Stellar Clusters: The Forgotten Middle Child

Theory and Simulations

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Introduction

What is a stellar cluster?

- When stars are born they develop from large clouds of molecular gas, they form in groups or clusters as molecular clouds are composed of hundreds of solar masses of material.
- After the remnant gas is heated and blow away, the stars collect together by gravity.
- During the exchange of energy between the stars, some stars reach escape velocity from the protocluster and become runaway stars. The rest become gravitationally bound, meaning they will exist as a stellar cluster, orbiting the 'core' consisting of densely packed stars.

Types of Clusters

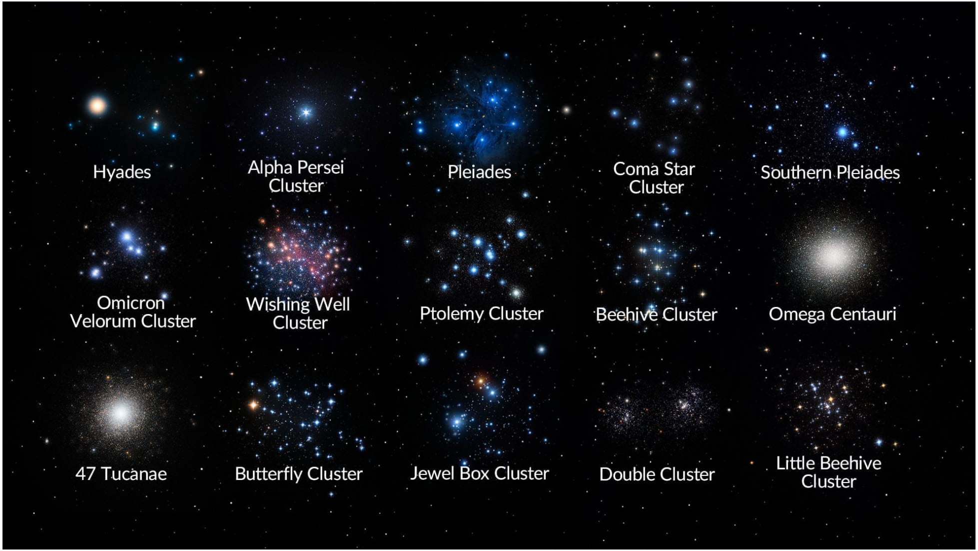
Open Clusters:

- When a cluster is young, the brightest members are O, B and A stars.
- Young clusters in our Galaxy are called open clusters due to their loose appearance.
- They usually contain between 100 and 1,000 members.
- Usually present in the spiral arms of the galaxy.

Globular Clusters:

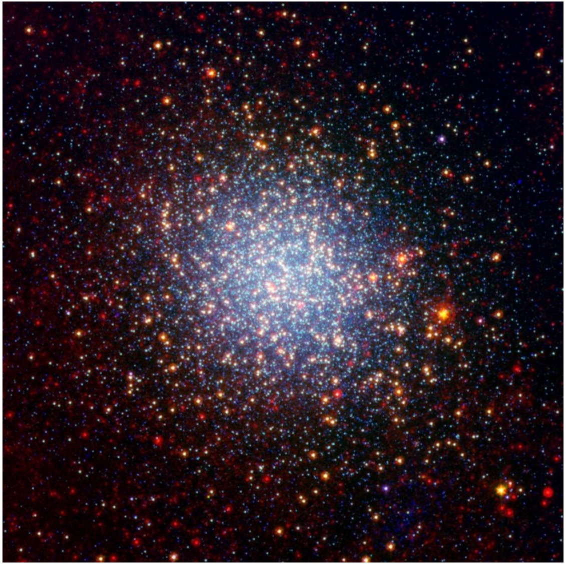
- Early in the formation of our Galaxy, very large, globular clusters formed from giant molecular clouds.
- Each contain over 10^4 members, appear very compact and have the oldest stars in the Universe.
- Usually present in the halo surrounding the galaxy.

Examples


Hyades
Alpha Persei Cluster
Pleiades
Coma Star Cluster
Southern Pleiades
Omicron Velorum Cluster
Wishing Well Cluster
Ptolemy Cluster
Beehive Cluster
Omega Centauri
47 Tucanae
Butterfly Cluster
Jewel Box Cluster
Double Cluster
Little Beehive Cluster

Pleidaes (open cluster)

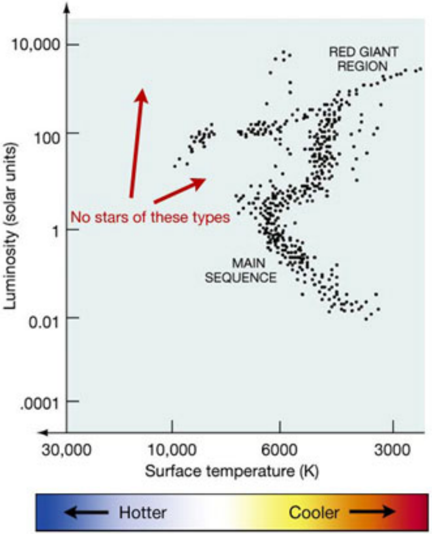
Omega Centauri (globular cluster)



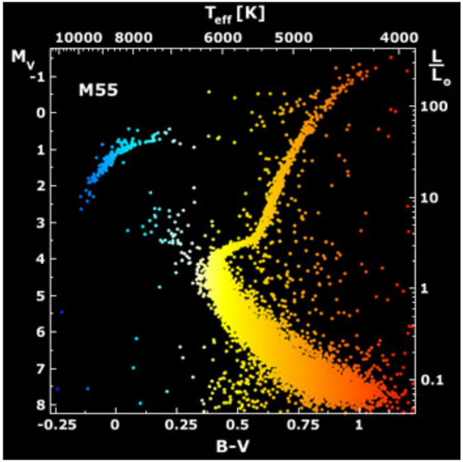
What do stellar clusters tell us?

- Stars in open clusters provide means to measure distances in space.
- Dynamical and evolutionary time scales $<$ or $<<$ than Galaxy's age, and a broad range of evolutionary states is present
- Great "laboratories" for stellar dynamics.
- Methods of determination of ages of various types of stars and their present stage of life.

HR Diagrams

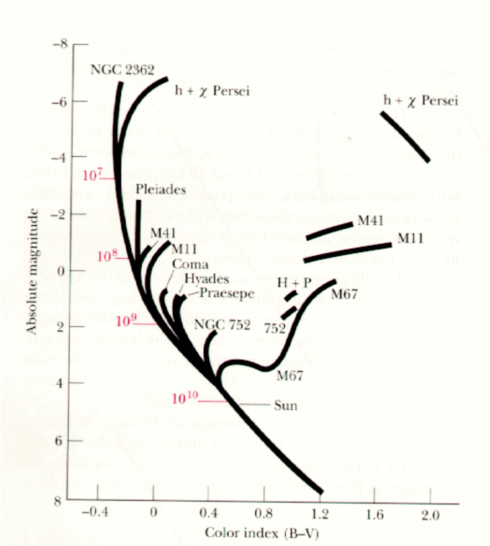


10,000
100
1
0.01
.0001
Luminosity (solar units)
RED GIANT REGION
No stars of these types
MAIN SEQUENCE
30,000
10,000
6000
3000
Surface temperature (K)
Hotter
Cooler



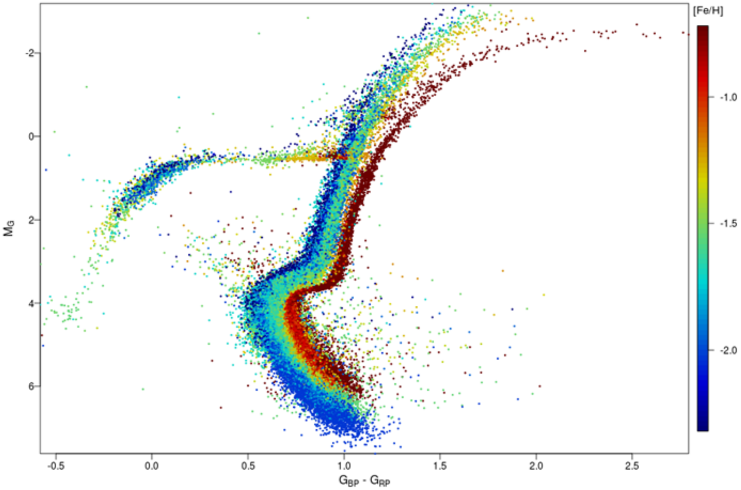
T_{eff} [K]
10000 8000 6000 5000 4000
M_V
M55
-1
0
1
2
3
4
5
6
7
8
$\frac{L}{L_\odot}$
100
10
1
0.1
-0.25 0 0.25 0.5 0.75 1
B-V

HR Diagrams

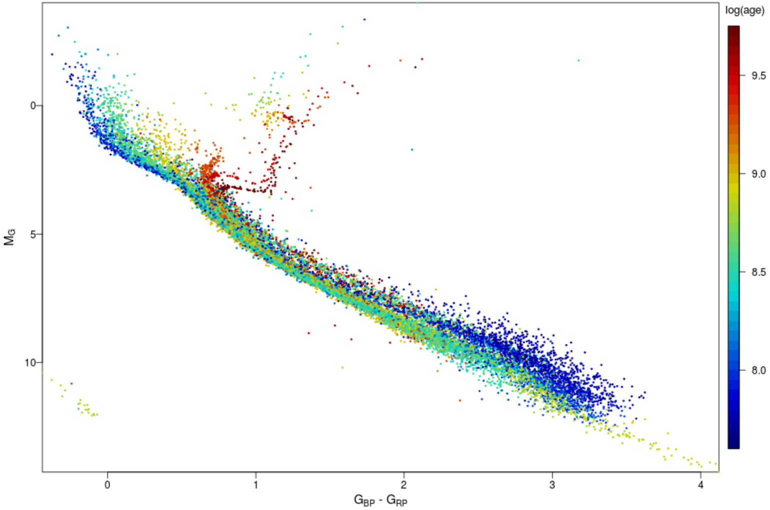


NGC 2362
h + χ Persei
h + χ Persei
Pleiades
M41
M11
Coma
Hyades
Praesepe
H + P
M67
M41
M11
NGC 752
752
M67
Sun
10^7
10^8
10^9
10^10
Absolute magnitude
−8
−6
−4
−2
0
2
4
6
8
−0.4
0
0.4
0.8
1.2
1.6
2.0
Color index (B–V)

HRD from Gaia DR2 - Globular Clusters

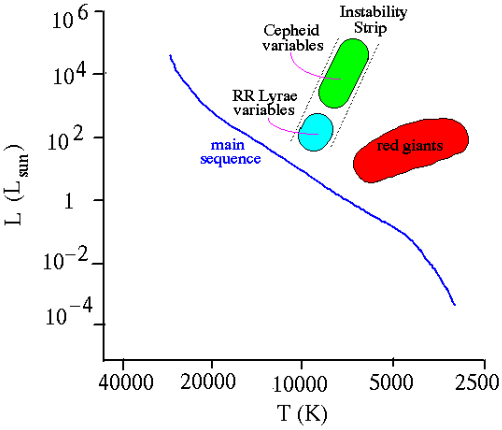


HRD from Gaia DR2 - Open Clusters

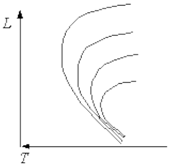
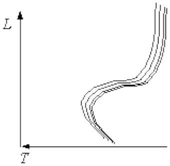


Instability Strip and Variable Stars

Two most famous types of variable stars: Cepheid variables and RR Lyrae variables.
Mechanism of variability: Pulsation.



Summary

	OPEN (GALACTIC) CLUSTERS	GLOBULAR CLUSTERS
Morphology	Loose, irregular collections of stars	Dense, spherically symmetric distribution of stars
Membership	$\sim 10^2$ stars, plus gas	$\sim 10^5 - 10^6$ stars, no gas
Distribution	restricted to galactic plane	roughly spherical distribution around Milky Way
H-R Diagram	 — all show main sequence — wide variety of turn-off points	 — all show short main sequence — all have very similar turn-off points

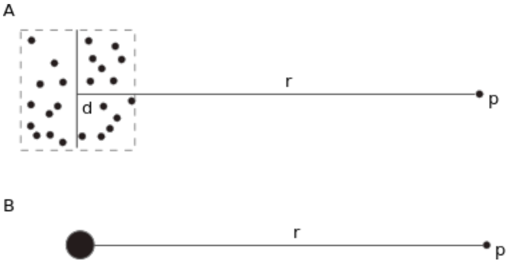
Computational Details: Barnes-Hut Algorithm

Brute Force vs Barnes-Hut Algorithm

- The algorithmic complexity of solving an N-body system through a brute-force approach is $\frac{N(N-1)}{2} \approx O(N^2)$
- Barnes-Hut Algorithm uses a tree-based approximation scheme which reduces the problem's computational complexity from $O(N^2)$ to $O(NlogN)$.
- The energy accuracy is high, with less than 1% change in total energy over a simulation for suitable time steps.

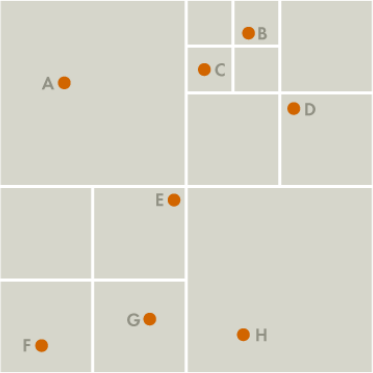
Barnes-Hut Algorithm

- The crucial idea in speeding up the force calculation is to group nearby bodies and approximate them as a single body.

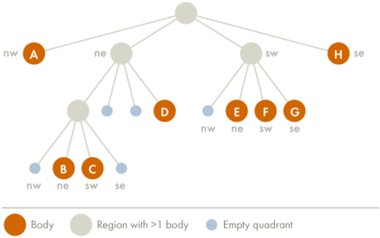


- This is accomplished by a data structure called a Quadtree/Octtree
- If $\frac{d}{r} < \theta$, consider a group of particles as a single body at CoM

Barnes-Hut Algorithm



(a) Space divided into quadrants



(b) Quad-tree

Figure: Arrangement of particles in Barnes-Hut Algorithm

Performance Analysis

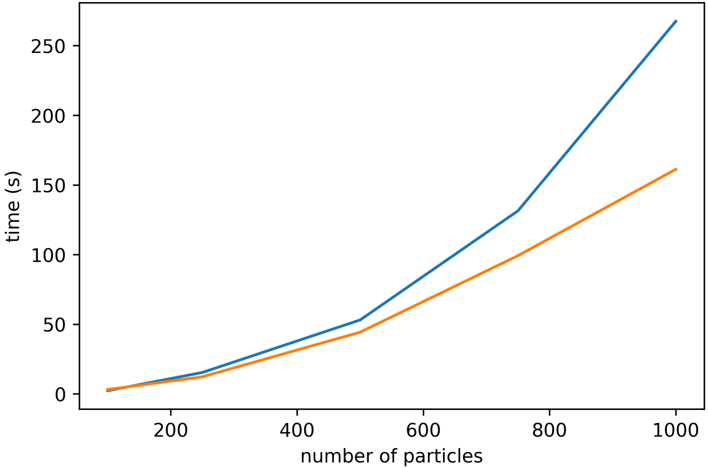


Figure: Barnes Hut (orange) vs Brute Force (blue)

Stellar Cluster Simulation

Plummer Model

The Plummer model uses the Schuster softened potential:

$$\Phi(r) = -GM \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} \tag{1}$$

Using Poisson's equation, we obtain the expression for the density distribution:

$$\rho(r) = \frac{M}{4\pi} \frac{3a^2}{(r^2 + a^2)^{\frac{5}{2}}} \tag{2}$$

Now, the mass distribution can be obtained by integrating the expression for density over concentric shells. Inverting this equation gives us the dependence of radius on cumulative mass:

$$r(m) = \frac{a}{\sqrt{(\frac{M}{m})^{\frac{2}{3}} - 1}} \tag{3}$$

Plummer model (continued)

Similarly, using energy conservation and the expression for the potential, we arrive at an equation expressing the max. velocity of a particle at radius r:

$$v_e(r) = \sqrt{2GM}(r^2 + a^2)^{\frac{-1}{4}} \tag{4}$$

We also use the expression for the phase space distribution of the Plummer model:

$$f(r, v)drdv = \frac{384\sqrt{2}}{7\pi m}(-E)^{\frac{7}{2}} r^2 v^2 drdv \tag{5}$$

We implement a rejection technique to assign velocities to each particle in the cluster. This completes our set of required initial conditions.

Some Parameters

We could use the following parameters for a typical globular cluster:

- Total Mass, m $= 3.977 \times 10^{35}$ kg (200,000 solar masses)
- Structural length, $a = 9.2 \times 10^{16}$ m (3 parsecs)
- No. of stars, $n = 10000$

But by my calculations, this system will take **20 hours** of computer time to simulate. So we will scale it down to the following:

- Total Mass, m $= 3.977 \times 10^{33}$ kg (2000 solar masses)
- Structural length, $a = 3.1 \times 10^{16}$ m (1 parsec)
- No. of stars, $n = 1000$

Latest Research/Cool Topics to look into

- Statistical studies of masses of stellar clusters
- Dissolution time, mass leakage
- Age distribution studies
- Binary formation leading to extended main sequence through mass transfer
- Dynamical mass segregation
- Intermediate mass black hole stirring up a stellar cluster - 47 Tucanae

Thank you!