Methods in Computational Linguistics
Master of Science *Computational Linguistics*
Exercise: Probability and Information Theory

apl. Prof. Dr. Sabine Schulte im Walde
Institut für Maschinelle Sprachverarbeitung
Universität Stuttgart

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1. ***Probabilities of throwing a fair die***:

   Calculate probabilities for throwing a fair die four times.

   (a) What is the size of the sample space?

   (b) The event A is defined as *four identical numbers*. List all elements in A, and calculate the probability $P(A)$.

   (c) How many elements are in the complementary event A^c? Provide the total number of elements as well as two examples.

   (d) The event B is defined as *exactly three identical numbers*. How many elements are in B? Provide the total number of elements as well as two examples.

   (e) How many elements are in $C = A \cup B$ and in $D = A \cap B$?

2. ***Probabilities of pairs of fair dice numbers***:

   Calculate probabilities for the simultaneous throwing of two fair dice.

   One die is white, and the other die is red. The elementary events are pairs of dice numbers $\langle i, k \rangle$, with i the number of the white die and k the number of the red die.

   (a) How many pairs are in the sample space?

   (b) The event A is defined as all pairs whose sum is 5. List the pairs in A.

   (c) Assume that throwing the dice is a Laplace experiment, i.e., all pairs have the same probability. Calculate the probability that the sum of the pair numbers is 8.

3. ***Conditional probabilities of colourful balls***:

   A box contains 10 white, 5 yellow and 10 black balls. You randomly choose one ball, which is not black. What is the probability that the chosen ball is yellow?

   (a) Calculate the probability of the events "a ball is yellow" and "a ball is not black".

   (b) Calculate the conditional probability that the chosen ball is yellow, given that it is not black.

4. ***Conditional probabilities of VP structures***:

   Suggest probabilities for verb phrase structure rules.

   A grammar contains the following rules for verb phrase structures
   (V: verb; VP: verb phrase; NP: noun phrase):

$$\Omega = \{\ \text{VP} \rightarrow \text{V},\ \text{VP} \rightarrow \text{V NP},\ \text{VP} \rightarrow \text{V NP NP}\ \}.$$

   (a) Suggest intuitively plausible probabilities for the three VP rules for English and justify your choices.

   (b) If you lexicalise the grammar rules regarding specific verbs, this will result in different conditional probabilities. Come up with three English verbs for which you expect different conditional probabilities across the three VP rules. Suggest intuitively plausible conditional probabilities for the three verbs and the three lexicalised VP rules.

5. ***Properties of probability distributions***:

   You are given an unfair die with a steal plate at number 1. The probability distribution of the die is as follows.
   $p(\omega = 1) = 0.1$,
   $p(\omega = 6) = 0.3$,
   $p(\omega = 2) = p(\omega = 3) = p(\omega = 4) = p(\omega = 5) = 0.15$.

   Calculate the properties of the probability distribution: expectation, variance, and standard deviation.

6. ***Entropy***:

   You are given 16 colourful balls and randomly choose one of them.

   (a) Calculate the entropy for a uniform probability distribution, i.e., assuming that the probability for each ball being chosen is equal.

   (b) How does the entropy change if the probabilities for two of the balls are three times as high as the probabilities for the other 14 balls? Calculate the entropy and explain the difference.