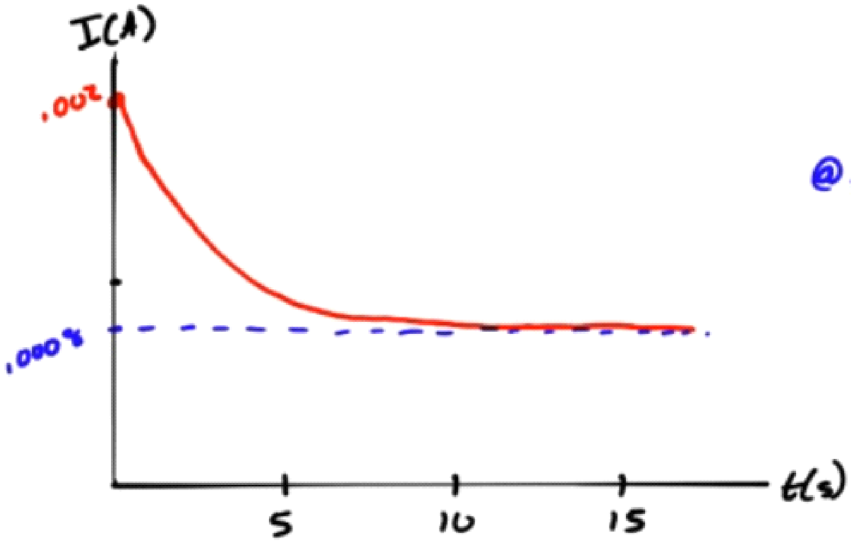
Resistor R_2 is removed and replaced with another resistor of lesser resistance. Switch S remains closed for a long time.

(f) Indicate below whether the energy stored in the capacitor is greater than, less than, or the same as it was with resistor R_2 in the circuit.

____ Greater than ____ Less than ____ The same as

Explain your reasoning.

$@ t=0, I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{20V}{10k\Omega} = .002\,A$



@ $t=0$, $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{20V}{10k\Omega} =$ .002 A

@ $t \to \infty$, $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{8V}{10k\Omega} = .00$

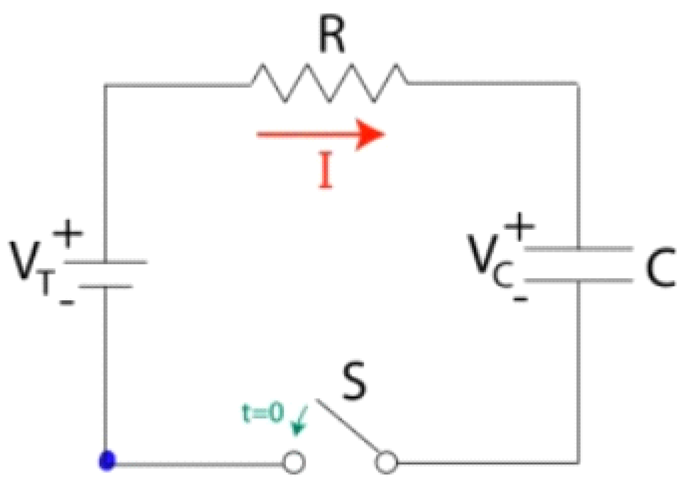
f) $U = \frac{1}{2}CV^2$, but V_c goes up if R_2 decreases, therefore $U \uparrow$

Greater Than

2.5 - RC Circuits: Transient Analysis

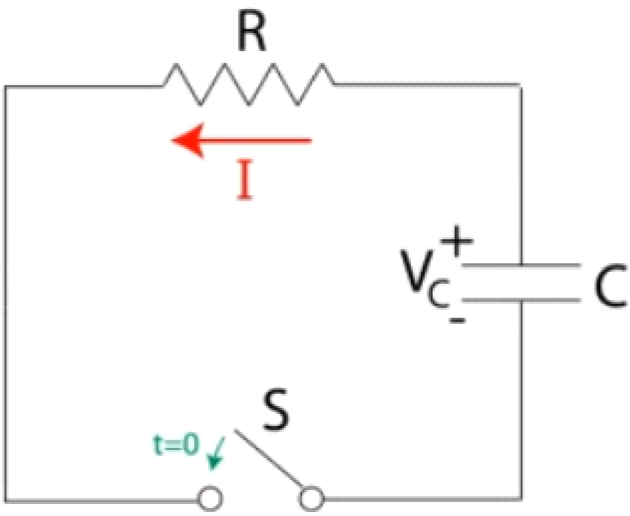
Thursday, March 2, 2017 9:55 AM

Charging an RC Circuit



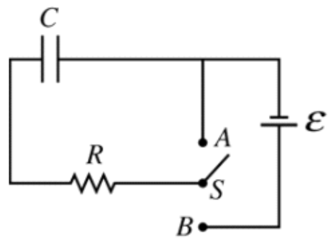
- $V_T = IR + V_C = R\dfrac{dQ}{dt} + \dfrac{Q}{C}$
- $\dfrac{dQ}{C \cdot V_T - Q} = \dfrac{dt}{RC}$
- $\displaystyle\int_{Q=0}^{Q} \frac{dQ}{C \cdot V_T - Q} = \int_{t=0}^{t} \frac{dt}{RC}$
- $-\ln(C \cdot V_T - Q)\Big|_0^Q = \dfrac{1}{RC}\,(t)\Big|_0^t$
- $-\ln(C \cdot V_T - Q) + \ln C \cdot V_T = \dfrac{t}{RC}$
- $\ln\left(1 - \dfrac{Q}{C \cdot V_T}\right) = -\dfrac{t}{RC}$
- $1 - \dfrac{Q}{C \cdot V_T} = e^{-\frac{t}{RC}}$
- $Q = C \cdot V_T\left(1 - e^{-\frac{t}{RC}}\right)$
- $V_c = \dfrac{Q}{C} = V_T\left(1 - e^{-\frac{t}{RC}}\right)$
- $I = \dfrac{dQ}{dt} = \dfrac{d}{dt}\left(C \cdot V_T\left(1 - e^{-\frac{t}{RC}}\right)\right) = \dfrac{V_T}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$

Discharging an RC Circuit



- $V_C = IR$
- $\frac{Q}{C} = -\frac{dQ}{dt}R$
- $\frac{dQ}{Q} = -\frac{dt}{RC}$
- $\int_{Q=Q_0}^{Q} \frac{dQ}{Q} = -\int_{t=0}^{t} \frac{dt}{RC}$
- $\ln(Q)\Big|_{Q_0}^{Q} = -\frac{1}{RC}\,(t)\Big|_{0}^{t}$
- $\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$
- $\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$
- $Q = Q_0 \cdot e^{-\frac{t}{RC}}$
- $V_c = \frac{Q}{C} = \frac{Q_0}{C} \cdot e^{-\frac{t}{RC}} = V_0 \cdot e^{-\frac{t}{RC}}$
- $I = -\frac{dQ}{dt} = -\frac{d}{dt}\left(Q_0 \cdot e^{-\frac{t}{RC}}\right) = \frac{Q_0}{RC} \cdot e^{-\frac{t}{RC}} = I_0 \cdot e^{-\frac{t}{RC}}$

2013 Free Response Question 2



E&M 2.

In a lab, you set up a circuit that contains a capacitor *C*, a resistor *R*, a switch *S*, and a power supply, as shown in the diagram above. The capacitor is initially uncharged. The switch, which is initially open, can be moved to positions *A* or *B*.

(a)

i. Indicate the position to which the switch should be moved to charge the capacitor.

__ *A* __ *B*

ii. On the diagram, draw a voltmeter that is properly connected to the circuit in a manner that will allow the voltage to be measured across the capacitor.

After a long time you move the switch to discharge the capacitor, and your lab partner starts a stopwatch. You collect the following measurements of the voltage across the capacitor at various times.

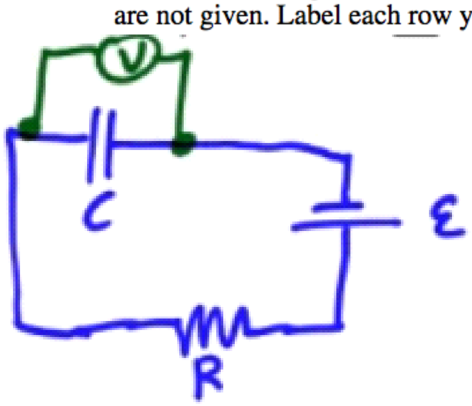
t (s)	6	18	30	42	54
V(*V*)	252	74	33	10	6

You wish to determine the time constant τ of the circuit from the slope of a linear graph.

(b)

i. Indicate two quantities you would plot to obtain a linear graph.

ii. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.



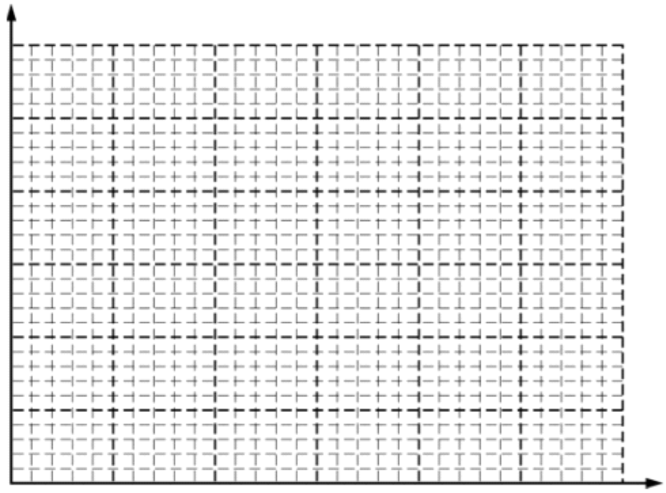
bi) $V = V_0 e^{-t/RC} \Rightarrow$

$\ln V = \ln V_0 + -t/RC$

$\ln V = \left(\frac{-1}{RC}\right) t + \ln V_0$

$y = m \; x + b$

(c) On the axes below, graph the data from the table that will produce a linear relationship. Clearly scale and label all axes including units, if appropriate. Draw a straight line that best fits your data points.

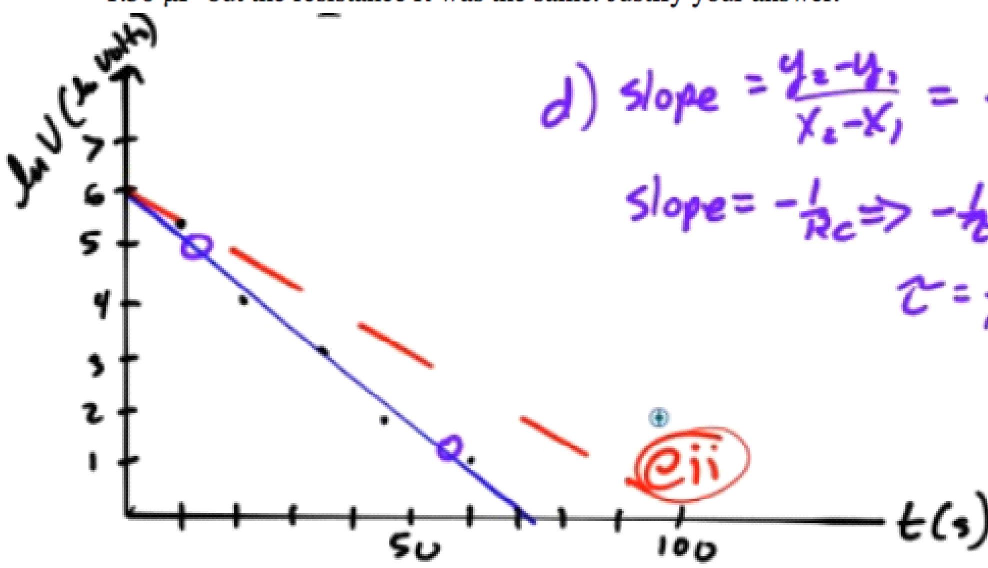


(d) From your line in part (c), obtain the value of the time constant τ of the circuit.

(e)

i. In the experiment, the capacitor C had a capacitance of 1.50 μF. Calculate an experimental value for the resistance R.

ii. On the axes in part (c), use a dashed line to sketch a possible graph if the capacitance was greater than 1.50 μF but the resistance R was the same. Justify your answer.



d) slope $= \frac{y_2 - y_1}{x_2 - x_1} = -.079$

slope $= -\frac{1}{RC} \Rightarrow -\frac{1}{\tau} = -.079$

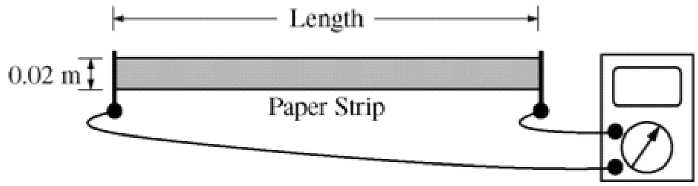
$\tau = \frac{1}{.079} = \boxed{12.7s}$

ei) $RC = 12.7s \Rightarrow R = \frac{12.7s}{1.5\times10^{-6}F} = \boxed{8.47\ M\Omega}$

eii) RC↑ C↑ so τ↑

Slope is inverse of τ ⇒ slope ↓

2012 Free Response Question 2

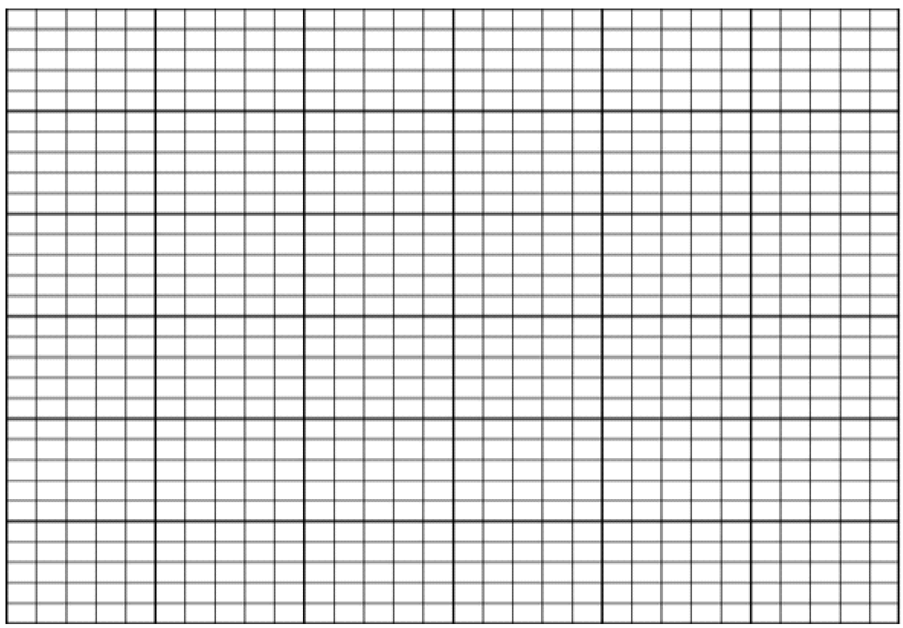


E&M. 2.

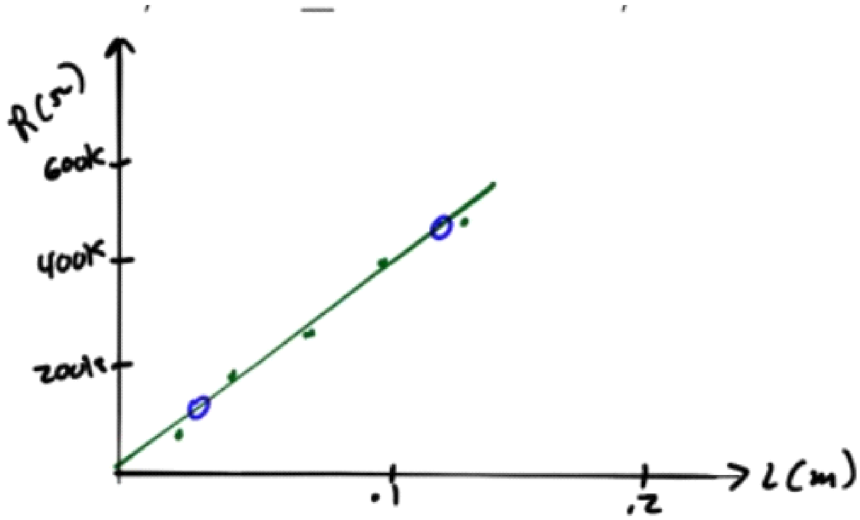
A physics student wishes to measure the resistivity of slightly conductive paper that has a thickness of 1.0×10^{-4} m. The student cuts a sheet of the conductive paper into strips of width 0.02 m and varying lengths, making five resistors labeled R1 to R5. Using an ohmmeter, the student measures the resistance of each strip, as shown above. The data are recorded below.

Resistor	R1	R2	R3	R4	R5
Length (m)	0.020	0.040	0.060	0.080	0.100
Resistance (Ω)	80,000	180,000	260,000	370,000	440,000

(a) Use the grid below to plot a linear graph of the data points from which the resistivity of the paper can be determined. Include labels and scales for both axes. Draw the straight line that best represents the data.

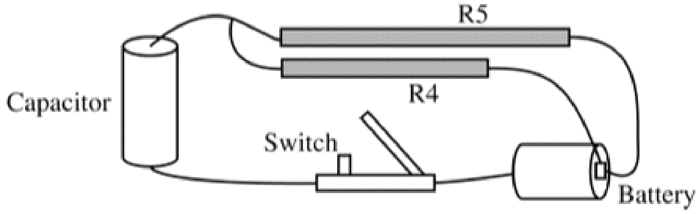


(b) Using the graph, calculate the resistivity of the paper.



$$R = \frac{\rho L}{A} = \frac{\rho L}{wt} \Rightarrow$$
$$R = \left(\frac{\rho}{wt}\right) L$$
$$y = m \quad x$$

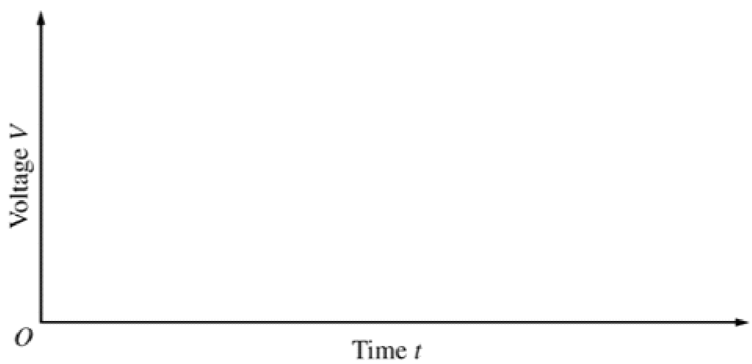
b) $\text{slope} = \frac{\rho}{wt} \Rightarrow \rho = wt\,\text{slope} = (.02)(10^{-4})(\text{slope}) = \boxed{9.75\,\Omega\cdot m}$



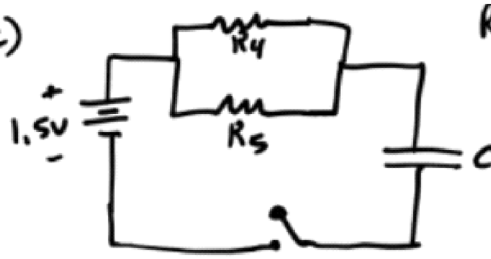
The student uses resistors R4 and R5 to build a circuit using wire, a 1.5 V battery, an uncharged 10 μF capacitor, and an open switch, as shown above.

(c) Calculate the time constant of the circuit.

(d) At time t = 0, the student closes the switch. On the axes below, sketch the magnitude of the voltage V_c across the capacitor and the magnitudes of the voltages V_{R4} and V_{R5} across each resistor as functions of time t. Clearly label each curve according to the circuit element it represents. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.



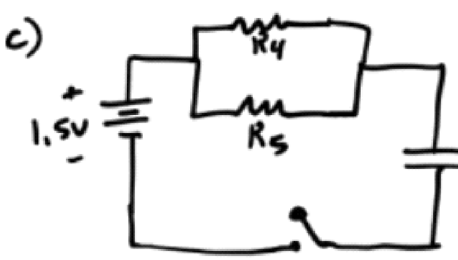
c)



$$R_{eq} = \frac{R_4 R_5}{R_4 + R_5} = \frac{(370{,}000)(440{,}000)}{810{,}000} \approx 200\,k\Omega$$

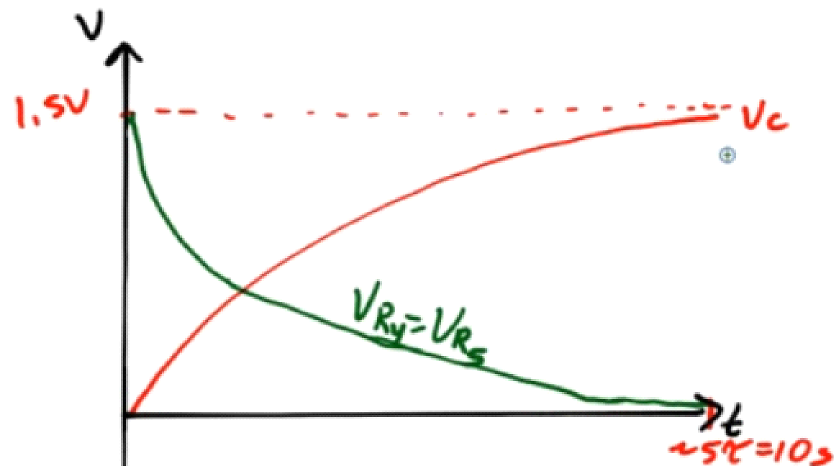
$$\tau = RC = (200{,}000\,\Omega)(10 \cdot 10^{-6}\,F) = \boxed{2\,s}$$

c)

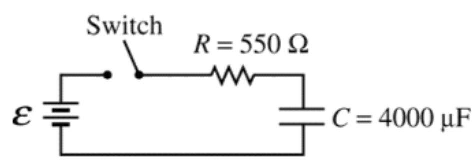


$$R_{eq} = \frac{R_4 R_5}{R_4 + R_5} = \frac{(370,000)(440,000)}{810,000} \approx 200\,k\Omega$$

$$\tau = RC = (200,000\,\Omega)(10 \cdot 10^{-6}\,F) = \boxed{2\,s}$$

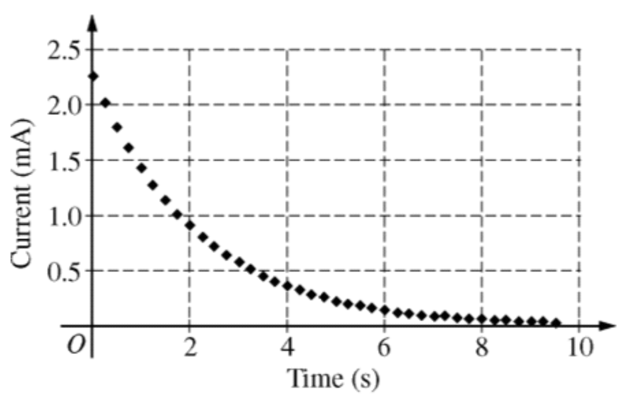


2007 Free Response Question 1



E&M 1.

A student sets up the circuit above in the lab. The values of the resistance and capacitance are as shown, but the constant voltage $\mathcal{E}$ delivered by the ideal battery is unknown. At time $t = 0$, the capacitor is uncharged and the student closes the switch. The current as a function of time is measured using a computer system, and the following graph is obtained.



(a) Using the data above, calculate the battery voltage $\mathcal{E}$.

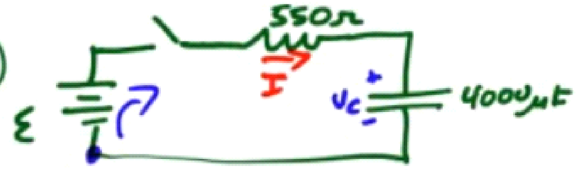
(b) Calculate the voltage across the capacitor at time $t = 4.0$ s.

(c) Calculate the charge on the capacitor at $t = 4.0$ s.

a) $I(t=0) = 2.25\ mA = .00225\ A$

$\varepsilon = IR = (.00225)(550) = \boxed{1.24V}$

b)



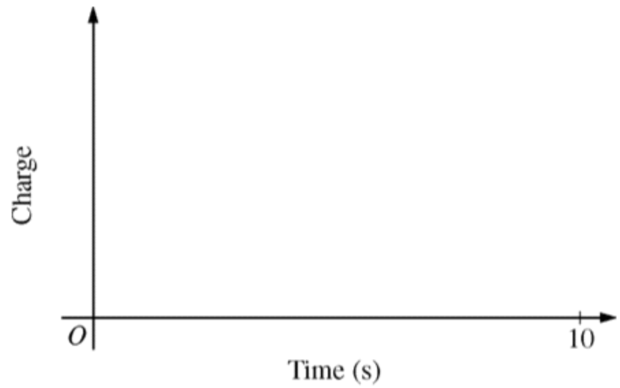
$-\varepsilon + IR + V_c = 0 \Rightarrow V_c = \varepsilon - IR$

$I(t=4s) = .35\ mA \Rightarrow V_c = 1.24V - (.00035)(550)$

$V_c = \boxed{1.05V}$

c) $Q = CV = (4000 \cdot 10^{-6} F)(1.05V) = \boxed{0.0042\ C}$

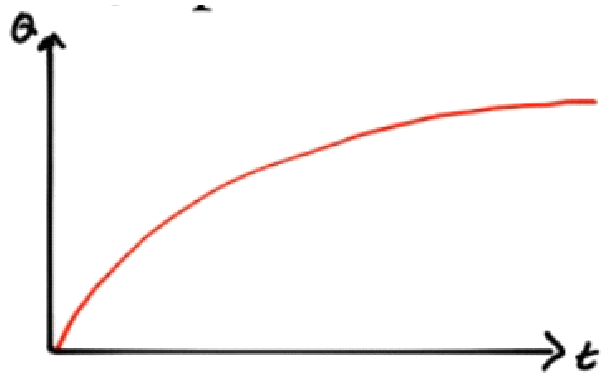
(d) On the axes below, sketch a graph of the charge on the capacitor as a function of time.



(e) Calculate the power being dissipated as heat in the resistor at $t = 4.0$ s.

(f) The capacitor is now discharged, its dielectric of constant $\kappa = 1$ is replaced by a dielectric of constant $\kappa = 3$, and the procedure is repeated. Is the amount of charge on one plate of the capacitor at $t = 4.0$ s now greater than, less than, or the same as before? Justify your answer.

_____Greater than _____Less than _____The same



e) $P = I^2R = (.00035A)^2(550\Omega) = \boxed{6.74 \cdot 10^{-5}\,W}$

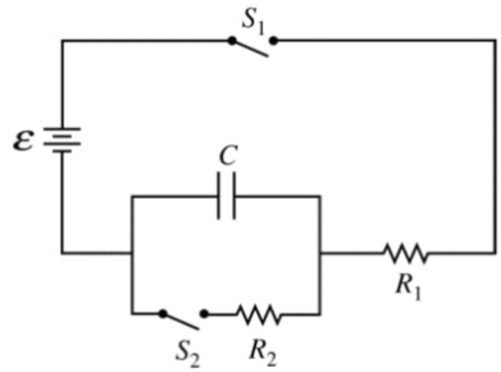
f) $C = \frac{K\epsilon_0 A}{d} \xrightarrow{K\text{ triples}} C$ triples $\Rightarrow$ Greater

$Q_3 = \varepsilon(3C)\left(1 - e^{-\frac{t}{3RC}}\right) = 3\varepsilon C\left(1 - e^{\frac{-4}{3 \cdot 550 \cdot 4000 \cdot 10^{-6}}}\right) \Rightarrow$

$Q_3 = (5454\mu F)(1.24V) = 6.76 \cdot 10^{-3}C$

.00676 C (greater than .0042 C)

2006 Free Response Question 2



E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

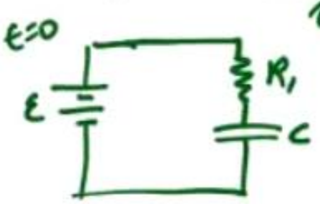
Numerical values for the components are given as follows:

$$\mathcal{E} = 12 \text{ V}$$
$$C = 0.060 \text{ F}$$
$$R_1 = R_2 = 4700\ \Omega$$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

a) $t=0$



$\tau = RC$

$$-\mathcal{E}+IR_1+V_c=0 \xRightarrow{V_c=\frac{Q}{C}} -\mathcal{E}+IR_1+\frac{Q}{C}=0 \xRightarrow{I=\frac{dQ}{dt}}$$

$$\boxed{-\mathcal{E}+R_1\frac{dQ}{dt}+\frac{Q}{C}=0}$$

b) $$-\frac{\mathcal{E}}{R_1}+\frac{dQ}{dt}+\frac{Q}{R_1C}=0 \Rightarrow \frac{dQ}{dt}=\frac{\mathcal{E}}{R_1}-\frac{Q}{R_1C} \Rightarrow \frac{dQ}{dt}=\frac{\mathcal{E}C-Q}{R_1C} \Rightarrow$$

$$\frac{dQ}{\mathcal{E}C-Q}=\frac{dt}{R_1C} \Rightarrow \int_{Q=0}^{Q}\frac{dQ}{Q-\mathcal{E}C}=\int_{t=0}^{t}-\frac{dt}{R_1C} \Rightarrow \ln(Q-\mathcal{E}C)\Big|_0^{Q}=-\frac{t}{R_1C} \Rightarrow$$

$$\ln(Q-\mathcal{E}C)-\ln(-\mathcal{E}C)=-\frac{t}{R_1C} \Rightarrow \ln\left(\frac{Q-\mathcal{E}C}{-\mathcal{E}C}\right)=-\frac{t}{R_1C} \Rightarrow$$

$$\frac{Q-\mathcal{E}C}{-\mathcal{E}C}=e^{-t/R_1C} \Rightarrow Q-\mathcal{E}C=-\mathcal{E}Ce^{-t/R_1C} \Rightarrow \boxed{Q=\mathcal{E}C\left(1-e^{-t/R_1C}\right)}$$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

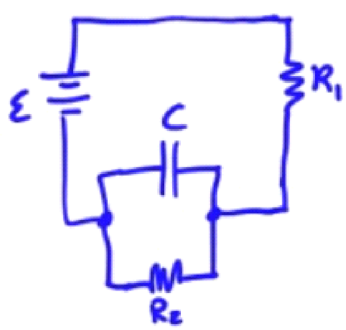
(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.

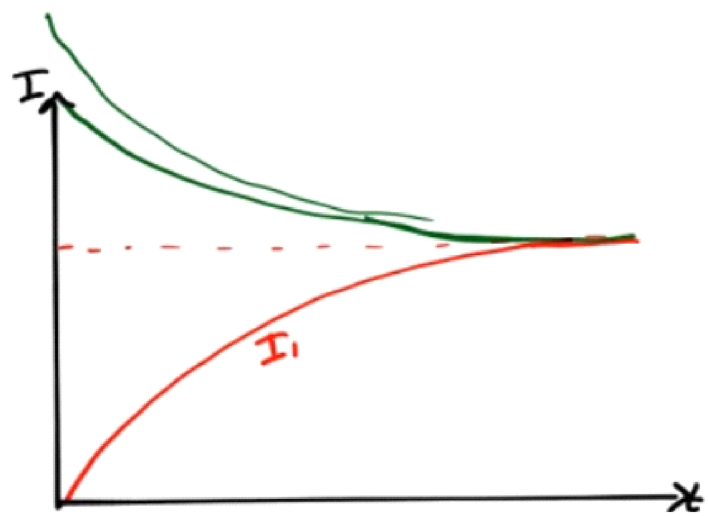
c) $Q = CV = \varepsilon C\left(1 - e^{-\frac{t}{R_1C}}\right) \Rightarrow V = \varepsilon\left(1 - e^{-\frac{t}{R_1C}}\right) \Rightarrow \frac{V}{\varepsilon} - 1 = -e^{-\frac{t}{R_1C}} \Rightarrow$

$1 - \frac{V}{\varepsilon} = e^{-\frac{t}{R_1C}} \Rightarrow \ln\left(1 - \frac{V}{\varepsilon}\right) = -\frac{t}{RC} \Rightarrow t = -RC\ln\left(1 - \frac{V}{\varepsilon}\right) \Rightarrow$

$t = (-4700)(.060)\ln\left(1 - \frac{4}{12}\right) = \boxed{114.3\,s}$

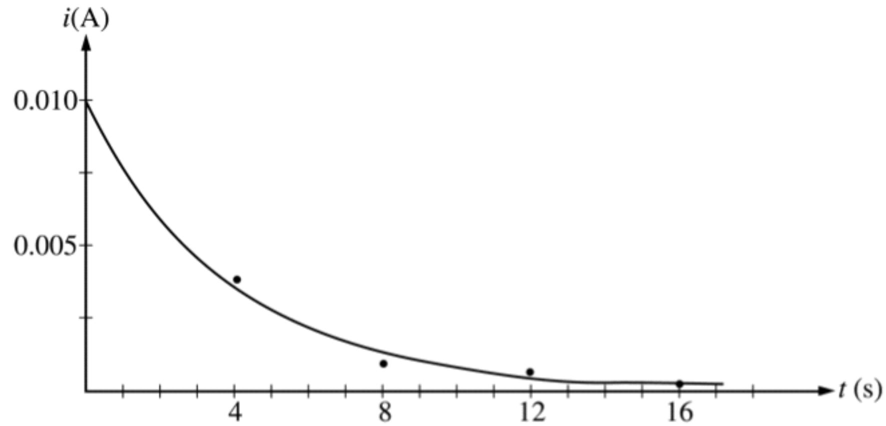
d) new $t=0$





2003 Response Question 2

In the laboratory, you connect a resistor and a capacitor with unknown values in series with a battery of emf $\mathcal{E} = 12\text{ V}$. You include a switch in the circuit. When the switch is closed at time $t = 0$, the circuit is completed, and you measure the current through the resistor as a function of time as plotted below.

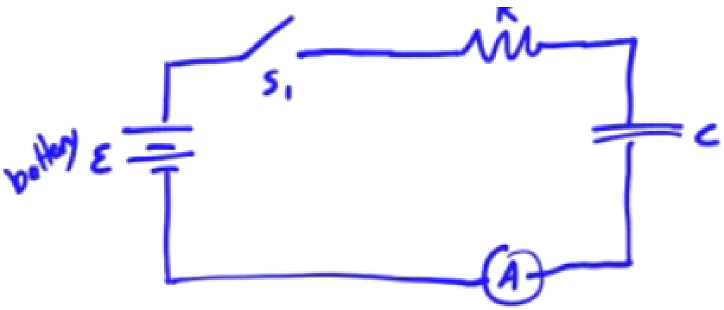


A data-fitting program finds that the current decays according to the equation $i(t) = \frac{\mathcal{E}}{R}e^{-t/4}$.

(a) Using common symbols for the battery, the resistor, the capacitor, and the switch, draw the circuit that you constructed. Show the circuit before the switch is closed and include whatever other devices you need to measure the current through the resistor to obtain the above plot. Label each component in your diagram.

(b) Having obtained the curve shown above, determine the value of the resistor that you placed in this circuit.

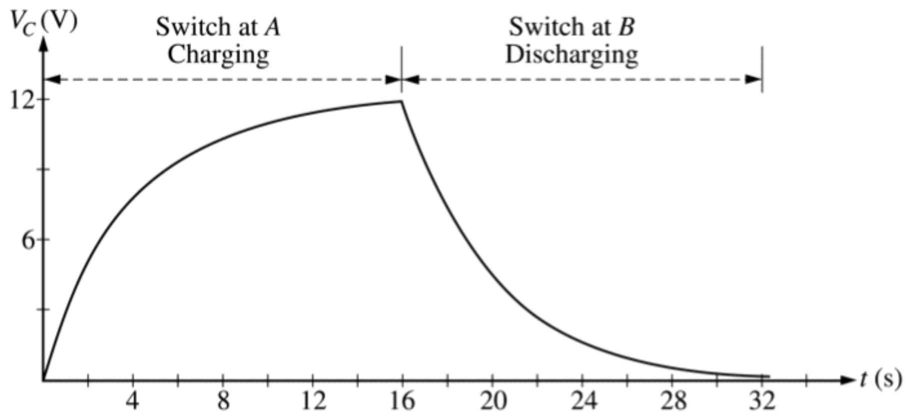
(c) What capacitance did you insert in the circuit to give the result above?



b) $R = \frac{V}{I}$ @ $t=0$, $V_c = 0 \Rightarrow V_R = \varepsilon \Rightarrow R = \frac{\varepsilon}{I} = \frac{12\text{V}}{.01\text{A}} = \boxed{1200\,\Omega}$

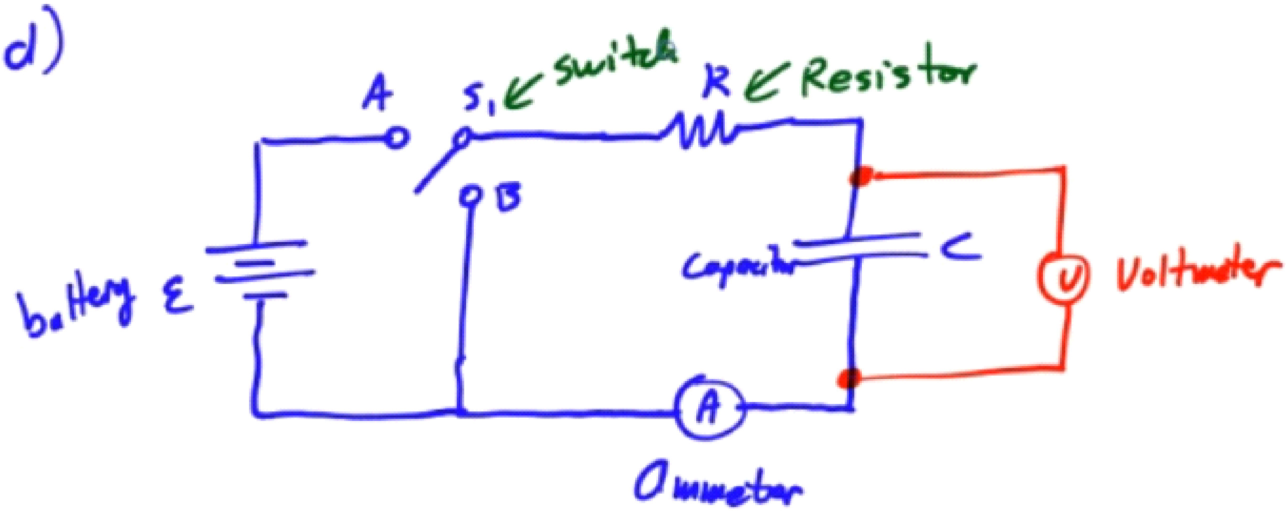
c) $\tau = RC = 4\text{s} \Rightarrow C = \frac{4\text{s}}{R} = \frac{4\text{s}}{1200\,\Omega} = \boxed{3.3 \cdot 10^{-3}\text{ F}}$

You are now asked to reconnect the circuit with a new switch in such a way as to charge and discharge the capacitor. When the switch in the circuit is in position A, the capacitor is charging; and when the switch is in position B, the capacitor is discharging, as represented by the graph below of voltage V_C across the capacitor as a function of time.



(d) Draw a schematic diagram of the RC circuit that you constructed that would produce the graph above. Clearly indicate switch positions A and B on your circuit diagram and include whatever other devices you need to measure the voltage across the capacitor to obtain the above plot. Label each component in your diagram.

d)

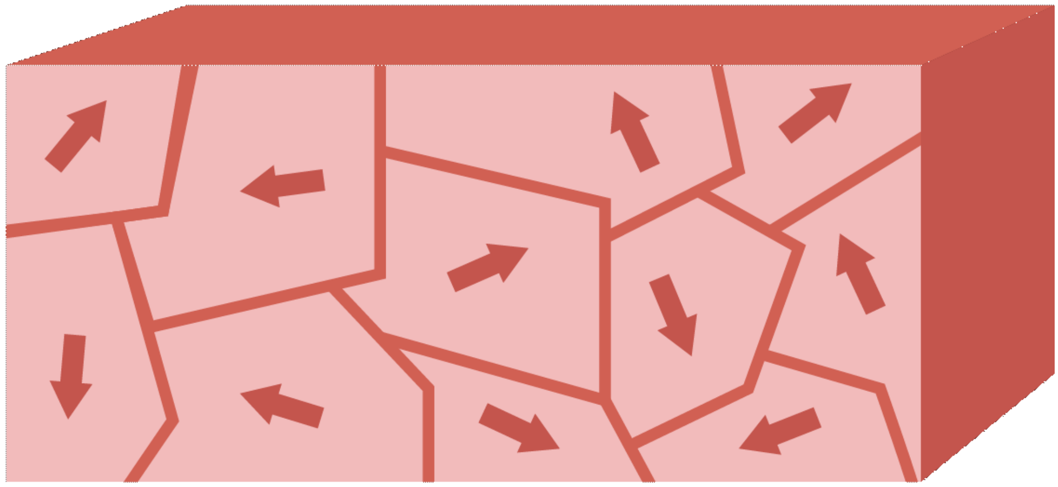


3.1 - Magnets

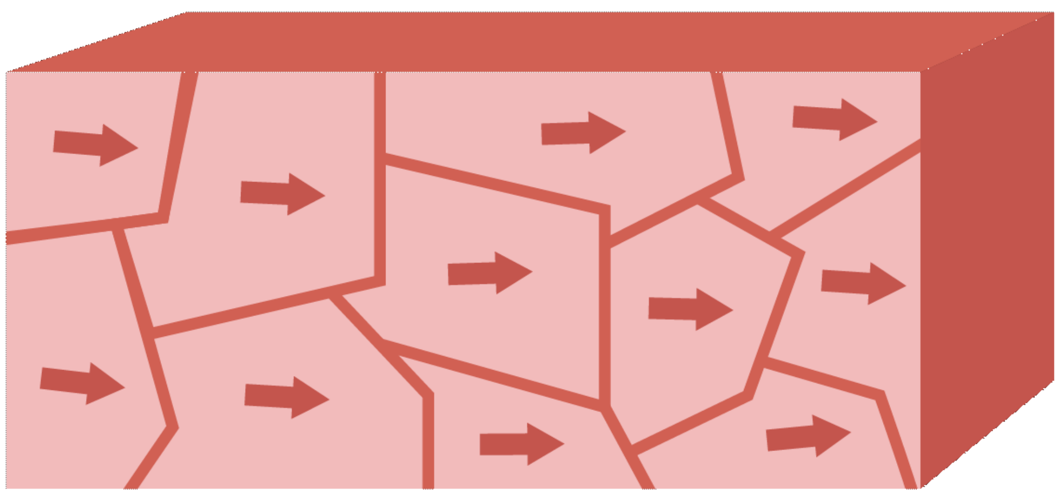
Friday, March 3, 2017 9:15 AM

Magnetism

- Magnetism is a force caused by moving charges
- Magnets are dipoles (they have a north and a south)
- Like poles repel, opposite poles attract
- Magnetic domains are clusters of atoms with electrons spinning in the same direction
  - Random domains: no net magnetic field
  - When many of the domains line up, you get a net magnetic field, creating a strong magnet



Domains Before Magnetization



Domains After Magnetization

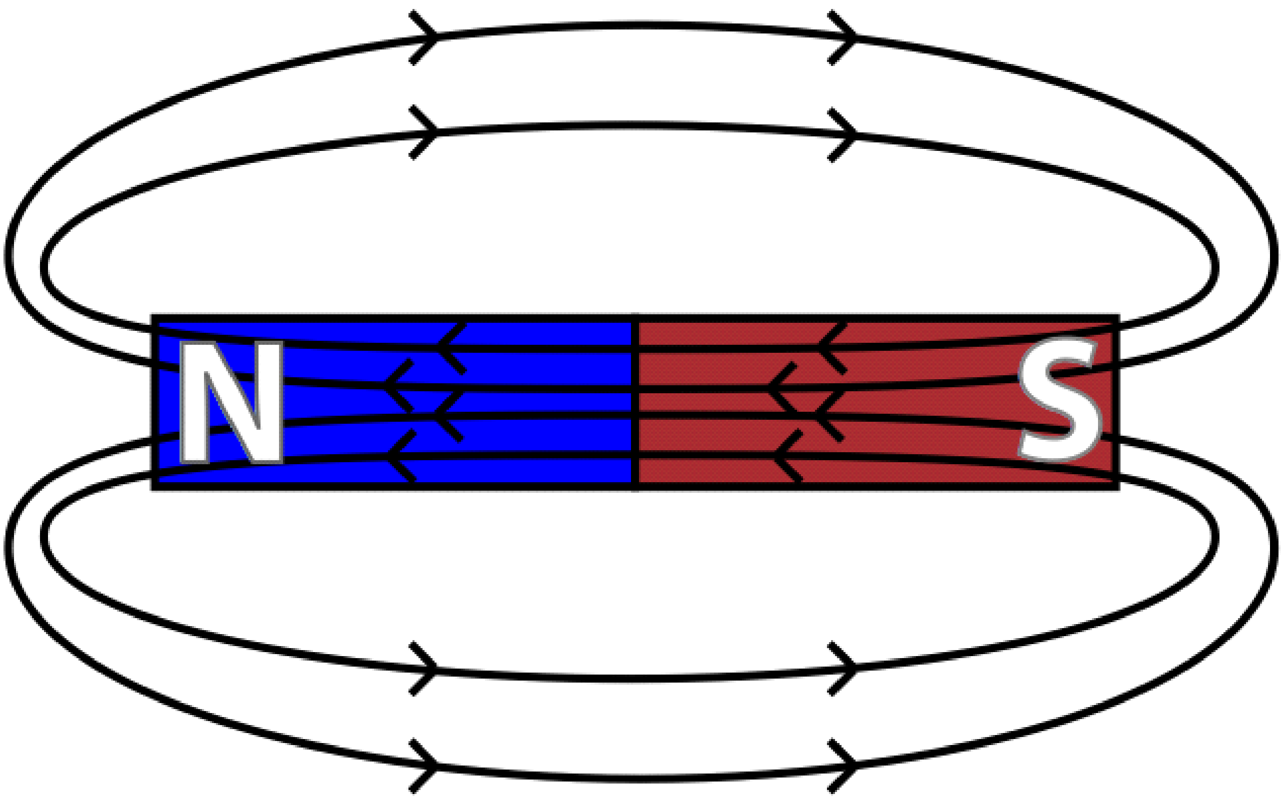
Magnetic Fields

- Magnetic Field Strength $\vec{B}$ is a vector quantity.
    - Units a Tesla (T)

- Magnets are polarized (each has two opposite ends)
  - End of a magnet that points toward the geographic north pole of the Earth is called the north pole of the magnet
  - There are no magnetic monopoles

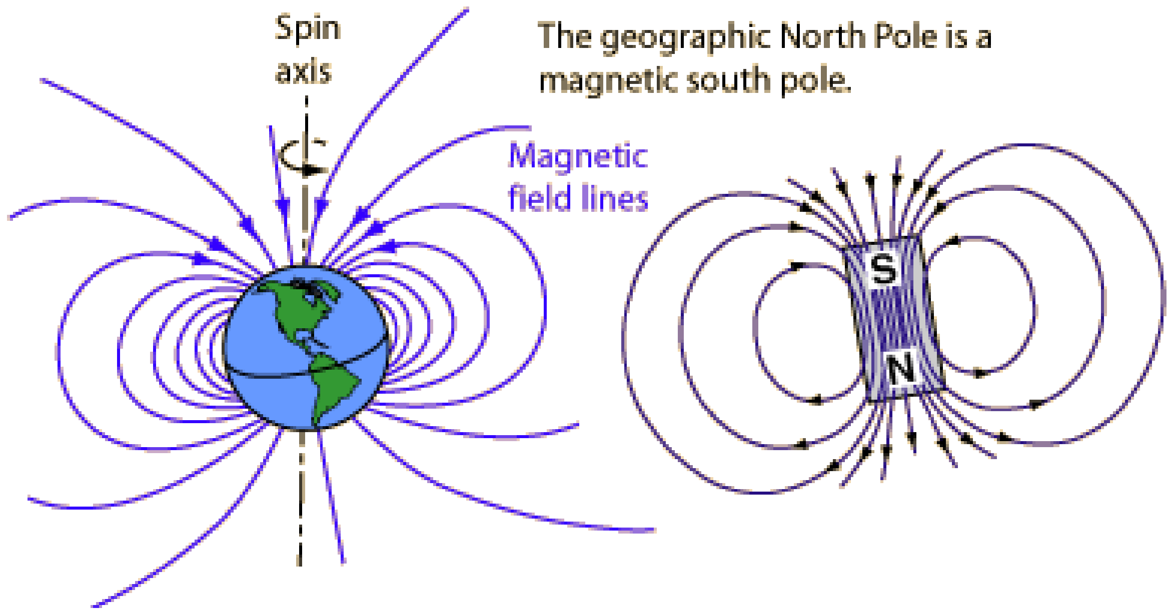
Magnetic Field Lines

- Magnetic Field Lines make closed loops and run from north to south outside the magnet
  - Magnet field lines show the direction the north pole of a magnet would tend to point if placed in the field
- Density of magnetic field is known as magnetic flux (Φ_B or Φ_M)



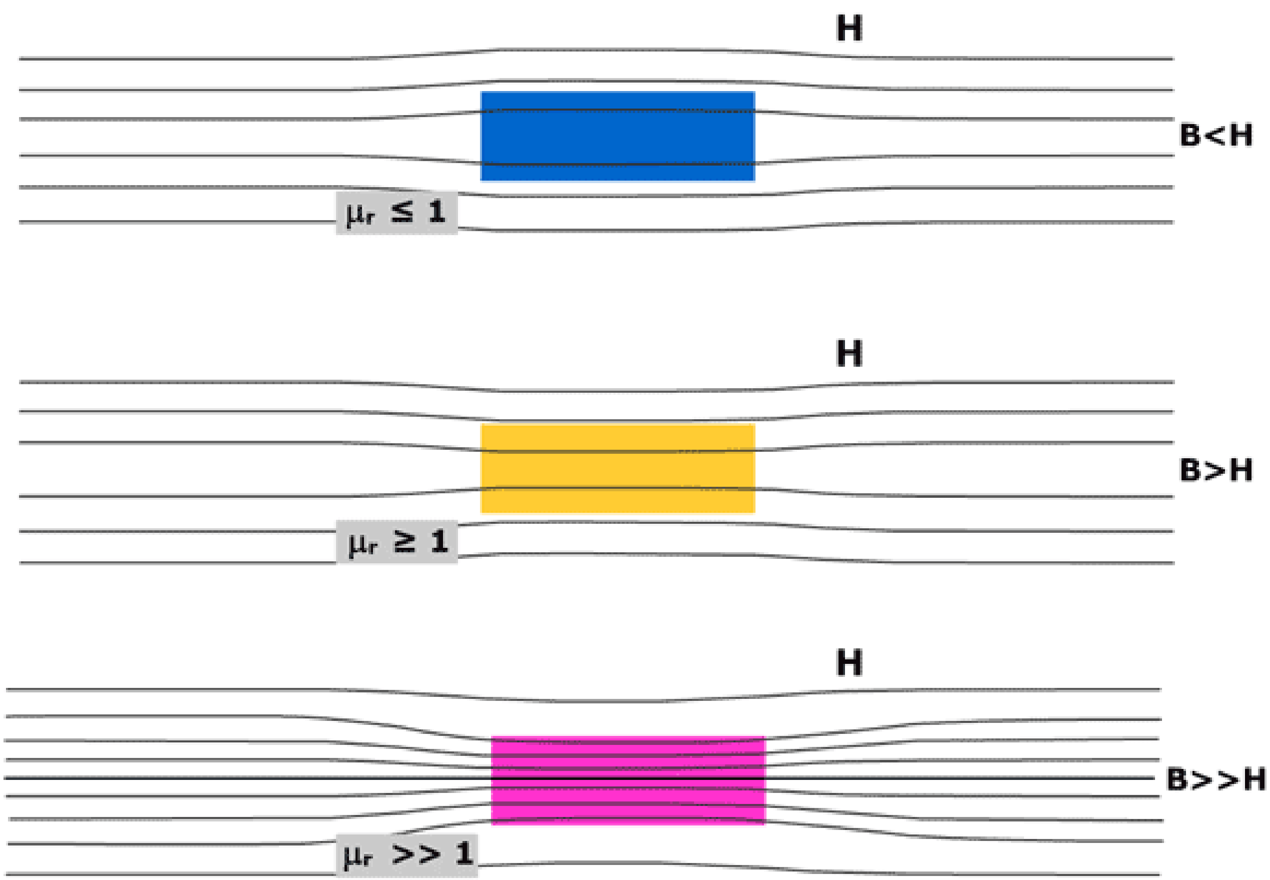
The Compass

- The Earth is a giant magnet
- The Earth's magnetic north pole is located near the geographic south pole, and vice versa
  - A compass's north magnet pole points toward the Earth's magnetic south pole (geographic north)
- A compass lines up with the net magnetic field



Magnetic Permeability

- Magnetic permeability refers to the ratio of magnetic field strength induced in a material to the magnetic field strength of the induced field



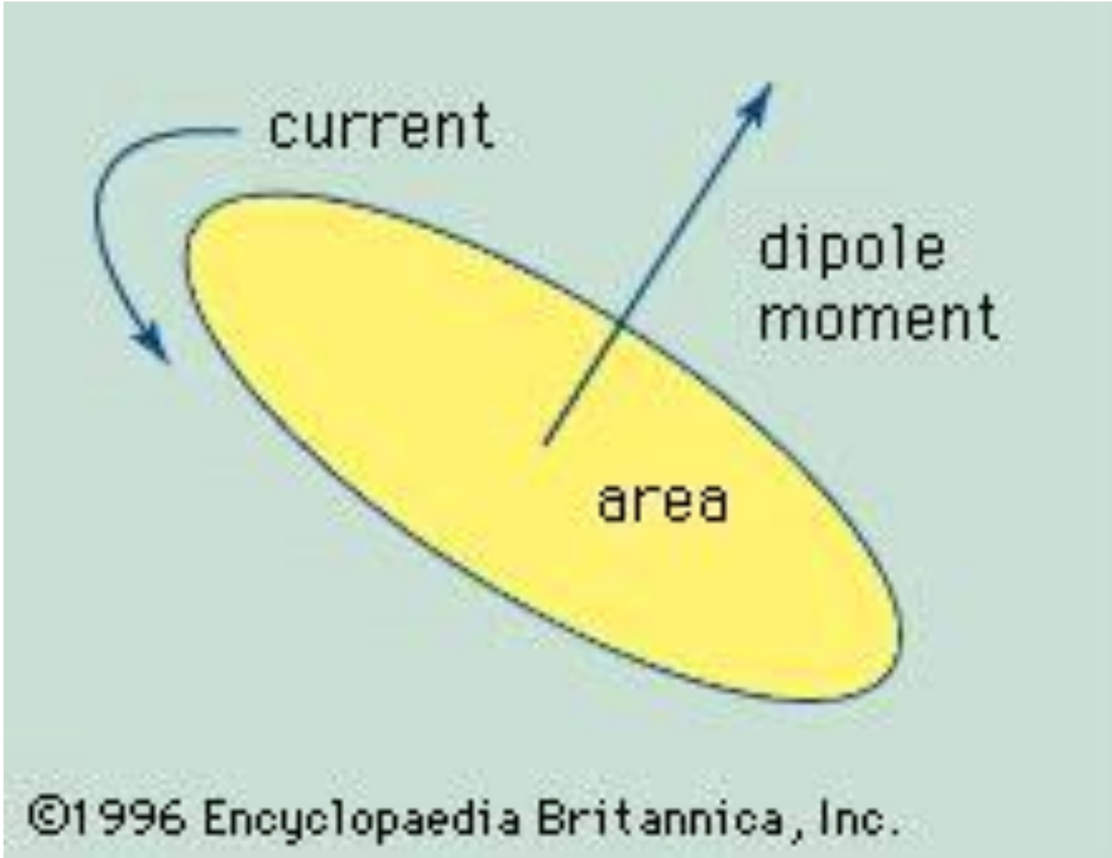
- Free space has a constant value of magnetic permeability that appears in physical relationships

$$\mu_0 = 4\pi \times 10^{-7}\ (\mathrm{T \cdot m})/\mathrm{A}$$

- The permeability of matter has a value different from that of free space
- Highly magnetic materials (such as iron) have higher values of magnetic permeability

Magnetic Dipole Moment

- The magnetic dipole moment of a magnet refers to the force that a magnet can exert on moving charges
- Can be thought of in simplistic terms as the relative strength of a magnet
    - Compare the magnetic dipole moment of a hydrogen atom to the magnetic dipole moment of a highly magnetized iron bar
    - Fe > H



3.2 - Moving Charges In Magnetic Fields

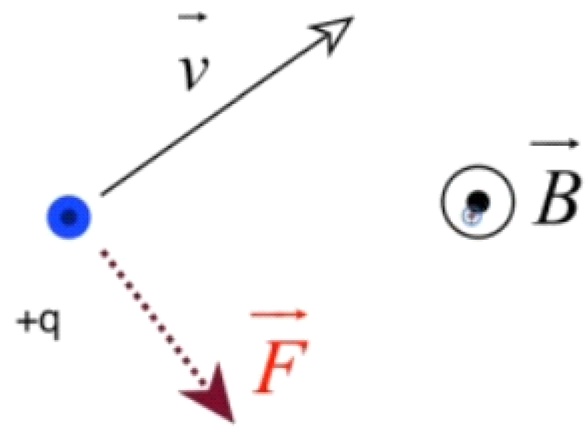
Friday, March 3, 2017 9:15 AM

Magnetic Fields

- Magnetic Field Strength ($\vec{B}$) is a vector quantity
- Units are Tesla: $1T = 1\ N \cdot s/(C \cdot m)$
- 1 Tesla is a very strong magnetic field
- More common non-SI units is the Gauss
- $1\ Gauss = 10^{-4}\ Tesla$
- Earth's magnetic field strength $\approx$ 0.5 Gauss

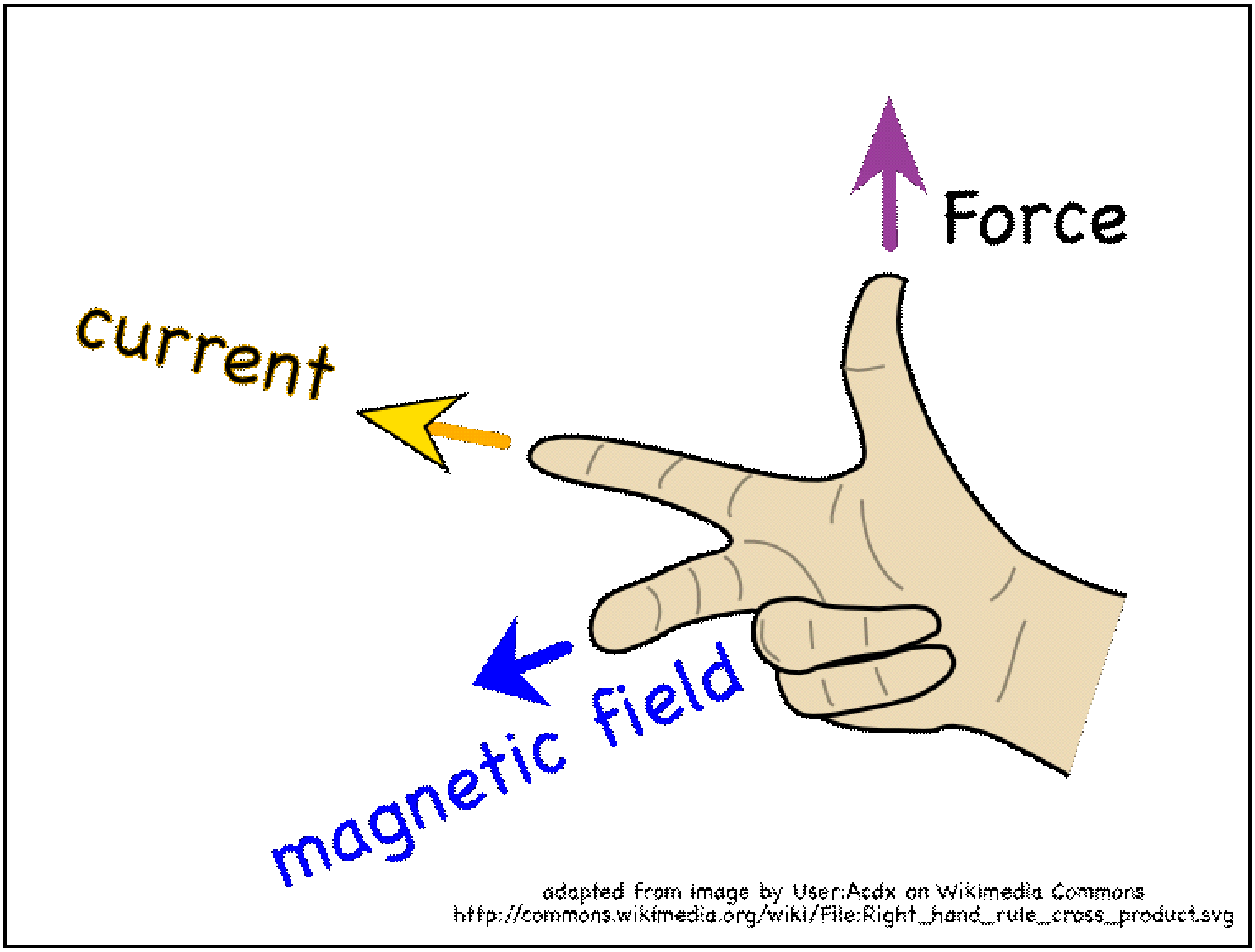
Forces on Moving Charges

- The magnetic force is always perpendicular to the charged object's velocity, therefore the magnetic force on a moving charge is never applied in the direction of the displacement, therefore a magnetic force can do no work on a moving charge (but it can change its direction)
- $\overrightarrow{F_M} = q\vec{v} \times \vec{B}$
- $\left|\overrightarrow{F_m}\right| = qvB\sin\theta$



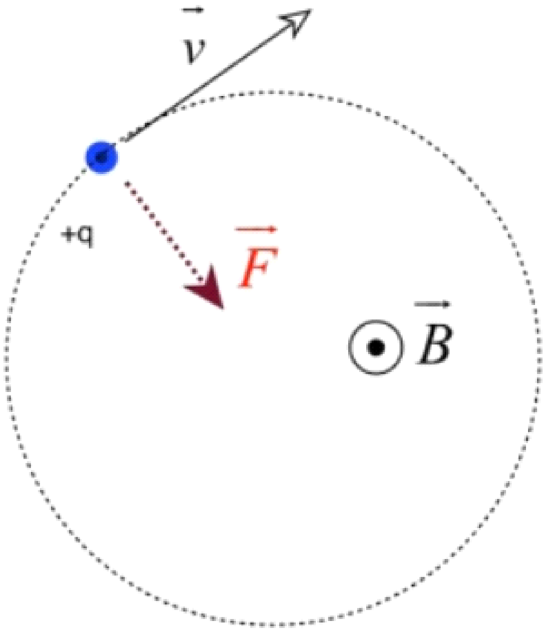
Direction of the Magnetic Force

- Direction of the force given by right-hand rule
  - Point fingers of right hand in direction of positive particles' velocity
  - Curl fingers inward in the direction of the magnetic field
  - Thumb points in the direction of the force on charged particle



Path of Charged Particles in B Fields

- Magnetic force cannot perform work on a moving charge
- Magnetic force can change its direction (moving it in a circle if the magnetic force is constant)



$$\left|\vec{F}_M\right| = qvB\sin\theta$$

$$F_m = F_c \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow$$

$$r = \frac{mv}{qB} \leftarrow \text{momentum of particle}$$

Total Force on a Moving Charged Particle

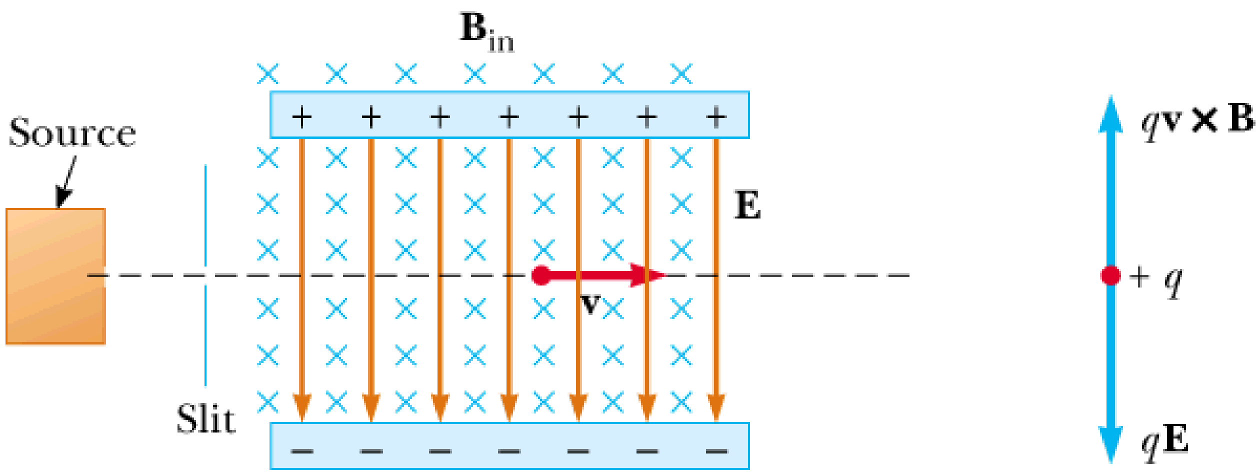
- E field can do work on a moving charge
- B filed can never do work on a moving charge

- Lorentz Force

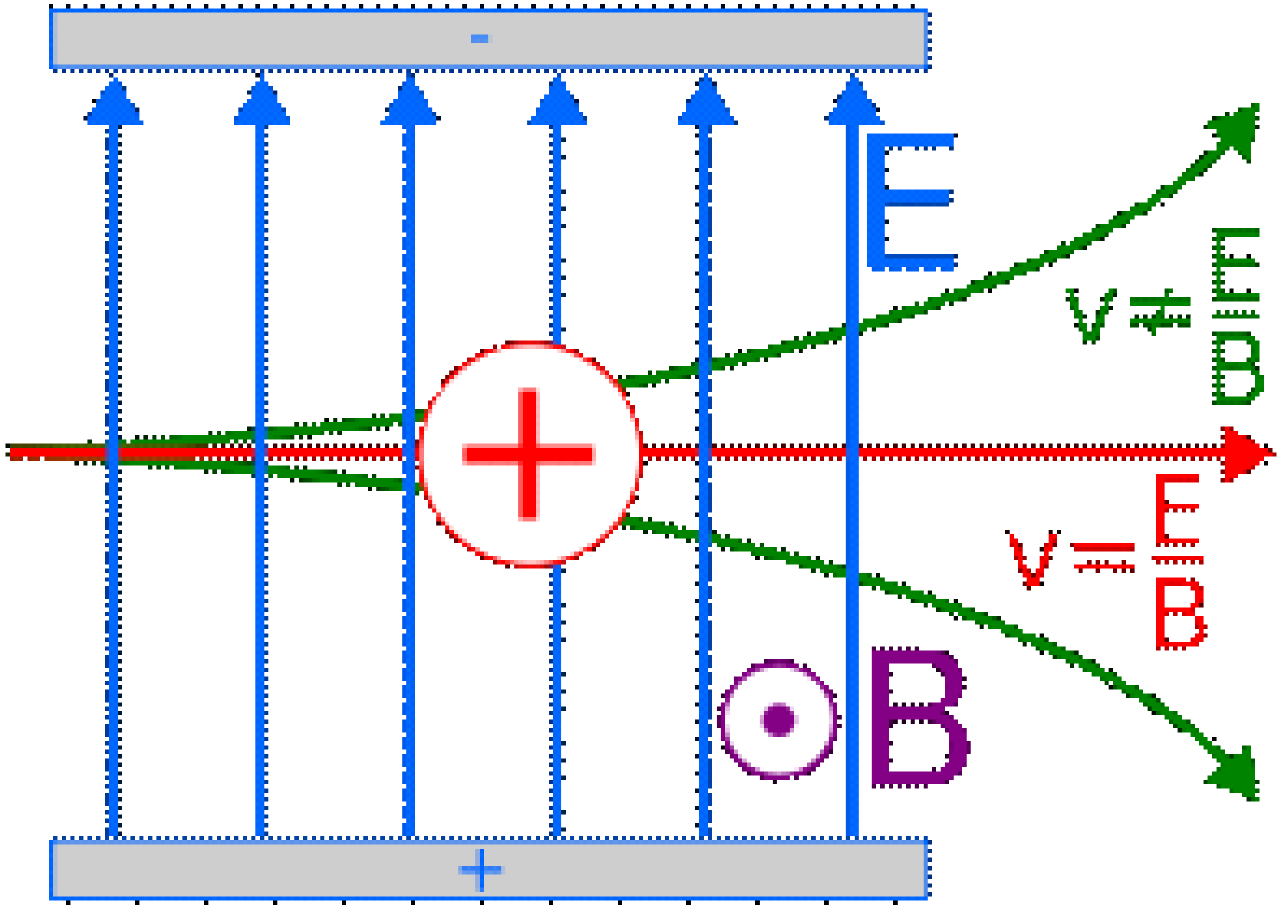
$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v}\ x\ \vec{B}}_{\text{Magnetic force}}$$

Velocity Selector

- A charged particle in crossed E and B fields can undergo constant velocity motion is v, B, and E are all selected perpendicular to each other
- If $v = \frac{E}{B}$, the particle can travel through the selector without any deflection, while particles with any other velocity are diverted

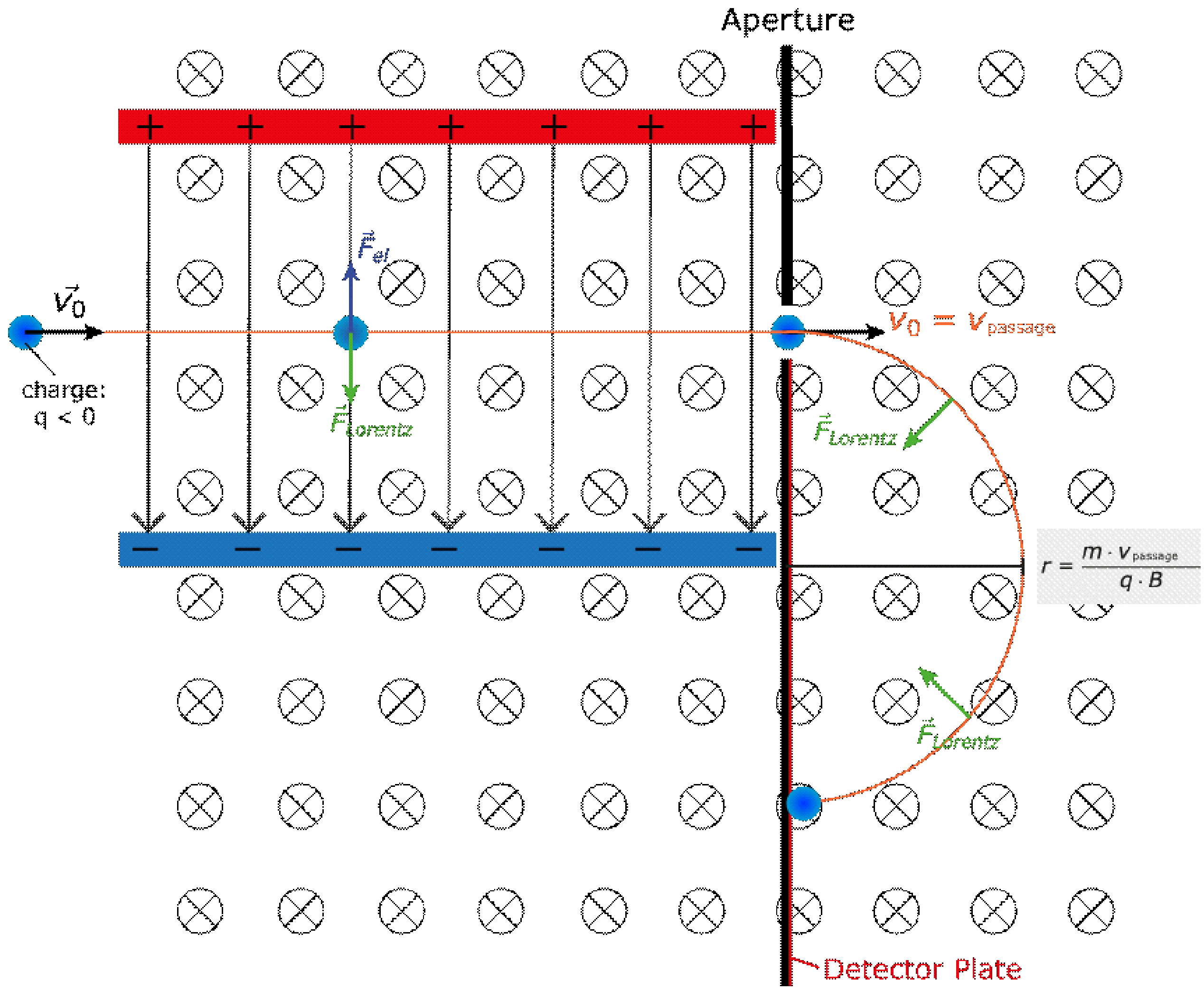


(a) (b)



Mass Spectrometer

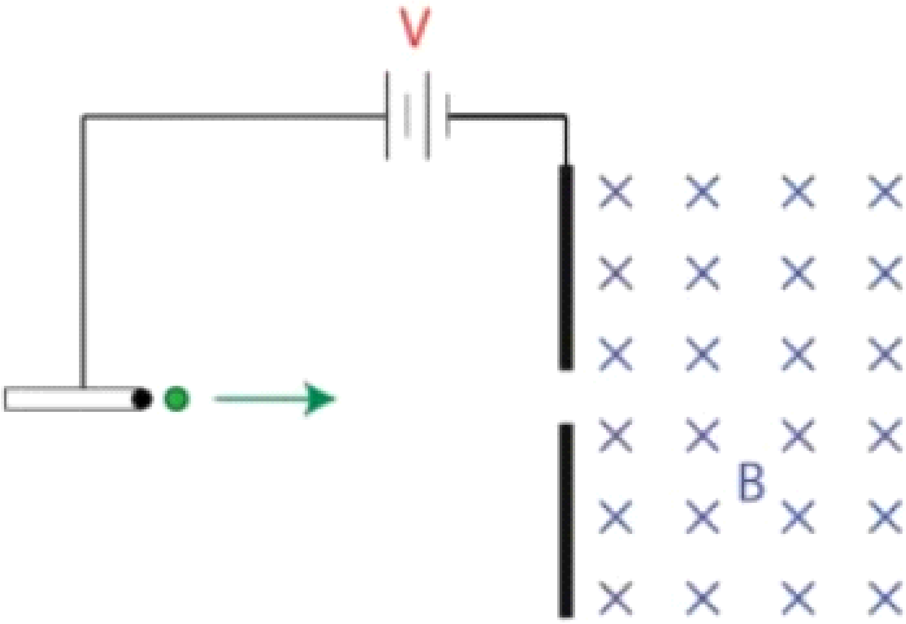
- Magnetic fields accelerate moving charges so that they travel in a circle
- This can be used to determine the mass of an unknown particle!



Example 1: Velocity Selector

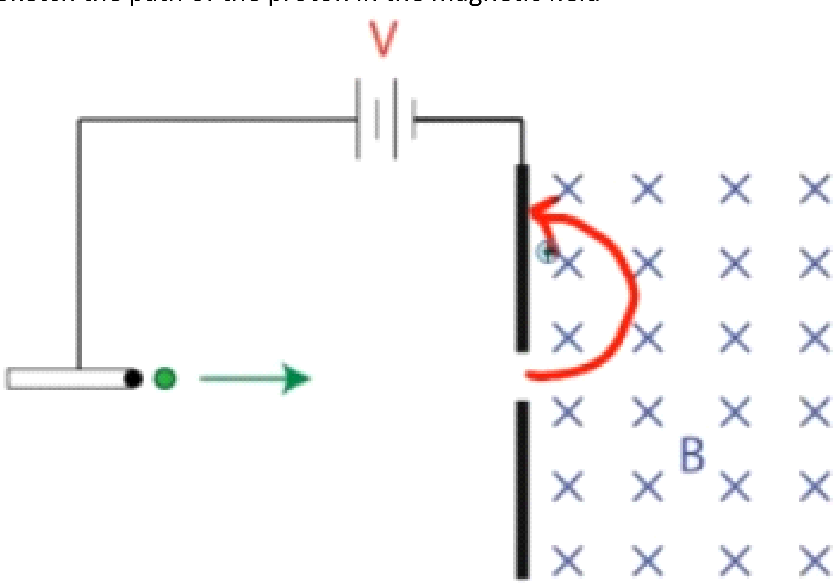
- Find the speed of a charged particle which passes through a velocity selector with magnetic field strength of 1 Tesla perpendicular to an electric field of 600,000 N/C
- $F_e = F_M$
- $qE = qvB$
- $v = \frac{E}{B} = \frac{600{,}000}{1} = 600{,}000\ m/s$

Example 2: Mass Spectrometer



- A proton is accelerated through a potential difference V before passing into a region of uniform magnetic field B as shown

1. Determine the voltage necessary to give the proton a speed v as it enters the magnetic field region in terms of the proton's mass m, its velocity v, and its charge, q.
   - $W = qV = \frac{1}{2}mv^2$
   - $V = \frac{mv^2}{2q}$
2. Determine an expression for the radius of the proton's motion in the uniform magnetic field region
   - $F_B = qvB = \frac{mv^2}{r}$
   - $r = \frac{mv^2}{qvB} = \frac{mv}{qB}$
3. Sketch the path of the proton in the magnetic field



4. An electric field is applied in the same region as the uniform magnetic field. Determine the magnitude and direction of electric field required so that the proton passes through the region in a straight line

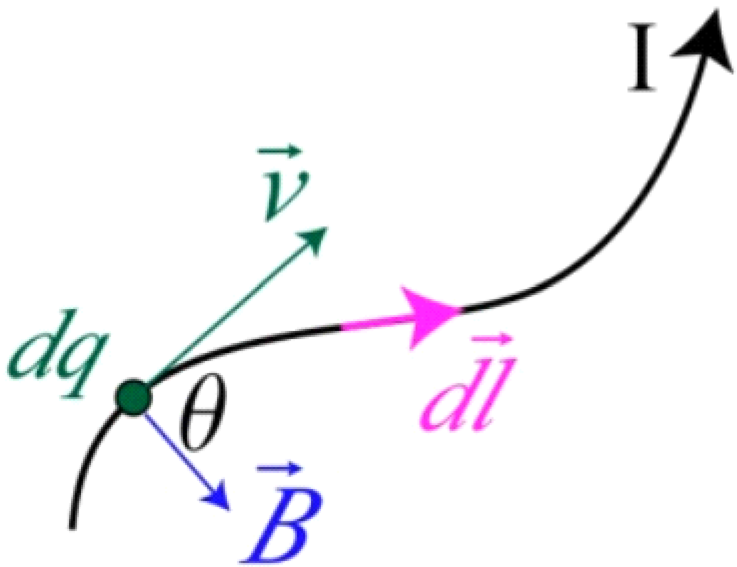
- $F_e = F_M$
- $qE = qvB$
- $E = vB$

3.3 - Forces on Current-Carrying Wires

Friday, March 3, 2017 9:15 AM

Forces on Current-Carrying Wires

- Moving charges in magnetic fields experience forces.
- Current in a wire is just a flow of charges.
- If charges are moving perpendicular to magnetic fields, they experience a force which is applied to the wire.



$$\overrightarrow{F_B} = q\left(\vec{v} \times \vec{B}\right)$$

$$d\overrightarrow{F_B} = dq\left(\vec{v} \times \vec{B}\right)$$

$$\because I = \frac{dq}{dt}$$

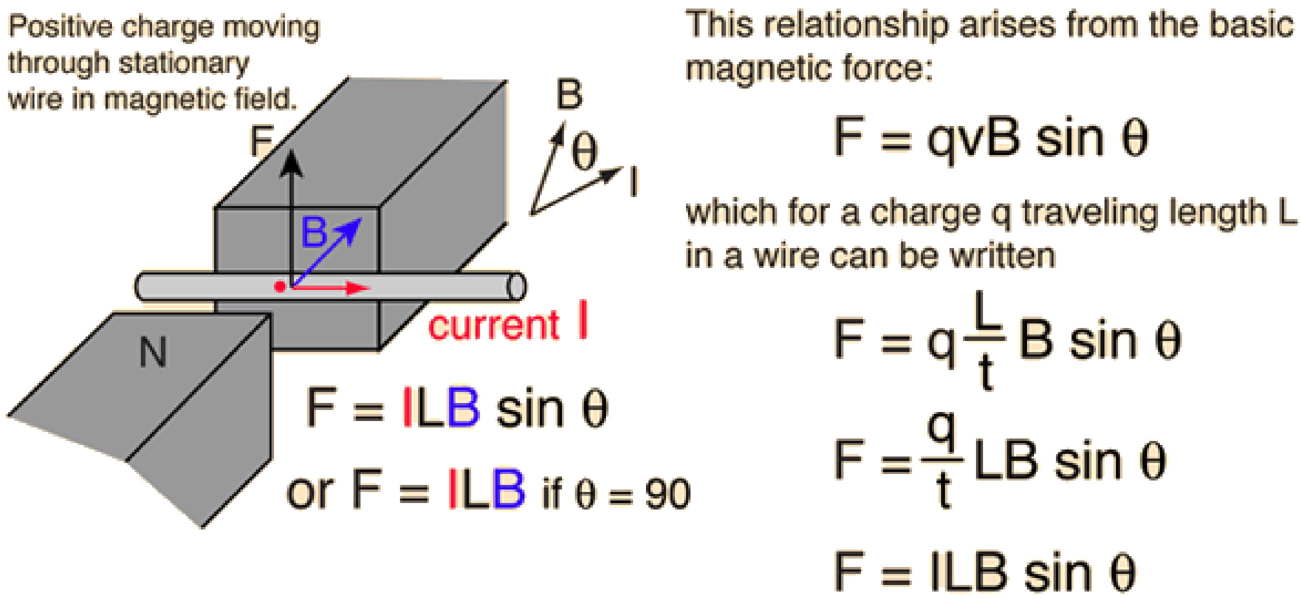
$$\therefore d\overrightarrow{F_B} = I \cdot dt \cdot \vec{v} \times \vec{B}$$

$$\because v = \frac{dl}{dt}$$

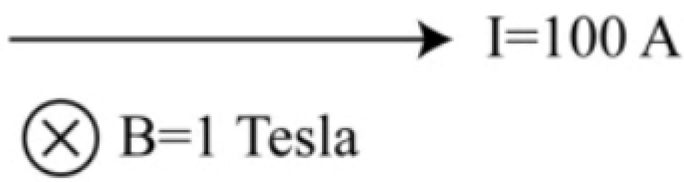
$$\therefore d\overrightarrow{F_B} = I \cdot d\vec{l} \times \vec{B}$$

$$\int d\overrightarrow{F_B} = \int I \cdot d\vec{l} \times \vec{B}$$

$$\overrightarrow{F_B} = \int I \cdot d\vec{l} \times \vec{B}$$

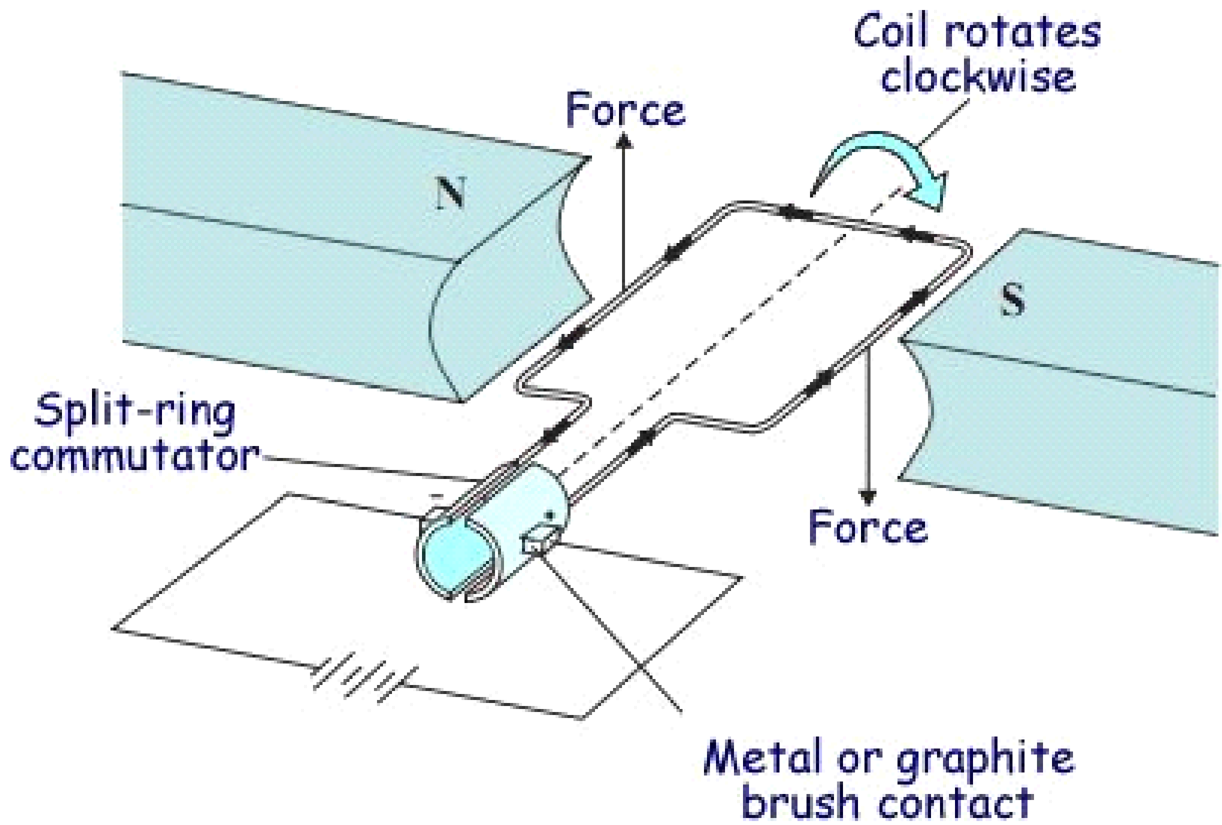


Example 1: Current Through a Wire



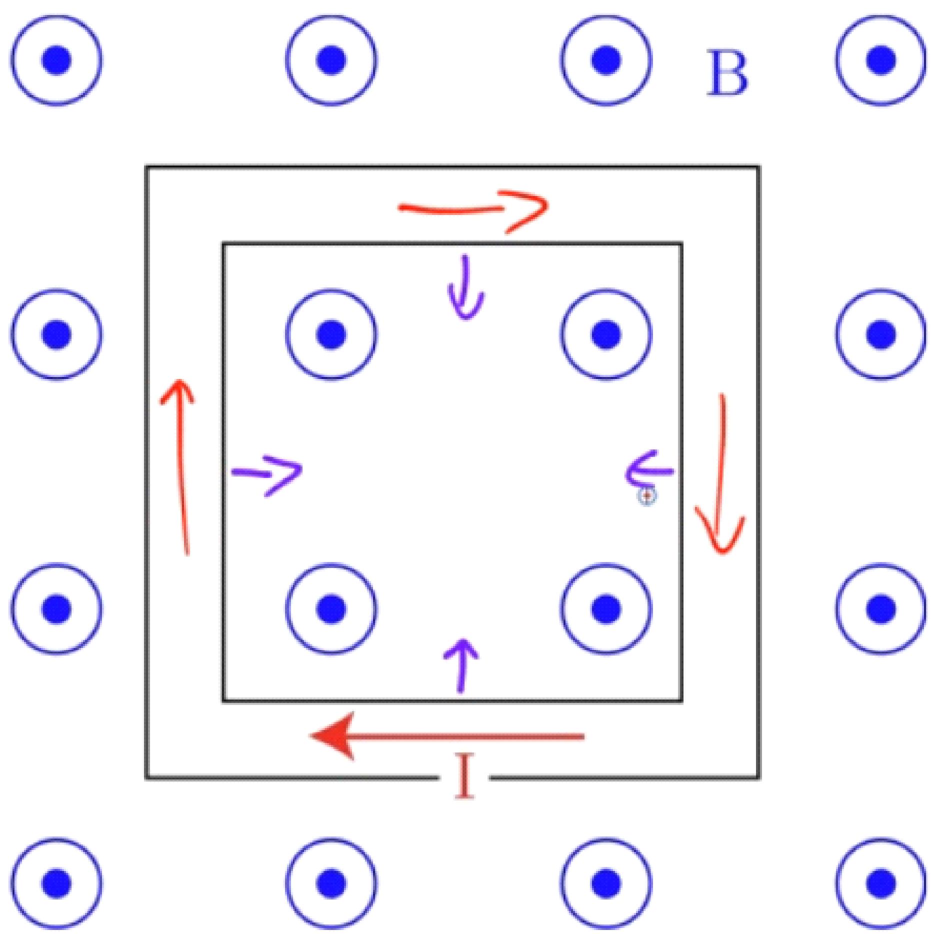
- A straight wire of length 1 m carries a current of 100 A through a magnetic field of 1 Tesla. Find the force on the wire
- $F_B = \int I\, dl\ \times B = IlB \sin\theta = 100 \times 1 \times 1 \times \sin 90° = 100N$

Electric Motors

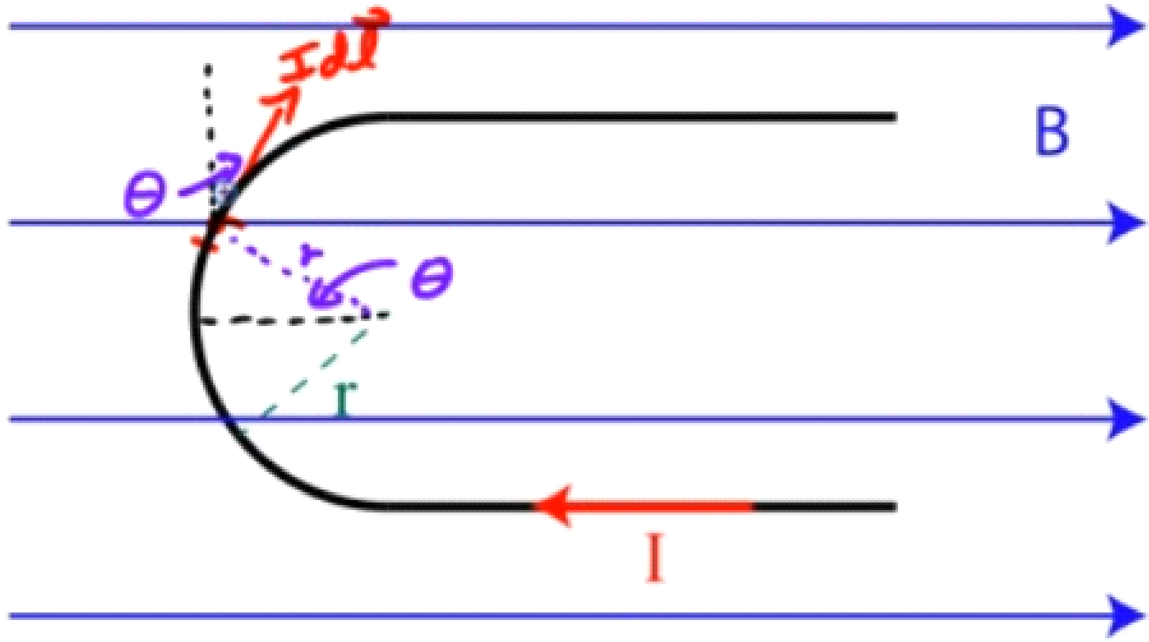


Example 2: Net Torque

- Determine the direction of the net torque on the current-carrying closed circuit



Example 3: Force on a Curved Wire



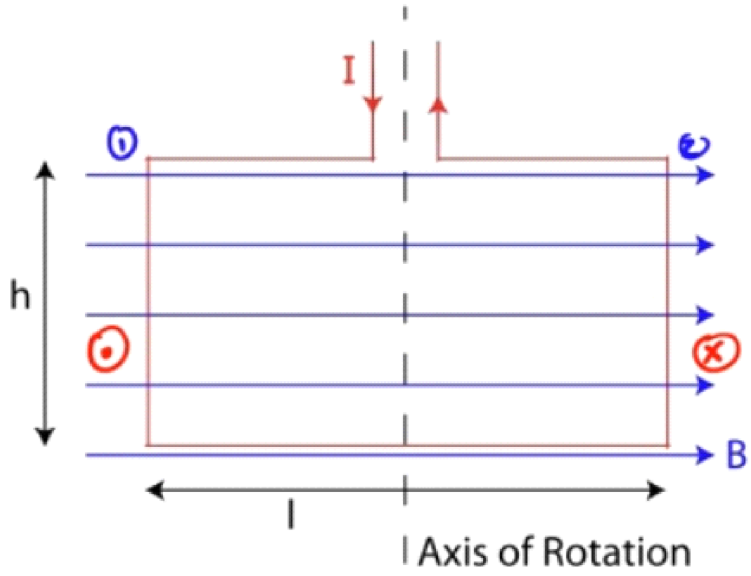
- The curved wire carries constant current I through a uniform magnetic field B as shown. Find the net force acting on the wire.
- $\mathrm{d}\vec{\mathrm{l}} = \mathrm{rd\theta} \sin\theta\, \hat{\imath} + \mathrm{rd\theta} \cos\theta\, \hat{\jmath}$
- $F_B = \int I\, d\vec{l} \times \vec{B} = \int I\, (\mathrm{rd\theta} \sin\theta\, \hat{\imath} + \mathrm{rd\theta} \cos\theta\, \hat{\jmath}) \times B\hat{\imath} = \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} -IBr\cos\theta\, d\theta\, \hat{k} = -IBr\hat{k} \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta$ $= -2IBr\hat{k}$

3.4 - Magnetic Fields Due to Current-Carrying Wires

Friday, March 3, 2017 11:50 AM

Example 1: Torque on a Loop of Wire

- A loop of wire carrying current I is placed in a magnetic field. Determine the net torque around the axis of rotation due to the current in the wire.



$F_1 = IlB = IhB$

$F_2 = IhB$

$\tau_1 = Fr\sin\theta = IhB\frac{l}{2}$

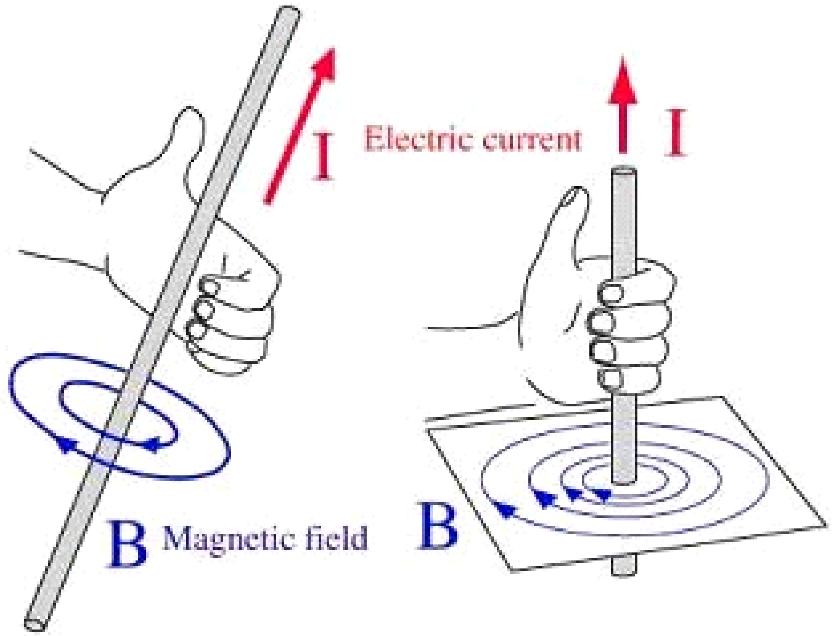
$\tau_2 = IhB\frac{l}{2}$

$\boxed{\tau_{NET} = IhBl}$

Magnetic Field due to a Current-Carrying Wire

- Moving charges create magnetic fields
- Current-carrying wires carry moving charges, therefore they create B fields
- Direction given by right-hand rule
- For multiple wires, determine B field from each and add them up using superposition
- B fields may interact with other moving charges, so current-carrying wires can exert forces upon each other.

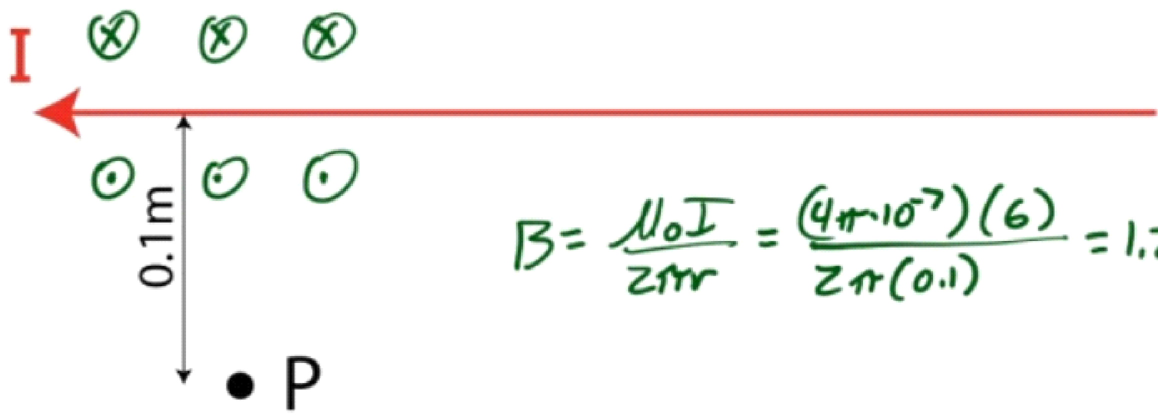
First Right Hand Rule



- "Hold" wire with your right hand, thumb in direction of positive current flow
- Your fingers wrap in the direction of the magnetic field
- $B = \frac{\mu_o}{2\pi} \frac{I}{r}$
- $\mu_0 = 4\pi \times 10^{-7}\ (T \cdot m)/A$

Example 2: Magnetic Field due to a Wire

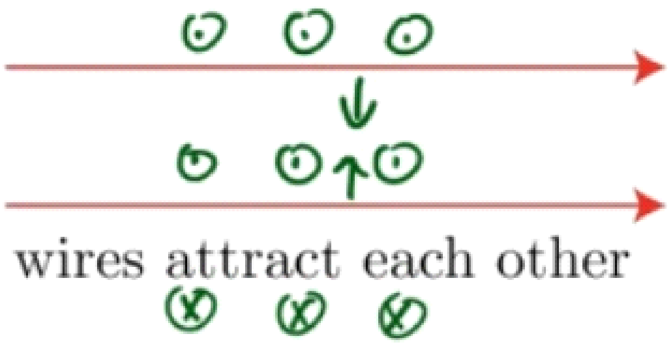
- A wire carries a current of 6 amperes to the left.
- Find the magnetic field at point P, located 0.1 meters below the wire



$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \cdot 10^{-7})(6)}{2\pi(0.1)} = 1.2 \cdot 10^{-5}\,T$

Force Between Parallel Current-Carrying Wires

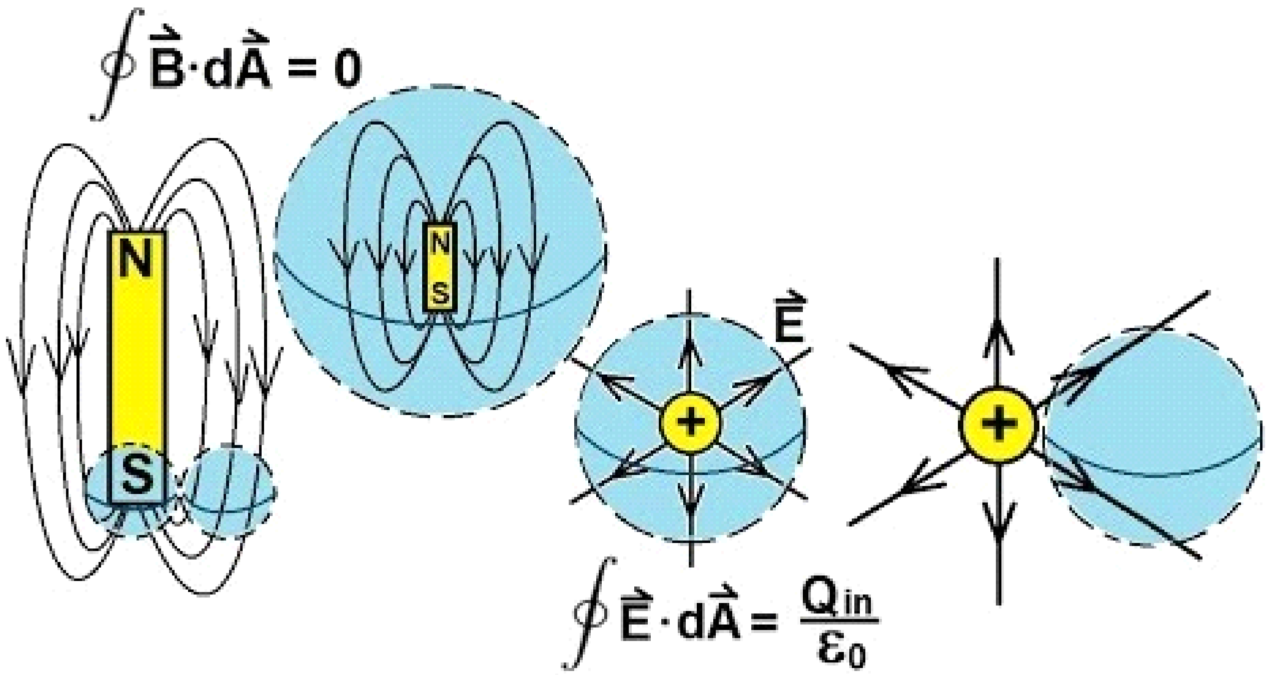
- Use right hand rules to determine force between parallel current-carrying wires
- Find magnetic field due to first wire. Draw it.
- Find direction of force on second wire due to current in second wire. Force on the first wire will be equal and opposite (Newton's third Law)



Gauss's Law for Magnetism

- You can never draw a closed surface with any net magnetic flux because there are no magnetic monopoles.
- This is the basis of Gauss's Law for Magnetism (Maxwell's second equation)
- $\Phi_{B_{net}} = \oint \vec{B} \cdot d\vec{A} = 0$

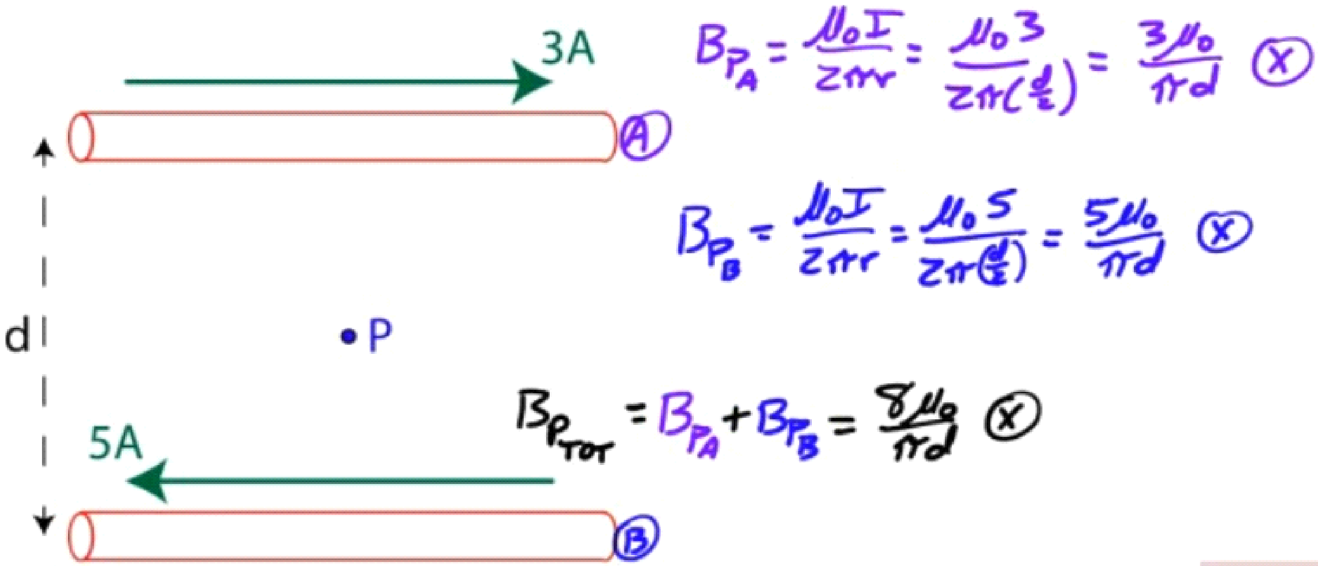
[redacted]



[redacted]

Example 3: Field due to Wires

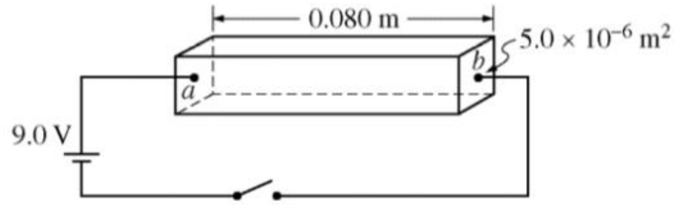
- Two long current-carrying wires are separated by a distance d as shown.
- What is the net magnetic field due to these wires at point P, located midway between the two wires, if the top wire carries a current of 3A and the bottom wire caries a current of 5A.

- 

Example 4: Force on a Wire

- A 5-m long straight wire runs at a 45-degree angle to a uniform magnetic field of 5 T. If the force on the wire is 1N, determine the current in the wire
- $F_B = IlB\, sin\, \theta$
- $I = \frac{F}{lB\, sin\, \theta} = \frac{1}{5 \times 5 \times sin\, 45°} = 0.0566A$

2009 Free Response Question 2

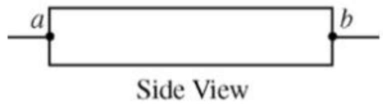


E&M. 2.

A 9.0 V battery is connected to a rectangular bar of length 0.080 m, uniform cross-sectional area $5.0 \times 10^{-6}\ \text{m}^2$, and resistivity $4.5 \times 10^{-4}\ \Omega\text{•m}$, as shown above. Electrons are the sole charge carriers in the bar. The wires have negligible resistance. The switch in the circuit is closed at time $t = 0$.

(a) Calculate the power delivered to the circuit by the battery.

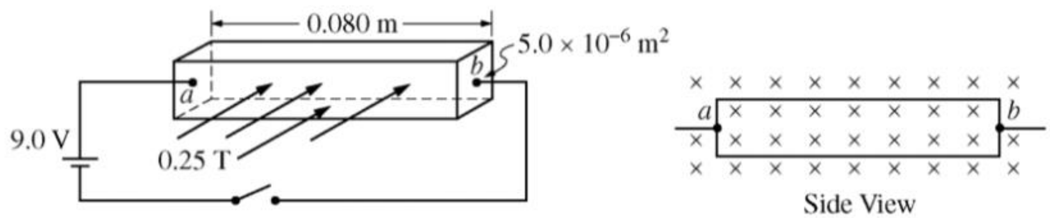
(b) On the diagram below, indicate the direction of the electric field in the bar.



Explain your answer.

(c) Calculate the strength of the electric field in the bar.

A uniform magnetic field of magnitude 0.25 T perpendicular to the bar is added to the region around the bar, as shown below.



(d) Calculate the magnetic force on the bar.

a) $P = IV = \frac{V^2}{R}$

$R = \frac{\rho L}{A} = \frac{(4.5 \cdot 10^{-4}\ \Omega \cdot m)(.08m)}{(5 \cdot 10^{-6} m^2)}$

$P = \frac{(9V)^2}{7.2\Omega} = \boxed{11.25\ W}$

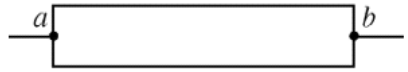
$R = 7.2\ \Omega$

b) $\rightarrow$

c) $E = \frac{V}{d} = \frac{9V}{.08m} = \boxed{112.5\ \frac{V}{m}}$

d) $F = I\ell B = \frac{V}{R}\ell B = \left(\frac{9V}{7.2\Omega}\right)(.08)(.25) = \boxed{.025N}$

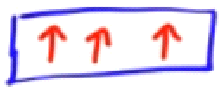
(e) The electrons moving through the bar are initially deflected by the external magnetic field. On the diagram below, indicate the direction of the additional electric field that is created in the bar by the deflected electrons.



Side View

(f) The electrons eventually experience no deflection and move through the bar at an average speed of 3.5×10^{-3} m/s . Calculate the strength of the additional electric field indicated in part (e).

e)



f) $F_e = F_B \Rightarrow qE = qvB \Rightarrow E = vB = (3.5\cdot 10^{-3})(.25)$

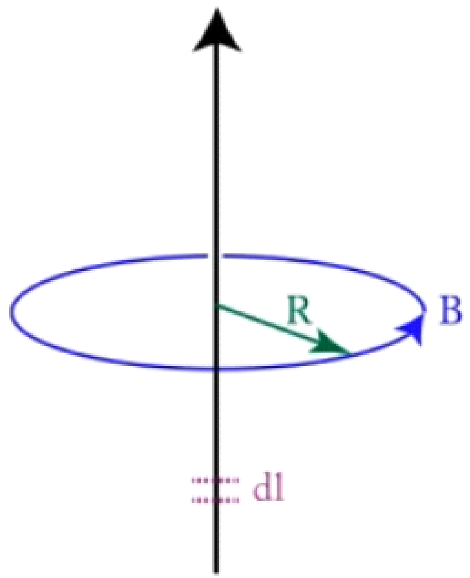
$$\boxed{E = 8.75\cdot 10^{-4}\ \text{V/m}}$$

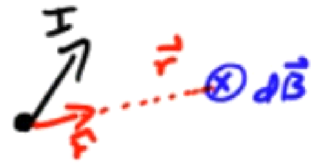
3.5 - The Biot-Savart Law

Friday, March 3, 2017 2:15 PM

Biot-Savart Law

- A "brute force" method of finding the magnetic field due to a length of current-carrying wire





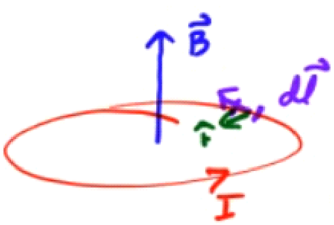
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l} \times \hat{r}}{r^2}\right)$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l} \times \vec{r}}{r^3}\right)$$

Example 1: Magnetic Field due to a Current Loop

- Derive the magnetic field due to a current loop at the center of the loop



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

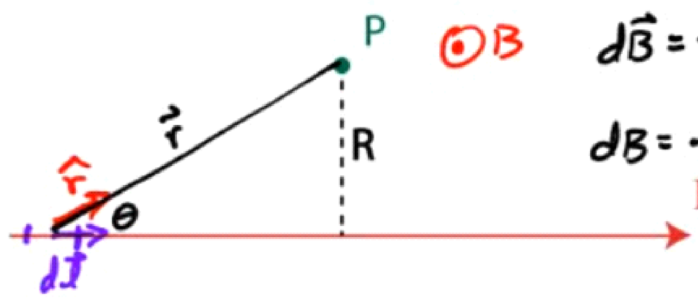
$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2} \xrightarrow[\sin\theta = 1]{d\vec{l} \times \hat{r} = dl \sin\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r^2}\int dl \xrightarrow{\int dl = 2\pi r} B = \frac{\mu_0 I}{4\pi r^2}(2\pi r) \Rightarrow$$

$$\boxed{B = \frac{\mu_0 I}{2r}}$$

Example 2: Magnetic Field due to Long Straight Current-Carrying Wire

- Derive the magnetic field strength at a point P located a distance R from an infinitely long current-carrying wire using the Biot-Savart Law



$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}(d\vec{l}\times\hat{r}) \xrightarrow{d\vec{l}\times\hat{r} = dx\sin\theta}$$

$$dB = \frac{\mu_0 I\,dx\sin\theta}{4\pi r^2} \xrightarrow[\sin\theta = \frac{R}{\sqrt{x^2+R^2}}]{r^2 = x^2+R^2}$$

$$dB = \frac{\mu_0 I\,dx\,R}{4\pi(x^2+R^2)^{\frac{3}{2}}} \Rightarrow B = \int dB = \int_{x=-\infty}^{\infty}\frac{\mu_0 I\,dx\,R}{4\pi(x^2+R^2)^{\frac{3}{2}}} \Longrightarrow$$

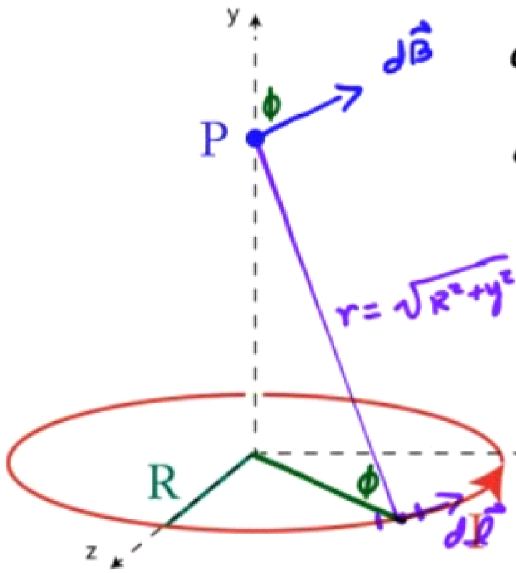
$$B = \frac{\mu_0 I R}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{(x^2+R^2)^{\frac{3}{2}}} \xrightarrow{\text{Table of Integrals}}$$

$$B = \frac{\mu_0 I R}{4\pi}\left(\frac{x}{R^2(R^2+x^2)^{\frac{1}{2}}}\right)\Bigg|_{-\infty}^{\infty} = \frac{\mu_0 I R}{4\pi}\left(\frac{\infty}{R^2(R^2+\infty^2)^{\frac{1}{2}}} - \frac{-\infty}{R^2(R^2+(-\infty)^2)^{\frac{1}{2}}}\right) \Rightarrow$$

$$B = \frac{\mu_0 I R}{4\pi}\left(\frac{1}{R^2}+\frac{1}{R^2}\right) = \frac{\mu_0 I R}{4\pi}\left(\frac{2}{R^2}\right) \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R}}$$

Example 3: Magnetic Field due to A Current Loop

- Derive the magnetic field due to a current loop at a point out of the plane of the loop but centered on the loop's axis as shown



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2} \xrightarrow{r^2 = R^2+y^2}$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl \sin\theta}{R^2+y^2} \xrightarrow[\sin\theta = 1]{\theta = 90^\circ}$$

$$dB = \frac{\mu_0 I \, dl}{4\pi(R^2+y^2)}$$

$$B = \int dB_y = \int dB\cos\phi \xrightarrow[\cos\phi = \frac{R}{\sqrt{R^2+y^2}}]{\cos\phi = \frac{adj}{hyp}}$$

$$B = \int \frac{\mu_0 I}{4\pi}\frac{R\,dl}{(R^2+y^2)^{\frac{3}{2}}} \Longrightarrow$$

$$B = \frac{\mu_0 I R}{4\pi(R^2+y^2)^{\frac{3}{2}}}\int dl \Longrightarrow$$

$$B = \frac{\mu_0 I R}{4\pi(R^2+y^2)^{\frac{3}{2}}}(2\pi R) \Longrightarrow$$

$$\boxed{B = \frac{\mu_0 I R^2}{2(R^2+y^2)^{\frac{3}{2}}}}$$

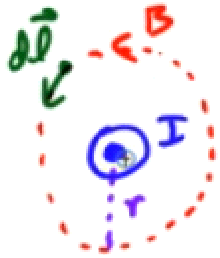
3.6 - Ampere's Law

Sunday, March 5, 2017 8:08 PM

Ampere's Law

- Ampere's Law provides an elegant method of finding the magnetic field due to current flowing in a wire in situations of planar and cylindrical symmetry

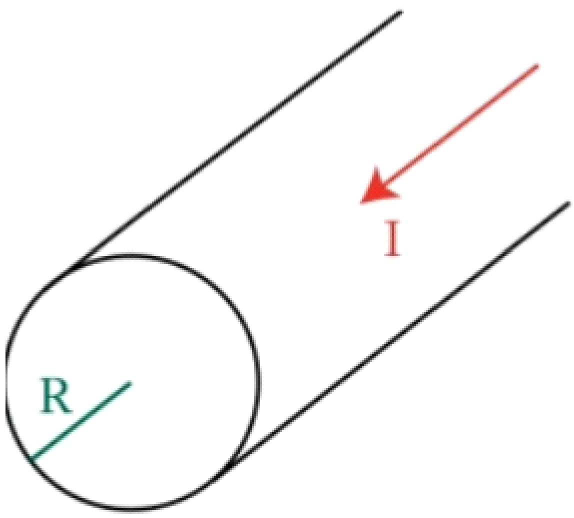
$$\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$$



$$\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Example 1: Magnetic Field of a Wire



1. Find the magnetic field outside a current-carrying wire

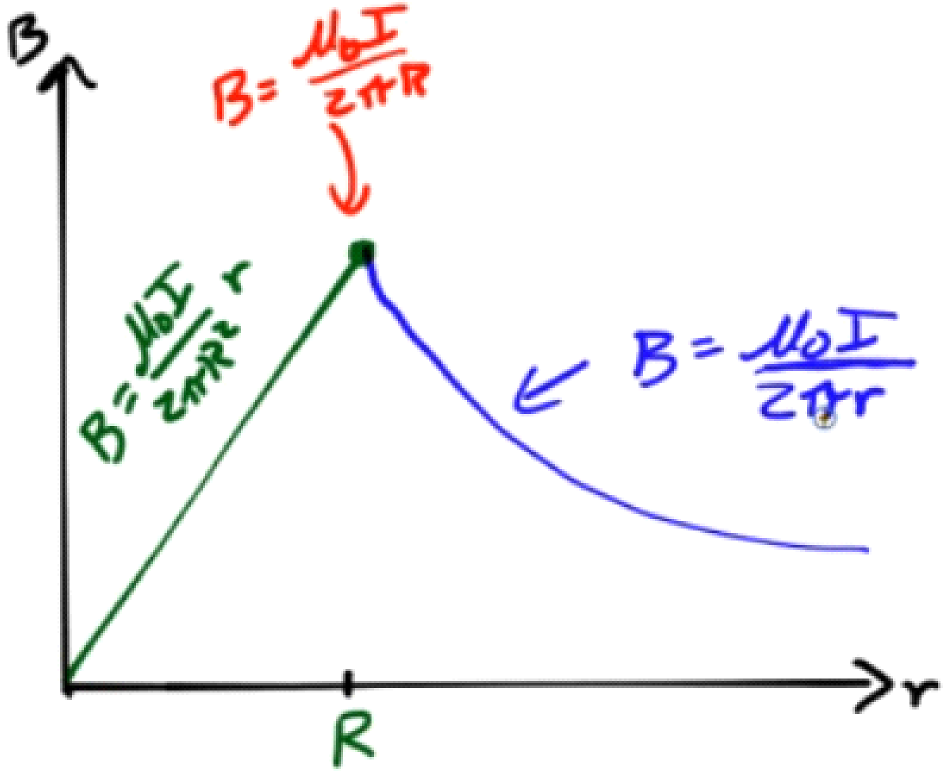
- $\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$
- $B \times 2\pi r = \mu_0 I$
- $B = \dfrac{\mu_0 I}{2\pi r}$

2. Find the magnetic field inside a current-carrying wire

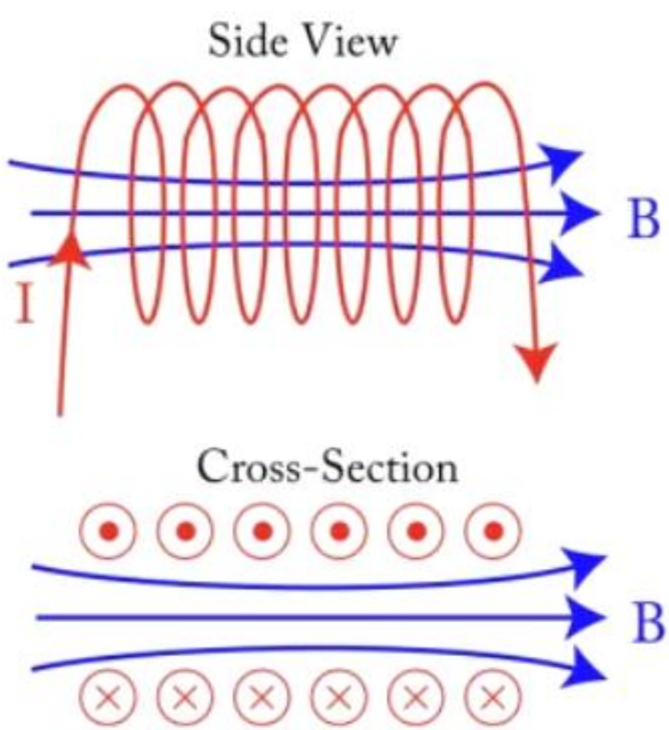
- $\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$
- $B \times 2\pi r = \mu_0 \left(\dfrac{\pi r^2}{\pi R^2}\right) I$

- $B = \dfrac{\mu_0 I r}{2\pi R^2}$

3. Graph the magnetic field of a current-carrying wire as a function of the distance from the center of the wire



Example 2: Magnetic Field in a Solenoid

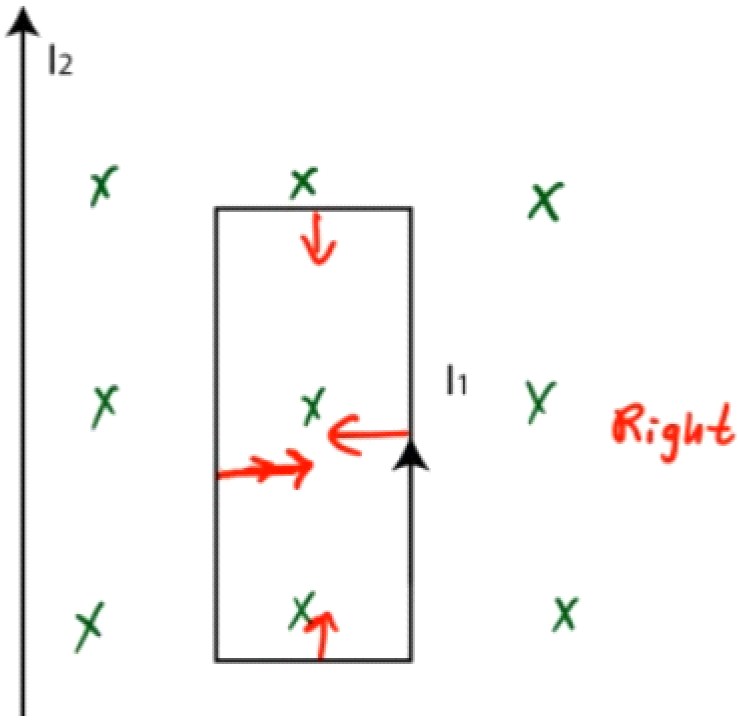


- Calculate the magnetic field in the middle of a solenoid (i.e. Slinky) using Ampere's Law
- $\oint_{\substack{closed \\ loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$

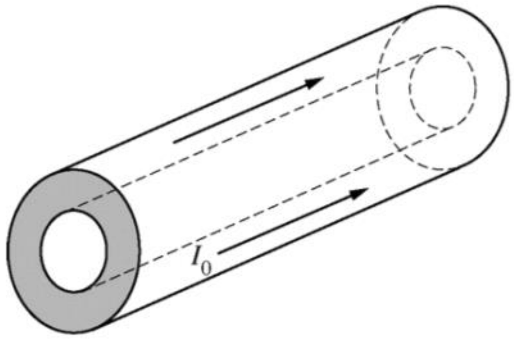
- $\because I_{penetrating} = \frac{l}{L} NI$
- $\therefore Bl = \mu_0 \frac{l}{L} NI$
- $\therefore B = \frac{N}{L} \mu_0 I$

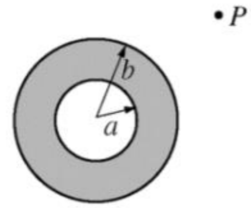
Example 3: Net Force on a Wire Loop

- A rectangular wire loop carrying current I_1 lies a distance r to the right of a long wire carrying current I_2.
- What is the direction of the net force on the loop?



2011 Free Response Question 3





Cross-sectional View
(current into page)

E&M. 3.

A section of a long conducting cylinder with inner radius a and outer radius b carries a current I_0 that has a uniform current density, as shown in the figure above.

(a) Using Ampère's law, derive an expression for the magnitude of the magnetic field in the following regions as a function of the distance r from the central axis.

i. $r < a$

____

ii. $a < r < b$

____

iii. $r = 2b$

(b) On the cross-sectional view in the diagram above, indicate the direction of the field at point P, which is at a distance $r = 2b$ from the axis of the cylinder.

(c) An electron is at rest at point P. Describe any electromagnetic forces acting on the electron. Justify your answer.

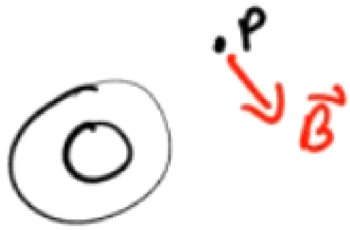
ai) $\oint_{\text{closed loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{pen} \xRightarrow{I_{pen}=0} \boxed{B=0}$

ii) $\oint_{\text{closed loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{pen} \Rightarrow B(2\pi r) = \mu_0 I_0\left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2}\right) \Rightarrow$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}\left(\frac{r^2-a^2}{b^2-a^2}\right)}$$

iii) $\oint_{\text{closed loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{pen} \xRightarrow{r=2b} B(2\pi\, 2b) = \mu_0 I_0 \Rightarrow$

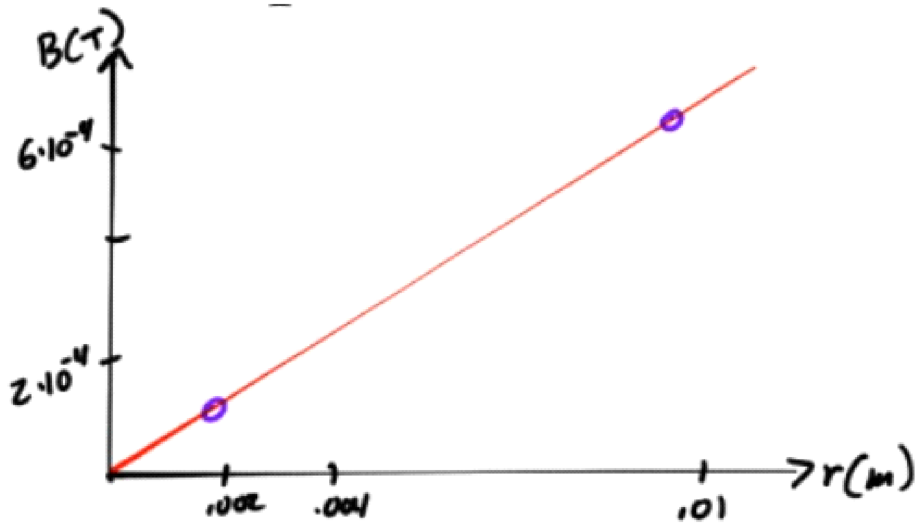
$$\boxed{B = \frac{\mu_0 I_0}{4\pi b}}$$



c) $F_E = qE \qquad F_E = 0$

$F_B = q(\vec{v} \times \vec{B}) \qquad v = 0 \Rightarrow F_B$

$\Big\}$ No EM Forces



$\text{Slope} = \frac{\text{rise}}{\text{run}} = .0635\ \text{T/m}$

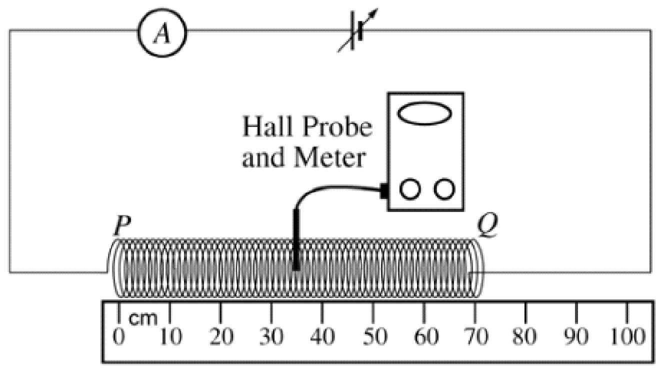
$B = \frac{\mu_0 I_0}{2\pi b^2} r$

$y = m x$

$\text{slope} = \frac{\mu_0 I_0}{2\pi b^2} \Rightarrow \mu_0 = \frac{2\pi b^2}{I_0}(\text{slope}) \Rightarrow$

$\mu_0 = \frac{2\pi (.01)^2}{25}(.0635) = 1.6 \cdot 10^{-6}\ \frac{T \cdot m}{A}$

2005 Free Response Question 3



E&M. 3.

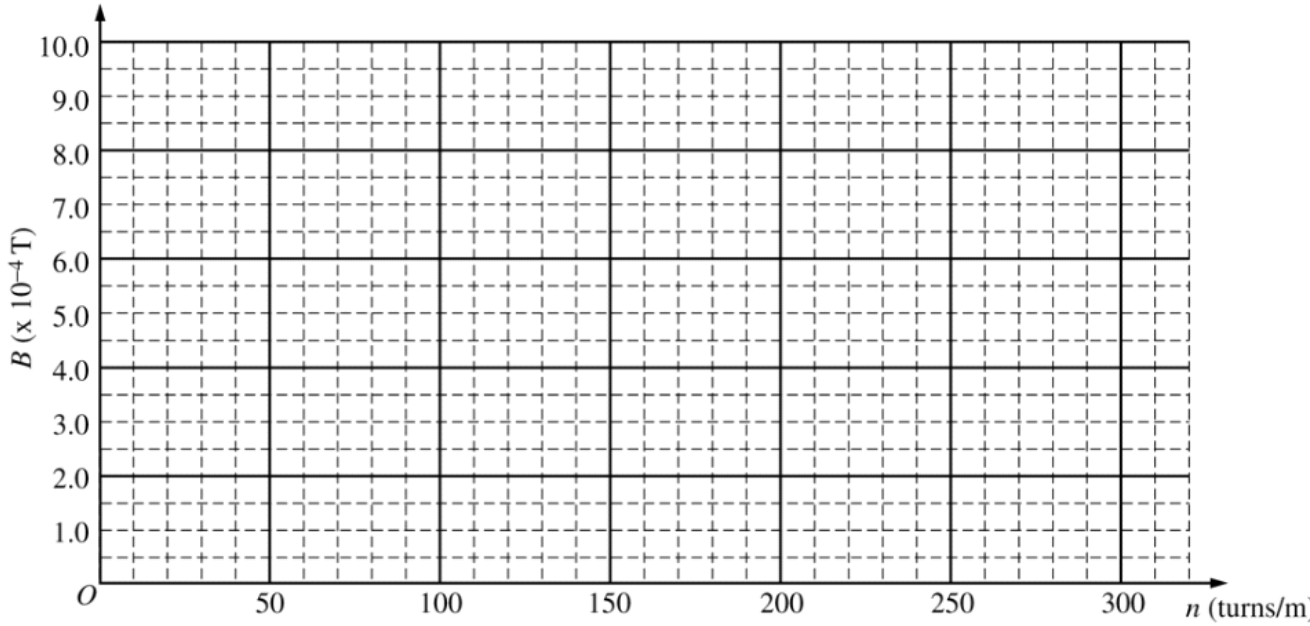
A student performs an experiment to measure the magnetic field along the axis of the long, 100-turn solenoid PQ shown above. She connects ends P and Q of the solenoid to a variable power supply and an ammeter as shown. End P of the solenoid is taped at the 0 cm mark of a meterstick. The solenoid can be stretched so that the position of end Q can be varied. The student then positions a Hall probe* in the center of the solenoid to measure the magnetic field along its axis. She measures the field for a fixed current of 3.0 A and various positions of the end Q. The data she obtains are shown below.

Trial	Position of End Q (cm)	Measured Magnetic Field (T) (directed from P to Q)	n (turns/m)
1	40	9.70×10^{-4}	
2	50	7.70×10^{-4}	
3	60	6.80×10^{-4}	
4	80	4.90×10^{-4}	
5	100	4.00×10^{-4}	

(a) Complete the last column of the table above by calculating the number of turns per meter.

*A Hall Probe is a device used to measure the magnetic field at a point.

(b) On the axes below, plot the measured magnetic field B versus n. Draw a best-fit straight line for the data points.

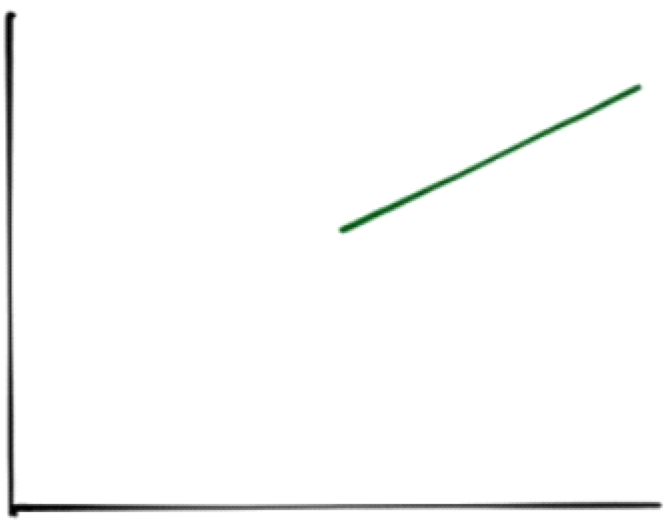


(c) From the graph, obtain the value of μ_0, the magnetic permeability of vacuum.

(d) Using the theoretical value of $\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A, determine the percent error in the experimental value of μ_0 computed in part (c).

$\frac{100 \text{ turns}}{.4 \text{m}} = 250 \frac{\text{turns}}{\text{m}}$

b)



c) $B = \mu_0 n I \Rightarrow \mu_0 = \frac{B}{nI} = \frac{\text{slope}}{I}$

slope $= 3.81 \cdot 10^{-6}$ T·m

c) $B = \mu_0 n I \Rightarrow \mu_0 = \frac{B}{nI} = \frac{\text{slope}}{I}$

$\text{slope} = 3.81 \cdot 10^{-6} \frac{T \cdot m}{\text{turn}}$

$\mu_0 = \frac{\text{slope}}{I} = \frac{3.81 \cdot 10^{-6}}{3} = 1.27 \cdot 10^{-6} \frac{T \cdot m}{A}$

d) $\% \text{ error} = \frac{|\text{actual} - \text{accepted}|}{\text{accepted}} \cdot 100\% = \frac{|4\pi \cdot 10^{-7} - 1.27 \cdot 10^{-6}|}{4\pi \cdot 10^{-7}} \cdot 100\%$

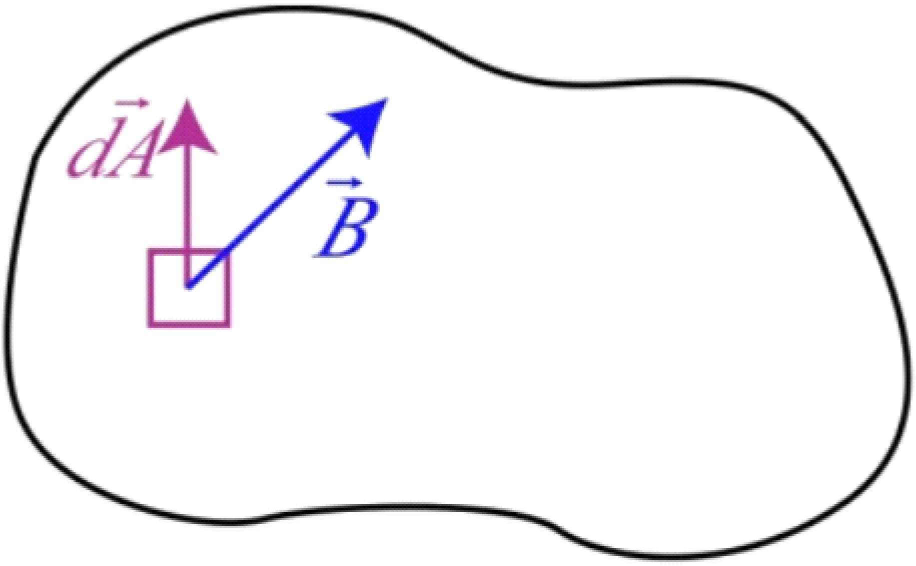
$= \boxed{1.06\%}$

3.7 - Magnetic Flux

Sunday, March 5, 2017 8:08 PM

Magnetic Flux

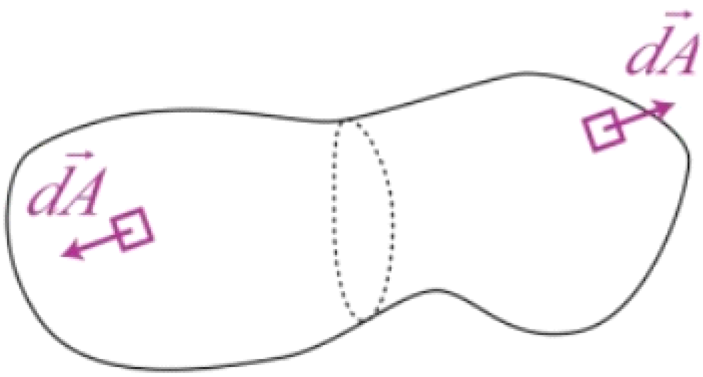
- Magnetic Flux (Φ_B or Φ_M) describes the amount of magnetic field penetratign a surface
- Units of magnetic flux are Weber (Wb)
- $1\, weber = 1\, tesla \cdot m^2$



- $d\Phi_B = \vec{B} \cdot d\vec{A} = B\, dA \cos\theta$
- $\Phi_B = \int d\Phi_B = \int_{\substack{open\\ surface}} \vec{B} \cdot d\vec{A}$

Magnetic Flux Through Closed Surfaces

- Normal to closed surfaces point from the inside to the outside

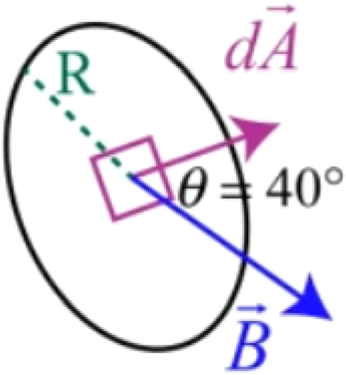


$\Phi_B = \int d\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$

Gauss's Law for Magnetism

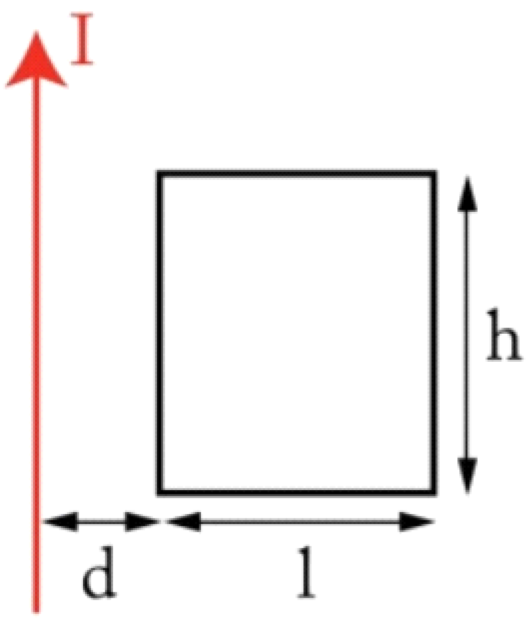
- The total magnetic flux through any closed surface is zero
- This would not be true if magnetic monopoles were found to exist
- $\Phi_B = \int_{\substack{closed \\ surface}} \vec{B} \cdot d\vec{A} = 0$

Example 1: Flux through a Circular Loop



- Calculate the flux of 3-Tesla uniform magnetic field through the circular loop of radius 0.2 meters with three turns of wire
- $\Phi_B = \int_{\substack{open \\ surface}} \vec{B} \cdot d\vec{A} = B\ A \cos\theta = B\pi R^2 \cos\theta = 3 \times \pi \times 0.2^2 \times \cos 40° = 0.866\ Wb$

Example 2: Flux Due to a Wire



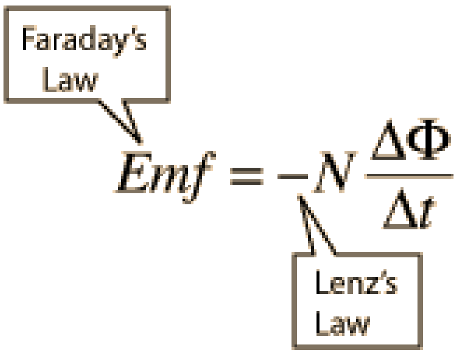
- A long straight wire carries a current I as shown.
- Calculate the magnetic flux through the loop
- $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int_{r=d}^{r=d+l} \frac{\mu_0 I}{2\pi r}\ h\ dr = \frac{\mu_0 Ih}{2\pi r} \int_{r=d}^{r=d+l} \frac{dr}{r} = \frac{\mu_0 Ih}{2\pi r}(ln(d+l) - ln\, d\,)$
$= \frac{\mu_0 Ih}{2\pi r}\ ln \frac{d+l}{d}$

3.8 - Faraday's Law & Lenz's Law

Sunday, March 5, 2017 9:54 PM

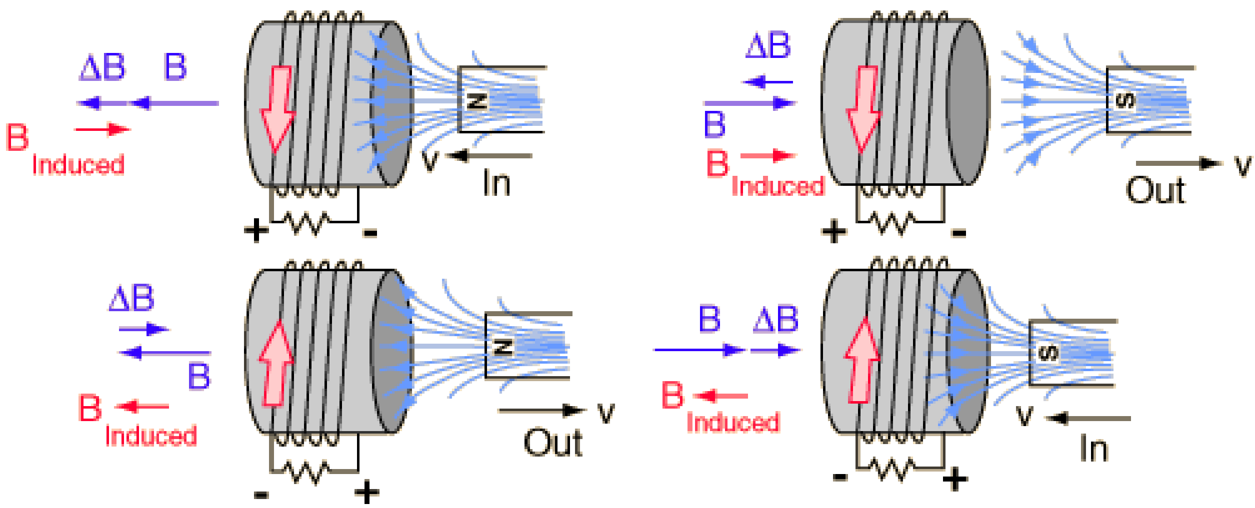
Faraday's Law

- The induced emf due to a changing magnetic field is equal in magnitude to the rate of change of the magnetic flux through a surface bounded by the circuit
- The direction of the induced current is given by Lenz's Law
- $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int\limits_{\substack{open\\surface}} \vec{B}\cdot d\vec{A} = \oint\limits_{\substack{closed\\loop}} \vec{E}\cdot d\vec{l}$



Lenz's Law

- The current induced by a changing magnetic flux creates a magnetic field opposing the change in flux



Maxwell's Equations

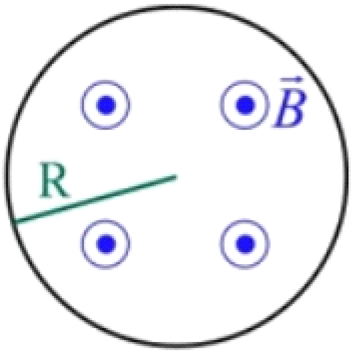
- Gauss's Law $$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

- Gauss's Law for Magnetism $$\oint \vec{B} \bullet d\vec{A} = 0$$

- Faraday's Law $$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_{\substack{open\\surface}} \vec{B} \bullet d\vec{A}$$

- Ampere's Law* $$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{l} = \mu_0 I$$

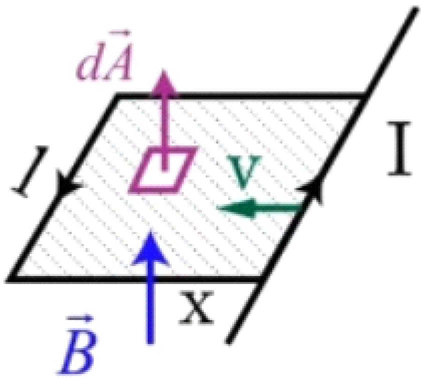
Example 1: Induced Current in a Loop



- A magnetic field of strength $B(t) = 3t^2 - 2t + 1$ is directed out of the plane of a circular loop of wire as shown. A lamp (not shown) is part of the loop

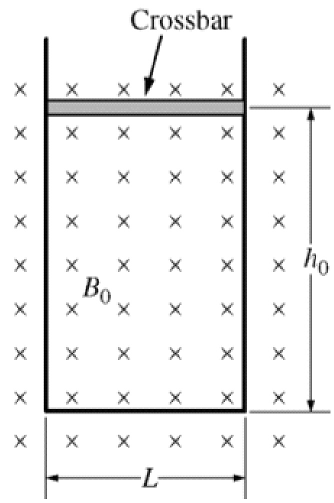
1. Find the generated emf as a function of time
   - $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt} \int_{\substack{open\\surface}} \vec{B} \cdot d\vec{A} = -\frac{d}{dt}(BA) = -A\frac{d}{dt}(3t^2 - 2t + 1) = -\pi R^2(6t - 2)$
2. Determine the current through the 100-ohm lamp as a function of time
   - $I = \frac{\varepsilon}{R} = -\frac{\pi R^2(6t-2)}{100}$
3. What is the direction of the current through the loop at time t=5s?
   - $\frac{dB}{dt} = 6 \times 5 - 2 = 28 > 0$
   - Clockwise, due to Lenz's Law

Example 2: Rod on Rails



- Consider a circuit in which a current-carrying rod on rails is moved to the left with constant velocity v. If the circuit is perpendicular to a constant magnetic field, determine the induced emf in the circuit
- $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_{\substack{open \\ surface}} \vec{B} \cdot d\vec{A} = -Bl\frac{dx}{dt} = -Blv$

2012 Free Response Question 3



E&M. 3.

A closed loop is made of a U-shaped metal wire of negligible resistance and a movable metal crossbar of resistance R. The crossbar has mass m and length L. It is initially located a distance h_0 from the other end of the loop. The loop is placed vertically in a uniform horizontal magnetic field of magnitude B_0 in the direction shown in the figure above. Express all algebraic answers to the questions below in terms of B_0, L, m, h_0, R, and fundamental constants, as appropriate.

(a) Determine the magnitude of the magnetic flux through the loop when the crossbar is in the position shown.

The crossbar is released from rest and slides with negligible friction down the U-shaped wire without losing electrical contact.

(b) On the figure below, indicate the direction of the current in the crossbar as it falls.

Justify your answer.

(c) Calculate the magnitude of the current in the crossbar as it falls as a function of the crossbar's speed v.

a) $\Phi_B = \int_{\text{open surface}} \vec{B}\cdot d\vec{A} = \boxed{B_0 L h_0}$

b)

c) $I = \frac{\varepsilon}{R} = \frac{-\frac{d\Phi_B}{dt}}{R} = \frac{-\frac{d}{dt}(B_0 L h)}{R} = -\frac{B_0 L}{R}\frac{dh}{dt} \xrightarrow{v=\frac{dh}{dt}}$

$I = -\frac{B_0 L v}{R} \xrightarrow{\text{mag}} |I| = \boxed{\frac{B_0 L v}{R}}$

(d) Derive, but do NOT solve, the differential equation that could be used to determine the speed v of the crossbar as a function of time t.

(e) Determine the terminal speed v_T of the crossbar.

(f) If the resistance R of the crossbar is increased, does the terminal speed increase, decrease, or remain the same?

____ Increases ____ Decreases ____ Remains the same

Give a physical justification for your answer in terms of the forces on the crossbar.

d) $\uparrow F_B$, $\downarrow mg$, $\downarrow +y$

$F_{NET_y} = mg - F_B = ma \qquad \xrightarrow{F_B = \int I(d\vec{l} \times \vec{B}) = ILB_0}$

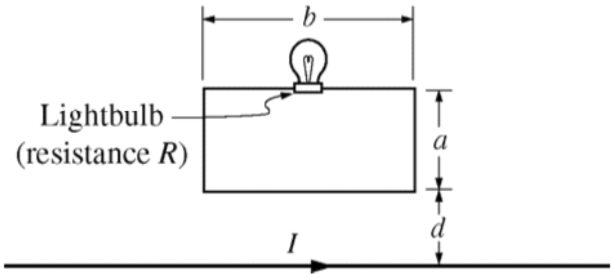
$mg - ILB_0 = ma \xrightarrow[a = \frac{dv}{dt}]{I = \frac{B_0 L v}{R}} mg - \frac{B_0^2 L^2 v}{R} = m\frac{dv}{dt} \Rightarrow$

$$\boxed{\frac{dv}{dt} + \frac{B_0^2 L^2}{mR} v - g = \emptyset}$$

e) $F_B = mg \Rightarrow ILB_0 = mg \Rightarrow \frac{B_0^2 L^2 v}{R} = mg \Rightarrow V_{term} = \boxed{\frac{mgR}{B_0^2 L^2}}$

f) Increase

2010 Free Response Question 3



E&M. 3.

The long straight wire illustrated above carries a current I to the right. The current varies with time t according to the equation $I = I_0 - Kt$, where I_0 and K are positive constants and I remains positive throughout the time period of interest. The bottom of a rectangular loop of wire of width b and height a is located a distance d above the long wire, with the long wire in the plane of the loop as shown. A lightbulb with resistance R is connected in the loop. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current in the loop.

____Clockwise ____Counterclockwise

Justify your answer.

(b) Indicate whether the lightbulb gets brighter, gets dimmer, or stays the same brightness over the time period of interest.

____Gets brighter ____Gets dimmer ____Remains the same

Justify your answer.

(c) Determine the magnetic field at $t = 0$ due to the current in the long wire at distance r from the long wire.

(d) Derive an expression for the magnetic flux through the loop as a function of time.

(e) Derive an expression for the power dissipated by the lightbulb.

a) ccw

b) $P = I^2 R$ constant remains the same

c)

$$B = \frac{\mu_0 I}{2\pi r} \xrightarrow[I=I_0]{t=0} \boxed{B = \frac{\mu_0 I_0}{2\pi r}}$$

$$\Phi_B = \int_{\substack{\text{open}\\ \text{surface}}} \vec{B}\cdot d\vec{A} = \int_{r=d}^{r=d+a} \frac{\mu_0 I}{2\pi r} b\,dr = \frac{\mu_0 I b}{2\pi}\int_d^{d+a}\frac{dr}{r} =$$

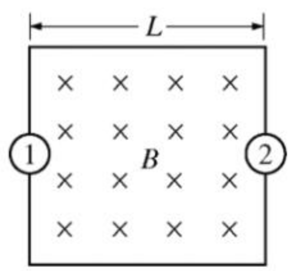
$$\frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{d}\right) \Rightarrow \boxed{\Phi_B = \frac{\mu_0 b}{2\pi}\ln\left(\frac{d+a}{d}\right)(I_0 - kt)}$$

e) $P = \frac{\varepsilon^2}{R}$ $\quad \varepsilon = -\frac{d\Phi_B}{dt} = \frac{-d}{dt}\left(\frac{\mu_0 b}{2\pi}\ln\left(\frac{d+a}{d}\right)(I_0 - kt)\right) =$

$$\varepsilon = \frac{k\mu_0 b}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

$$\boxed{P = \frac{\left[\frac{\mu_0 k b}{2\pi}\ln\left(\frac{d+a}{d}\right)\right]^2}{R}}$$

2009 Free Response Question 3



E&M. 3.

A square conducting loop of side L contains two identical lightbulbs, 1 and 2, as shown above. There is a magnetic field directed into the page in the region inside the loop with magnitude as a function of time t given by $B(t) = at + b$, where a and b are positive constants. The lightbulbs each have constant resistance R_0. Express all answers in terms of the given quantities and fundamental constants.

(a) Derive an expression for the magnitude of the emf generated in the loop.

(b) i. Determine an expression for the current through bulb 2.

ii. Indicate on the diagram above the direction of the current through bulb 2.

(c) Derive an expression for the power dissipated in bulb 1.

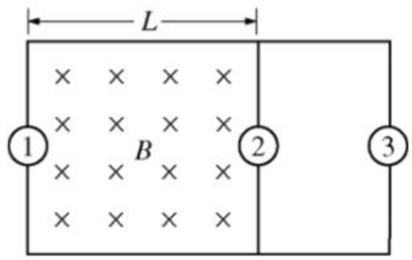
a) $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(AB) = -A\frac{dB}{dt} = -A\frac{d}{dt}(at+b) = -Aa \xrightarrow{A=L^2}$

$$\boxed{|\varepsilon| = aL^2}$$

b) $I = \frac{V}{R} = \boxed{\frac{aL^2}{2R_0}}$ $\boxed{\uparrow}$

c) $P = IV = \frac{aL^2}{2R_0}\,\frac{aL^2}{2} = \boxed{\frac{a^2L^4}{4R_0}}$

Another identical bulb 3 is now connected in parallel with bulb 2, but it is entirely outside the magnetic field, as shown below.

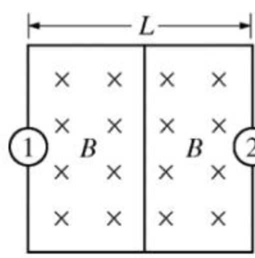


(d) How does the brightness of bulb 1 compare to what it was in the previous circuit?

____ Brighter ____ Dimmer ____ The same

Justify your answer.

Now the portion of the circuit containing bulb 3 is removed, and a wire is added to connect the midpoints of the top and bottom of the original loop, as shown below.



(e) How does the brightness of bulb 1 compare to what it was in the first circuit?

____ Brighter ____ Dimmer ____ The same

Justify your answer.

d) Brighter

e) Same

2008 Free Response Question 3

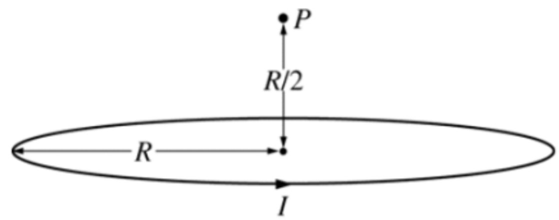


Figure 1

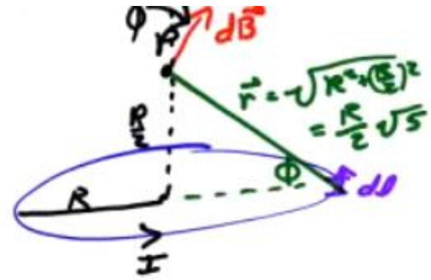
E&M. 3.

The circular loop of wire in Figure 1 above has a radius of R and carries a current I. Point P is a distance of $R/2$ above the center of the loop. Express algebraic answers to parts (a) and (b) in terms of R, I, and fundamental constants.

(a)

i. State the direction of the magnetic field B_1 at point P due to the current in the loop.

ii. Calculate the magnitude of the magnetic field B_1 at point P.



a i) UP

ii) $d\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l}\times\vec{r}}{r^3}\right)$

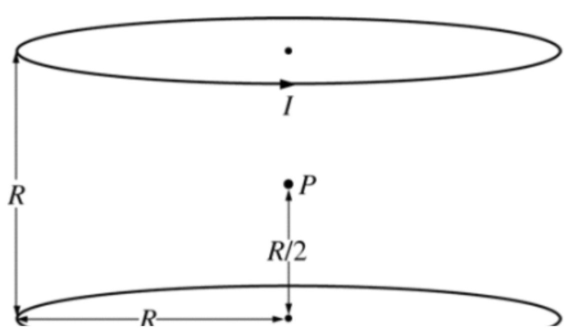
$$dB = \frac{\mu_0 I}{4\pi r^3}\, dl\, r\sin\theta \Rightarrow dB = \frac{\mu_0 I}{4\pi r^2}\, dl$$

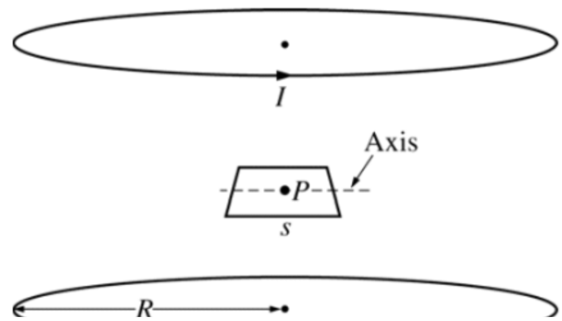
$$B_{vert} = \int dB_{vert} = \int dB\cos\phi \xrightarrow{\cos\phi = \frac{R}{r}} \int \frac{R}{r}\, dB \Rightarrow$$

$$B_{vert} = \int \frac{R}{r}\frac{\mu_0 I}{4\pi r^2}\, dl = \frac{\mu_0 I R}{4\pi r^3}\int_{wire} dl = \frac{\mu_0 I R}{4\pi r^3}(2\pi R) \Rightarrow$$

$$B_{vert} = \frac{\mu_0 I R^2}{2r^3} \xrightarrow{r=\frac{R}{2}\sqrt{5}} B_{vert} = \frac{\mu_0 I R^2}{2\left(\frac{R^3}{8}5\sqrt{5}\right)} = \frac{\mu_0 I}{\frac{5}{4}\sqrt{5}R} \Rightarrow$$

$$B_{vert} = \boxed{\frac{4\mu_0 I}{5\sqrt{5}R}}$$





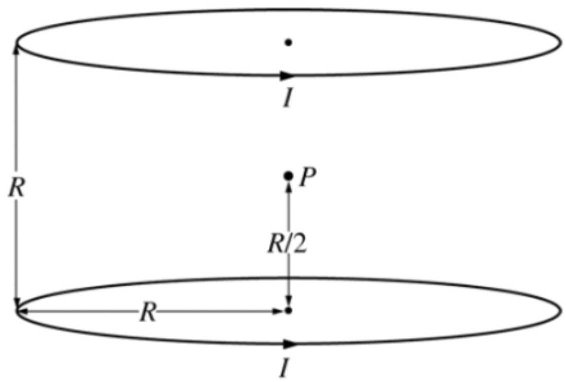


Figure 2

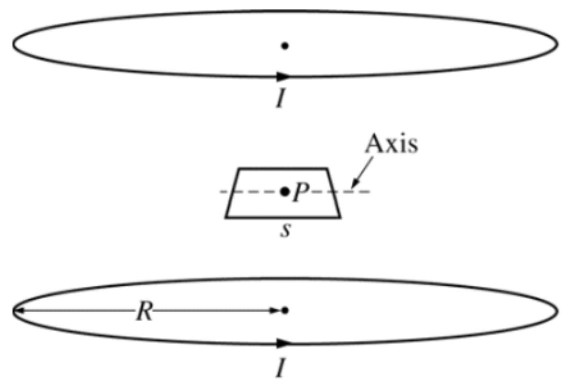


Figure 3

A second identical loop also carrying a current I is added at a distance of R above the first loop, as shown in Figure 2 above.

(b) Determine the magnitude of the net magnetic field B_{net} at point P.

A small square loop of wire in which each side has a length s is now placed at point P with its plane parallel to the plane of each loop, as shown in Figure 3 above. For parts (c) and (d), assume that the magnetic field between the two circular loops is uniform in the region of the square loop and has magnitude B_{net}.

(c) In terms of B_{net} and s, determine the magnetic flux through the square loop.

(d) The square loop is now rotated about an axis in its plane at an angular speed ω. In terms of B_{net}, s, and ω, calculate the induced emf in the loop as a function of time t, assuming that the loop is horizontal at $t = 0$.

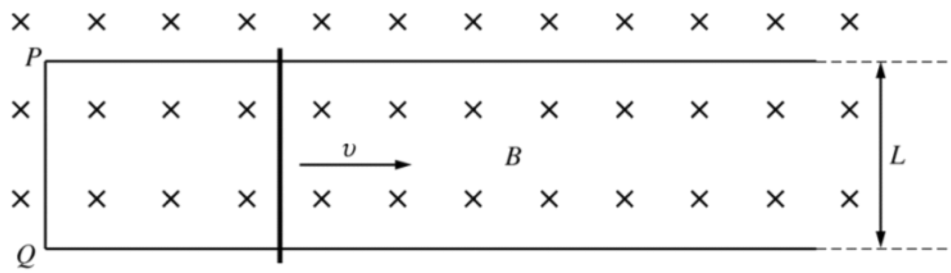
b) $B_p = 2B_{vert} = \boxed{\frac{8\mu_0 I}{5\sqrt{5}R}} = B_{net}$

c) $\Phi_B = \int \vec{B}\cdot d\vec{A} = BA = \frac{8\mu_0 I}{5\sqrt{5}R}s^2 = \boxed{B_{net}s^2}$

d) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B_{net}s^2\cos\omega t\right) = -B_{net}s^2\frac{d}{dt}(\cos\omega t)$

$\Rightarrow \boxed{\mathcal{E} = \omega B_{net}s^2\sin(\omega t)}$

2007 Free Response Question 3



Top View

E&M 3.

In the diagram above, a nichrome wire of resistance per unit length λ is bent at points P and Q to form horizontal conducting rails that are a distance L apart. The wire is placed within a uniform magnetic field of magnitude B pointing into the page. A conducting rod of negligible resistance, which was aligned with end PQ at time $t = 0$, slides to the right with constant speed v and negligible friction. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current induced in the circuit.

______Clockwise   ______Counterclockwise

Justify your answer.

(b) Derive an expression for the magnitude of the induced current as a function of time t.

(c) Derive an expression for the magnitude of the magnetic force on the rod as a function of time.

a) CCW

b) $\mathcal{E} = BLv \qquad R = \lambda(L + 2vt)$

$$I = \frac{\mathcal{E}}{R} = \boxed{\frac{BLv}{\lambda(L+2vt)}}$$

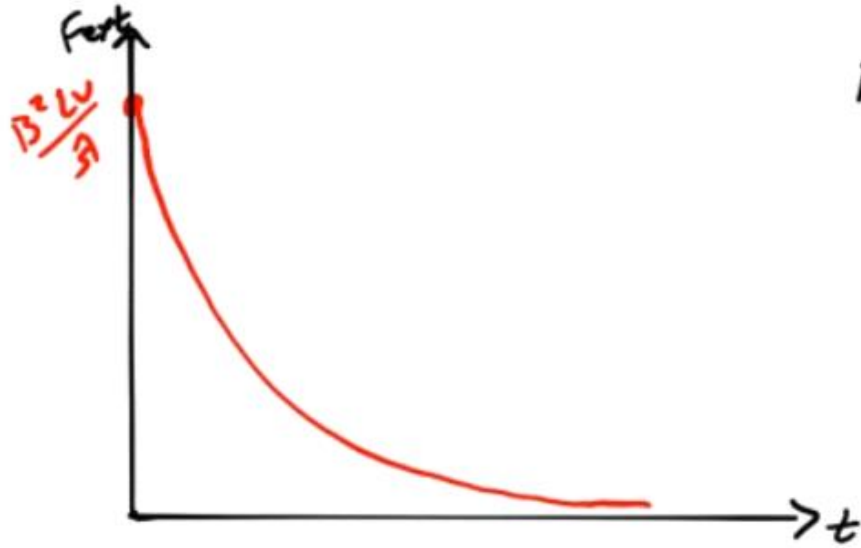
c) $$F_B = ILB = \frac{BLv}{\lambda(L+2vt)} LB = \boxed{\frac{B^2L^2v}{\lambda(L+2vt)}}$$

(d) On the axes below, sketch a graph of the external force F_{ext} as a function of time that must be applied to the rod to keep it moving at constant speed while in the field. Label the values of any intercepts.

(e) The force pulling the rod is now removed. Indicate whether the speed of the rod increases, decreases, or remains the same.

______Increases   _____Decreases   ______Remains the same

Justify your answer.



$F_{ext} = F_B$

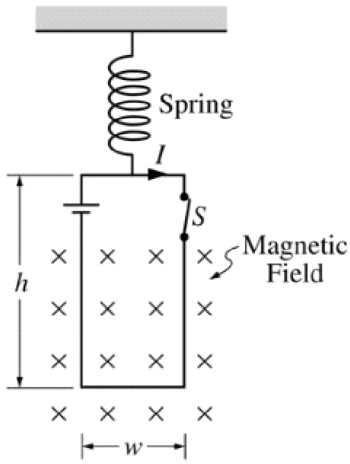
$F_B(t=0) = \frac{B^2L^2v}{RL}$

e) No $F_{ext} \Rightarrow F_B$ opposes motion

N2: $F=ma$, rod must be accelerating in a direction opposite its velocity

Decreases

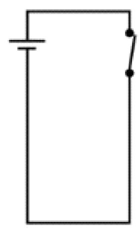
2006 Free Response Question 3



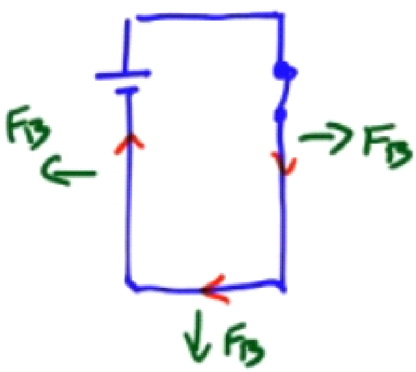
E&M 3.

A loop of wire of width w and height h contains a switch and a battery and is connected to a spring of force constant k, as shown above. The loop carries a current I in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

(a) On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.



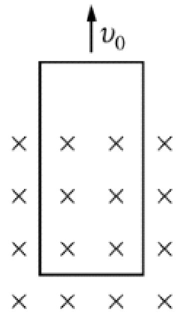
b)

$$F_s = F_B \Rightarrow kx = I\ell B \Rightarrow B = \frac{kx}{I\ell} = \boxed{\frac{kx}{Iw}}$$

The spring and loop are replaced with a loop of the same dimensions and resistance R but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed v_0. Express algebraic answers to the following questions in terms of B_0, v_0, R, and the dimensions of the loop.

(c)

i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



ii. Derive an expression for the magnitude of this current.

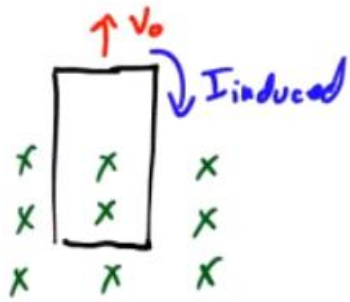
(d) Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.

(e) Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed v_0 increase, decrease, or remain the same?

_____ Increases _____ Decreases _____ Remains the same

Justify your answer.

c i)



c ii) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{-d}{dt}\int \vec{B}\cdot d\vec{A} = -B\frac{d}{dt}\int dA$

$= -B\frac{dA}{dt} = -B\frac{d}{dt}(wy) = -Bw\frac{dy}{dt} \Rightarrow$

$\mathcal{E} = -BwV_o$

$|I| = \left|\frac{\mathcal{E}}{R}\right| = \left|-\frac{BwV_o}{R}\right| = \boxed{\frac{BwV_o}{R}}$

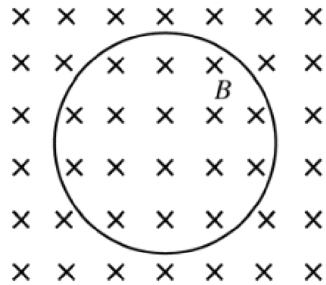
d) $P = IV = \left(\frac{BwV_o}{R}\right)(BwV_o) = \boxed{\frac{B^2w^2V_o^2}{R}}$

e) Stronger $B \Rightarrow$ Stronger $F_B = I\ell B$ + larger $I_{ind} = \frac{BwV_o}{R}$

all lead to stronger F_{ext}

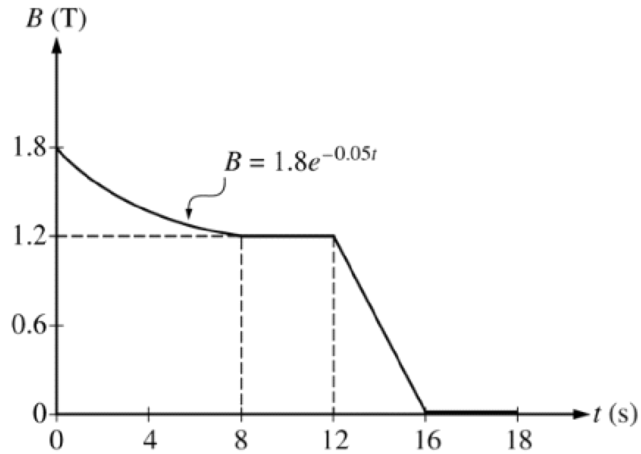
$\boxed{\text{Increase}}$

2013 Free Response Question 3



E&M 3.

The figure above shows a circular loop of area $0.25\ \text{m}^2$ and resistance $12\ \Omega$ that lies in the plane of the page. A magnetic field of magnitude B directed into the page exists in the area of the loop. The field varies with time t, as shown in the graph below.



(a)

i. Derive an expression for the magnitude of the induced emf in the loop as a function of time for the interval $t = 0$ s to $t = 8$ s.

ii. Calculate the magnitude of the induced current I in the loop at time $t = 4$ s.

a i) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{-d}{dt}AB \xrightarrow{A = \text{constant}} -A\frac{dB}{dt} = -A\frac{d}{dt}\left(1.8e^{-.05t}\right) \Rightarrow$

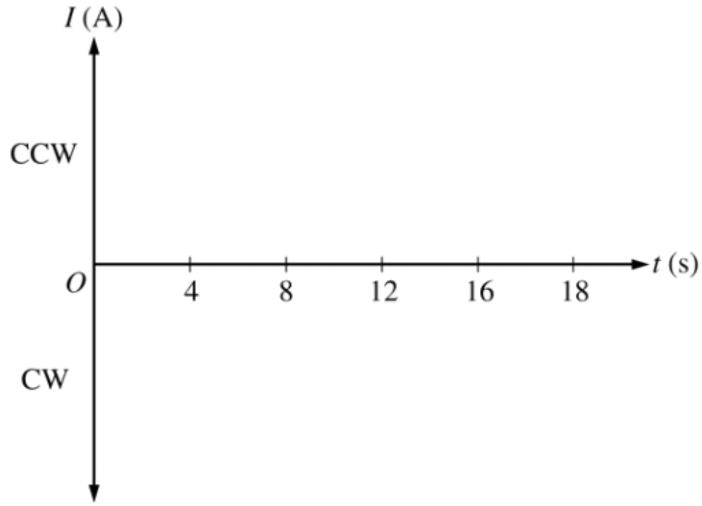
$\mathcal{E} = -A\left(1.8e^{-.05t} \cdot (-.05)\right) = -(.25)(1.8e^{-.05t})(-.05) \Rightarrow$

$\boxed{\mathcal{E} = 0.0225e^{-.05t}}$

a ii) $I = \frac{\mathcal{E}}{R} = \frac{.0225e^{-.05t}}{12} \xrightarrow{t = 4\text{ s}} \boxed{0.00154\text{ A}}$

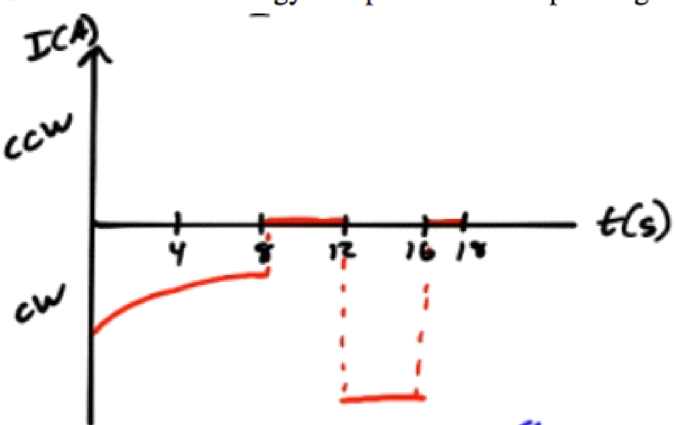
(b)

i. Sketch a graph of the induced current I in the loop as a function of time t from $t = 0$ s to $t = 18$ s on the axes below, assuming that a counterclockwise (CCW) current is positive.



ii. For the time interval 12 s to 16 s, justify the direction of the current you have indicated in your graph.

(c) Calculate the total energy dissipated in the loop during the first 8 s shown.



c) $E=\int P\,dt=\int \frac{E^2}{R}\,dt=\int_{t=0}^{8s}\frac{(.0225e^{-.05t})^2}{12}\,dt=\frac{(.0225)^2}{12}\int_0^8 e^{-.1t}\,dt$

$\Rightarrow E=4.22\cdot10^{-5}\left(\frac{1}{-.1}\right)\int_0^8 e^{-.1t}(-.1)\,dt=-4.22\cdot10^{-4}\left[e^{-.1t}\right]\Big|_0^8\Rightarrow$

$E=-4.22\cdot10^{-4}\left(e^{-.8}-e^{0}\right)=-4.22\cdot10^{-4}(.449-1)=\boxed{2.32\cdot10^{-4}\,J}$

4.1 - Inductance

Monday, March 6, 2017 3:38 PM

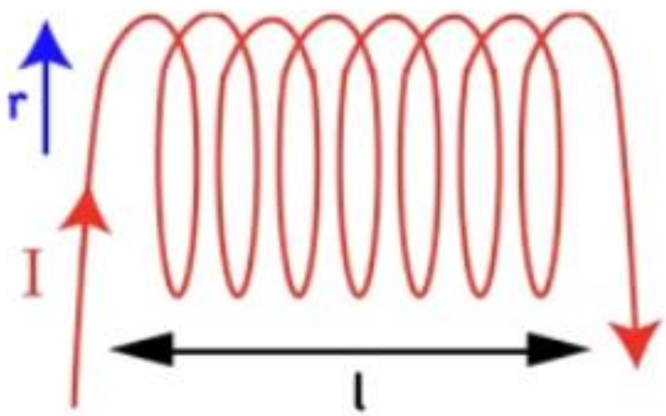
Self-Inductance

- Self-inductance (L) is the ability of a circuit to oppose the magnetic flux that is produced by the circuit itself
- Running a changing current through a circuit creates a changing magnetic field, which creates an induced emf that fights the change
- Units are henrys (H)
  - $1\ \mathrm{H} = 1\ (\mathrm{V} \cdot \mathrm{s})/\mathrm{A}$
- Self-inductance is purely a function of the circuit's geometry

Calculating Self Inductance

- $\mathrm{L} = \frac{\Phi_\mathrm{B}}{\mathrm{I}}$
- Ratio of magnetic flux to current flow
- $\varepsilon_{ind} = -\frac{d}{dt}\phi_B = -\frac{d}{dt}LI = -L\frac{dI}{dt}$
- For inductor:
  - $U_L = \frac{1}{2}LI^2$
- For capacitor:
  - $U_C = \frac{1}{2}CV^2$

Example 1: Self Inductance of s Solenoid

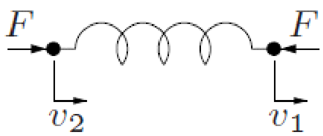
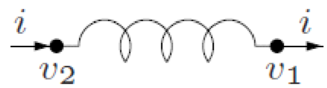
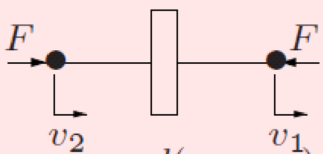
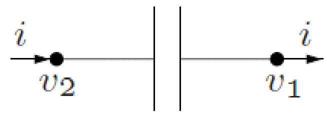
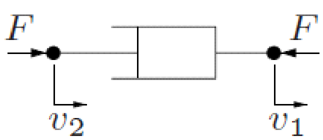
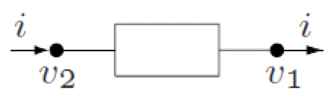


- Calculate the self-inductance of a solenoid of radius r and length L with N windings
- $B_{inside} = \frac{N}{l}\mu_0 I$
- $L = \frac{\Phi_\mathrm{B}}{\mathrm{I}} = \frac{\mathrm{NB}\pi\mathrm{r}^2}{\mathrm{I}} = \frac{\mathrm{N}\frac{N}{l}\mu_0 I\pi\mathrm{r}^2}{\mathrm{I}} = \frac{N^2}{l}\mu_0\pi\mathrm{r}^2$

Example 2: Calculating Self Inductance

- Calculate the self-inductance of a solenoid with 3400 turns of wire if the solenoid is 9 cm long and has a diameter of 11 cm.
- $L = \frac{N^2}{l}\mu_0 \pi r^2 = \frac{3400^2}{0.04}\mu\pi(0.055)^2 = 1.53\ \mathrm{H}$

Inductor, Capacitor, and Resistor

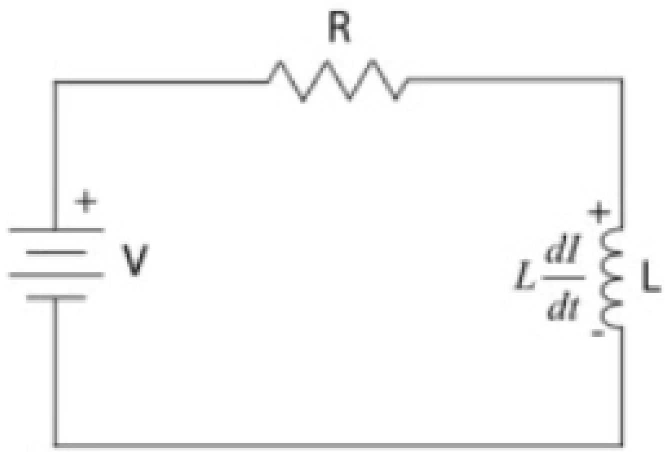
Mechanical	Electrical
 $Y(s) = \frac{k}{s}$ $\frac{dF}{dt} = k(v_2 - v_1)$ spring	 $Y(s) = \frac{1}{Ls}$ $\frac{di}{dt} = \frac{1}{L}(v_2 - v_1)$ inductor
 $Y(s) = bs$ $F = b\frac{d(v_2 - v_1)}{dt}$ inerter	 $Y(s) = Cs$ $i = C\frac{d(v_2 - v_1)}{dt}$ capacitor
 $Y(s) = c$ $F = c(v_2 - v_1)$ damper	 $Y(s) = \frac{1}{R}$ $i = \frac{1}{R}(v_2 - v_1)$ resistor

4.2 - RL Circuits

Monday, March 6, 2017 4:11 PM

Inductors in Circuits

- When circuit is first turned on, inductor opposes current flow and act like an open circuit
- After a time, inductor keeps current going and acts as a short
- After a long time, if the battery is removed, the inductor acts as an emf source to keep the current going
- As the resistor dissipates power, the current will decay exponentially to zero.



Current in RL Circuits

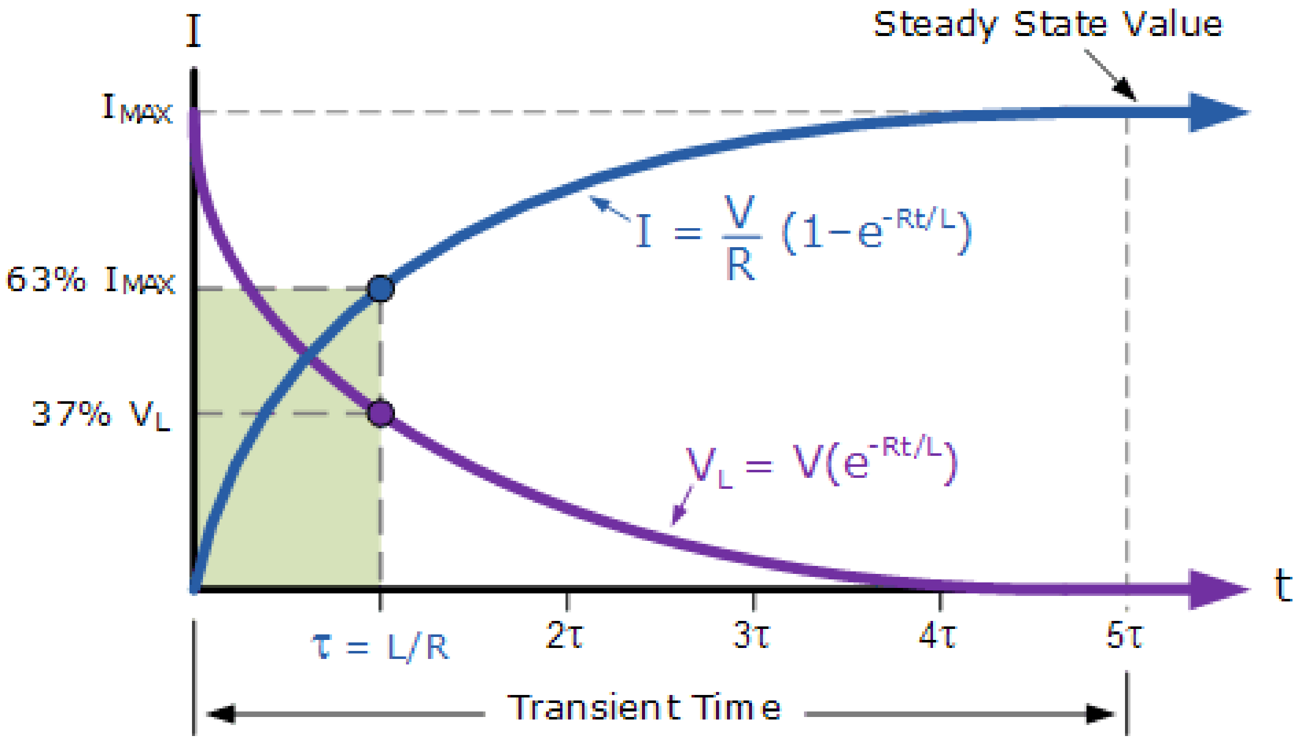
- $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$
- $-V + IR = -L\frac{dI}{dt}$
- $\frac{dI}{I - \frac{V}{R}} = -\frac{R}{L}dt$
- $\int_{I=0}^{I=I} \frac{dI}{I - \frac{V}{R}} = \int_{t=0}^{t=t} -\frac{R}{L}dt$
- $\ln\left(I - \frac{V}{R}\right)\Big|_0^I = -\frac{R}{L}t$
- $\ln\frac{I - V/R}{-V/R} = -\frac{R}{L}t$
- $I - \frac{V}{R} = -\frac{V}{R}e^{-\frac{R}{L}t}$
- $I = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$
- $\because \tau = \frac{L}{R}$

- $\therefore I = \frac{V}{R}(1 - e^{-\frac{t}{\tau}})$

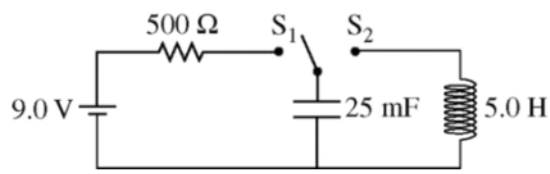
Voltage in RL Circuits

- $V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left(\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right) = \frac{LV}{R} \times \left(-e^{-\frac{R}{L}t}\right) \times \left(-\frac{R}{L}\right) = Ve^{-\frac{R}{L}t} = Ve^{-\frac{t}{\tau}}$

Current and Voltage Graphs



2011 Free Response Question 2



E&M. 2.

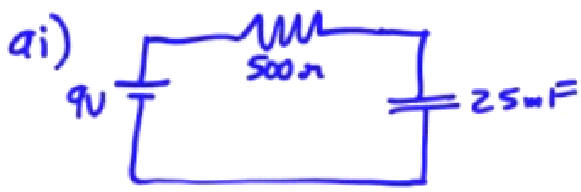
The circuit represented above contains a 9.0 V battery, a 25 mF capacitor, a 5.0 H inductor, a 500 Ω resistor, and a switch with two positions, S_1 and S_2. Initially the capacitor is uncharged and the switch is open.

(a) In experiment 1 the switch is closed to position S_1 at time t_1 and left there for a long time.

i. Calculate the value of the charge on the bottom plate of the capacitor a long time after the switch is closed.

ii. On the axes below, sketch a graph of the magnitude of the charge on the bottom plate of the capacitor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

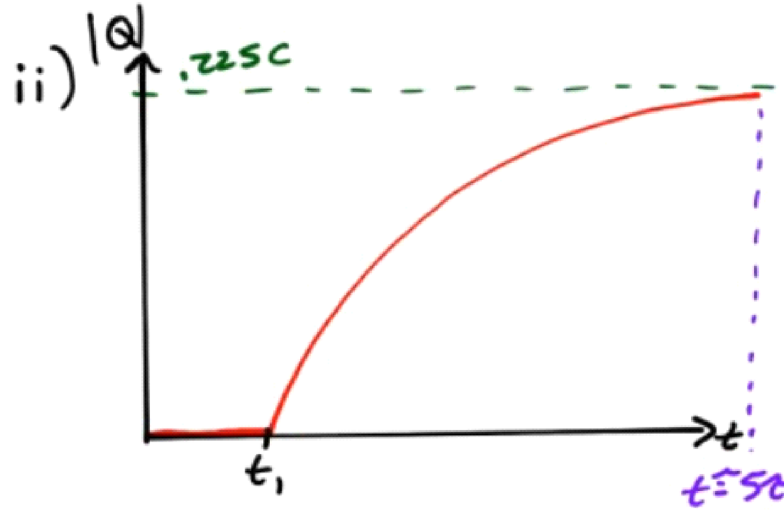
a i)



$C = \frac{Q}{V} \Rightarrow Q = CV = (.025F)(9V) \Rightarrow$

$Q = .225C \Rightarrow \boxed{Q_{bottom} = -.225C}$

ii)



$t \approx 5\tau = 5(500)(.025) = 62.5s$

iii. On the axes below, sketch a graph of the current through the resistor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

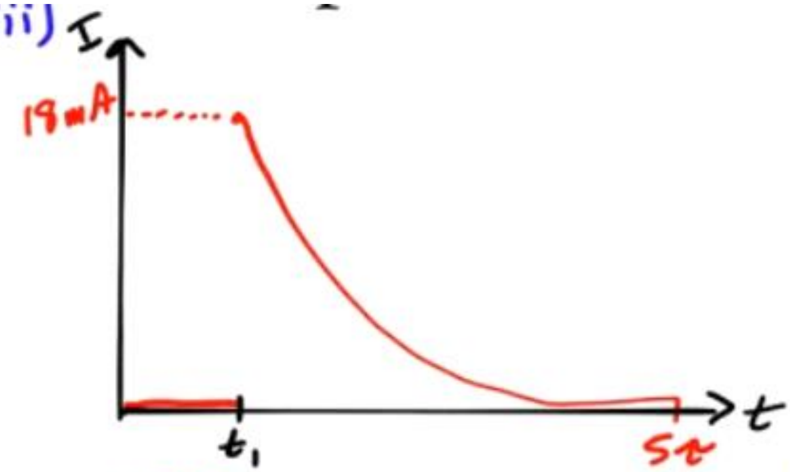
(b) In experiment 2 the capacitor is again uncharged when the switch is closed to position S_1 at time t_1. The switch is then moved to position S_2 at time t_2 when the magnitude of the charge on the capacitor plate is 105 mC, allowing electromagnetic oscillations in the LC circuit.

i. Calculate the energy stored in the capacitor at time t_2.

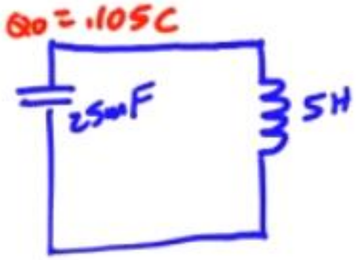
ii. Calculate the maximum current that will be present during the oscillations.

iii. Calculate the time rate of change of the current when the charge on the capacitor plate is 50 mC.

aiii)



b)



$$U_c = \frac{1}{2}CV^2 \xrightarrow[V=\frac{Q}{C}]{C=\frac{Q}{V}} \frac{1}{2}C\left(\frac{Q}{C}\right)^2 \Rightarrow$$

$$U_c = \frac{1}{2}\frac{Q^2}{C} = \frac{(.105C)^2}{2(.025F)} = \boxed{0.221\ J}$$

ii) $Q_c = 0 \Rightarrow U_L = .221J = \frac{1}{2}LI^2 \Rightarrow \sqrt{I^2} = \sqrt{\frac{2(.221)}{5}} \Rightarrow$

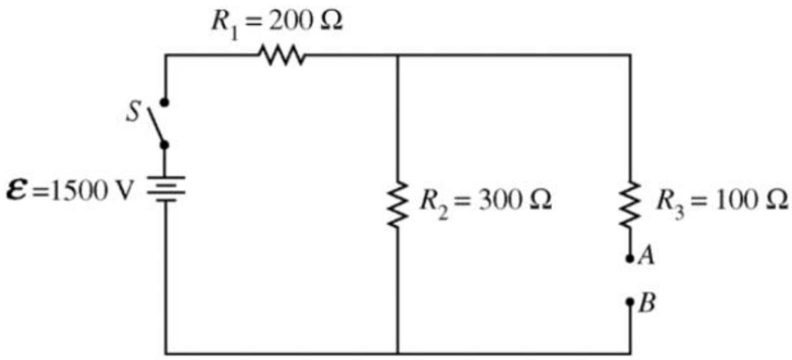
$$\boxed{I = 0.297\ A}$$

biii) Find $\frac{dI}{dt}$ when $Q_c = .05C$

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \Rightarrow \frac{Q}{C} = -L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{-Q}{LC} \Rightarrow$$

$$\frac{dI}{dt} = \frac{-(.05C)}{(5H)(.025F)} \Rightarrow \boxed{\frac{dI}{dt} = -0.4\ A/s}$$

2008 Free Response Question 2



E&M. 2.

In the circuit shown above, A and B are terminals to which different circuit components can be connected.

(a) Calculate the potential difference across R_2 immediately after the switch S is closed in each of the following cases.

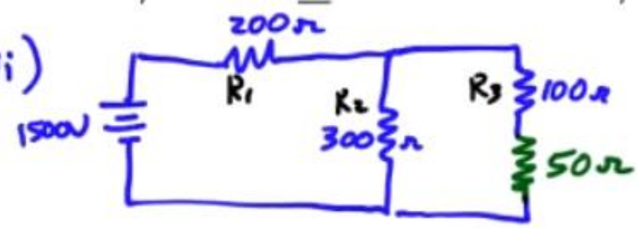
i. A 50 Ω resistor connects A and B.

ii. A 40 mH inductor connects A and B.

iii. An initially uncharged 0.80 μF capacitor connects A and B.

(b) The switch gets closed at time $t = 0$. On the axes below, sketch the graphs of the current in the 100 Ω resistor R_3 versus time t for the three cases. Label the graphs R for the resistor, L for the inductor, and C for the capacitor.

ai)

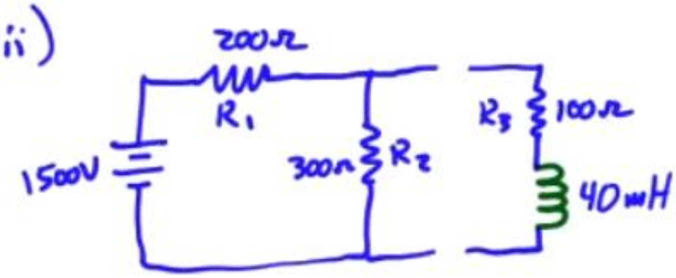


$$R_{23} = \frac{(300)(150)}{450} = 100\Omega$$

$$I = \frac{V}{R} = \frac{1500V}{300\Omega} = 5A$$

$$V_{R_2} = (5A)(100\Omega) = \boxed{500V}$$

aii)



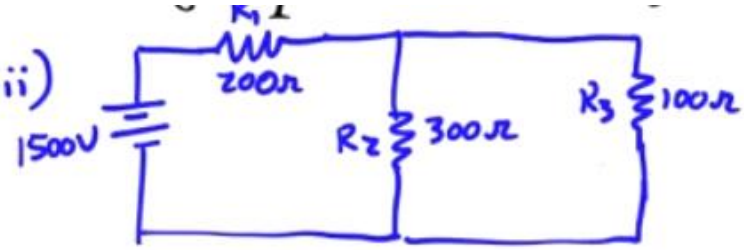
$$I_{ind_{t=0}} = 0$$

$$R_{eq_{t=0}} = 500\Omega$$

$$I = \frac{V}{R} = \frac{1500V}{500\Omega} = 3A$$

$$V_{R_2} = I_2 R_2 = (3A)(300\Omega) = \boxed{900V}$$

aiii)



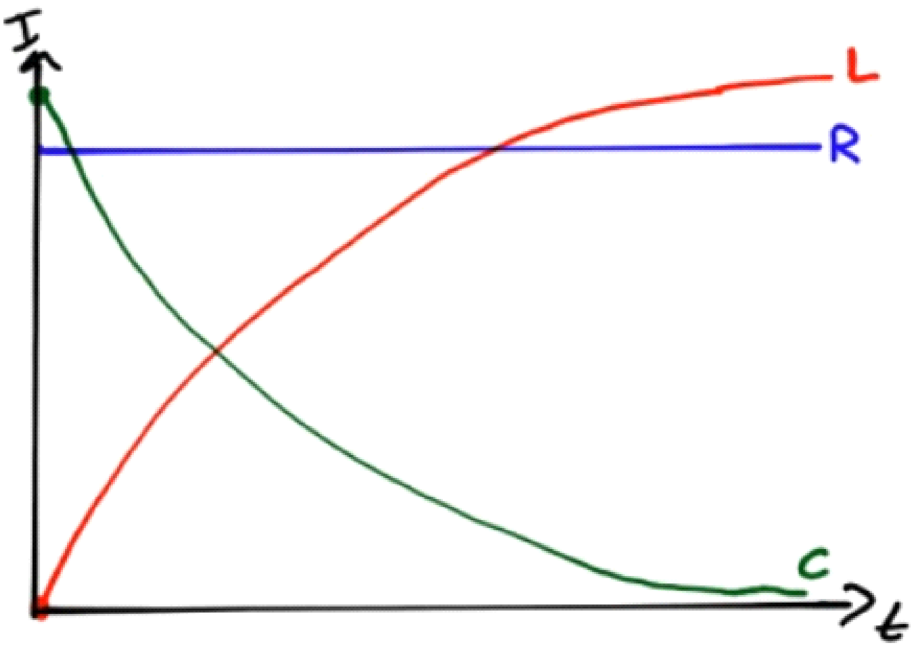
$$R_{eq_{23}} = \frac{(300)(100)}{400} = 75\,\Omega$$

$$R_{TOT} = 275\,\Omega$$

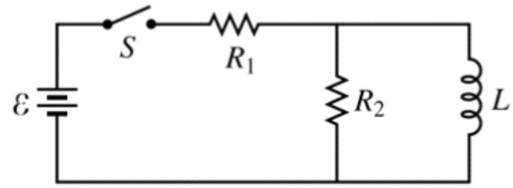
$$I_{TOT} = \frac{V}{R} = \frac{1500\,V}{275\,\Omega} = 5.45\,A$$

$$V_{R_2} = I R_{eq_{23}} = (5.45)(75)$$

$$= \boxed{409\,V}$$



2005 Free Response Question 2



E&M. 2.

In the circuit shown above, resistors 1 and 2 of resistance R_1 and R_2, respectively, and an inductor of inductance L are connected to a battery of emf $\mathcal{E}$ and a switch S. The switch is closed at time $t = 0$. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the current through resistor 1 immediately after the switch is closed.

(b) Determine the magnitude of the initial rate of change of current, dI/dt, in the inductor.

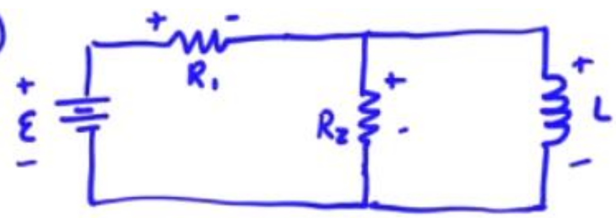
(c) Determine the current through the battery a long time after the switch has been closed.

(d) On the axes below, sketch a graph of the current through the battery as a function of time.

Some time after steady state has been reached, the switch is opened.

(e) Determine the voltage across resistor 2 just after the switch has been opened.

a i)



@ $t=0$, $I_L=0$

$I = \frac{V}{R} = \boxed{\frac{\mathcal{E}}{R_1+R_2}}$

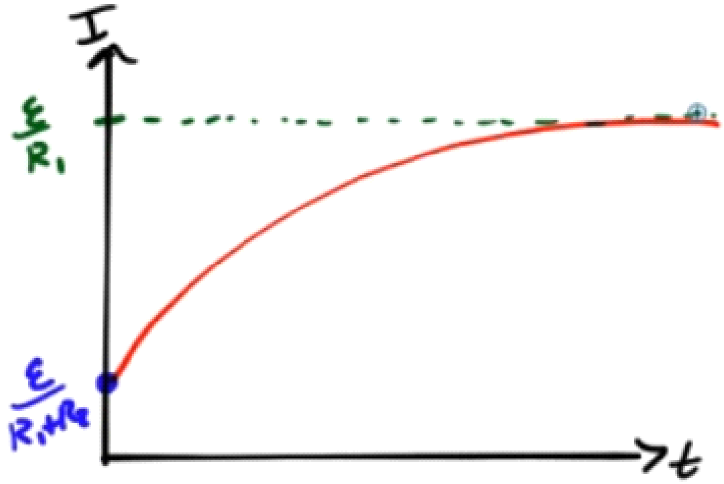
b) $-\mathcal{E} + I_0 R_1 = -L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}}{L} - \frac{I_0 R_1}{L} \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}}{L} - \left(\frac{\mathcal{E}}{R_1+R_2}\right)\frac{R_1}{L} \Rightarrow$

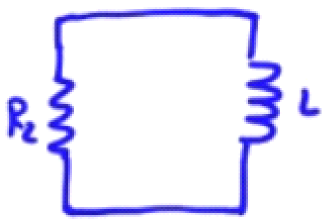
$$\boxed{\frac{dI}{dt} = \frac{\mathcal{E}}{L}\left(1 - \frac{R_1}{R_1+R_2}\right)}$$

c) @ $t \to \infty$, $V_L = 0$

$I = \frac{V}{R} = \boxed{\frac{\varepsilon}{R_1}}$

d)





$I_{R_2} = I_L = \frac{\varepsilon}{R_1}$

$V_{R_2} = IR = I_{R_2} R_2 = \frac{\varepsilon}{R_1} R_2 = \boxed{\frac{\varepsilon R_2}{R_1}}$

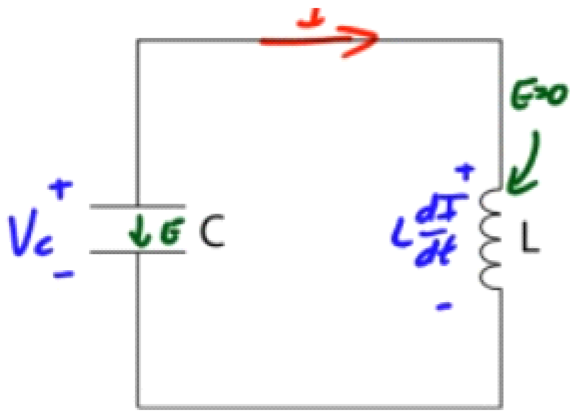
4.3 - LC Circuits

Monday, March 6, 2017 5:50 PM

LC Circuits

- When circuit is first turned on, assume capacitor is fully charged.
- Interplay of capacitor and inductor creates an oscillating system, modeled in similar fashion to SHM

Charge in LC Circuits

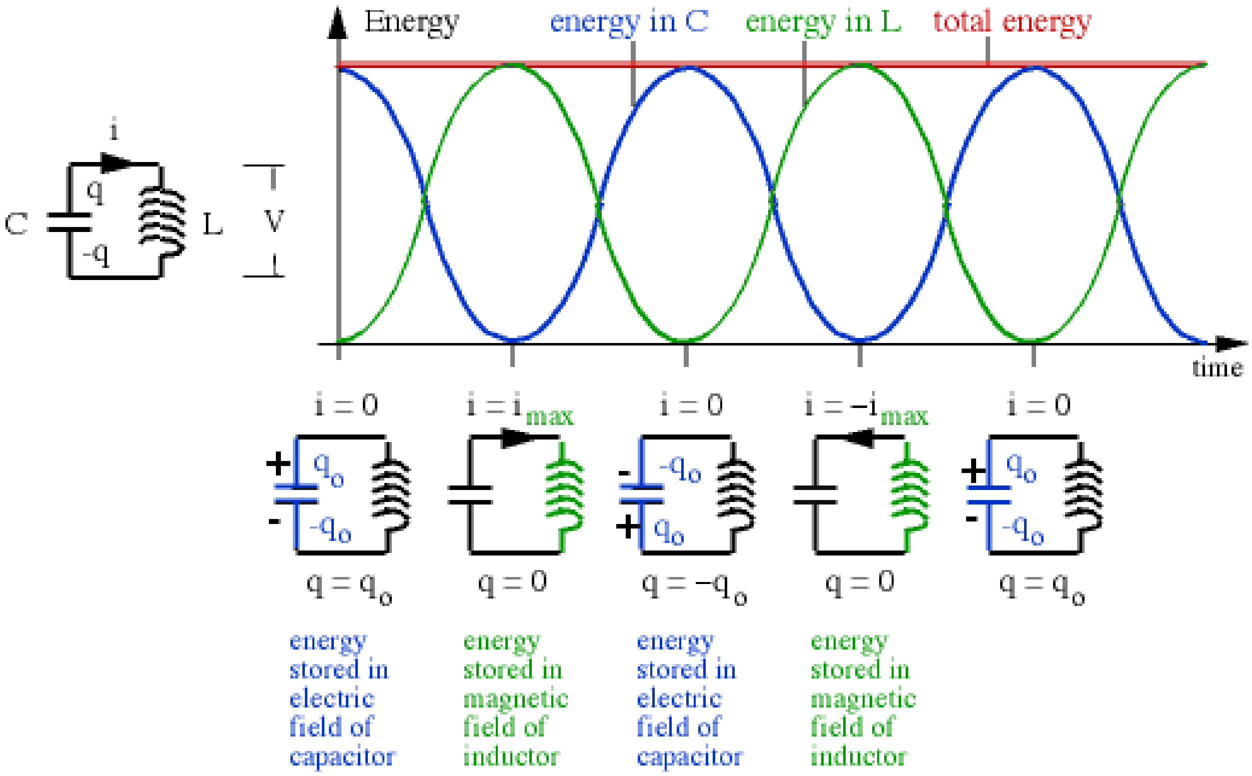


- $\frac{Q}{C} - L\frac{dI}{dt} = 0$
- $\because I = -\frac{dQ}{dt}$
- $\therefore \frac{dI}{dt} = -\frac{d^2Q}{dt^2}$
- $\therefore \frac{Q}{C} - L\left(-\frac{d^2Q}{dt^2}\right) = 0 \Rightarrow \frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$
- $Let\ \omega = \frac{1}{\sqrt{LC}}$
- $Q(t) = A\cos\omega t + B\sin\omega t$
- $From\ the\ bondary\ condition, we\ get\ A = Q_0\ and\ B = 0$
- $\therefore Q(t) = Q_0\cos\omega t$

Current and Potential in LC Circuits

- $V_c = \frac{Q}{C} = \frac{Q_0}{C}\cos\omega t$
- $I = -\frac{dQ}{dt} = \omega Q_0 \sin\omega t = \frac{Q_0}{\sqrt{LC}}\sin\omega t$

Graphs of LC Circuits



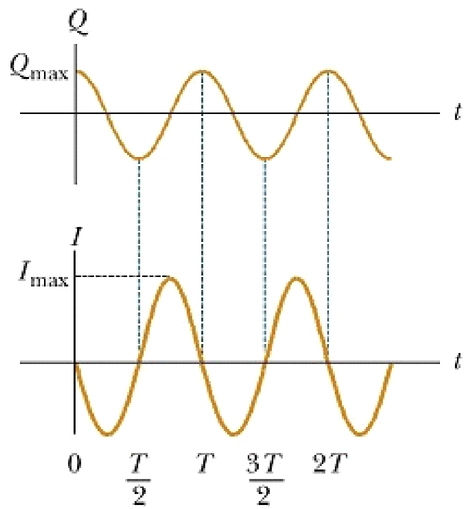


Figure 32.16 Graphs of charge versus time and current versus time for a resistanceless, nonradiating LC circuit. Note that Q and I are 90° out of phase with each other.

- $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

5.1 - Maxwell's Equations

Monday, March 6, 2017 6:18 PM

Maxwell's Equations

Gauss's Law

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss's Law for Magnetism

$$\oint \vec{B} \bullet d\vec{A} = 0$$

Faraday's Law

$$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_{\substack{open\\surface}} \vec{B} \bullet d\vec{A}$$

Ampere's Law*

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{l} = \mu_0 I$$

Revisiting Ampere's Law

- Ampere's Law as written allows us to calculate the magnetic field due to an electric current.
- We also know that a changing electric field produces a magnetic field
- Combine effects of electric current and changing E field on magnetic field to obtain a more complete version of Ampere's Law
- Contribution due to the penetrating current is known a conduction current.

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{l} = \mu_0 I$$

- Contribution due to changing electric field is known as the displacement current

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_{\substack{open\\surface}} \vec{E} \bullet d\vec{A}$$

- Final Ampere's Law

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Sample Questions

Wednesday, April 5, 2017 5:57 PM

Question 2

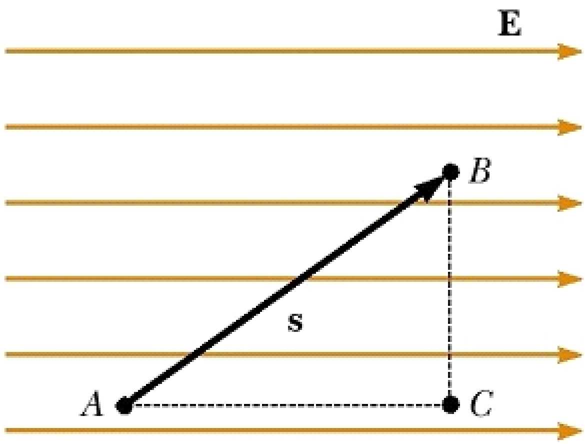
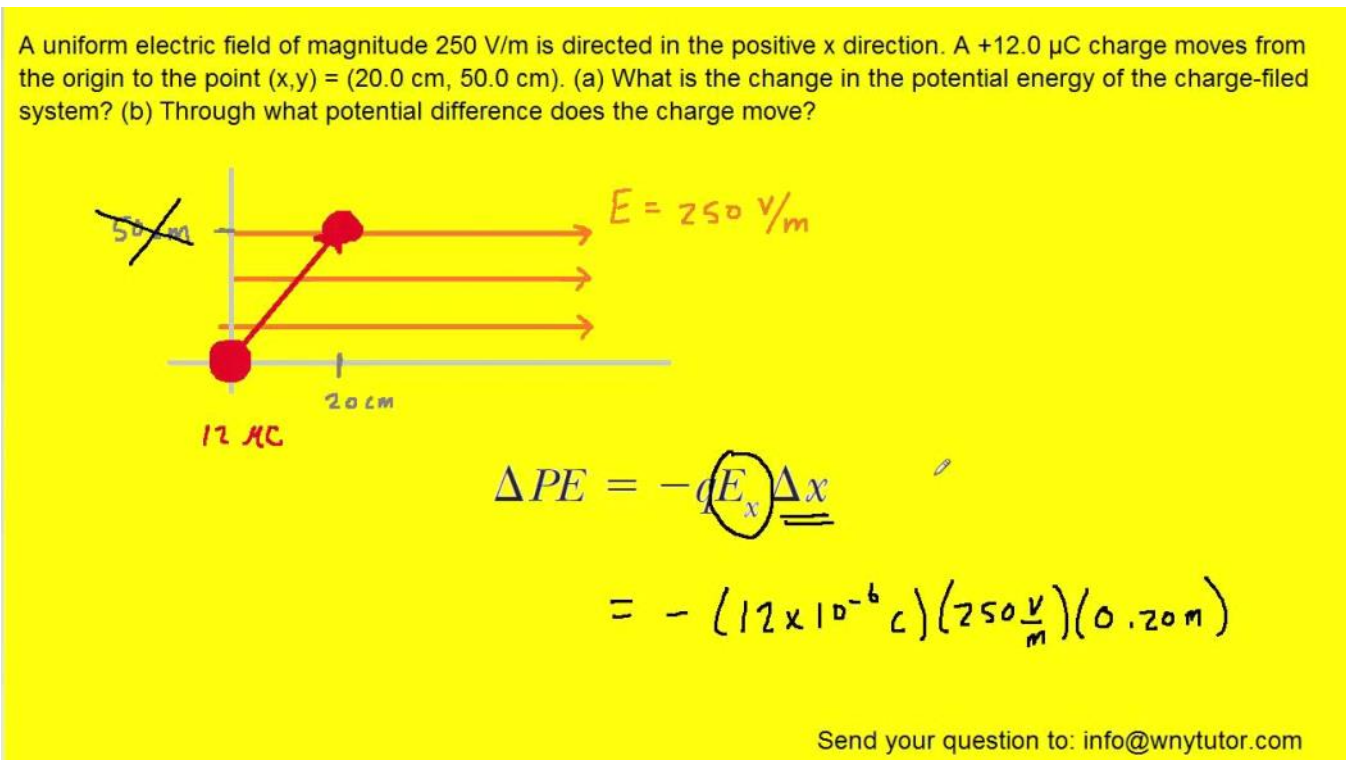


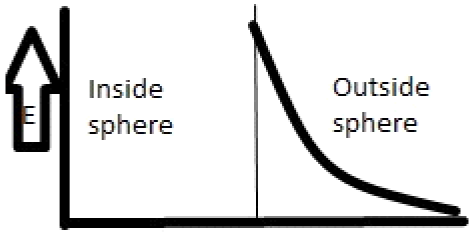
Figure 25.2 A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the *same* electric potential.

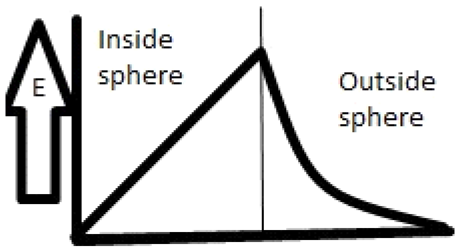


Question 3

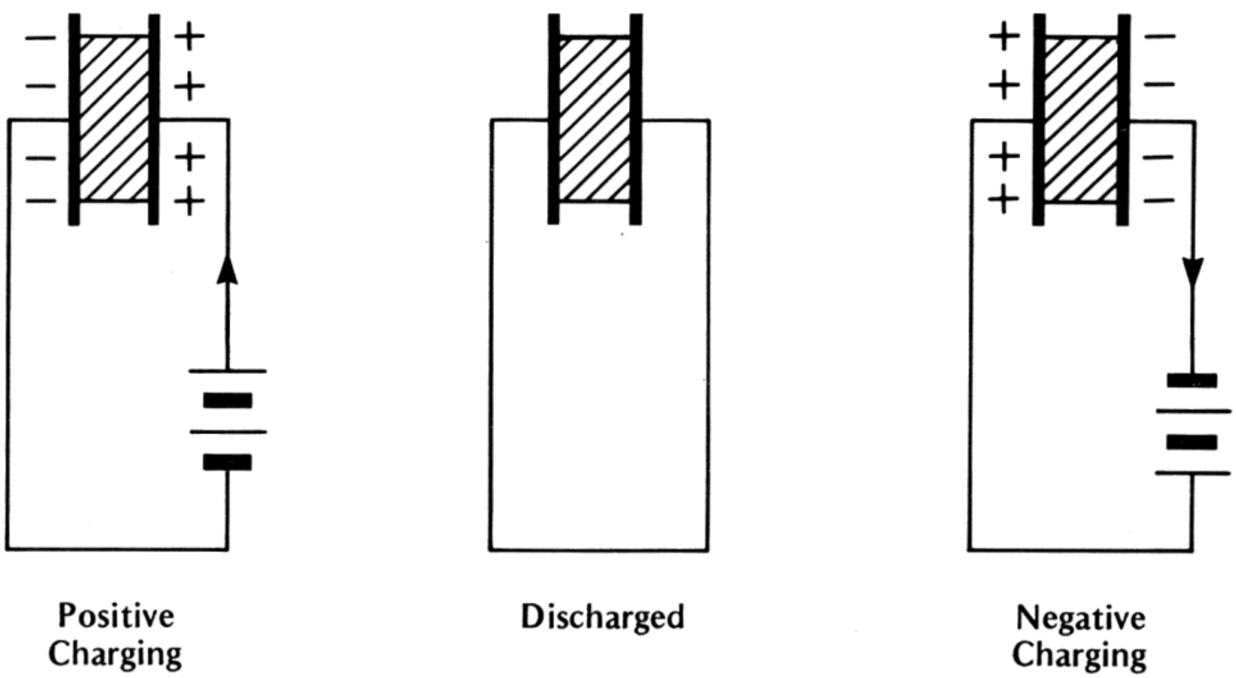
- Solid conducting mettalic sphere and hollow sphere are same as charge resides on the outer surface of the sphere.
- Inside the sphere, field is zero. Outside the sphere, it behaves as a point charge placed at the centre of sphere.
- But the case of non-conducting sphere is different.

- Whole charge doesn't move to surface. Inside the sphere it is directly proportional to distance from centre. Outside it behaves as point charge at centre of sphere.





Question 5

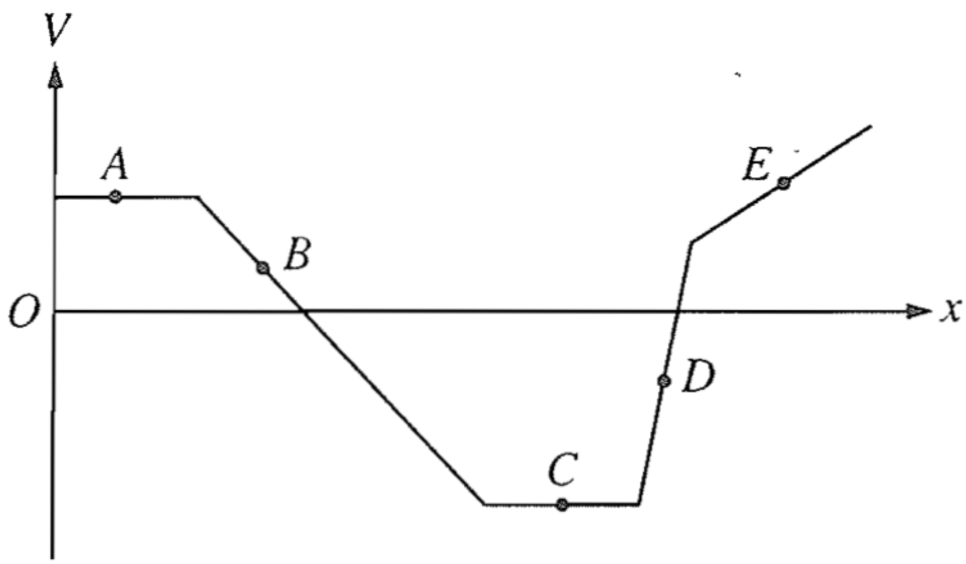


1998 Multiple Choice

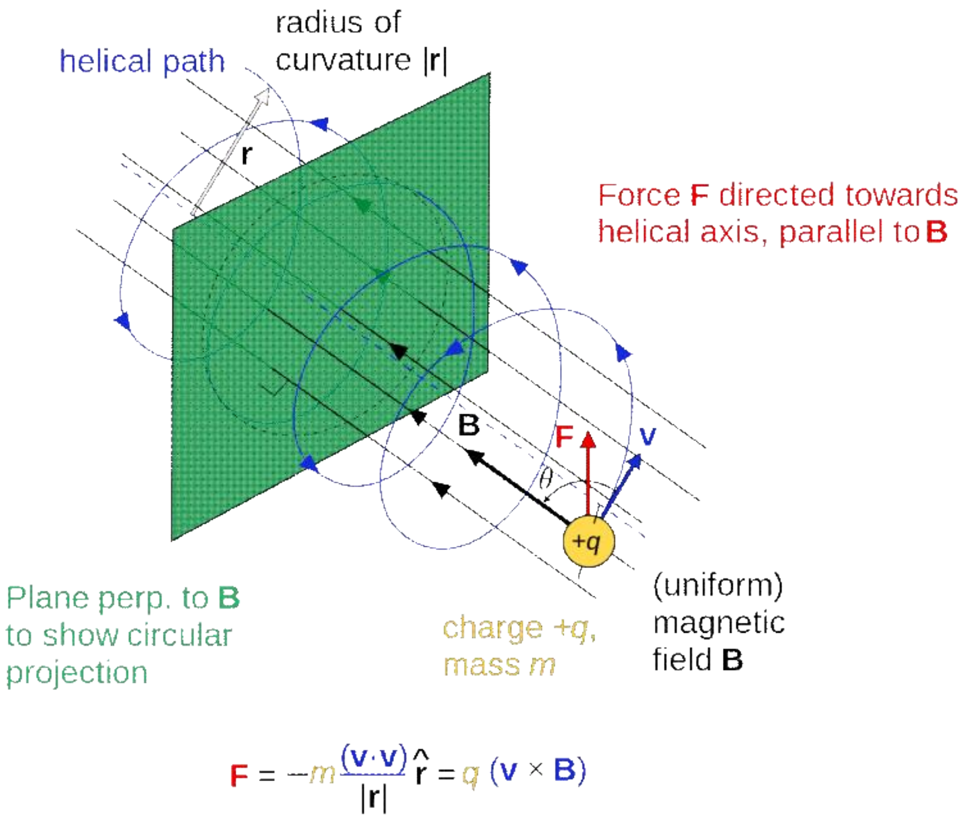
Wednesday, March 8, 2017 1:54 PM

Question 47

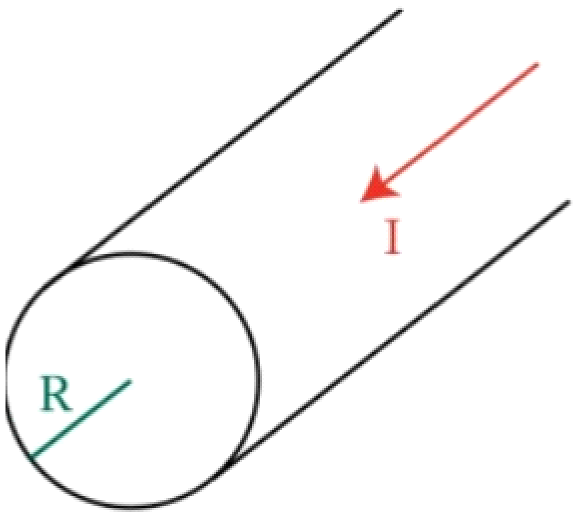
- $F = qE$
- ∴ The greatest force happens at the greatest E.
- $E = -\frac{dV}{dr}$
- ∴ The greatest E happens at the greatest slope the graph



Question 50



Question 52



1. Find the magnetic field outside a current-carrying wire

- $\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$
- $B \times 2\pi r = \mu_0 I$
- $B = \dfrac{\mu_0 I}{2\pi r}$

2. Find the magnetic field inside a current-carrying wire

- $\oint_{\substack{closed\\loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{penetrating}$

- $B \times 2\pi r = \mu_0 \left(\frac{\pi r^2}{\pi R^2}\right) I$
- $B = \frac{\mu_0 I r}{2\pi R^2}$

Question 55

- $F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2}$
- $F_c = \frac{mv^2}{R}$
- $\because F_e = F_c$
- $\therefore K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 R}$

Question 63

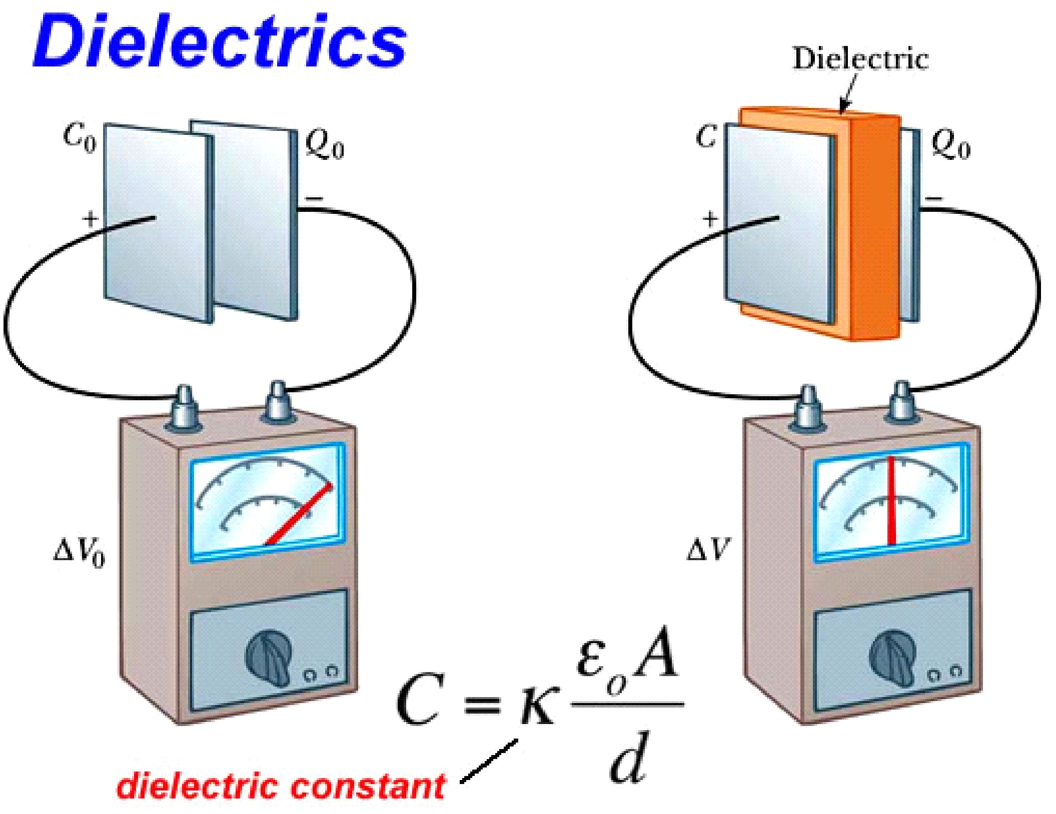
- $V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

Question 65

- $V_1 = \frac{Q_{total}}{C_1} = \frac{V_{total}C_{total}}{C_1} = V_{total}\frac{C_{total}}{C_1}$

Question 70

- When a dielectric is inserted between the plates of a capacitor, the capacitance increases.



2004 Multiple Choice

2017年5月7日 星期日 下午9:54

Question 43

43. A potential difference V is maintained between two large, parallel conducting plates. An electron starts from rest on the surface of one plate and accelerates toward the other. Its speed as it reaches the second plate is proportional to

(A) $1/V$

(B) $1/\sqrt{V}$

(C) $\boxed{\sqrt{V}}$

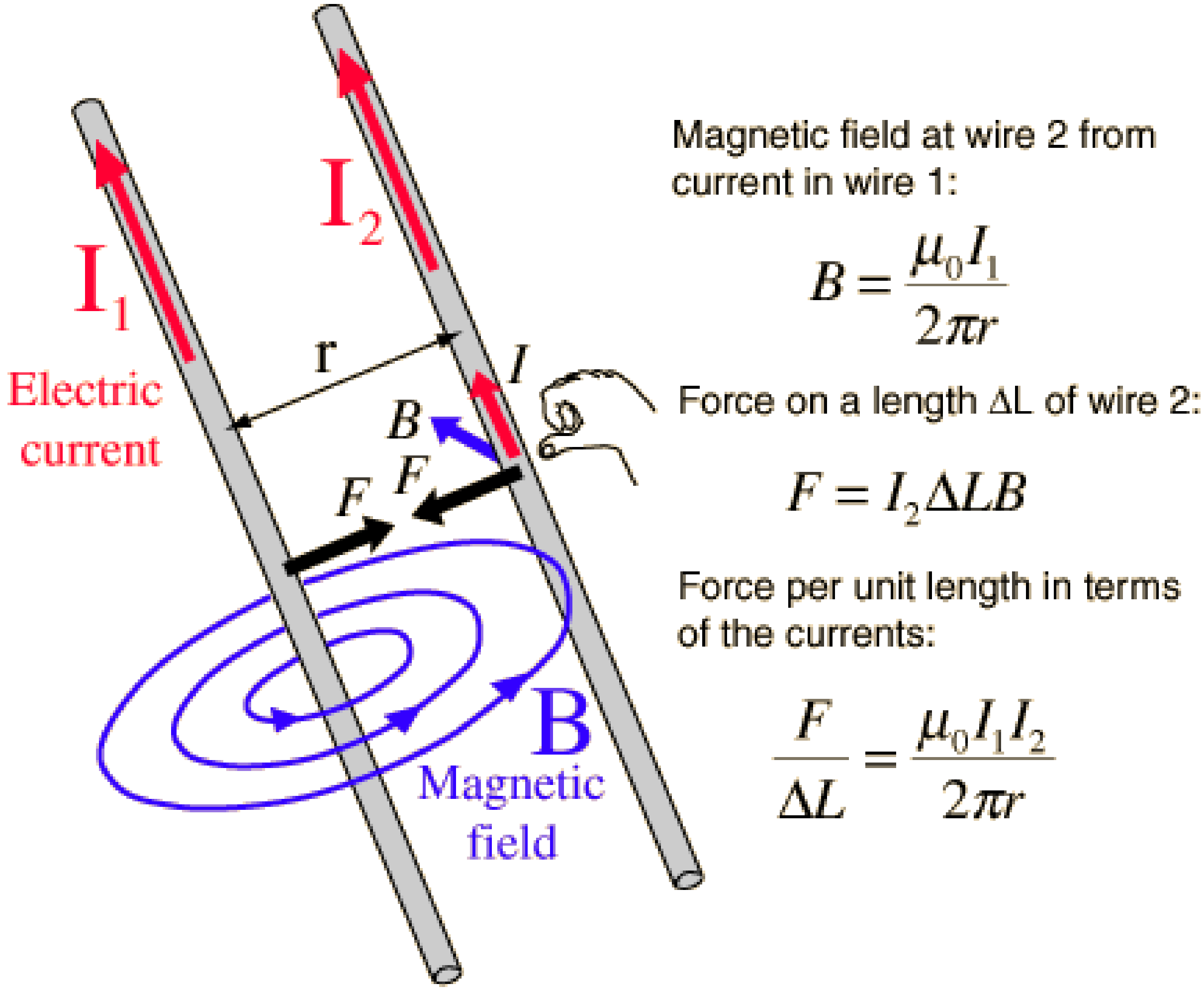
(D) V

(E) V^2

$$\Delta U + \Delta K = -qV + \frac{1}{2}mv^2 = 0; \quad -eV + \frac{1}{2}m_e v^2 = 0$$

$$v = \sqrt{\frac{2eV}{m_e}}$$

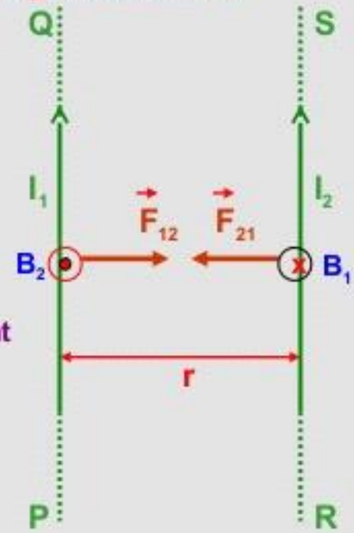
Question 51


I_1
Electric
current
I_2
r
I
B
F
F
B
Magnetic
field
Magnetic field at wire 2 from current in wire 1:
$B = \frac{\mu_0 I_1}{2\pi r}$
Force on a length ΔL of wire 2:
$F = I_2 \Delta L B$
Force per unit length in terms of the currents:
$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$

Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{(in magnitude)}$$



Force acting on RS due to current I_2 through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 \, l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 \, l}{2\pi r}$$

B_1 acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between l and B_1 is 90°. l is length of the conductor.

Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{(in magnitude)}$$

Force acting on PQ due to current I_1 through it is

$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 \, l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 \, l}{2\pi r}$$

(The angle between l and B_2 is 90° and B_2 is emerging out)

$$\boxed{F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 \, l}{2\pi r}}$$

Force per unit length of the conductor is $\boxed{F / l = \frac{\mu_0 I_1 I_2}{2\pi r}}$ N / m

Question 53

53. A charged particle can move with constant velocity through a region containing both an electric field and a magnetic field only if the

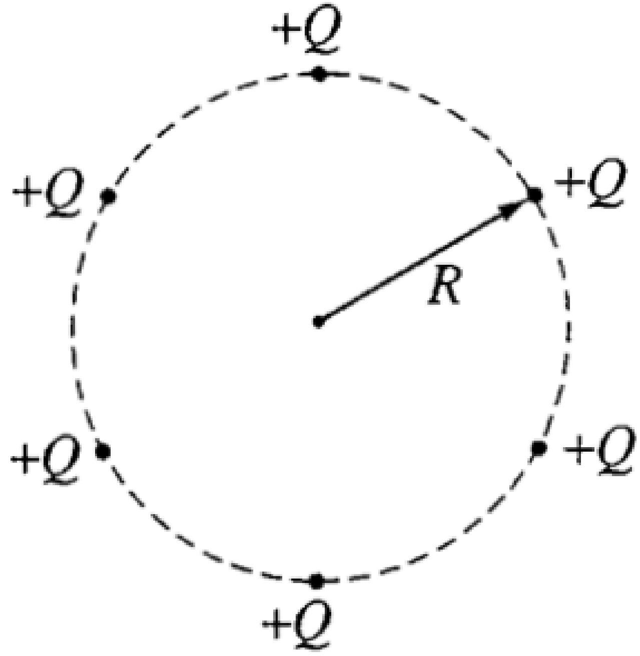
(A) electric field is parallel to the magnetic field
(B) electric field is perpendicular to the magnetic field
(C) electric field is parallel to the velocity vector
(D) magnetic field is parallel to the velocity vector
(E) magnetic field is perpendicular to the velocity vector

Because the forces caused by the two fields must be in opposite directions in order to add to zero, and since the force of the magnetic field is perpendicular to the magnetic field and the force of the electric field is in the same line as that of the field, the two fields must be perpendicular.

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{E}$$

Question 58



As shown in the figure above, six particles, each with charge $+Q$, are held fixed and are equally spaced around the circumference of a circle of radius R.

58. With the six particles held fixed, how much work would be required to bring a seventh particle of charge $+Q$ from very far away and place it at the center of the circle?

(A) 0

(B) $\dfrac{\sqrt{6}}{4\pi\epsilon_0}\dfrac{Q}{R}$

(C) $\dfrac{3}{2\pi\epsilon_0}\dfrac{Q^2}{R^2}$

(D) $\dfrac{3}{2\pi\epsilon_0}\dfrac{Q^2}{R}$

(E) $\dfrac{9}{\pi\epsilon_0}\dfrac{Q^2}{R}$

The potential at the center of the circle is $V = \dfrac{1}{4\pi\varepsilon_o}\dfrac{6Q}{R}$ so the potential energy of a sixth charge will be $U = \dfrac{1}{4\pi\varepsilon_o}\dfrac{6Q^2}{R} = \dfrac{3}{2\pi\varepsilon_o}\dfrac{Q^2}{R}$ so this equals the work required to bring it from "infinity" where the U = 0.

Question 65

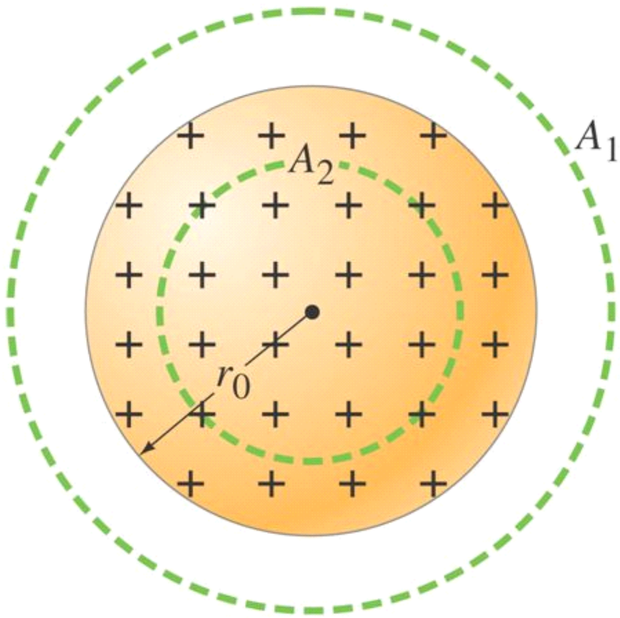
65. A physics problem starts: "A solid sphere has charge distributed uniformly throughout . . ." It may be correctly concluded that the

(A) electric field is zero everywhere inside the sphere
(B) electric field inside the sphere is the same as the electric field outside
(C) electric potential on the surface of the sphere is not constant
(D) electric potential in the center of the sphere is zero
(E) sphere is not made of metal

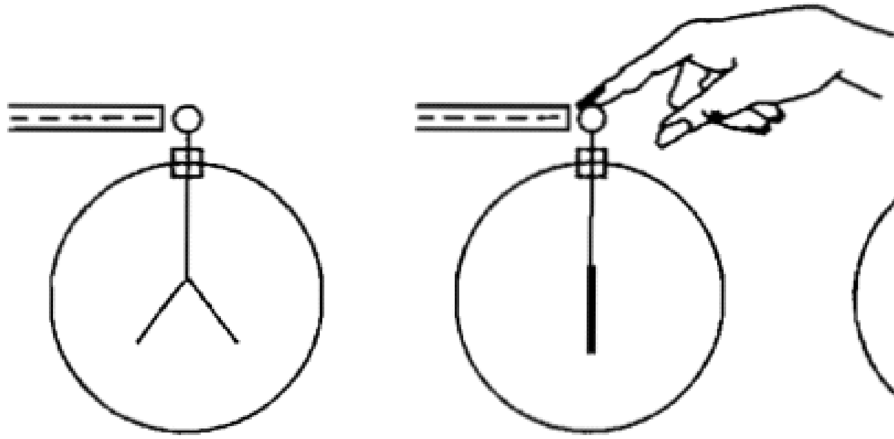
22-3 Applications of Gauss's Law

Example 22-4: Solid sphere of charge.

An electric charge Q is distributed uniformly throughout a nonconducting sphere of radius r_0. Determine the electric field (a) outside the sphere ($r > r_0$) and (b) inside the sphere ($r < r_0$).



Question 69

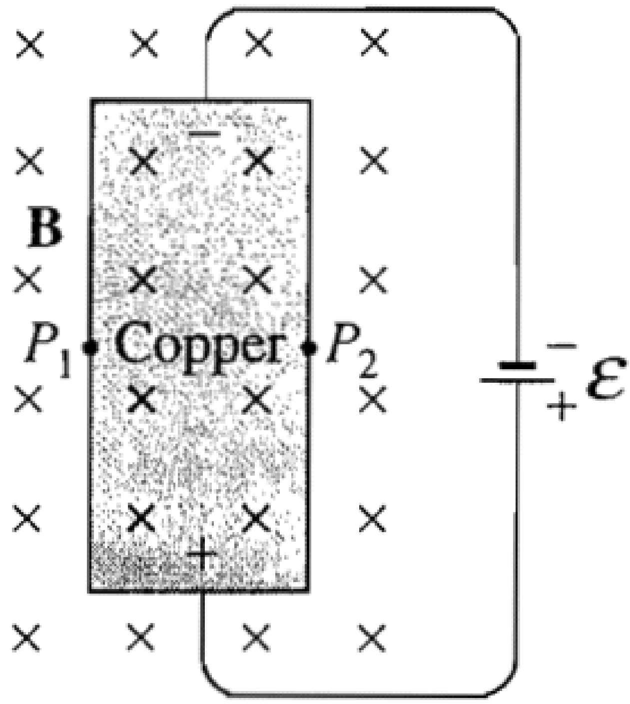


59. When a negatively charged rod is brought near, but does not touch, the initially uncharged electroscope shown above, the leaves spring apart (I). When the electroscope is then touched with a finger, the leaves collapse (II). When next the finger and finally the rod are removed, the leaves spring apart a second time (III). The charge on the leaves is

(A) positive in both I and III
(B) negative in both I and III
(C) positive in I, negative in III
(D) negative in I, positive in III
(E) impossible to determine in either I or III

In I the electrons are driven onto the leaves. In II the electrons are allowed to go to ground, so in III the leaves have a net positive charge.

Question 70



'0. A sheet of copper in the plane of the page is connected to a battery as shown above, causing electrons to drift through the copper toward the bottom of the page. The copper sheet is in a magnetic field **B** directed into the page. P_1 and P_2 are points at the edges of the strip. Which of the following statements is true?

(A) P_1 is at a higher potential than P_2.

(B) P_2 is at a higher potential than P_1.

(C) P_1 and P_2 arc at cqual positive potential.

(D) P_1 and P_2 are at equal negative potential.

(E) Current will cease to flow in the copper sheet.

Negative charge will shift to the left causing the right side to be at a higher potential, since potential is defined in terms of the positive charge.

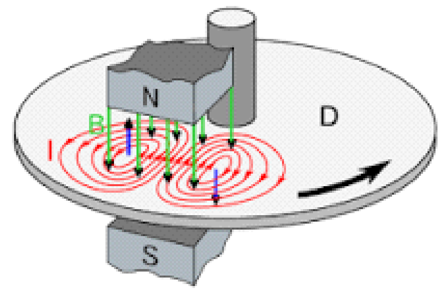
2009 Multiple Choice

2017年5月7日 星期日 下午8:47

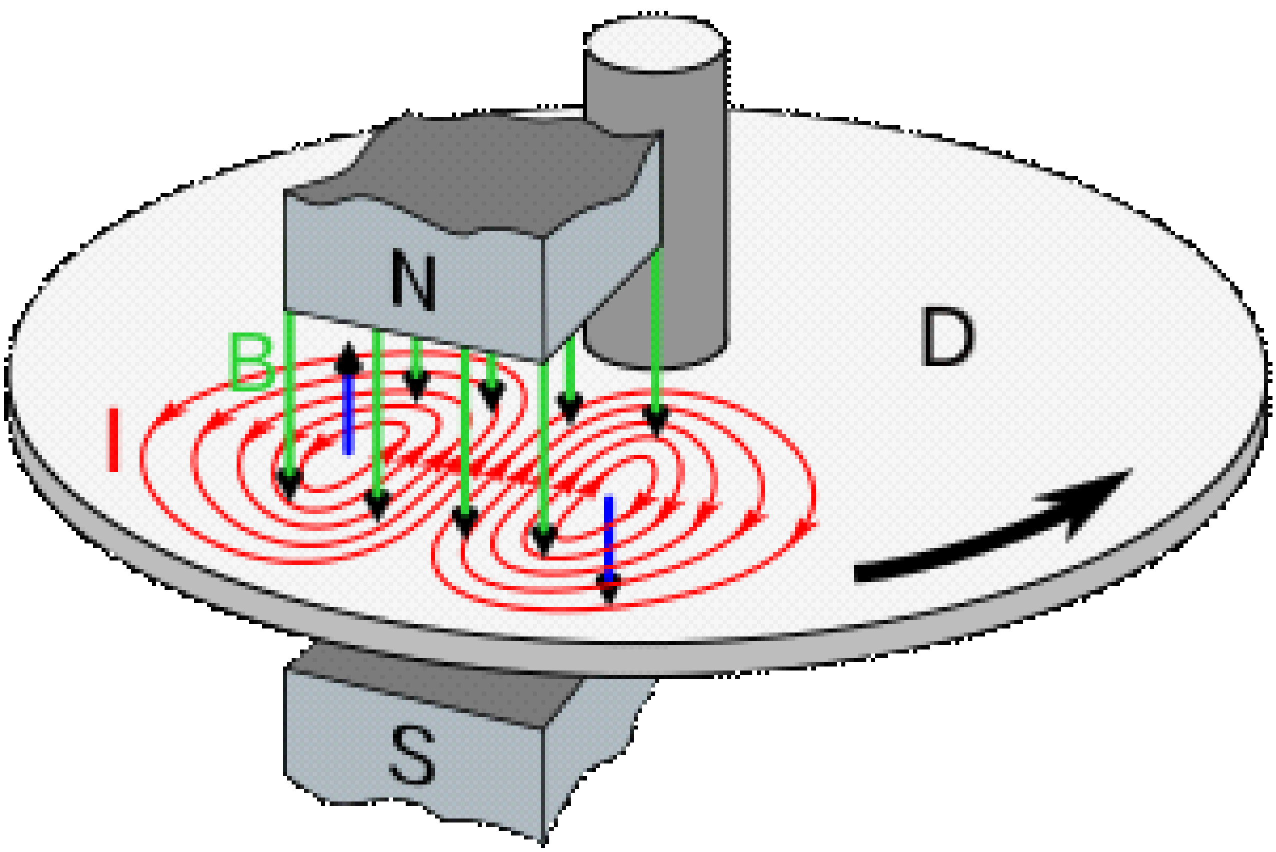
Question 4

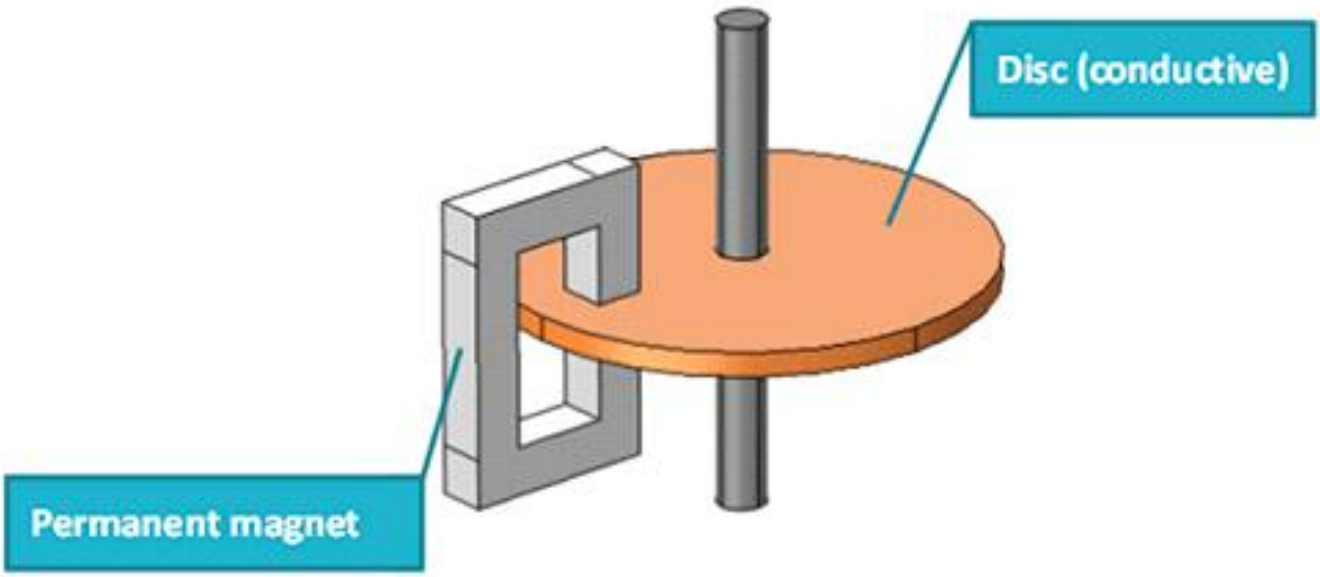
- Eddy current brake

However, unlike electro-mechanical **brakes**, in which the drag force used to stop the moving object is provided by friction between two surfaces pressed together, the drag force in an **eddy current brake** is an electromagnetic force between a magnet and a nearby conductive object in relative motion, due to **eddy currents** ...



Eddy current brake - Wikipedia
https://en.wikipedia.org/wiki/Eddy_current_brake





- Eddy

Report inappropriate predictions

ed·dy

/ˈedē/

noun

1. a circular movement of water, counter to a main current, causing a small whirlpool.
   synonyms: swirl, whirlpool, vortex, maelstrom
   "small eddies at the river's edge"

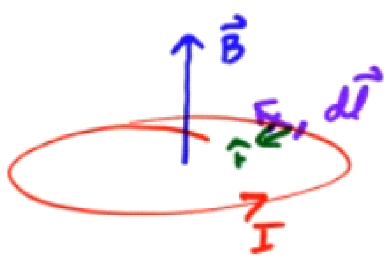
verb

1. (of water, air, or smoke) move in a circular way.
   "the mists from the river eddied around the banks"
   synonyms: swirl, whirl, spiral, wind, circulate, twist; More

Translations, word origin, and more definitions

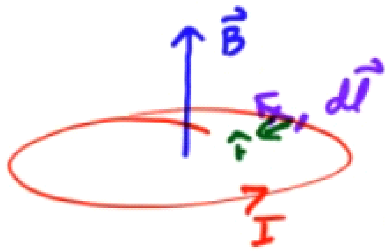
Question 9

- Magnetic Field due to a Current Loop



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \qquad \frac{d\vec{l} \times \hat{r} = dl \sin\theta}{\sin\theta = 1} \longrightarrow$$



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2} \xrightarrow[\sin\theta = 1]{d\vec{l}\times\hat{r} = dl\sin\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r^2}\int dl \xrightarrow{\int dl = 2\pi r} B = \frac{\mu_0 I}{4\pi r^2}(2\pi r) \Rightarrow$$

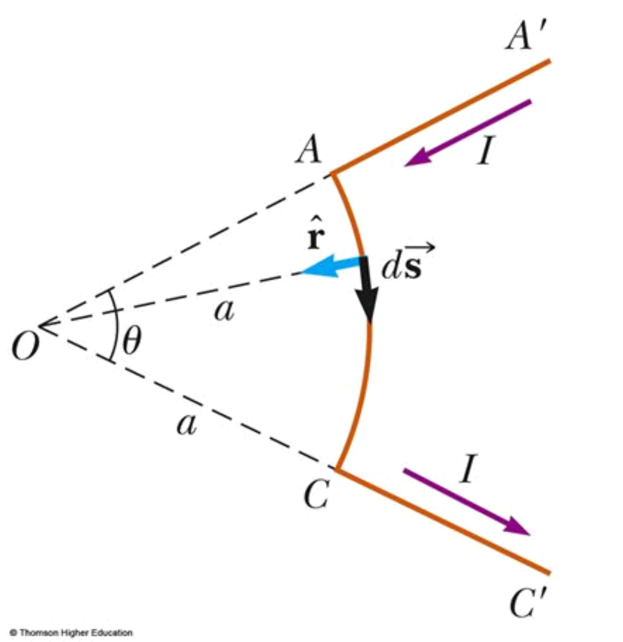
$$\boxed{B = \frac{\mu_0 I}{2r}}$$

$\vec{B}$ for a Curved Wire Segment

- Find the field at point *O* due to the wire segment
- *I* and *R* are constants

$$B = \frac{\mu_o I}{4\pi R}\theta$$

  - θ will be in radians



$\vec{B}$ for a Circular Loop of Wire

$\vec{B}$ for a Circular Loop of Wire

- Consider the previous result, with a full circle
  - $\theta = 2\pi$

$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

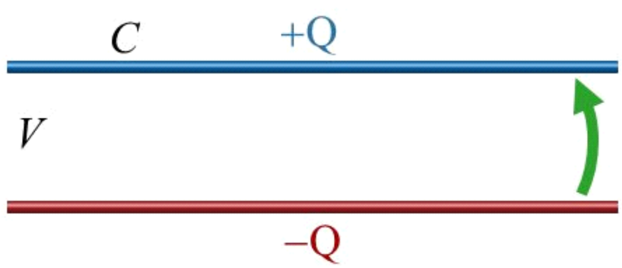
- This is the field at the *center* of the loop

Question 10

- Energy in a capacitor

Energy in a Capacitor

In Prelecture 7 we calculated the work done to move charge Q from one plate to another:



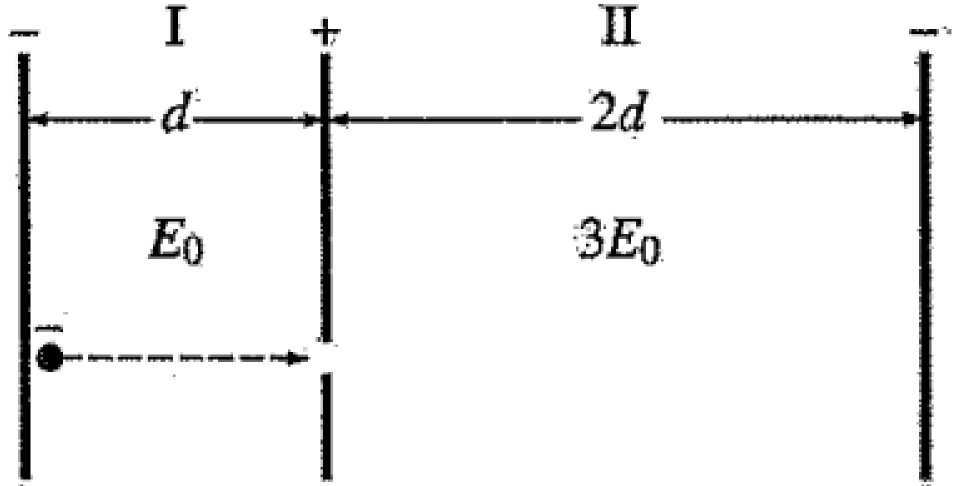
$$U = {}^1\!/_2 QV = {}^1\!/_2 CV^2 = {}^1\!/_2 Q^2/C$$

Since $Q = VC$

This is potential energy waiting to be used…

This is potential energy waiting to be used...

Question 21



21. A negatively charged particle initially in region I, as shown above, is accelerated from rest by an electric field of magnitude E_0 between two parallel plates separated by a distance d. The particle then enters region II, the space between two parallel plates with a separation $2d$ and an electric field of magnitude $3E_0$ in the opposite direction. How far into region II does the particle travel before reversing direction?

(A) $\frac{d}{3}$ $\quad \Delta V_I = E_0 d = (3E_0)x$

(B) $\frac{d}{2}$ $\quad x = d$

(B) $\frac{}{2}$

(C) $\frac{2d}{3}$

(D) d

(E) $\frac{3d}{2}$

$x = \frac{d}{3}$

Question 22

Solenoid

22. A student wants to construct an inductor of a given inductance using copper wire and a plastic tube. If a sufficient supply of copper wire is available, the student will also need a 2

$L = \mu_0 n^2 l A$

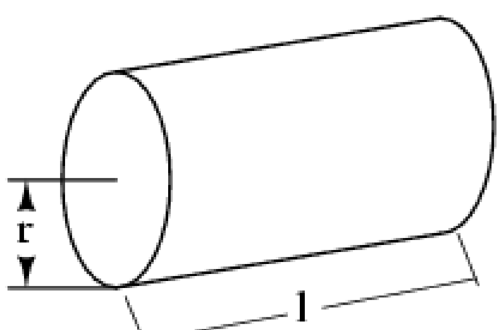
(A) meterstick only
(B) secondary coil and a meterstick
(C) resistor of known resistance and a meterstick
(D) voltmeter and a meterstick
(E) voltmeter and a DC power supply

only need to measure l and r for πr^2

- Formula for inductance of simple solenoids

$$L = \frac{N^2 \mu A}{l}$$

$$\mu = \mu_r \mu_0$$



Where,

L   Inductance of coil in Henrys

Where,

L = Inductance of coil in Henrys

N = Number of turns in wire coil (straight wire = 1)

μ = Permeability of core material (absolute, not relative)

μ_r = Relative permeability, dimensionless (μ_0=1 for air)

μ_0 = 1.26 x 10 $^{-6}$ T-m/At permeability of free space

A = Area of coil in square meters = πr^2

l = Average length of coil in meters

- Derivation

As the current in a coil of wire increases or decreases -- aka, changes -- the flux through the coil changes as well. This changing flux induces a back emf in the same coil. Since the change in flux is proportional to the change in the current, and opposes that change, we can write the relationship

$$\mathcal{E} = -L\frac{\Delta i}{\Delta t}$$

where L, the **inductance**, represents a proportionality constant that reflects the coil's geometry. Inductance is measured in a unit called a Henry (H) which is a V sec/amp or J sec^2/C^2.

The most common inductor is a solenoid. Let's derive an expression for its inductance. An induced emf can be written as

$$\mathcal{E} = -N\frac{\Delta \phi}{\Delta t} \quad \text{or} \quad \mathcal{E} = -L\frac{\Delta i}{\Delta t}$$

Setting these two expressions equal to each other, yields

$$-L\frac{\Delta i}{\Delta t} = -N\frac{\Delta \phi}{\Delta t}$$

$$L = N\frac{\Delta \phi}{\Delta i}$$

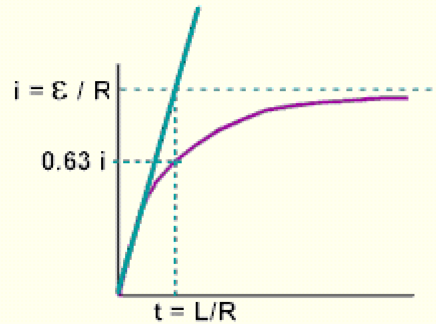
As the current changes from zero to I, the flux changes from zero to f, therefore Df = f and Di = I and our expression becomes

$$L = N\frac{\phi}{I} = N\frac{BA}{I} = N\frac{\mu_o n I}{I}A = \mu_o nNA$$

$$L = \mu_o \frac{N}{} NA = \mu_o A\frac{N^2}{}$$

$$L = \mu_o \frac{N}{L} NA = \mu_o A \frac{N^2}{L}$$

Notice that this expression, as predicted, deals only with the geometry of the solenoid: its cross-sectional area, length, and total number of coils.



When an inductor is part of a circuit, the current does not instantaneously reach its maximum value of I = e/R, instead the current builds gradually depending on the ratio of L/R.

Question 23

23. Correct statements about a constant magnetic field acting on a charged particle include which of the following?

I. The field can accelerate the particle. Radially

II. The field can change the kinetic energy of the particle. No

III. The field can do positive work on the particle. No

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Magnetic Field and Work

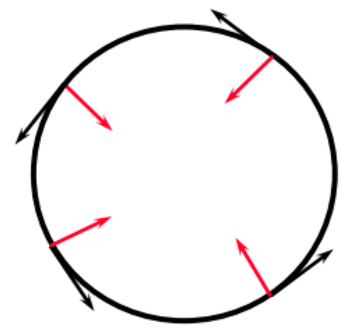
→ Magnetic force is *always* perpendicular to velocity

➔Magnetic force is *always* perpendicular to velocity

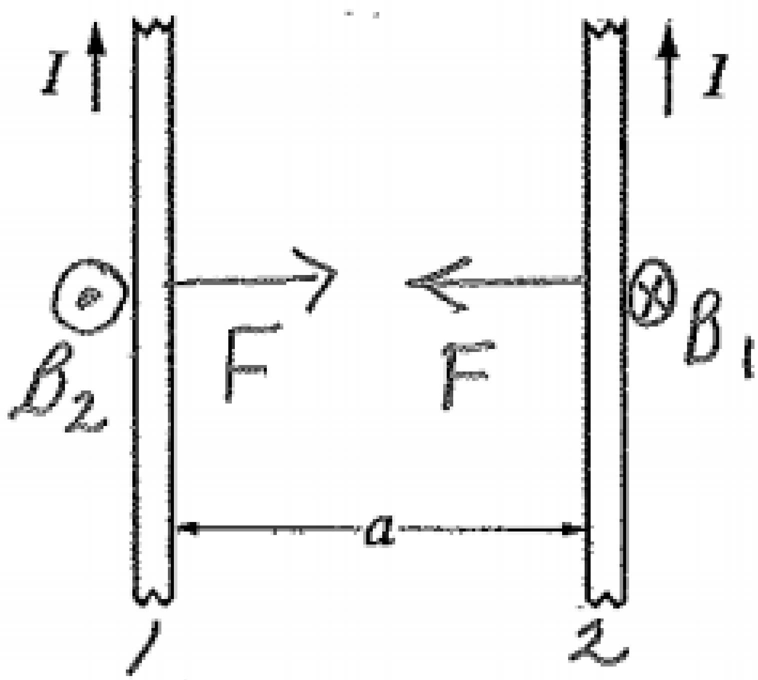
- Therefore B field does no work!
- Why? Because $\Delta K = \vec{F} \cdot \Delta\vec{x} = \vec{F} \cdot \left(\vec{v}\Delta t\right) = 0$

➔Consequences

- Kinetic energy does not change
- Speed does not change
- Only direction changes
- Particle moves in a circle (if $\vec{v} \perp \vec{B}$)



Question 28



28. Two long, straight, current-carrying wires are parallel to each other in the plane of the page and separated by a distance a, as shown above. The direction of the current I in each wire is toward

separated by a distance a, as shown above. The direction of the current I in each wire is toward the top of the page. Which of the following best represents the force per unit length acting on the wires?

(A) A repulsive force of magnitude $\mu_0 I^2/2\pi a$

(B) A repulsive force of magnitude $\mu_0 I/2\pi a$

(C) An attractive force of magnitude $\mu_0 I^2/2\pi a$

(D) An attractive force of magnitude $\mu_0 I/2\pi a$

(E) Zero

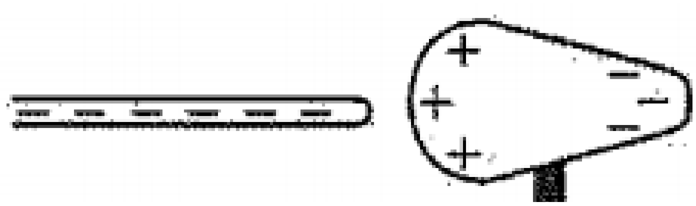
$$B = \frac{\mu_0 I}{2\pi a}$$

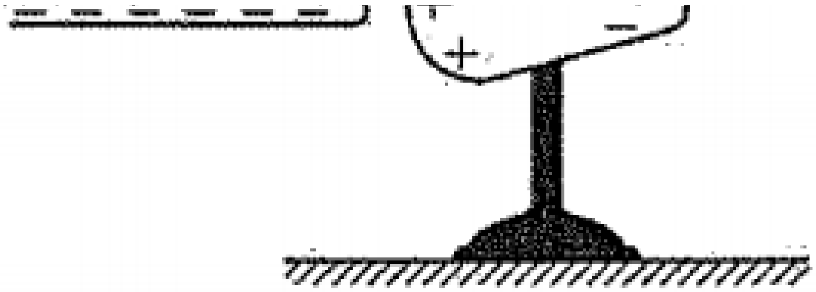
$\sin 90°$

$$F = I l \times B$$

$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi a}$$

Question 29





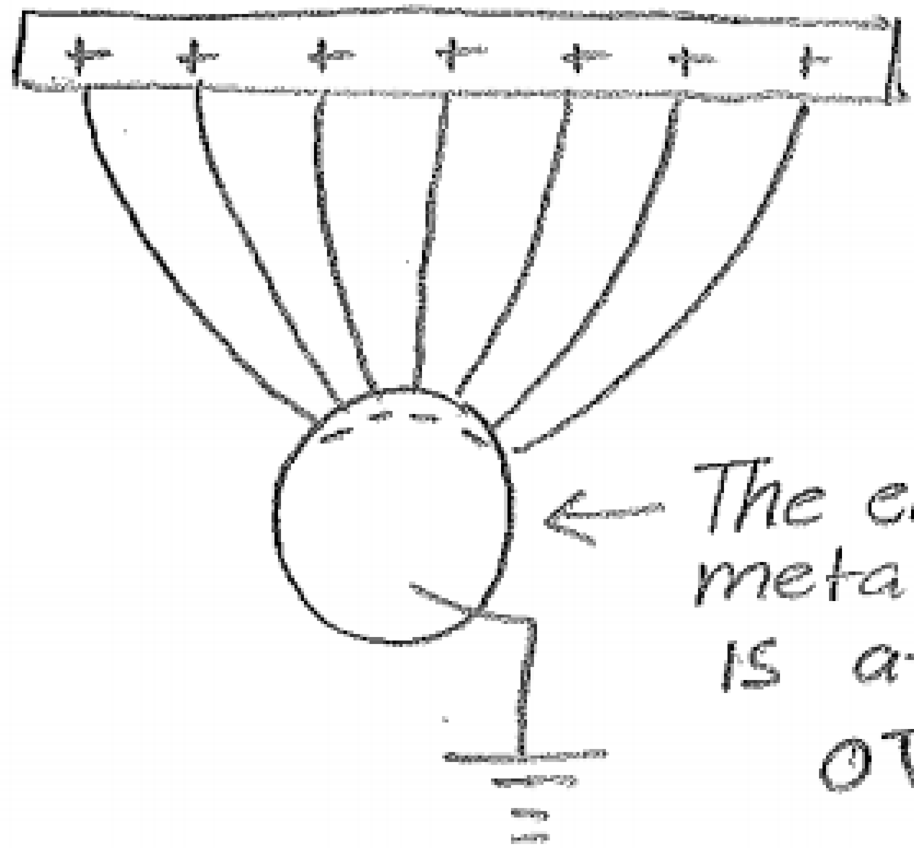
29. A negatively charged rod is brought near a metal object on an insulating stand, as shown above. When charges stop moving, the left side of the object has an excess of positive charge, and the right side of the object, where the radius of curvature is less, has an excess of negative charge. Which of the following best describes the electric potential on the metal object?

(A) It is greatest on the positively charged side of the object.
(B) It is greatest on the negatively charged side of the object.
(C) It is greatest at the center of the object.
(D) It is the same everywhere on the object.
(E) It cannot be determined from the information given.

Electric Potential = Voltage

Voltage is independent of charge distribution

For Example:



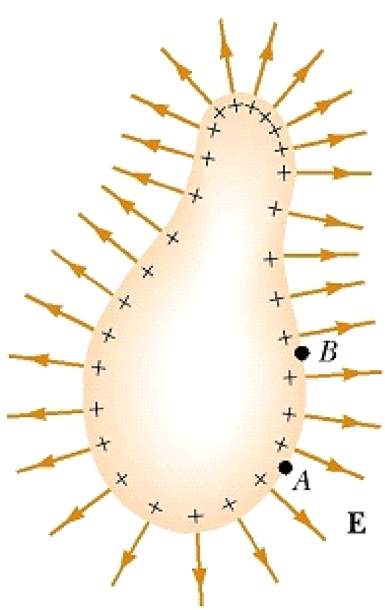
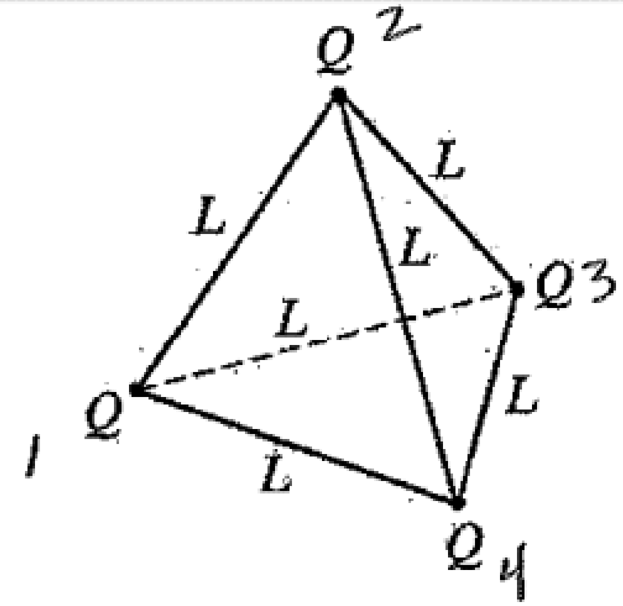


Figure 25.20 An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the plus signs that the surface charge density is nonuniform.

Question 30

Question 30



Four identical charged particles, each with charge Q, are fixed in place in the shape of an equilateral pyramid with sides of length L, as shown above.

30. What is the electrical potential energy of this arrangement of charges?

(A) $\frac{1}{4\pi\epsilon_0}\frac{Q^2}{L}$

(B) $\frac{2}{4\pi\epsilon_0}\frac{Q^2}{L}$

(C) $\frac{3}{4\pi\epsilon_0}\frac{Q^2}{L}$

(D) $\frac{4}{4\pi\epsilon_0}\frac{Q^2}{L}$

$U_{total} = \sum \frac{k q_1 q_2}{r}$

12 + 13 + 14

+ 24 + 23 + 34

//

(D) $\frac{4}{4\pi\epsilon_0}\frac{Q^2}{L}$ //

(E) $\frac{6}{4\pi\epsilon_0}\frac{Q^2}{L}$ ← 6 pairs

Question 31

31. One of the four charged particles is released and allowed to move away under the influence of the electrostatic force from the other three charges. How much kinetic energy will it have when it is very far away?

(A) $\frac{1}{4\pi\epsilon_0}\frac{Q^2}{L}$

(B) $\frac{2}{4\pi\epsilon_0}\frac{Q^2}{L}$

(C) $\frac{3}{4\pi\epsilon_0}\frac{Q^2}{L}$

(D) $\frac{4}{4\pi\epsilon_0}\frac{Q^2}{L}$

(E) $\frac{6}{4\pi\epsilon_0}\frac{Q^2}{L}$

$\Delta KE = -\Delta PE$

$PE_i = \frac{6}{4\pi\epsilon_0}\frac{Q^2}{L}$

$PE_f = \frac{3}{4\pi\epsilon_0}\frac{Q^2}{L}$

↓

13 + 14 + 34

$\therefore |KE_f| = 6 - 3 = \frac{3}{4\pi\epsilon_0}\frac{Q^2}{L}$

Question 32

Question 33

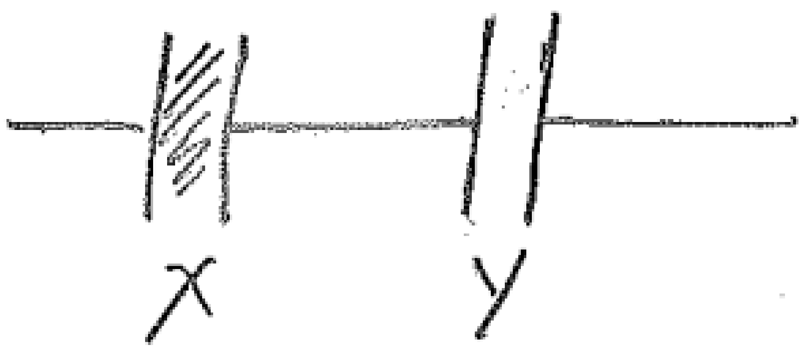
33. Two identical capacitors, X and Y, are connected in series across a battery. A dielectric material with $\kappa = 5$ is placed in capacitor X. Once equilibrium is reached, how do the potential differences across the two capacitors and their charges compare?

Series = Same q

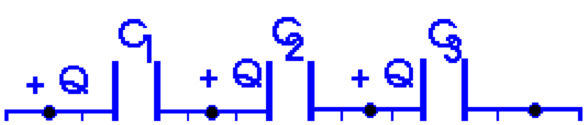
	Potential Difference	Charge
(A)	$V_x > V_y$	$Q_x > Q_y$
(B)	$V_x > V_y$	$Q_x = Q_y$
(C)	$V_x = V_y$	$Q_x > Q_y$
(D)	$V_x < V_y$	$Q_x = Q_y$
(E)	$V_x < V_y$	$Q_x < Q_y$

$C = \dfrac{q}{V}$

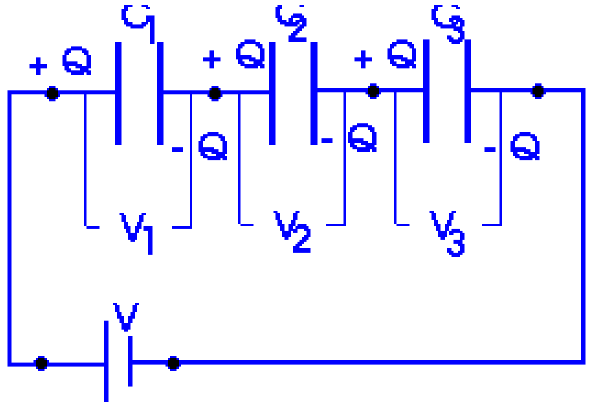
$V = \dfrac{q}{C}$



$C_x > C_y \rightarrow \therefore V_x < V_y$



The same charge is on each capacitor.



Series

The same charge is on each capacitor.

$$V = V_1 + V_2 + V_3$$

$$V = Q/C_1 + Q/C_2 + Q/C_3$$

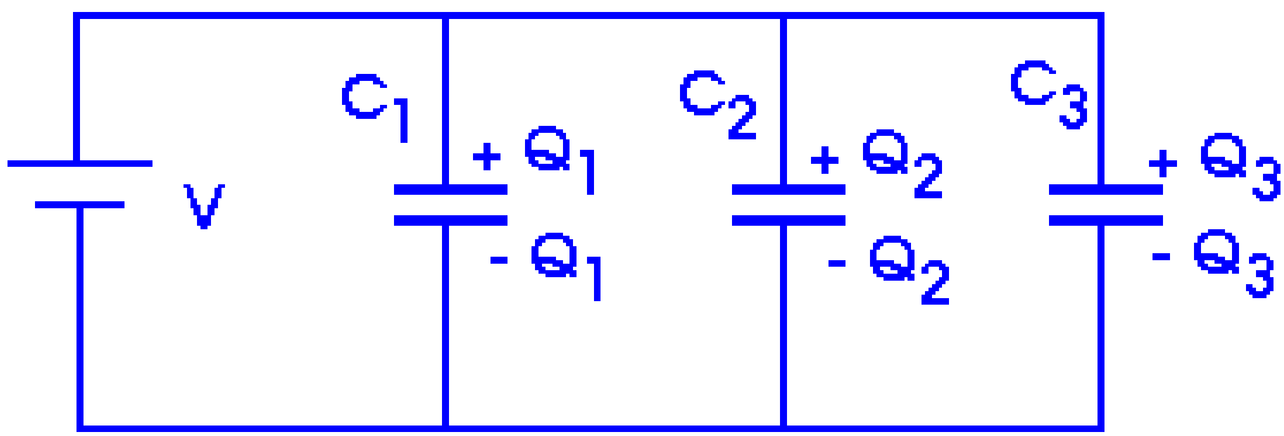
$$V = Q(1/C_1 + 1/C_2 + 1/C_3)$$

$$V = Q(1/C_{eq})$$

$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$$

Parallel

Now the charges are not the same. But the voltages are!



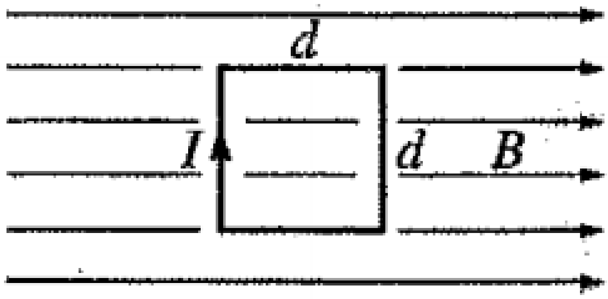
$$Q_{eq} = Q_{tot} = Q_1 + Q_2 + Q_3$$

$$Q_{eq} = C_1\Delta V + C_2\Delta V + C_3\Delta V$$

$$Q_{eq} = (C_1 + C_2 + C_3)\Delta V$$

$$Q_{eq} = (C_1 + C_2 + C_3)\Delta V$$
$$Q_{eq} = (C_{eq})\Delta V$$
$$C_{eq} = C_1 + C_2 + C_3$$

Question 35



35. When a square wire loop of side d carrying current I is in a uniform magnetic field of magnitude B in the position shown above, there is a torque on the loop. The magnitude of this torque is

(A) directly proportional to d
(B) directly proportional to d^2
(C) inversely proportional to d
(D) inversely proportional to d^2
(E) independent of d

$F = I\vec{\ell} \otimes \vec{B}$, $\ell = d$

$\tau = \vec{r} \otimes \vec{F}$, $r = \frac{d}{2}$, $F = 2(IdB \sin\theta)$

) P

) MAGNETISM SECTION I

2

$\tau \alpha d^2$

sin

Practice Exam Multiple Choice

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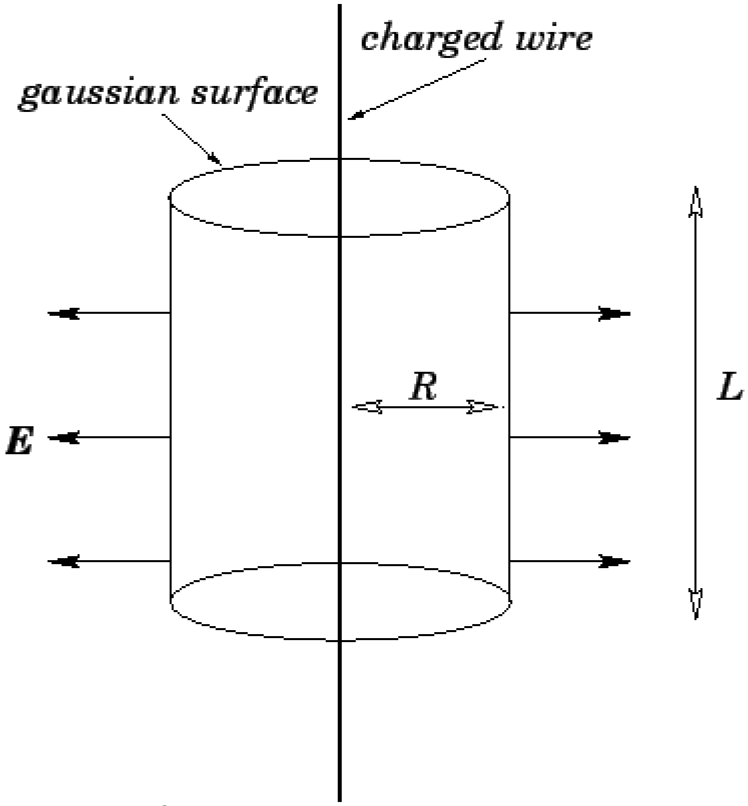
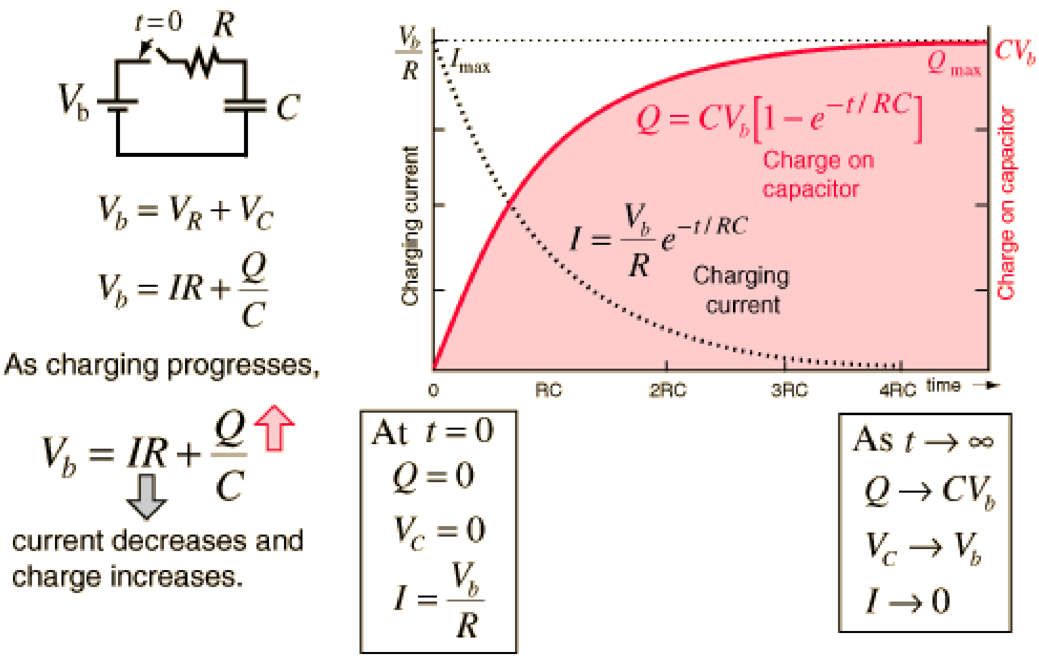
Question 2

- Electric Field of a Uniformly Charged Wire

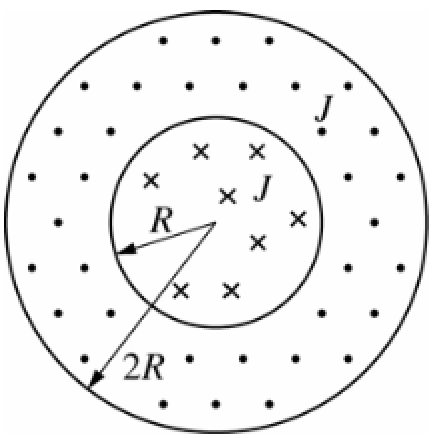


- $\oint E \cdot dA = \frac{Q}{\epsilon_0}$
- $E2\pi dL = \frac{\lambda L}{\epsilon_0}$
- $E = \frac{\lambda}{2\pi d\epsilon_0} \propto \frac{\lambda}{d}$

Question 9

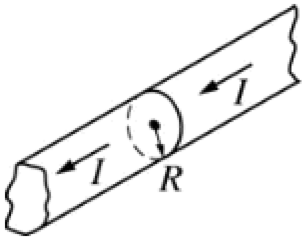


Question 31



- What is the magnitude of the magnetic field at the surface of the outer conductor?
- $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
- $B 2\pi(2R) = \mu_0\left[\left(4\pi R^2 - \pi R^2\right)J - \pi R^2 J\right]$
- $B = \frac{1}{2}\mu_0 R J$

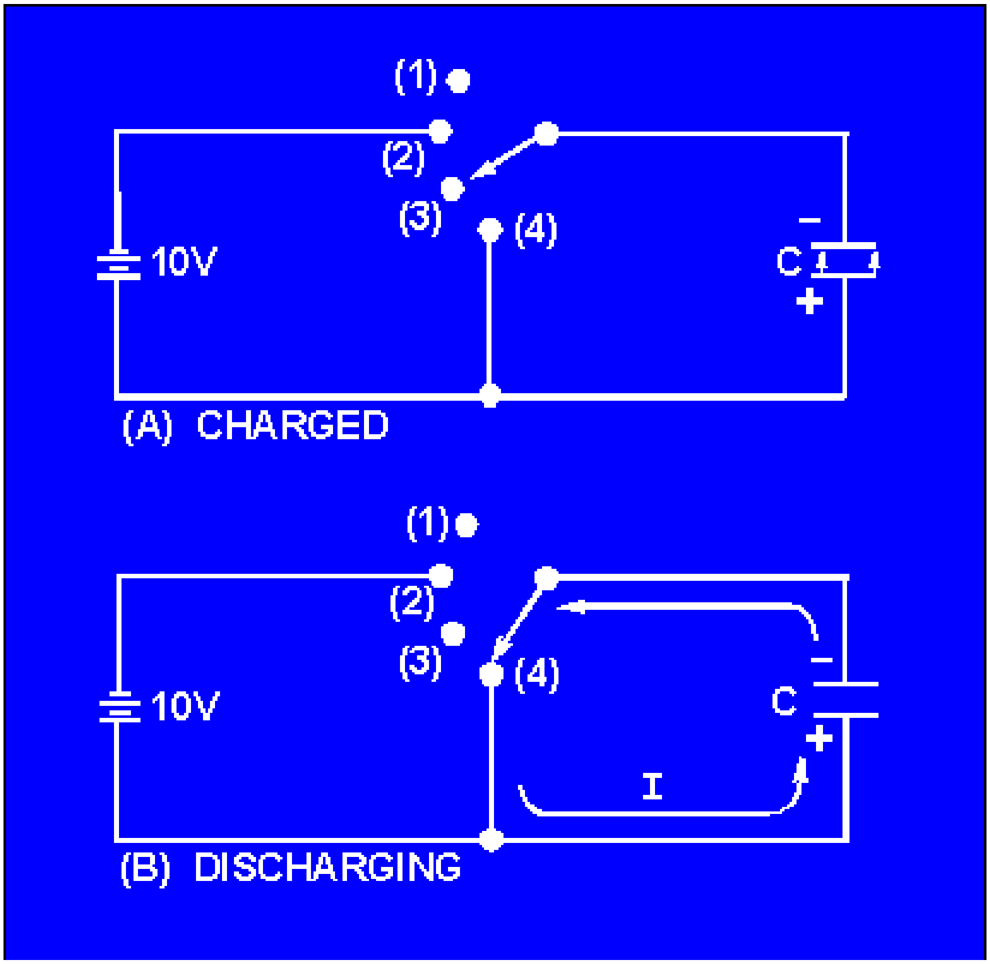
Question 32



A long wire of radius R carries a current I, as shown above, with a current density $J = \alpha r$ that increases linearly with the distance r from the center of the wire. Which of the following graphs best represents the magnitude of the magnetic field B as a function r ?

- $r < R$
  - $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
  - $B 2\pi r = \mu_0 \pi r^2 \cdot \alpha r$
  - $B = \frac{1}{2}\mu_0 r^2 \alpha \propto r^2$
- $r \geq R$
  - $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
  - $B 2\pi r = \mu_0 \pi R^2 \cdot \alpha R$
  - $B = \frac{\mu_0 R^3 \alpha}{2r} \propto \frac{1}{r}$

Question 33



Does the direction of the current change when the capacitor goes from charging to discharging?

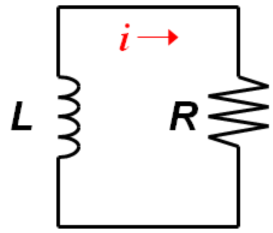
Yes. When a capacitor is charging, current flows towards the positive plate (as positive charge is added to that plate) and away from the negative plate. When the capacitor is discharging, current flows away from the positive and towards the negative plate, in the opposite direction.

Question 34

- Gauss's Law for Magnetism
    - The total magnetic flux through any closed surface is zero
    - This would not be true if magnetic monopoles were found to exist
    - $\Phi_B = \int_{\substack{closed \\ surface}} \vec{B} \cdot d\vec{A} = 0$

Question 35

RL Circuit



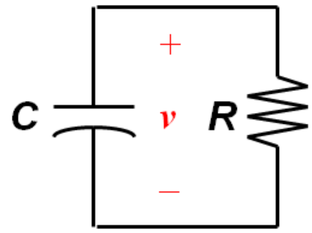
Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

time constant $\tau = \dfrac{L}{R}$

RC Circuit



- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

Resonant Frequency Formula

$$f = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

where 'f' = frequency in hertz
'L' = inductance in henrys
'C' = capacitance in farads

The formula also holds true if used with the units of megahertz, microhenries and microfarads and many other combinations of units.

Princeton 1 Multiple Choice

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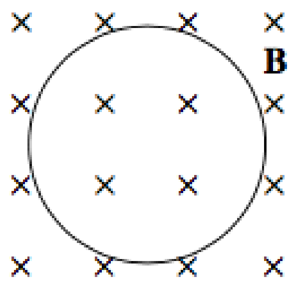
Question 41

- $N = m \cdot kg \cdot s^{-2}$
- $J = m^2 \cdot kg \cdot s^{-2}$
- $C = A \cdot s$
- $V = m^2 \cdot kg \cdot s^{-3} A^{-1}$
- $T = kg \cdot s^{-2} A^{-1}$

Question 48

- $\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt} \int_{\substack{open \\ surface}} \vec{B} \cdot d\vec{A} = \oint_{\substack{closed \\ loop}} \vec{E} \cdot d\vec{l}$



A copper wire in the shape of a circle of radius 1 m, lying in the plane of the page, is immersed in a magnetic field, **B**, that points into the plane of the page. The strength of **B** varies with time, *t*, according to the equation

$$B(t) = 2t(1 - t)$$

where B is given in teslas when t is measured in seconds. What is the magnitude of the induced electric field in the wire at time $t = 1$ s?

Apply Faraday's Law of Electromagnetic Induction:

$$\oint E \cdot d\ell = -\frac{d\Phi_B}{dt}$$

$$E(2\pi r) = -\frac{d}{dt}(BA)$$

$$= -A\frac{dB}{dt}$$

$$= \pi r^2 \cdot \frac{d}{dt}(2t - 2t^2)$$

$$= \pi r^2 (2 - 4t)$$

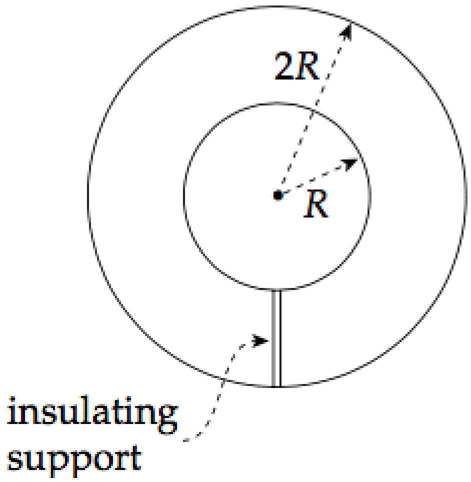
$$E = -r(1 - 2t)$$

Since r = 1 m, the value of E at t = 1 s is E = 1 N/C.

Question 53

- Parallel capacitors:
  - $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

Question 56



- Calculating Capacitance
  1. Assume a charge of +Q and -Q on each conductor
  2. Find the electric field between the conductors (Gauss's Law)
     - $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
  3. Calculate V by intergrating the electric field ($V = -\int \vec{E} \cdot d\vec{l}$)

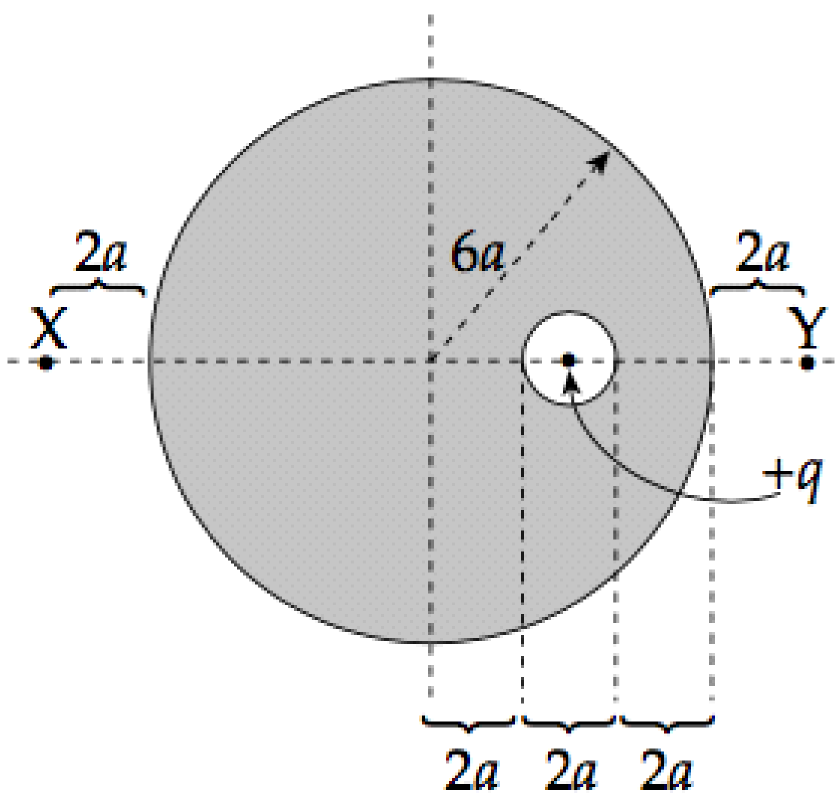
- $V = -\int_R^{2R} \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0} \int_R^{2R} \frac{dr}{r^2} = -\frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{r}\right)_R^{2R}$
  $= \frac{Q}{8\pi\epsilon_0 R}$

4. Utilize $C = \frac{Q}{V}$ to solve for capacitance.
   - $C = \frac{Q}{V} = 8\pi\epsilon_0 R$

Question 57

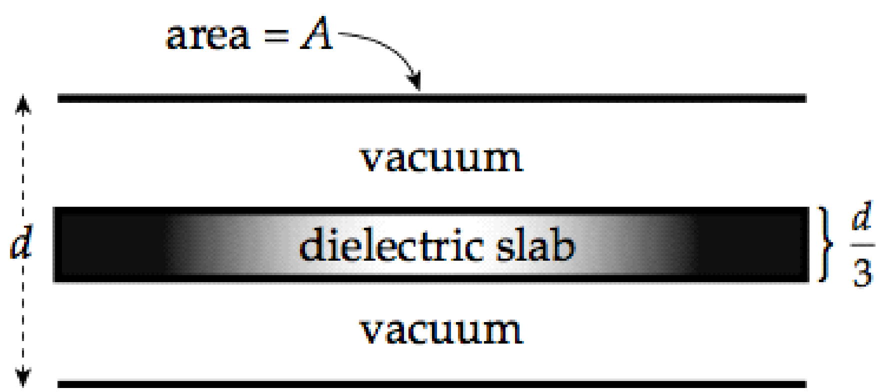
- Since the magnetic force is always perpendicular to the object's velocity, it does zero work on any charged particle.
- Zero work means zero change in kinetic energy, so the speed remains the same.
- Remember: The magnetic force can only change the direction of a charged particle's velocity, not its speed.

Question 61



If a conducting sphere contains a charge of $+q$ within an inner cavity, a charge of $-q$ will move to the wall of the cavity to "guard" the interior of the sphere from an electrostatic field, regardless of the size, shape, or location of the cavity. As a result, a charge of $+q$ is left on the exterior of the sphere (and it will be uniform). So, at points outside the sphere, the sphere behaves as if this charge $+q$ were concentrated at its center, so the electric field outside is simply kQ/r^2. Since points X and Y are at the same distance from the center of the sphere, the electric field strength at Y will be the same as at X.

Question 66



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{\left(\frac{3\varepsilon_o A}{d}\right)} + \frac{1}{\left(\frac{3K\varepsilon_o A}{d}\right)} + \frac{1}{\left(\frac{3\varepsilon_o A}{d}\right)}$$

$$\frac{1}{C_{eq}} = \frac{d}{3\varepsilon_o A} + \frac{d}{3K\varepsilon_o A} + \frac{d}{3\varepsilon_o A}$$

$$\frac{1}{C_{eq}} = \frac{2Kd + d}{3K\varepsilon_o A}$$

$$C_{eq} = \frac{3K\varepsilon_o A}{d(2K+1)}$$

Princeton 2 Multiple Choice

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Question 43

The electric field is uniform in between the plates, so the force and the acceleration are constant. Relating the potential difference to the electric field gives us $\Delta V = -Ex$, where x is the separation of the plates. The definition of the electric field is the force divided by the charge. Use these two pieces of information and Newton's Second Law to solve for the acceleration in terms of V.

$$E = \frac{F}{q}$$

$$\frac{V}{x} = \frac{F}{q}$$

$$\frac{Vq}{x} = ma$$

$$\frac{Vq}{mx} = a$$

This shows the acceleration is directly proportional to the potential difference V.

Question 50

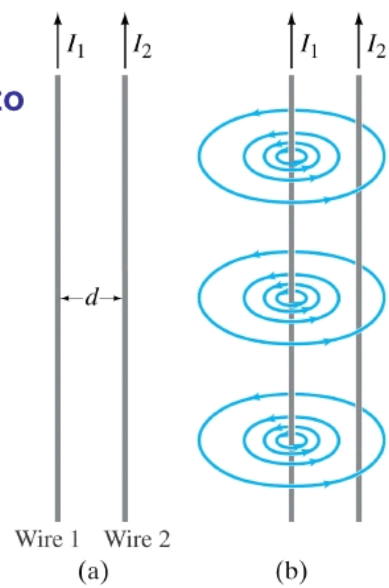
20.6 Force between Two Parallel Wires

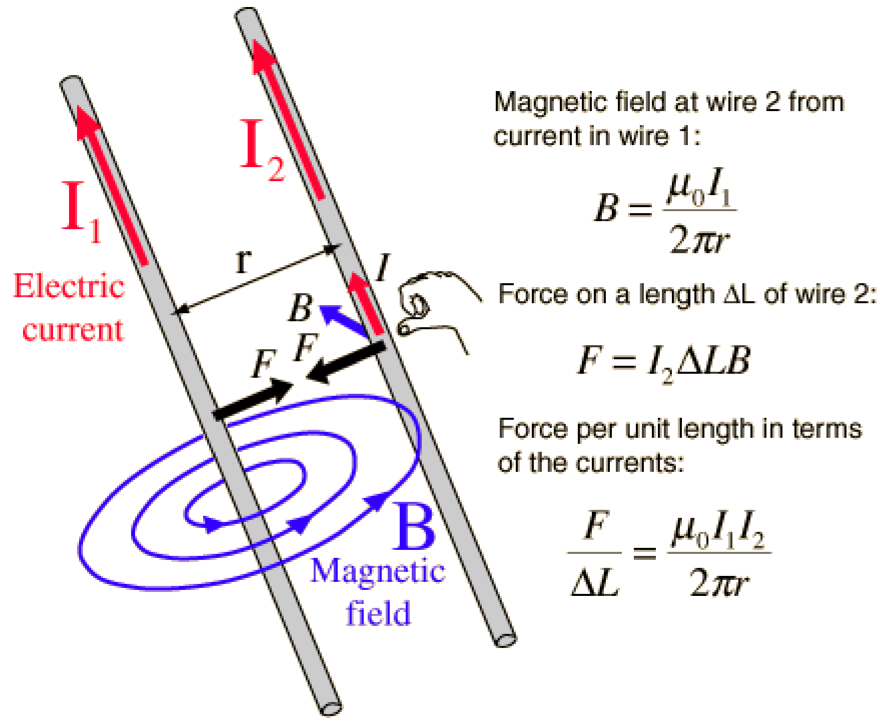
The magnetic field produced at the position of wire 2 due to the current in wire 1 is:

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

The force this field exerts on a length l_2 of wire 2 is:

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l_2 \qquad \text{(20-7)}$$





Question 61

$$\because \Delta V = \frac{\Delta U}{q}$$
$$\therefore \Delta U = q\Delta V$$

Question 62

- Ampère-Maxwell's Law -- A changing electric field would create a changing electric flux, which in turn would induce a magnetic field.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 I_{d,enc}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Question 65

A solid, metal object is isolated from other charges and has charge distributed on its surface. The charge distribution is not uniform. It may be correctly concluded that the

(A) electric field outside the object is zero

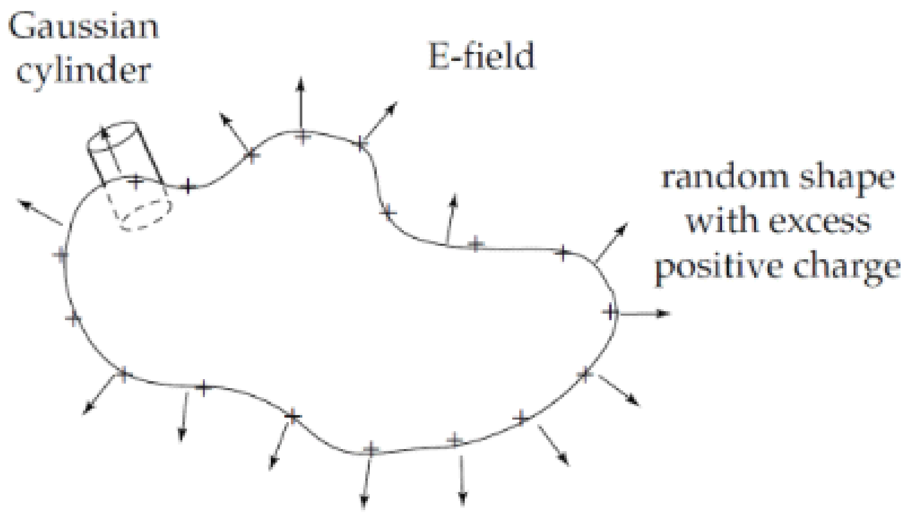
(B) the electric field outside the object is equal to the electric field inside the object

(C) the electric field outside the object is directly proportional to the distance away from the center of mass of the object

(D) the electric field outside the object, but very close to the surface, is equal to the surface charge density at any location divided by the permittivity of free space

(E) the electric potential on the surface of the object is not constant

For a solid, metal object the electric field inside is equal to zero. The electric field outside the object will not be zero because some excess charge is contained on the object and Gauss's Law can be applied to show that Q_{in} would generate an electric field. This information eliminates (A) and (B). The surface of a conductor is an equipotential, so (E) is not correct. The strength of the electric field should decrease as the distance away from the center increases, so (C) is not correct. (D) is correct, and we can apply Gauss's Law as shown below to the odd-shaped figure as indicated. The Q_{in} will be equal to the surface charge density at the location times the area of the endcap of the Gaussian cylinder. This is only true very close to the surface of the object so that the Gaussian cylinder is perpendicular to the surface.



$$\oint E \bullet dA = \frac{Q_{in}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Question 68