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Question - 01 :

Answer

Here,

$f(z) = \ln(1+z)$, where, $z = x^T x$, $x \in R^d$

If $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$ then $X^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix}$

$\therefore X^T X = \begin{bmatrix} x_1^2 + x_2^2 + \cdots + x_d^2 \end{bmatrix}$

Applying Chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\ln(1+z)\right) \frac{d}{dx}(X^T X)$$

$$= \frac{1}{1+z} \frac{d}{dz} z \cdot \frac{d}{dx}(x_1^2 + x_2^2 + \dots x_d^2)$$

$$= \frac{1}{1+z} \cdot (2x_1 + 2x_2 + \dots 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \dots x_d) = \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

Question - 02

Answer

$f(z) = e^{-z/2}$, where, $z = g(y)$

$$g(y) = y^T \cdot S^{-1} y$$

then $y = h(x) = x - \mu$

By applying chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Here,

$$\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{1}{2}e^{-z/2}$$

$$\frac{dz}{dy} = \frac{d}{dy}\left(g(y)\right)$$

$$= \frac{d}{dy}\left(y^T S^{-1} y\right)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) \cdot S^{-1}(y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T S^{-1} h + h S^{-1} y + h^2 S^{-1}}{h}$$

$$= \lim_{h \to 0} \frac{h(y^T S^{-1} + S^{-1} y + h S^{-1})}{h}$$

$$= \lim_{h \to 0} \cancel{h}\, y^T S^{-1} + S^{-1} y + h S^{-1}$$

$$= y^T S^{-1} + S^{-1} y + 0$$

$$= y^T S^{-1} + S^{-1} y$$

$$\frac{dy}{dx} = \frac{d}{dx}(x - \mu) = 1$$

$$\therefore \quad \frac{df}{dx} = -\frac{1}{2} e^{-z/2} \cdot (y^T S^{-1} + S^{-1} y)$$

$$= -\frac{1}{2} e^{-z/2} \cdot \frac{1}{S} (y^T + y)$$