Annotated Answers to the 2009 AP Physics C Multiple Choice Examination—Mechanics

1\. C. Galileo I: $v = v_o + at = 0 + 4\,\mathrm{m/s^2} \times 5\,\mathrm{s} = 20\,\mathrm{m/s}$.

2\. E. Galileo II: $x = x_o + v_o t + \frac{1}{2}at^2 = \frac{1}{2}(4\,\mathrm{m/s^2})(5\,\mathrm{s})^2 = 50\,\mathrm{m}$.

3\. E. Power is in units of (energy)/(time); 1 Watt = 1 Joule/s. So A, B and D are all Watts (1 Joule = 1 Newton-meter). 1 eV = 1.6×10^{-19} J, so C is also energy per second. (Recall that electric work is qV.) E is *not* energy/time.

4\. C. Newton's First Law: $F_{\text{net}} = ma$, so $M_1 a_1 = M_2 a_2$ or

$$\frac{M_1}{M_2} = \frac{a_2}{a_1}$$

That's the best that can be done here because F is unknown.

5\. C. Galileo I: $v = v_o + at = v_o - gt$. The slope is constant and negative, equal to $-g$.

6\. B. Galileo IV: $2a\Delta x = v^2 - v_o^2$, so

$$v = \sqrt{2(-10\,\mathrm{m/s^2})(-0.2\,\mathrm{m})} = 2\,\mathrm{m/s}$$

7\. B. $F_{\text{net}} = \dfrac{\Delta p}{\Delta t} = \dfrac{\Delta(mv)}{\Delta t} = v\dfrac{\Delta m}{\Delta t} = 6.0\,\mathrm{m/s} \times 1.0\,\mathrm{kg/s} = 6.0\,\mathrm{N}$

8\. D. Rotational Galileo II: $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$, so

$$\Delta\theta = \theta - \theta_o = \tfrac{1}{2}(3\,\mathrm{radians/s^2})(4\,\mathrm{s})^2 = 24\,\mathrm{radians}$$

9\. C. $T_{\text{spring}} = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2\,\mathrm{kg}}{50\,\mathrm{N/m}}} = \dfrac{2\pi}{5}\,\mathrm{s} = 0.4\pi\,\mathrm{s}$.

10\. E. The maximum velocity occurs when the displacement from equilibrium is zero; there the energy is all kinetic. A and B describe the same positions (the endpoints); there the energy is all potential. C also describes the endpoints. Halfway between equilibrium and the amplitude the mass-spring system has both kinetic and potential energy. All the energy is kinetic at $x = 0$, the equilibrium point.

11\. E. An application of Kepler's Third Law;

$$\frac{r^3}{T^2} = \frac{GM}{4\pi}$$

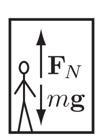
If you know both r and T, you can find the mass M. Alternatively, use the satellite equation (for a circular orbit, which may or may not be true):

$$F_{\text{net}} = \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r}$$

For a circular orbit, $v = 2\pi r/T$, so you recover Kepler's Third Law (which is true even for elliptical orbits, with a, the ellipse's semimajor axis, replacing the radius r.)

12\. C. Angular momentum is conserved because there are no external torques; momentum is conserved because there are no external forces (like a peg to which the rod might be attached). The kinetic energy *could* be conserved, but it isn't absolutely required to be. But **p** and **L** *must* be conserved.

13\. E. This probably requires a diagram. Consider a passenger in the elevator as shown:



Newton's Second Law says $F_N - mg = ma$ or $F_N = mg + ma$. Clearly the normal force F_N will be least

13. cont'd

when the acceleration is negative (when the elevator is nearing its destination at an upper floor.) The normal force will be greatest at the beginning, when the elevator and passengers need to be accelerated upwards; in the middle of the trip, the elevator is coasting (and the normal force equals the weight); at the end of the trip, the elevator accelerates down, to reduce the speed of the elevator and passenger to zero. The acceleration is most negative around $t = 6$ s; when the velocity graph has a tangent whose slope is most negative.

14. D. A standard problem: $a = g\sin\theta - \mu_k \cos\theta$ and $F_N = mg\cos\theta$. Here, there is no friction, so $a = g\sin\theta$.

15. D. If the final kinetic energy K' equals one-fourth of the initial K, then

$$K' = \tfrac{1}{2}mv'^2 = \tfrac{1}{4}K = \tfrac{1}{4} \times \tfrac{1}{2}mv^2$$

which means $v' = \frac{1}{2}v$. But the direction is reversed after hitting the wall, so

$$\mathbf{v}' = -\tfrac{1}{2}\mathbf{v}$$

Hard to know why the exam didn't use any sort of vector notation; an arrow on top of the variable, bold type, something... (as in questions 27 and 34.)

16. C. The parametric set C has $\mathbf{v} = (v_x, v_y) = (dx/dt, dy/dt) = (A, B)$. The acceleration is $d\mathbf{v}/dt = (0, 0)$.

17. E. Neither set A nor set B have an acceleration constant in direction. From question 16, set C has **a** equal to zero. The only options are D and E. The acceleration in D is constant; $\mathbf{a} = (2A, 2B)$. The velocity in D is $(dx/dt, dy/dt) = (2At, 2Bt)$; at $t = 0$ this is zero. Now, one could argue that the zero vector is perpendicular to *all* vectors (it has, after all, a zero dot product with every vector.) If so, D would suffice. But a better choice is E. Its acceleration $\mathbf{a} = (0, 2B)$ is constant and clearly perpendicular to the initial velocity, $\mathbf{v}_o = (A, 0)$.

18. A. Set A clearly describes uniform circular motion, in which the speed is constant and the acceleration is non-zero but constant in size.

19. E. See question 14;

$$a = g\sin\theta - \mu_k g\cos\theta$$

If the acceleration is zero (the skier is traveling with constant velocity) then

$$g\sin\theta = \mu_k g\cos\theta \quad \Rightarrow \quad \mu_k = \tan\theta$$

20. E. There are a couple of ways to do this. The most straightforward is to recall the definition that power is the rate at which work is done. The work done is the change in kinetic energy;

$$W = \Delta K = K' - K = \tfrac{1}{2}(2000\,\mathrm{kg})(20\,\mathrm{m/s})^2 = 0.4\,\mathrm{MJ}$$

The time required to deliver this work is the time the car spends accelerating from rest to 20 m/s;

$$\Delta t = \frac{20\,\mathrm{m/s} - 0}{3\,\mathrm{m/s}^2} = \tfrac{20}{3}\,\mathrm{s}$$

Then

$$P = \frac{0.4\,\mathrm{MJ}}{\frac{20}{3}\,\mathrm{s}} = 6 \times 10^4\,\mathrm{W}$$

Alternatively, use the formula $P = Fv_{\mathrm{avg}} = mav_{\mathrm{avg}} = 2000\,\mathrm{kg} \times 3\,\mathrm{m/s}^2 \times 10\,\mathrm{m/s} = 6 \times 10^4\,\mathrm{W}$.

21. E. After the person has walked to the other end, the situation must be the mirror image of this one; the center of mass will be shifted to a distance d to the left of the center of the raft. But the center of mass *cannot* move as the person walks on the boat. Therefore, the boat must have moved a distance $2d$ to the right. (This elegant solution comes from Zach Reneau-Wedeen, Lab School '10).

22. A. A little calculus. Newton's Second Law says

$$ma = mg - kv$$

(taking the positive direction as downward.) But $a = dv/dt$, so

$$m\frac{dv}{dt} = mg - kv \quad \Rightarrow \quad \frac{dv}{mg - kv} = \frac{dt}{m}$$

Integrate both sides with respect to v and t and you're done. Note, however, the differences between answers A and B: The limits in A are fine. In B they describe the wrong variable. Talk about fine print...

23. E. This problem and the next are two of the most challenging I've encountered on the C Mechanics exam. The key is that Δt is not zero: The velocity *remains* zero for a few moments. During this non-instantaneous interval, the acceleration also must be zero; otherwise, the velocity would change. By Newton's Second Law, the net force must be zero;

$$F_{\text{net}} = T - mg = ma = 0$$

Then, as we'll need for the next question, $T = mg$.

24. B. There is an associated, famous problem about dropping a spool (with the cord held by a *motionless* hand). In that case, you write Newton's Second Law for the forces

$$F_{\text{net}} = mg - T = ma$$

and for the torques,

$$\tau = TR = I\alpha = I(a/R)$$

where one associates a with a_{cm}. Then $T = Ia/R^2$ and

$$ma = mg - Ia/R^2 \quad \Rightarrow \quad a = \frac{mg}{m + (I/R^2)}$$

The problem here is, of course, that the center of mass acceleration is *zero.* In fact you have to think a little more carefully. The typical relationship for rolling without slipping, $a = \alpha R$, still holds, but now it is the *tangential* acceleration of the rim, which is equal to the upward acceleration of the cord (and the hand.) Generally in rolling without slipping (on a horizontal surface, for example), the rim's acceleration is the same as the center of mass's; that isn't true here. Here, the center of mass doesn't move at all during the time interval Δt. But we can still say, with a the acceleration of the rim (equal to the acceleration of the hand),

$$\tau = TR = I\alpha = I(a/R)$$

From the previous problem, we know $T = mg$. Then

$$mgR = I(a/R) = \tfrac{1}{2}mR^2 \times (a/R) = \tfrac{1}{2}maR$$

Cancel mR on both sides, multiply both sides by 2 and obtain $a = 2g$. Unsurprisingly, 90% of those who took the test got this one wrong.

25. C. The angular momentum cannot change, because there are no external torques. On the other hand, we have the useful formulas

$$K = \frac{p^2}{2m} \quad \text{and} \quad K = \frac{L^2}{2I}$$

As the skater's arms come in, I, which depends on the square of the average distance of mass from the axis, is getting smaller. Since the kinetic energy depends directly on the square of the angular momentum and inversely on the moment of inertia, and I here decreases, it follows K increases. Also, work has to be done to draw the arms in; this work goes into increased kinetic energy.

26. E. A variation on the theme of question 25. Now set $p^2 = 2mK$. If $p_1 = p_2$, then $p_1^2 = p_2^2$, and

$$2m_1K_1 = 2m_2K_2 \;\Rightarrow\; \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

If $K_1 > K_2$, then $m_2 > m_1$. But $p_1 = p_2$, so we have to have $v_2 < v_1$; which is answer E.

27. B. The acceleration $\mathbf{a} = \mathbf{F}/m = 0.5t/5 = 0.1\,t$. To find the velocity at time $t = 4\,\text{s}$, integrate:

$$v_4 - v_o = v_4 = \int_0^4 0.1\,t\,dt = \tfrac{1}{20}\,t^2\Big|_0^4 = \tfrac{16}{20} = 0.8\,\text{m/s}$$

because $v_o = 0$.

28. D. Momentum is conserved. The initial momentum is $m \cdot v + 2m \cdot (-v) = -mv$ in the x-direction. In A, the final momentum has no x-component. In B and C it has a non-zero y-direction. E has its x-component to the right. The answer has to be D.

29. E. The projectile motion equation for the y-component of the object's position is

$$y = y_o + v_o \sin\theta t - \tfrac{1}{2}gt^2 = v_o \sin\theta t - \tfrac{1}{2}gt^2$$

Set this equal to zero and solve for t;

$$0 = t(v_o \sin\theta - \tfrac{1}{2}gt) \;\Rightarrow\; t = 0 \text{ or } t = \frac{2v_o \sin\theta}{g}$$

The object leaves the ground at $t = 0$; it hits the ground at the later time.

30. E. The centripetal force is $F_c = \dfrac{mv^2}{r}$. This is the force provided by the string; $F_c = F_s$. But $v = \omega r = 2\pi f r$. Then

$$F_c = \frac{m(2\pi f r)^2}{r} = 4\pi^2 f^2 m r$$

If both r and f are doubled, F_c will be increased by $2^2 \cdot 2 = 8$.

31. C. When the object is dropped,

$$E_{\text{top}} = mgh$$

and when it hits the ground, $E_{\text{bottom}} = \frac{1}{2}mv_1^2$. Setting these two equal leads to $v_1^2 = 2gh$ or $v_1 = \sqrt{2gh}$. In the second case,

$$E_{\text{top}} = mgh + \tfrac{1}{2}mv_1^2 = mgh + \tfrac{1}{2}m\left(\sqrt{2gh}\right)^2 = 2mgh$$

and when it hits the ground,

$$E_{\text{bottom}} = \tfrac{1}{2}mv_2^2 = 2mgh \;\Rightarrow\; v_2 = 2\sqrt{gh} = \sqrt{2}\sqrt{2gh} = \sqrt{2}\,v_1$$

32. E. The key is that the center of mass cannot move. Imagine putting the masses on a seesaw, and the pivot at the triangle. In A, B, C and D, it's clear that the seesaw would not balance at the pivot. In E, though, it might; you've got twice as much mass at half the distance (horizontally), and the masses are symmetric about the x-axis.

33. A. For circular orbits,

$$F_{\text{net}} = \frac{mv^2}{r} = \frac{GMm^2}{r} \;\Rightarrow\; v^2 = \frac{GM}{r}$$

Compare kinetic energies;

$$\frac{K_2}{K_3} = \frac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_3^2} = \frac{v_2^2}{v_3^2} = \frac{GM/r_2}{GM/r_3} = \frac{r_3}{r_2} = \frac{3}{2}$$

so the kinetic energy of S_2 is larger than that of S_3; $K_2 > K_3$ For angular momentum, $L = mvr$, so

33. cont'd

$$\frac{L_2}{L_3} = \frac{mv_2r_2}{mv_3r_3} = \frac{v_2r_2}{v_3r_3}$$

From the previous ratio,

$$\frac{v_2}{v_3} = \sqrt{\frac{3}{2}}$$

so

$$\frac{L_2}{L_3} = \frac{v_2r_2}{v_3r_3} = \frac{v_2}{v_3} \cdot \frac{r_2}{r_3} = \sqrt{\frac{3}{2}} \cdot \frac{2}{3} = \sqrt{\frac{2}{3}}$$

hence $L_2 < L_3$. The correct answer is A.

34. D. As $t \to \infty$, the speed $v \to 0$ exponentially (the problem is just like air resistance, but without the driving effect of gravity.) The position increases so long as the velocity is positive, but as the velocity tends to zero, the position tends to a limiting value; the appropriate values are shown by D.

35. B. For the mass not to fall, the net force in the y-direction must be zero. That is,

$$mg = f_s \leq \mu_s F_N$$

For the mass to move in a circle, the net force in the x-direction must be F_{centrip}; this force is provided uniquely by the normal force:

$$F_{\text{net},\,x} = F_N = \frac{mv^2}{R}$$

Substituting this into the previous equation gives

$$mg \leq \mu_s \frac{mv^2}{R}$$

or

$$g \leq \mu_s \frac{v^2}{R}$$

But $v = \omega r$, so plugging this in,

$$g \leq \mu_s \frac{\omega^2 R^2}{R} = \mu_s \omega^2 R$$

or, solving for μ_s,

$$\mu_s \geq \frac{g}{\omega^2 R}$$

which is answer B.

Discussion

There are many admirable questions on this test: questions 13 (elevator and graph), 21 (person walking on boat) and 34 (air resistance) are simultaneously very physical but require at least some mathematical reasoning. A number of questions are elementary (the first six are nothing a reasonably good AP Physics B student couldn't answer; only about ten require techniques or concepts beyond the B syllabus). A handful (24, 30 and 33) were easy to get wrong, either from inherent difficulty or from algebraic intricacy. At some point, though, what is being tested by a problem like 33 is not physical understanding but algebraic technique; maybe this is not the best sort of problem on a physics test. In question 22, answer B with the switched limits is a little petty (but at least it wasn't put first, at A, where students in a hurry might well have chosen it without reading on to a correct answer at B.) Nothing on elliptical orbits, calculation of moments of inertia (parallel axis theorem, say, or simple integration) or pendulums, but on the whole a very comprehensive and fair examination.

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