CS-7641 Problem Set 1

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Problem 2 - Design a two-input perceptron that implements the boolean function $A \wedge \sim B$. Design a two-layer network of perceptrons that implements $A \oplus B$ ($\oplus$ is XOR).

1. $A \wedge \sim B$
Equation for two-input perceptron = w0 + w1A + w2B
input1 = A
input2 = B
w0,w1,w2 = -0.5, 1, -1
1 == True
-1 == False

Table 1: $A \wedge \sim B$

A	B	w0 + w1A + w2B	Result
1	1	-0.5	-1
1	-1	1.5	1
-1	1	-2.5	-1
-1	-1	-0.5	-1

2. $A \oplus B$
input1 = A
input2 = B
Hidden Layer 1: w0, w1, w2 = -0.5, 1, -1
Hidden Layer 2: w0, w1, w2 = -0.5, -1, 1
Result(Use output of hiddenlayers): w0, w1, w2 = 0.5, 1, 1
1 == True
-1 == False

Table 2: $A \oplus B$

A	B	HiddenLayer1 Result	HiddenLayer2 Result	Result
1	1	-1	-1	-1
1	-1	1	-1	1
-1	1	-1	1	1
-1	-1	-1	-1	-1

Problem 6 - Imagine you had a learning problem with an instance space of points on the plane and a target function that you knew took the form of a line on the plane where all points on one side of the line are positive and all those on the other are negative. If you were constrained to only use decision tree or nearest-neighbor learning, which would you use? Why?

A decision tree would be best to use in this learning problem because decision trees, by nature, separate instance spaces linearly. For example a decision tree may split elements with the rule x>0. On a 2D plane this would mean all points on the right of the y axis would be positive and all of the points on the left would be negative. Nearest neighbors model, by nature, do not separate instance spaces linearly. Nearest neighbor boundaries are more complex and based on proximity to other points. Therefore a decision tree would be best suited to represent a target function that linearly separates points on a plane.

Problem 7 - Give the VC dimension of the following hypothesis spaces. Briefly explain your answers.
1. An origin-centered circle (2D)
2. An origin-centered sphere (3D)

The VC dimension of both an origin-centered circle and an origin-centered sphere is 2. Shown below:

VC 1) + or - :a point can be classified + or - just by changing the radius of the circle or sphere to be greater or lesser than the radius of the point.

VC 2) ++ or - - : 2 points can be classified ++ or - - just by changing the radius of the circle or sphere to be greater than the greatest radius of the 2 points or lesser than the smaller radius of the 2 points.
-+ or +-: The circle and sphere can classify both points differently by having a radius that is greater than one point and lesser than the other point

VC 3) Set of 3 points (p1,p2,p3) with radii $r1 \leq r2 \leq r3$ labeled(+,-,+). It is impossible for a circle or sphere to separate the space in a way that would label p1 +, p2 -, and then p3 +.

Therefore the VC of both an origin-centered circle and an origin-centered sphere is 2