

①

(a) size of sample space $= 6^4 = 1296$

(b) $A = \left\{ \begin{aligned} &\{1, 1, 1, 1\} \\ &\{2, 2, 2, 2\} \\ &\{3, 3, 3, 3\} \\ &\{4, 4, 4, 4\} \\ &\{5, 5, 5, 5\} \\ &\{6, 6, 6, 6\} \end{aligned} \right\}$

$$\therefore P(A) = \frac{6}{1296} = \frac{1}{216}$$

(c) $(1296 - 6) = 1290$ elements in A^c

(d) $B = 6 \times 5 = 30$
 $\rightarrow 6$ possibilities

\rightarrow since exactly 3, exclude the one with 4

(e) $n(A \cup B) = 30 + 6 = 36$
 $n(A \cap B) = 0$

(2)

(a) $6 \times 6 = 36$ pairs

(b) $A = \left\{ (1, 4), (2, 3), (3, 2), (4, 1) \right\}$

(c) $B = \{ \text{sum} = 8 \}$
 $= \left\{ (3, 5), (4, 4), (5, 3), (2, 6) \right\}$

$P(B) = \frac{1}{4}$ [since laplace experiment]

(3)

$w = 10, y = 5, b = 10$

(a) $P(y) = \frac{5}{25} = \frac{1}{5}$

$P(b^c) = 1 - \frac{10}{25} = 1 - \frac{2}{5} = \frac{3}{5}$

$$\begin{aligned}
 (6) \quad P(Y|b^c) &= \frac{P(Y \cap b^c)}{P(b^c)} \rightarrow P(Y) \\
 &= \frac{1 \times 5}{5 \times 3} = 1/3
 \end{aligned}$$

$$\begin{aligned}
 (5) \\
 E = \mu &= 1 \times 0.1 + 6 \times 0.3 + 0.15 \times (2+3+4+5) \\
 &= 2.1 + 0.1 + 1.8 \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \sigma^2 &= \frac{1}{4} \times (3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2) \\
 &= \frac{1}{4} (9 + 4 + 1 + 1 + 2) \\
 &= \frac{17}{4} = 4.25
 \end{aligned}$$

$$\sigma = \sqrt{17/4} = \frac{\sqrt{17}}{2} = 2.062$$

⑥

$$n = 16$$

$$\begin{aligned} (a) \quad H(x) &= -\frac{1}{16} \sum_{i=1}^{16} \log_2 \frac{1}{16} \\ &= -\log_2 2^{-4} \\ &= 4 \times 1 = 4 \end{aligned}$$

$$\begin{aligned} (b) \quad H(x) &= -\frac{1}{14} \sum_{i=1}^{14} \log_2 \frac{1}{14} - \frac{3}{14} \sum_{i=1}^2 \log_2 \frac{3}{14} \\ &= -\log_2 1/14 - \frac{3}{7} \log_2 3/14 \\ &= 4.76 \end{aligned}$$

Since probability has changed the entropy will change too and in this case the outcome will be more uncertain.

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