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Analysis of the inverse kinematics for 5 DOF robot arm using D-H parameters

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***Abstract* - This paper proposes an algorithm to develop the kinematic model of a 5 DOF robot arm. The formulation of the problem is based on finding the D-H parameters of the arm. Brute Force iterative method is employed to solve the system of non linear equations. The focus of the paper is to obtain the accurate solutions by reducing the root mean square error. The result obtained will be implemented to grip the objects. The trajectories followed by the end effector for the required workspace coordinates are plotted. The methodology used here can be used in solving the problem for any other kinematic chain of up to six DOF.**

Keywords - 5 DOF robot arm; D-H parameters; Inverse kinematics; Iterative method; Trajectories

I. INTRODUCTION

This paper presents an approach to control the position of a 5 DOF robot arm using kinematics between rigid links. It is desirable to control the end effector in the Cartesian coordinate space but motor system needs the reference inputs in the joint space. Hence to know the conversion between Cartesian and joint space, inverse and forward kinematics are essential. Forward kinematics gives the location of the end effector for the given joint angles. Inverse kinematics is employed to reach the desired position of the end effector. Solving the system of equations for the joint angles will move the end effector to a particular position.

There are many solutions to solve the inverse kinematics problem, such as geometric, algebraic, and numerical hit and trial, iterative, FABRIC methods [1]. But it is hard to find a consistent solution using analytical method [2]. Hence often iterative methods are used [3]. One of the iterative methods uses the inverse Jacobian matrix approach. But it faces problems of singularity and computational complexity [4]. A closed-form joint solution is derived for a 6-DOF humanoid robot arm in [5]. The geometric method used in [6] uses geometric perception. The inverse transform technique is another method used in [7] to obtain the joint solution of a 6-DOF robot arm whereas the method discussed in [8] uses technique of minimization of error vector. In this paper a simple and efficient technique to develop the kinematic model of a robot manipulator is discussed.

II. TRANSFORMATION USING D-H PARAMETERS

In any robot arm the position and orientation of the base are known. To locate the end effector the relationship between the base frame and the end effector frame is found out by transformation. The frames are attached to all the links of the robot arm.

Z axis: along the joint axis
X axis: along common normal
Y axis: according to the RH frame

$^{(i-1)}T_i$ is the homogeneous transformation matrix between frame R_{i-1} and R_i which takes into account the rotation of the frame R_i w.r.t. frame R_{i-1} and the translation of the origin of frame R_i w.r.t origin of frame R_{i-1}. The transformation matrix $^{(i-1)}T_i$ is obtained by four parameters a_i,α_i,d_i and θ_i which are called as Denavit-Hatenberg (D-H) parameters.

a_i : distance (z_i,z_{i+1}) along x_i
α_i : angle (z_i,z_{i+1}) about x_i
d_i : distance (x_{i-1},x_i) along z_i
θ_i : angle (x_{i-1},x_i) about z_i

To attach the frames to the first and the last links, it is better to try to make $a_0 = a_n = \alpha_0 = \alpha_n = 0$ and to keep the constant parameters among θ_1, θ_n, d_1, d_n zero. This reduces the complexity. Thus, the following analysis makes use of this approach.

$^{i-1}T_i = R_x(\alpha_{i-1})D_x(a_{i-1})R_z(\theta_i)D_z(d_i)$
where,

$$R_z(\theta_i) = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_x(a_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\alpha_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & -c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The final transformation matrix from the base to the end effector is obtained as

$$T = {}^0T_1{}^1T_2{}^2T_3{}^3T_4...{}^{n-1}T_n \tag{2}$$

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

This matrix depends on the joint angles of the links between the base and the end effector.

$$T = f(\theta_i) \text{ where, } 1 \leq i \leq n \tag{4}$$

At the given configuration the encoder values of the joint angles are read and the transformation matrix T is calculated. The end effector configuration parameters are obtained from this matrix. These parameters are used to find the trajectories, the velocities and the acceleration.

III. METHODOLOGY TO SOLVE INVERSE KINEMATICS PROBLEM

Inverse kinematics can be solved by following two ways:

1. Analytical method: The solution is obtained with closed form of equations.
2. Numerical method: The solution is obtained with an iterative approximation method.

It is complex to solve the system of non linear equations having trigonometric functions analytically. Hence, the analysis carried out uses iterative numerical method.

The analysis includes the following steps:
1. Construct the line diagram of the arm with all the joints and links connecting the joints
2. Assign the frame to each of the joints and find the D-H parameters

3. Construct the transformation matrices between the successive 2 joints
4. Multiply all these matrices to form the total transformation matrix (symbolic) between the base and the end effector
5. Construct the (numeric) transformation matrix to the target point
6. Form the system of equations by equating these two matrices.
7. Solve the system of equations using iterative approximation method to get the values of θ_0, θ_1,...,θ_n
8. Find root mean sqaure error for each obtained solution using forward kinematics and select suitable solution accordingly

This methodology being quite generic can be used for any other kinematic chain of up to six DOF.

IV. KINEMATICS SOLUTION OF ARM

The schematic representation of kinematics of arm is as shown in fig. 1.

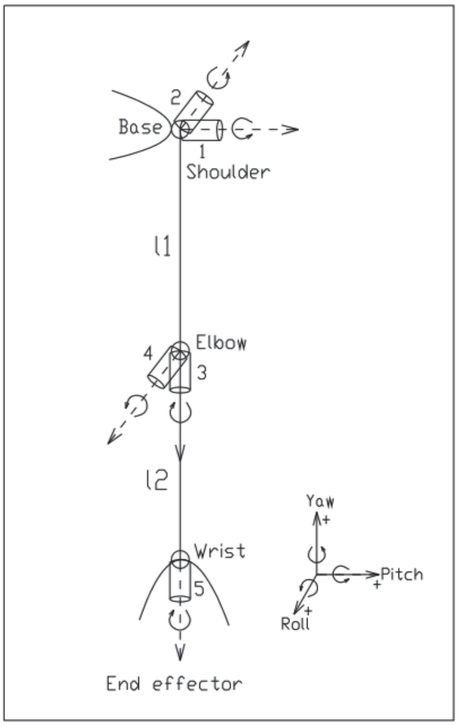

Figure 1: Kinematic diagram of Arm

The values of l_1 and l_2 are 28 cm and 30 cm respectively.
The CAD model of 5 DOF arm along with its different joints is as shown in fig. 2.

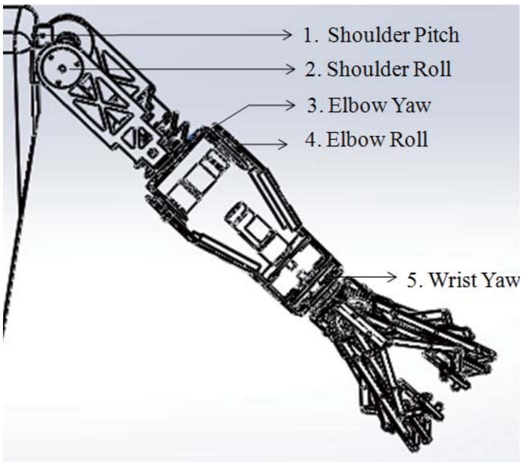

Figure 2: Arm Design

The frame attachment to 5 joints of the arm is as shown in fig. 3.

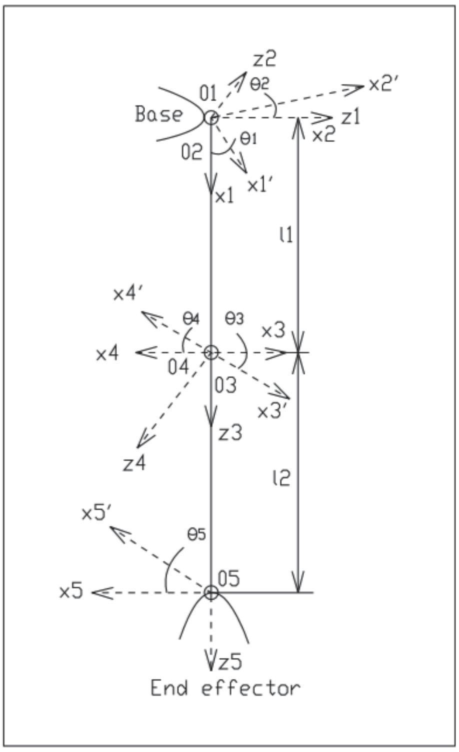

Figure 3: Frame Attachment of Arm

Frame 0 is aligned with frame 1 and frame of the gripper is considered to be aligned with frame 5.
The D-H parameters found using this coordinate system are as shown in table I.

TABLE I: D-H PARAMETERS OF ROBOT ARM

Joint	**Frame i**	a_i	α_i	d_i	θ_i
Base	Frame 0	0	0	–	–
Shoulder pitch	Frame 1	0	$-\pi/2$	0	θ_1
Shoulder roll	Frame 2	0	$\pi/2$	0	$-\theta_2+\pi/2$
Elbow yaw	Frame 3	0	$\pi/2$	l_1	θ_3
Elbow roll	Frame 4	0	$-\pi/2$	0	θ_4
Wrist yaw	Frame 5	0	0	l_2	θ_5

V. ANALYSIS OF FORWARD AND INVERSE KINEMATICS

These DH parameters are substituted in transformation matrix and LHS and RHS of equation(3) are equated using equations (1) and (2) to obtain:

$$\begin{aligned} n_x &= -c\theta_5 c\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_3 s\theta_2) - c\theta_1 s\theta_4 c\theta_2 - \\ &\quad s\theta_5(c\theta_3 s\theta_1 - c\theta_1 s\theta_2 s\theta_3) \\ n_y &= c\theta_5 c\theta_4(c\theta_1 s\theta_3 - s\theta_1 c\theta_3 s\theta_2) + s\theta_1 s\theta_4 c\theta_2 + \\ &\quad s\theta_5(c\theta_3 c\theta_1 + s\theta_1 s\theta_2 s\theta_3) \\ n_z &= c\theta_5(-s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4) + \\ &\quad s\theta_3 s\theta_5 c\theta_2 \end{aligned} \tag{5}$$

$$\begin{aligned} o_x &= s\theta_5(c\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_3 s\theta_2) - c\theta_1 s\theta_4 c\theta_2) - \\ &\quad c\theta_5(c\theta_3 s\theta_1 - c\theta_1 s\theta_2 s\theta_3) \\ o_y &= c\theta_5(c\theta_1 c\theta_3 + s\theta_2 s\theta_1 s\theta_3) - s\theta_5(c\theta_4(c\theta_1 s\theta_3 - \\ &\quad c\theta_3 s\theta_2 s\theta_1) + s\theta_1 s\theta_4 c\theta_2) \\ o_z &= c\theta_5 s\theta_3 c\theta_2 + s\theta_5(s\theta_2 s\theta_4 + c\theta_3 c\theta_4 c\theta_2) \end{aligned} \tag{6}$$

$$\begin{aligned} a_x &= s\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_4 c\theta_2 \\ a_y &= c\theta_4 s\theta_1 c\theta_2 - s\theta_4(c\theta_1 s\theta_3 - c\theta_3 s\theta_2) s\theta_1 \\ a_z &= -c\theta_4 s\theta_2 + c\theta_3 s\theta_4 c\theta_2 \end{aligned} \tag{7}$$

$$\begin{aligned} p_x &= 0.28c\theta_1 c\theta_2 + 0.3s\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_3 s\theta_2) + \\ &\quad 0.3c\theta_1 c\theta_4 c\theta_2 \\ p_y &= 0.28s\theta_1 c\theta_2 - 0.3s\theta_4(c\theta_1 s\theta_3 - c\theta_3) s\theta_2 s\theta_1 + \\ &\quad 0.3c\theta_4 s\theta_1 c\theta_2 \\ p_z &= -0.28s\theta_2 - 0.3c\theta_4 s\theta_2 + 0.3c\theta_3 s\theta_4 c\theta_2 \end{aligned} \tag{8}$$

The desired coordinates of the end effector p_x, p_y and p_z are substituted in (8). This set of equations is used to find θ_1, θ_2, θ_3 and θ_4.

The values of θ_1, θ_2, θ_3 and θ_4 are found using iterartive methd. The problem is to solve set of three equations for four unknowns θ_1, θ_2, θ_3 and θ_4.

The solution of position coordinates is obtained using brute force iterative method in embedded c and the algorithm followed is as shown in fig. 4. The complexity of the algorithm is N^4.

The variables funx, funy or funz represent difference between calculated value of px, py or pz and the desired (actual) value of px, py or pz. Thus, their values should be as close to zero as possible.

The root mean square error for each solution found is calculated and the appropriate solution is selected.

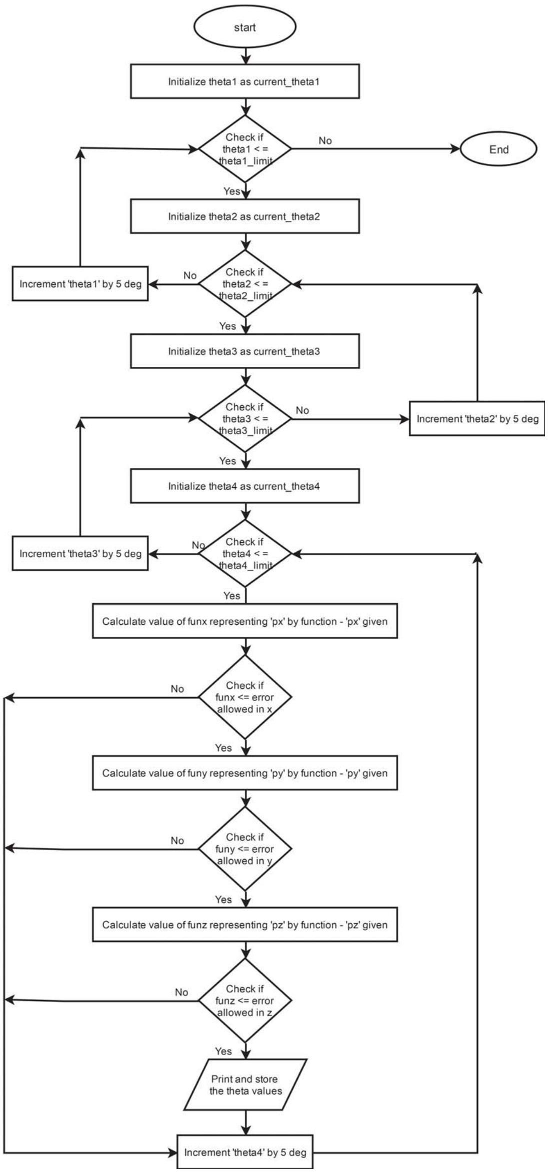

Figure 4: Flowchart of Algorithm

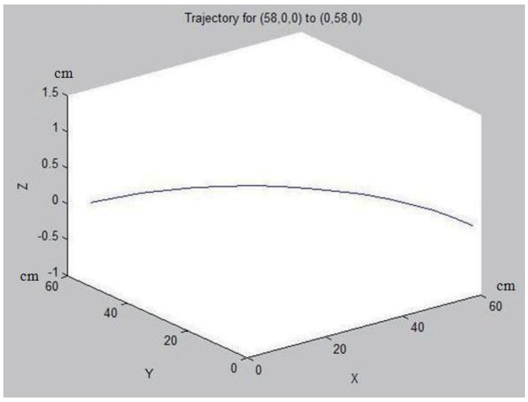

Figure 5: Graph 1

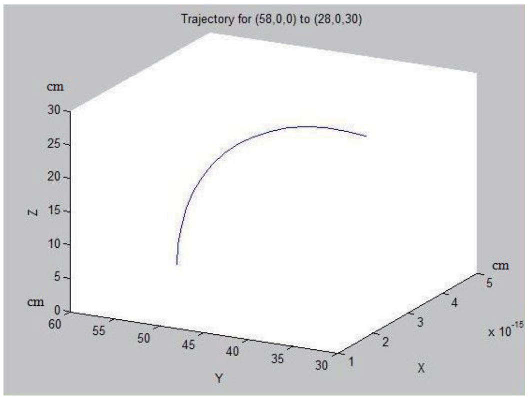

Figure 6: Graph 2

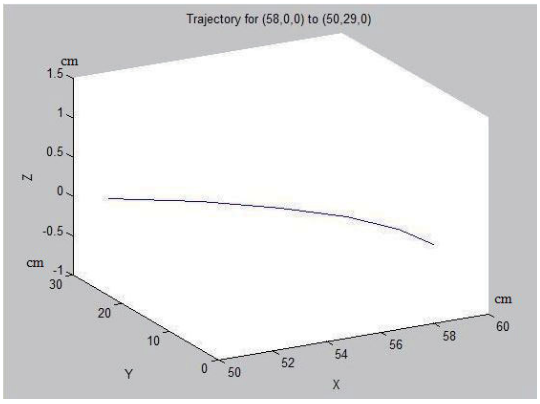

Figure 7: Graph 3

VI. RESULT AND ANALYSIS

The algorithm is tested for over 20 sample test cases and is found to be working efficiently.

Initially, θ_i=0 $\forall$ 1$\leq i \leq$ 5 and hence, the default position of the arm is (58,0,0). The three sample trajectories of end effector shown in fig. 5, 6 and 7 are for desired coordinates of end effector as
$(p_x, p_y, p_z) = (0{,}58{,}0)$, $(p_x, p_y, p_z) = (28{,}0{,}30)$ and $(p_x, p_y, p_z) = (50{,}29{,}0)$.

These solution trajectories are found in joint space. Here, target coordinates are known. Inverse kinematics is solved and coordinates of each joint are found. By joining these points to initial points, the trajectory is created.

VII. CONCLUSION

The kinematic model of a robot arm having 5 DOF has been developed in this paper. The kinematic analysis of the arm is carried out considering D-H parameters. A consistent joint solution for the arm is found using the inverse transform. The arm can be moved to the desired position by global planning using two shoulder and two elbow actuators. The local planning can be done by the fifth actuator fitted at the wrist which can be used to change the orientation of the arm. Finally, the trajectories followed by the end effector to attain some desired positions are plotted.

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