HW 15

Shaya Engelman

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1.

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

$$(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)$$

```r
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)

lm(y ~ x)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

The equation of the regression line is $y = 4.26x - 14.8$

2.

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$$f(x, y) = 24x - 6xy^2 - 8y^3$$

To solve this we find the critical points by taking the partial derivatives of the function and setting them equal to zero.

$$f_x = 24 - 6y^2 = 0$$
$$24 = 6y^2$$
$$y^2 = 4$$
$$y = \pm 2$$

$$f_y = -12xy - 24y^2 = 0$$

$$-12xy - 24y^2 = 0$$
$$-12x - 24y = 0$$
$$-12x = 24y$$
$$x = -2y$$
$$x = -2(\pm 2)$$
$$x = \pm 4$$

Now we plug the critical points into the original function to find the z values.

```
equation = function(x,y){
  z = 24*x - 6*x*y^2 - 8*y^3
  return(c(x,y,z))
}
print(rbind(equation(-4,2),equation(4,-2)))
```

```
##      [,1] [,2] [,3]
## [1,]   -4    2  -64
## [2,]    4   -2   64
```

So the critical points are $(-4, 2, -64)$ and $(4, -2, 64)$.

Now we need to find the second partial derivatives to determine the nature of the critical points.

$ f_{xx} = 0$ and $f_{yy} = -12x - 48y$ and $f_{xy} = -12y$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$$D = 0(-12x - 48y) - (-12y)^2$$
$$D = 144y^2$$

Since $D < 0$, the critical points are saddle points.

3.

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1. Find the revenue function $R(x, y)$.

The revenue function is given by the product of the price and the quantity sold.

$$R(x, y) = x(81 - 21x + 17y) + y(40 + 11x - 23y)$$

$$R(x, y) = 81x - 21x^2 + 17xy + 40y + 11xy - 23y^2$$
$$R(x, y) = -21x^2 + 28xy - 23y^2 + 81x + 40y$$

Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

```r
x <- 2.3
y <- 4.1

revenue <- - (21 * x^2) + (28 * y * x) - (23 * y^2) + (81 * x) + (40 * y)
cat("The revenue is $", revenue, "\n")
```

```
## The revenue is $ 116.62
```

4.

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

This is a univariate function. To find the minimum, we take the partial derivatives of the function and set them equal to zero.

$$C_x = \frac{1}{3}x + 7 = 0$$
$$\frac{1}{3}x = -7$$
$$x = -21$$
$$C_y = \frac{1}{3}y + 25 = 0$$
$$\frac{1}{3}y = -25$$
$$y = -75$$

Since the number of units produced cannot be negative, the minimum cost occurs when 21 units are produced in Los Angeles and 75 units are produced in Denver.

5.

Evaluate the double integral on the given region.

$$\int\int_R (e^{8x+3y})\, dA; R : 2 \leq x \leq 4 \text{ and } 2 \leq y \leq 4$$

Write your answer in exact form without decimals.

$$f_x = 24 - 6y^2 = 0$$
$$24 = 6y^2$$
$$y^2 = 4$$
$$y = \pm 2$$

$$f_y = -12xy - 24y^2 = 0$$
$$-12xy - 24y^2 = 0$$

$$-12x - 24y = 0$$
$$-12x = 24y$$
$$x = -2y$$
$$x = -2(\pm 2)$$
$$x = \pm 4$$

5) Evaluate the double integral on the given region. Write your answer in exact form without decimals.

$$\int_2^4 \int_2^4 e^{8x+3y}\,dx\,dy$$

Apply u-substitution

$u = 8x + 3y$

Take derivative with respect to x:

$$8 + 0$$

$\frac{du}{dx} = 8 + 0$

$dx = \frac{1}{8}du$

Now plug in 2 and 4 to u for adjusted boundaries:

$8(2) + 3y = 16 + 3y$

$8(4) + 3y = 32 + 3y$

The updated double integral becomes:

$$\int_{16+3y}^{32+3y} e^u \cdot \frac{1}{8} du$$

Move the constant to the front:

$$\frac{1}{8}\int_{16+3y}^{32+3y} e^u\,du$$

$$\frac{1}{8}(e^{32+3y} - e^{16+3y})\,du$$

$$\int_2^4 \frac{1}{8}(e^{32+3y} - e^{16+3y})\,dy$$

Apply sum rule

$$\frac{1}{8}\int_2^4 e^{32+3y} - \int_2^4 e^{16+3y}\,dy$$

Apply u-substitution

$u = 32 + 3y$

Take derivative with respect to y:

$$0 + 3$$

$\frac{du}{dy} = 0 + 3$

$dy = \frac{1}{3}du$

Now plug in 2 and 4 to u for adjusted boundaries:

$32 + 3(2) = 38$

$32 + 3(4) = 44$

The updated double integral becomes:

$$\int_{38}^{44} e^u \cdot \frac{1}{3}\, du$$

$$\frac{1}{3}(e^{44} - e^{38})$$

Apply u-substitution

$u = 16 + 3y$

Take derivative with respect to y:

$$0 + 3$$

$\frac{du}{dy} = 0 + 3$

$dy = \frac{1}{3}du$

Now plug in 2 and 4 to u for adjusted boundaries:

$16 + 3(2) = 22$

$16 + 3(4) = 28$

The updated double integral becomes:

$$\int_{22}^{28} e^u \cdot \frac{1}{3}\, du$$

$$\frac{1}{3}(e^{28} - e^{22})$$

Now combine:

$$(e^{44} - e^{38}) - (e^{28} - e^{22})$$

$$e^{44} - e^{38} - e^{28} + e^{22}$$

Insert the constants:

$$\frac{1}{8} \cdot \frac{1}{3}(e^{44} - e^{38} - e^{28} + e^{22})$$

$$\frac{1}{24}(e^{44} - e^{38} - e^{28} + e^{22})$$

$$\frac{e^{44} - e^{38} - e^{28} + e^{22}}{24}$$