

# AP<sup>®</sup> Statistics 2010 Free-Response Questions Form B

#### The College Board

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the College Board is composed of more than 5,700 schools, colleges, universities and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,800 colleges through major programs and services in college readiness, college admission, guidance, assessment, financial aid and enrollment. Among its widely recognized programs are the SAT®, the PSAT/NMSQT®, the Advanced Placement Program® (AP®), SpringBoard® and ACCUPLACER®. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities and concerns.

© 2010 The College Board. College Board, ACCUPLACER, Advanced Placement Program, AP, AP Central, SAT, SpringBoard and the acorn logo are registered trademarks of the College Board. Admitted Class Evaluation Service is a trademark owned by the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation. All other products and services may be trademarks of their respective owners. Permission to use copyrighted College Board materials may be requested online at: www.collegeboard.com/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.com. AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

Formulas begin on page 3. Questions begin on page 6. Tables begin on page 12.

## **Formulas**

(I) Descriptive Statistics

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum \left(x_i - \overline{x}\right)^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \overline{x}}{s_x} \right) \left( \frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - 2}}}{\sqrt{\sum (x_i - \overline{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_{\mathcal{X}} = \sum x_i p_i$$

$$Var(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu_{\chi} = np$$

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size n from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

#### (III) Inferential Statistics

 $Standardized \ test \ statistic: \frac{statistic - parameter}{standard \ deviation \ of \ statistic}$ 

Confidence interval: statistic  $\pm$  (critical value) • (standard deviation of statistic)

Single-Sample

| Statistic         | Standard Deviation of Statistic |  |  |  |  |
|-------------------|---------------------------------|--|--|--|--|
| Sample Mean       | $\frac{\sigma}{\sqrt{n}}$       |  |  |  |  |
| Sample Proportion | $\sqrt{\frac{p(1-p)}{n}}$       |  |  |  |  |

Two-Sample

| Statistic                        | Standard Deviation of Statistic   |
|----------------------------------|---|
| Difference of sample means       | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$                              |
|                                  | Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ |
| Difference of sample proportions | $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$                              |
|                                  | Special case when $p_1 = p_2$ $\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$    |
|                                  |   |

Chi-square test statistic = 
$$\sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

## STATISTICS SECTION II Part A

# Ouestions 1-5

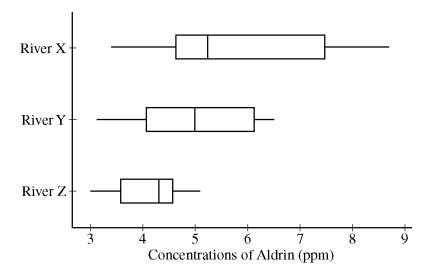
#### Spend about 65 minutes on this part of the exam.

#### Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. As a part of the United States Department of Agriculture's Super Dump cleanup efforts in the early 1990s, various sites in the country were targeted for cleanup. Three of the targeted sites—River X, River Y, and River Z—had become contaminated with pesticides because they were located near abandoned pesticide dump sites. Measurements of the concentration of aldrin (a commonly used pesticide) were taken at twenty randomly selected locations in each river near the dump sites.

The boxplots shown below display the five-number summaries for the concentrations, in parts per million (ppm) of aldrin, for the twenty locations that were sampled in each of the three rivers.



- (a) Compare the distributions of the concentration of aldrin among the three rivers.
- (b) The twenty concentrations of aldrin for River X are given below.

Construct a stemplot that displays the concentrations of aldrin for River X.

(c) Describe a characteristic of the distribution of aldrin concentrations in River X that can be seen in the stemplot but cannot be seen in the boxplot.

2. In response to nutrition concerns raised last year about food served in school cafeterias, the Smallville School District entered into a one-year contract with the Healthy Alternative Meals (HAM) company. Under this contract, the company plans and prepares meals for 2,500 elementary, middle, and high school students, with a focus on good nutrition. The school administration would like to survey the students in the district to estimate the proportion of students who are satisfied with the food under this contract.

Two sampling plans for selecting the students to be surveyed are under consideration by the administration. One plan is to take a simple random sample of students in the district and then survey those students. The other plan is to take a stratified random sample of students in the district and then survey those students.

- (a) Describe a simple random sampling procedure that the administrators could use to select 200 students from the 2,500 students in the district.
- (b) If a stratified random sampling procedure is used, give one example of an effective variable on which to stratify in this survey. Explain your reasoning.
- (c) Describe one statistical advantage of using a stratified random sample over a simple random sample in the context of this study.
- 3. A test consisting of 25 multiple-choice questions with 5 answer choices for each question is administered. For each question, there is only 1 correct answer.
  - (a) Let *X* be the number of correct answers if a student guesses randomly from the 5 choices for each of the 25 questions. What is the probability distribution of *X*?

This test, like many multiple-choice tests, is scored using a penalty for guessing. The test score is determined by awarding 1 point for each question answered correctly, deducting 0.25 point for each question answered incorrectly, and ignoring any question that is omitted. That is, the test score is calculated using the following formula.

Score =  $(1 \times \text{number of correct answers}) - (0.25 \times \text{number of incorrect answers}) + (0 \times \text{number of omits})$ 

For example, the score for a student who answers 17 questions correctly, answers 3 questions incorrectly, and omits 5 questions is

Score = 
$$(1 \times 17) - (0.25 \times 3) + (0 \times 5) = 16.25$$
.

- (b) Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. Show that the expected value of the student's score is 18 when using the scoring formula above.
- (c) A score of at least 20 is needed to pass the test. Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. What is the probability that the student will pass the test?

- 4. A husband and wife, Mike and Lori, share a digital music player that has a feature that randomly selects which song to play. A total of 2,384 songs were loaded onto the player, some by Mike and the rest by Lori. Suppose that when the player was in the random-selection mode, 13 of the first 50 songs selected were songs loaded by Lori.
  - (a) Construct and interpret a 90 percent confidence interval for the proportion of songs on the player that were loaded by Lori.
  - (b) Mike and Lori are unsure about whether the player samples the songs with replacement or without replacement when the player is in random-selection mode. Explain why this distinction is not important for the construction of the interval in part (a).
- 5. An advertising agency in a large city is conducting a survey of adults to investigate whether there is an association between highest level of educational achievement and primary source for news. The company takes a random sample of 2,500 adults in the city. The results are shown in the table below.

|                            | HIGHEST LEV                    | HIGHEST LEVEL OF EDUCATIONAL ACHIEVEMENT            |                  |       |  |  |  |  |  |  |
|----------------------------|--------------------------------|---|------------------|-------|--|--|--|--|--|--|
| Primary Source<br>for News | Not High<br>School<br>Graduate | High School Graduate<br>But Not College<br>Graduate | College Graduate | Total |  |  |  |  |  |  |
| Newspapers                 | 49                             | 205   | 188              | 442   |  |  |  |  |  |  |
| Local television           | 90                             | 170   | 75               | 335   |  |  |  |  |  |  |
| Cable television           | 113                            | 496   | 147              | 756   |  |  |  |  |  |  |
| Internet                   | 41                             | 401   | 245              | 687   |  |  |  |  |  |  |
| None                       | 77                             | 165   | 38               | 280   |  |  |  |  |  |  |
| Total                      | 370                            | 1,437   | 693              | 2,500 |  |  |  |  |  |  |

- (a) If an adult is to be selected at random from this sample, what is the probability that the selected adult is a college graduate or obtains news primarily from the internet?
- (b) If an adult who is a college graduate is to be selected at random from this sample, what is the probability that the selected adult obtains news primarily from the internet?
- (c) When selecting an adult at random from the sample of 2,500 adults, are the events "is a college graduate" and "obtains news primarily from the internet" independent? Justify your answer.
- (d) The company wants to conduct a statistical test to investigate whether there is an association between educational achievement and primary source for news for adults in the city. What is the name of the statistical test that should be used?

What are the appropriate degrees of freedom for this test?

## STATISTICS SECTION II Part B

#### **Question 6**

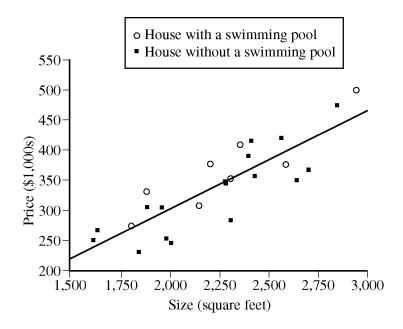
# Spend about 25 minutes on this part of the exam. Percent of Section II score—25

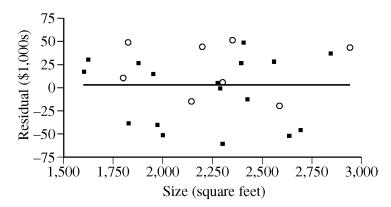
**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A real estate agent is interested in developing a model to estimate the prices of houses in a particular part of a large city. She takes a random sample of 25 recent sales and, for each house, records the price (in thousands of dollars), the size of the house (in square feet), and whether or not the house has a swimming pool. This information, along with regression output for a linear model using size to predict price, is shown below and on the next page.

| Price      | Size          | Dool | Residual   |
|------------|---------------|------|------------|
| (\$1,000s) | (square feet) | Pool | (\$1,000s) |
| 274        | 1,799         | yes  | 6          |
| 330        | 1,875         | yes  | 49         |
| 307        | 2,145         | yes  | -18        |
| 376        | 2,200         | yes  | 42         |
| 352        | 2,300         | yes  | 1          |
| 409        | 2,350         | yes  | 50         |
| 375        | 2,589         | yes  | -23        |
| 498        | 2,943         | yes  | 42         |
| 248        | 1,600         | no   | 13         |
| 265        | 1,623         | no   | 26         |
| 228        | 1,829         | no   | -45        |
| 303        | 1,875         | no   | 22         |
| 303        | 1,950         | no   | 10         |
| 251        | 1,975         | no   | -46        |
| 244        | 2,000         | no   | -57        |
| 347        | 2,274         | no   | 1          |
| 345        | 2,279         | no   | -2         |
| 282        | 2,300         | no   | -69        |
| 389        | 2,392         | no   | 23         |
| 413        | 2,410         | no   | 44         |
| 353        | 2,428         | no   | -19        |
| 419        | 2,560         | no   | 26         |
| 348        | 2,639         | no   | -58        |
| 365        | 2,701         | no   | -52        |
| 474        | 2,849         | no   | 33         |

© 2010 The College Board. Visit the College Board on the Web: www.collegeboard.com.





| Linear Fit Price = -28.144 + 0.165 Size          |                              |                               |                          |                              |  |  |  |  |  |  |
|--|------------------------------|-------------------------------|--------------------------|------------------------------|--|--|--|--|--|--|
| Summary of Fit RSquare 0.722 Parameter Estimates |                              |                               |                          |                              |  |  |  |  |  |  |
| Term Intercept Size                              | Estimate<br>-28.144<br>0.165 | Std Error<br>48.259<br>0.0213 | t Ratio<br>-0.58<br>7.72 | Prob> t <br>0.5654<br><.0001 |  |  |  |  |  |  |

(a) Interpret the slope of the least squares regression line in the context of the study.

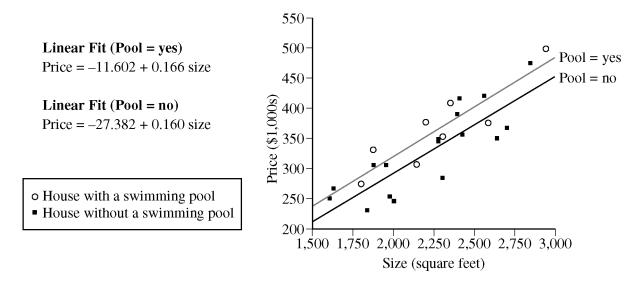
 $\ \$  © 2010 The College Board. Visit the College Board on the Web: www.collegeboard.com.

(b) The second house in the table has a residual of 49. Interpret this residual value in the context of the study.

The real estate agent is interested in investigating the effect of having a swimming pool on the price of a house.

(c) Use the residuals from all 25 houses to estimate how much greater the price for a house with a swimming pool would be, on average, than the price for a house of the same size without a swimming pool.

To further investigate the effect of having a swimming pool on the price of a house, the real estate agent creates two regression models, one for houses with a swimming pool and one for houses without a swimming pool. Regression output for these two models is shown below.



- (d) The conditions for inference have been checked and verified, and a 95 percent confidence interval for the true difference in the two slopes is (-0.099, 0.110). Based on this interval, is there a significant difference in the two slopes? Explain your answer.
- (e) Use the regression model for houses with a swimming pool and the regression model for houses without a swimming pool to estimate how much greater the price for a house with a swimming pool would be than the price for a house of the same size without a swimming pool. How does this estimate compare with your result from part (c)?

#### **STOP**

#### **END OF EXAM**

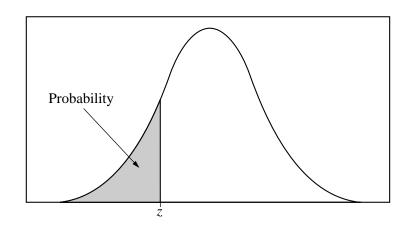


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

| z              | .00            | .01   | .02            | .03   | .04   | .05            | .06   | .07   | .08   | .09   |
|----------------|----------------|-------|----------------|-------|-------|----------------|-------|-------|-------|-------|
| -3.4           | .0003          | .0003 | .0003          | .0003 | .0003 | .0003          | .0003 | .0003 | .0003 | .0002 |
| -3.3           | .0005          | .0005 | .0005          | .0004 | .0004 | .0004          | .0004 | .0004 | .0004 | .0003 |
| -3.2           | .0007          | .0007 | .0006          | .0006 | .0006 | .0006          | .0006 | .0005 | .0005 | .0005 |
| -3.1           | .0010          | .0009 | .0009          | .0009 | .0008 | .0008          | .0008 | .0008 | .0007 | .0007 |
| -3.0           | .0013          | .0013 | .0013          | .0012 | .0012 | .0011          | .0011 | .0011 | .0010 | .0010 |
| -2.9           | .0019          | .0018 | .0018          | .0017 | .0016 | .0016          | .0015 | .0015 | .0014 | .0014 |
| -2.8           | .0026          | .0025 | .0024          | .0023 | .0023 | .0022          | .0021 | .0021 | .0020 | .0019 |
| -2.7           | .0035          | .0034 | .0033          | .0032 | .0031 | .0030          | .0029 | .0028 | .0027 | .0026 |
| -2.6           | .0047          | .0045 | .0044          | .0043 | .0041 | .0040          | .0039 | .0038 | .0037 | .0036 |
| -2.5           | .0062          | .0060 | .0059          | .0057 | .0055 | .0054          | .0052 | .0051 | .0049 | .0048 |
| -2.4           | .0082          | .0080 | .0078          | .0075 | .0073 | .0071          | .0069 | .0068 | .0066 | .0064 |
| -2.3           | .0107          | .0104 | .0102          | .0099 | .0096 | .0094          | .0091 | .0089 | .0087 | .0084 |
| -2.2           | .0139          | .0136 | .0132          | .0129 | .0125 | .0122          | .0119 | .0116 | .0113 | .0110 |
| -2.1           | .0179          | .0174 | .0170          | .0166 | .0162 | .0158          | .0154 | .0150 | .0146 | .0143 |
| -2.0           | .0228          | .0222 | .0217          | .0212 | .0207 | .0202          | .0197 | .0192 | .0188 | .0183 |
| -1.9           | .0287          | .0281 | .0274          | .0268 | .0262 | .0256          | .0250 | .0244 | .0239 | .0233 |
| -1.8           | .0359          | .0351 | .0344          | .0336 | .0329 | .0322          | .0314 | .0307 | .0301 | .0294 |
| -1.7           | .0446          | .0436 | .0427          | .0418 | .0409 | .0401          | .0392 | .0384 | .0375 | .0367 |
| -1.6           | .0548          | .0537 | .0526          | .0516 | .0505 | .0495          | .0485 | .0475 | .0465 | .0455 |
| -1.5           | .0668          | .0655 | .0643          | .0630 | .0618 | .0606          | .0594 | .0582 | .0571 | .0559 |
| -1.4           | .0808          | .0793 | .0778          | .0764 | .0749 | .0735          | .0721 | .0708 | .0694 | .0681 |
| -1.3           | .0968          | .0951 | .0934          | .0918 | .0901 | .0885          | .0869 | .0853 | .0838 | .0823 |
| -1.2           | .1151          | .1131 | .1112          | .1093 | .1075 | .1056          | .1038 | .1020 | .1003 | .0985 |
| -1.1           | .1357          | .1335 | .1314          | .1292 | .1271 | .1251          | .1230 | .1210 | .1190 | .1170 |
| -1.0           | .1587          | .1562 | .1539          | .1515 | .1492 | .1469          | .1446 | .1423 | .1401 | .1379 |
| -0.9           | .1841          | .1814 | .1788          | .1762 | .1736 | .1711          | .1685 | .1660 | .1635 | .1611 |
| -0.8           | .2119          | .2090 | .2061          | .2033 | .2005 | .1977          | .1949 | .1922 | .1894 | .1867 |
| -0.7           | .2420          | .2389 | .2358          | .2327 | .2296 | .2266          | .2236 | .2206 | .2177 | .2148 |
| -0.6           | .2743          | .2709 | .2676          | .2643 | .2611 | .2578          | .2546 | .2514 | .2483 | .2451 |
| -0.5           | .3085          | .3050 | .3015          | .2981 | .2946 | .2912<br>.3264 | .2877 | .2843 | .2810 | .2776 |
| -0.4           | .3446          | .3409 | .3372          | .3336 | .3300 |                | .3228 | .3192 | .3156 | .3121 |
| -0.3           | .3821          | .3783 | .3745          | .3707 | .3669 | .3632          | .3594 | .3557 | .3520 | .3483 |
| -0.2           | .4207<br>.4602 | .4168 | .4129<br>.4522 | .4090 | .4052 | .4013          | .3974 | .3936 | .3897 | .3859 |
| $-0.1 \\ -0.0$ |                | .4562 |                | .4483 | .4443 | .4404          | .4364 | .4325 | .4286 | .4247 |
| -0.0           | .5000          | .4960 | .4920          | .4880 | .4840 | .4801          | .4761 | .4721 | .4681 | .4641 |

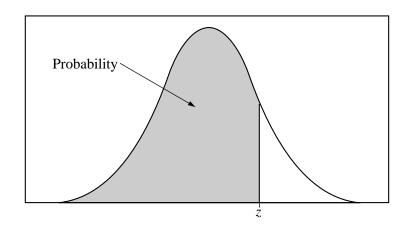


Table entry for z is the probability lying below z.

Table A (Continued)

| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

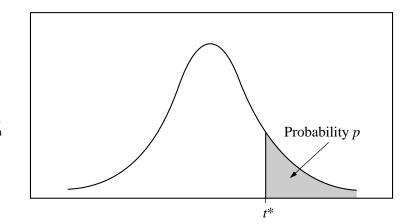


Table entry for p and C is the point  $t^*$  with probability p lying above it and probability C lying between  $-t^*$  and  $t^*$ .

Table B t distribution critical values

|          | Tail probability p |              |                |                |                |                |                |       |       |       |                |       |
|----------|--------------------|--------------|----------------|----------------|----------------|----------------|----------------|-------|-------|-------|----------------|-------|
| df       | .25                | .20          | .15            | .10            | .05            | .025           | .02            | .01   | .005  | .0025 | .001           | .0005 |
| 1        | 1.000              | 1.376        | 1.963          | 3.078          | 6.314          | 12.71          | 15.89          | 31.82 | 63.66 | 127.3 | 318.3          | 636.6 |
| 2        | .816               | 1.061        | 1.386          | 1.886          | 2.920          | 4.303          | 4.849          | 6.965 | 9.925 | 14.09 | 22.33          | 31.60 |
| 3        | .765               | .978         | 1.250          | 1.638          | 2.353          | 3.182          | 3.482          | 4.541 | 5.841 | 7.453 | 10.21          | 12.92 |
| 4        | .741               | .941         | 1.190          | 1.533          | 2.132          | 2.776          | 2.999          | 3.747 | 4.604 | 5.598 | 7.173          | 8.610 |
| 5        | .727               | .920         | 1.156          | 1.476          | 2.015          | 2.571          | 2.757          | 3.365 | 4.032 | 4.773 | 5.893          | 6.869 |
| 6        | .718               | .906         | 1.134          | 1.440          | 1.943          | 2.447          | 2.612          | 3.143 | 3.707 | 4.317 | 5.208          | 5.959 |
| 7        | .711               | .896         | 1.119          | 1.415          | 1.895          | 2.365          | 2.517          | 2.998 | 3.499 | 4.029 | 4.785          | 5.408 |
| 8        | .706               | .889         | 1.108          | 1.397          | 1.860          | 2.306          | 2.449          | 2.896 | 3.355 | 3.833 | 4.501          | 5.041 |
| 9        | .703               | .883         | 1.100          | 1.383          | 1.833          | 2.262          | 2.398          | 2.821 | 3.250 | 3.690 | 4.297          | 4.781 |
| 10       | .700               | .879         | 1.093          | 1.372          | 1.812          | 2.228          | 2.359          | 2.764 | 3.169 | 3.581 | 4.144          | 4.587 |
| 11       | .697               | .876         | 1.088          | 1.363          | 1.796          | 2.201          | 2.328          | 2.718 | 3.106 | 3.497 | 4.025          | 4.437 |
| 12       | .695               | .873         | 1.083          | 1.356          | 1.782          | 2.179          | 2.303          | 2.681 | 3.055 | 3.428 | 3.930          | 4.318 |
| 13       | .694               | .870         | 1.079          | 1.350          | 1.771          | 2.160          | 2.282          | 2.650 | 3.012 | 3.372 | 3.852          | 4.221 |
| 14       | .692               | .868         | 1.076          | 1.345          | 1.761          | 2.145          | 2.264          | 2.624 | 2.977 | 3.326 | 3.787          | 4.140 |
| 15       | .691               | .866         | 1.074          | 1.341          | 1.753          | 2.131          | 2.249          | 2.602 | 2.947 | 3.286 | 3.733          | 4.073 |
| 16       | .690               | .865         | 1.071          | 1.337          | 1.746          | 2.120          | 2.235          | 2.583 | 2.921 | 3.252 | 3.686          | 4.015 |
| 17       | .689               | .863         | 1.069          | 1.333          | 1.740          | 2.110          | 2.224          | 2.567 | 2.898 | 3.222 | 3.646          | 3.965 |
| 18       | .688               | .862         | 1.067          | 1.330          | 1.734          | 2.101          | 2.214          | 2.552 | 2.878 | 3.197 | 3.611          | 3.922 |
| 19       | .688               | .861         | 1.066          | 1.328          | 1.729          | 2.093          | 2.205          | 2.539 | 2.861 | 3.174 | 3.579          | 3.883 |
| 20       | .687               | .860         | 1.064          | 1.325          | 1.725          | 2.086          | 2.197          | 2.528 | 2.845 | 3.153 | 3.552          | 3.850 |
| 21       | .686               | .859         | 1.063          | 1.323          | 1.721          | 2.080          | 2.189          | 2.518 | 2.831 | 3.135 | 3.527          | 3.819 |
| 22       | .686               | .858         | 1.061          | 1.321          | 1.717          | 2.074          | 2.183          | 2.508 | 2.819 | 3.119 | 3.505          | 3.792 |
| 23       | .685               | .858         | 1.060          | 1.319          | 1.714          | 2.069          | 2.177          | 2.500 | 2.807 | 3.104 | 3.485          | 3.768 |
| 24       | .685               | .857         | 1.059          | 1.318          | 1.711          | 2.064          | 2.172          | 2.492 | 2.797 | 3.091 | 3.467          | 3.745 |
| 25       | .684               | .856         | 1.058          | 1.316          | 1.708          | 2.060          | 2.167          | 2.485 | 2.787 | 3.078 | 3.450          | 3.725 |
| 26       | .684               | .856         | 1.058          | 1.315          | 1.706          | 2.056          | 2.162          | 2.479 | 2.779 | 3.067 | 3.435          | 3.707 |
| 27       | .684               | .855         | 1.057          | 1.314          | 1.703          | 2.052          | 2.158          | 2.473 | 2.771 | 3.057 | 3.421          | 3.690 |
| 28       | .683               | .855         | 1.056          | 1.313          | 1.701          | 2.048          | 2.154          | 2.467 | 2.763 | 3.047 | 3.408          | 3.674 |
| 29       | .683               | .854         | 1.055          | 1.311          | 1.699          | 2.045          | 2.150          | 2.462 | 2.756 | 3.038 | 3.396          | 3.659 |
| 30       | .683               | .854         | 1.055<br>1.050 | 1.310          | 1.697          | 2.042          | 2.147          | 2.457 | 2.750 | 3.030 | 3.385          | 3.646 |
| 40       | .681               | .851         |                | 1.303          | 1.684          | 2.021          | 2.123          | 2.423 | 2.704 | 2.971 | 3.307          | 3.551 |
| 50       | .679<br>.679       | .849         | 1.047          | 1.299          | 1.676          | 2.009          | 2.109          | 2.403 | 2.678 | 2.937 | 3.261          | 3.496 |
| 60<br>80 |                    | .848         | 1.045<br>1.043 | 1.296          | 1.671          | 2.000          | 2.099          | 2.390 | 2.660 | 2.915 | 3.232          | 3.460 |
|          | .678               | .846         |                | 1.292          | 1.664          | 1.990          | 2.088          | 2.374 | 2.639 | 2.887 | 3.195          | 3.416 |
| 100      | .677               | .845         | 1.042          | 1.290          | 1.660          | 1.984          | 2.081          | 2.364 | 2.626 | 2.871 | 3.174          | 3.390 |
| 1000     | .675<br>.674       | .842<br>.841 | 1.037<br>1.036 | 1.282<br>1.282 | 1.646<br>1.645 | 1.962<br>1.960 | 2.056<br>2.054 | 2.330 | 2.581 | 2.813 | 3.098<br>3.091 | 3.300 |
| 8        | .074               | .841         | 1.030          | 1.282          | 1.043          | 1.900          | 2.034          | 2.326 | 2.576 | 2.807 | 3.091          | 3.291 |
|          | 50%                | 60%          | 70%            | 80%            | 90%            | 95%            | 96%            | 98%   | 99%   | 99.5% | 99.8%          | 99.9% |

Confidence level C

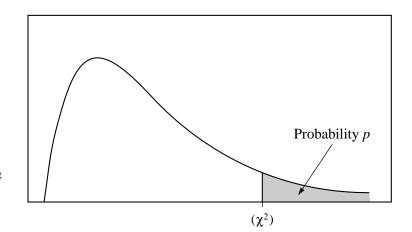


Table entry for p is the point  $(\chi^2)$  with probability p lying above it.

**Table C**  $\chi^2$  critical values

| -   | Tail probability p |       |       |       |       |       |       |       |       |       |       |       |
|-----|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| df  | .25                | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1   | 1.32               | 1.64  | 2.07  | 2.71  | 3.84  | 5.02  | 5.41  | 6.63  | 7.88  | 9.14  | 10.83 | 12.12 |
| 2   | 2.77               | 3.22  | 3.79  | 4.61  | 5.99  | 7.38  | 7.82  | 9.21  | 10.60 | 11.98 | 13.82 | 15.20 |
| 3   | 4.11               | 4.64  | 5.32  | 6.25  | 7.81  | 9.35  | 9.84  | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4   | 5.39               | 5.99  | 6.74  | 7.78  | 9.49  | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5   | 6.63               | 7.29  | 8.12  | 9.24  | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6   | 7.84               | 8.56  | 9.45  | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7   | 9.04               | 9.80  | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8   | 10.22              | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9   | 11.39              | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10  | 12.55              | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11  | 13.70              | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12  | 14.85              | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13  | 15.98              | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14  | 17.12              | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15  | 18.25              | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16  | 19.37              | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17  | 20.49              | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18  | 21.60              | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19  | 22.72              | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20  | 23.83              | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21  | 24.93              | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22  | 26.04              | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23  | 27.14              | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24  | 28.24              | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25  | 29.34              | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26  | 30.43              | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27  | 31.53              | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28  | 32.62              | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29  | 33.71              | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30  | 34.80              | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40  | 45.62              | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50  | 56.33              | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60  | 66.98              | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80  | 88.13              | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1              | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |