Untitled

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1

Use integration by substitution to solve the integral below:

$$\int 4e\, -^{7x}\, dx$$

$$u = -7x$$
$$du = -7dx$$
$$dx = \frac{du}{-7}$$

$$\int 4e^u \frac{-1}{7} du$$
$$\frac{4}{-7} \int e^u du$$
$$\frac{4}{-7} e^u + C$$
$$\frac{4}{-7} e^{-7x} + C$$

2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of dNdt=−3150t4−220 bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter

$$\int (-3150t^4 - 220)dt$$
$$-630t^5 - 220t + C$$

$$N(t) = -630t^5 - 220t + C$$
$$6530 = -630(1)^5 - 220(1) + C$$
$$6530 = -630 - 220 + C$$
$$6530 = -850 + C$$
$$C = 7380$$

$$N(t) = -630t^5 - 220t + 7380$$

3

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

There are 4 rectangles in the plot of this function within the closed interval [4.5,8.5]. The width of each rectangle is 1. The height of each rectangle is the value of the function at the midpoint of the rectangle.

That leads us to get the following:

$$\int_{4.5}^{8.5} (2x - 9)\, dx$$
$$[x^2 - 9x]_{4.5}^{8.5}$$
$$(8.5^2 - 9(8.5)) - (4.5^2 - 9(4.5))$$
$$(72.25 - 76.5) - (20.25 - 40.5)$$
$$-4.25 - (-20.25)$$
$$16$$

16

4

Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2$, $y = x + 2$

In order to find the area of the region bounded by the graphs of the given equations, we need to find the points of intersection of the two functions. We can do this by setting the two equations equal to each other and solving for x. $x^2 - 2x - 2 = x + 2$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ or $x = -1$

The area of the region bounded by the graphs of the given equations is given by the integral of the difference of the two functions over the interval from -1 to 4.

$$\int_{-1}^{4} (x + 2) - (x^2 - 2x - 2)\, dx$$
$$= \int_{-1}^{4} (x + 2 - x^2 + 2x + 2)\, dx$$
$$= \int_{-1}^{4} (-x^2 + 3x + 4)\, dx$$

We'll use R to calculate the integral.

```r
# Define the integrand
integrand <- function(x) {
  -x^2 + 3*x + 4
}

# Compute the integral from -1 to 4
result <- integrate(integrand, -1, 4)

# Print the result
print(result$value)
```

```
## [1] 20.83333
```

The area of the region bounded by the graphs of the given equations is **20.833**

5

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

The ordering cost (DS/Q) is the cost associated with placing orders. Each order has a fixed cost S, and if you need D units per year, then you will need to place D/Q orders per year. So, the total ordering cost per year is DS/Q.

The holding cost ($QH/2$) is the cost to hold one unit in inventory for one year. If you order Q units each time, then on average (assuming constant demand and no shortages), you will have $Q/2$ units in inventory. So, the total holding cost per year is $QH/2$.

The total cost C is the sum of the ordering cost and the holding cost:

$$C = \frac{DS}{Q} + \frac{QH}{2}$$

To find the minimum of this function, we can take the derivative of C with respect to Q, set it equal to zero, and solve for Q..

Take the derivative of C with respect to Q: The derivative of C with respect to Q is $-DS/Q^2 + H/2$.

$$\frac{dC}{dQ} = -\frac{DS}{Q^2} + \frac{H}{2}$$

Set the derivative equal to zero and solve for Q: Setting the derivative equal to zero gives us the equation $-DS/Q^2 + H/2 = 0$.

$$-\frac{DS}{Q^2} + \frac{H}{2} = 0$$

Multiplying through by Q^2 gives:

$$-DS + \frac{HQ^2}{2} = 0$$

Rearranging terms gives:

$$\frac{HQ^2}{2} = DS$$

Multiplying through by 2 gives:

$$HQ^2 = 2DS$$

Finally, solving for Q gives:

$$Q = \sqrt{\frac{2DS}{H}}$$

This is the calculus way of finding the order quantity that minimizes the total cost. This quantity is known as the Economic Order Quantitiy, often abbreviated as EOQ, and is the standard formula used in inventory management.

Let's calculate the EOQ and the number of orders per year using the given values of D, S, and H.

$$
\begin{aligned}
EOQ &= \sqrt{\frac{2 \times 110 \times 8.25}{3.75}} \\
&= \sqrt{\frac{1815}{3.75}} \\
&= \sqrt{484} \\
&= 22
\end{aligned}
$$

To find the optimal amount of orders, we divide the annual demand by the EOQ:

$$
\begin{aligned}
\text{Number of Orders per Year} &= \frac{D}{EOQ} \\
&= \frac{110}{22} \\
&= 5
\end{aligned}
$$

So the optimal order quantity (EOQ) is **22** flat irons, and the number of orders per year is **5**.

We can check our work using R:

```r
# Define the parameters
D <- 110       # Annual demand
S <- 8.25      # Cost per order
H <- 3.75      # Holding cost per unit per year

# Calculate EOQ
EOQ <- sqrt((2 * D * S) / H)

# Calculate the number of orders per year
num_orders <- D / EOQ

# Print the results
print(paste("Optimal order quantity (EOQ):", EOQ))
```

```
## [1] "Optimal order quantity (EOQ): 22"
```

```r
print(paste("Number of orders per year:", num_orders))
```

```
## [1] "Number of orders per year: 5"
```

6

Use integration by parts to solve the integral below.

$$\int ln(9x) \cdot x^6 dx$$

The formula for integration by parts is:

$$\int udv = uv - \int vdu$$

In this case, we can choose $u = ln(9x)$ and $dv = x^6dx$. Then, we need to find du and v.

$$u = ln(9x)$$
$$du = \frac{1}{9x} \cdot 9dx$$
$$du = \frac{1}{x}dx$$

$$dv = x^6dx$$
$$v = \frac{1}{7}x^7$$

Now, we can apply the formula for integration by parts:

$$\int ln(9x) \cdot x^6dx = ln(9x) \cdot \frac{1}{7}x^7 - \int \frac{1}{x} \cdot \frac{1}{7}x^7dx$$
$$= \frac{1}{7}x^7ln(9x) - \frac{1}{7}\int x^6dx$$
$$= \frac{1}{7}x^7ln(9x) - \frac{1}{7}\frac{1}{7}x^7 + C$$
$$= \frac{1}{7}x^7ln(9x) - \frac{1}{49}x^7 + C$$

So, the solution to the integral is:

$$\frac{1}{7}x^7ln(9x) - \frac{1}{49}x^7 + C$$

7

Determine whether f (x) is a probability density function on the interval [1, e^6] . If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

To determine if $f(x)$ is a probability density function on the interval $[1, e^6]$, we need to check if the integral of $f(x)$ over the interval equals 1.

$$\int_1^{e^6} \frac{1}{6x}dx$$

$$= \frac{1}{6}\int_1^{e^6} \frac{1}{x}dx$$

$$= \frac{1}{6}[ln(x)]_1^{e^6}$$

$$= \frac{1}{6}[ln(e^6) - ln(1)]$$

$$= \frac{1}{6}[6 - 0]$$

$$= 1$$

Since the integral of $f(x)$ over the interval $[1, e^6]$ equals 1, $f(x)$ is a probability density function on that interval.