HW Week10

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Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability .4 and loses A dollars with probability .6. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives Smith the better chance of getting out of jail?

Solution using simulations

First we'll define a function to simulate each round of betting:

```r
game <- function(bet, amount){
  if (runif(1) < 0.4){
    amount <- amount + bet
  } else {
    amount <- amount - bet
  }
  return(amount)
}
```

Now we will use this function to simulate the two strategies and calculate the probability of reaching $8.

```r
timid <- function(){
  amount <- 1
  while (amount > 0 & amount < 8){
    amount <- game(1, amount)
  }
  return(amount)
}

bold <- function(){
    amount <- 1
    while (amount > 0 & amount < 8){
        amount <- game(min(8 - amount, amount), amount)
    }
    return(amount)
}


# Simulate running both strategies 1000 times each
```

```r
num_simulations <- 10000

timid_success_count <- 0
bold_success_count <- 0

for (i in 1:num_simulations) {
  if (timid() == 8) {
    timid_success_count <- timid_success_count + 1
  }
  if (bold() == 8) {
    bold_success_count <- bold_success_count + 1
  }
}

# Calculate the percentage of times each strategy reached the target goal
timid_success_percentage <- timid_success_count / num_simulations
bold_success_percentage <- bold_success_count / num_simulations

# Print the results
cat("Timid strategy success rate:", timid_success_percentage,"\n")
```

```
## Timid strategy success rate: 0.0217
```

```r
cat("Bold strategy success rate:", bold_success_percentage,"\n")
```

```
## Bold strategy success rate: 0.0668
```

Solution using Markov Chains

Timid strategy

We can solve this using Markov Chains and the markovchain package.

```r
library(markovchain)
```

```
## Warning: package 'markovchain' was built under R version 4.3.3
```

```
## Package:  markovchain
## Version:  0.9.5
## Date:     2023-09-24 09:20:02 UTC
## BugReport: https://github.com/spedygiorgio/markovchain/issues
```

```r
# Define the transition matrix for the timid strategy
timid_mat <- matrix(c(1, 0, 0, 0, 0, 0, 0, 0, 0,
                      0.6, 0, 0.4, 0, 0, 0, 0, 0, 0,
                      0, 0.6, 0, 0.4, 0, 0, 0, 0, 0,
                      0, 0, 0.6, 0, 0.4, 0, 0, 0, 0,
                      0, 0, 0, 0.6, 0, 0.4, 0, 0, 0,
                      0, 0, 0, 0, 0.6, 0, 0.4, 0, 0,
```

```r
                      0, 0, 0, 0, 0, 0.6, 0, 0.4, 0,
                      0, 0, 0, 0, 0, 0, 0.6, 0, 0.4,
                      0, 0, 0, 0, 0, 0, 0, 0, 1), byrow=TRUE, nrow=9)

states <- as.character(0:8)

timid_chain <- new("markovchain", states = states, transitionMatrix = timid_mat)

abs_prob_timid <- absorptionProbabilities(timid_chain)
print(paste("The probability of Smith reaching $8 is", round(abs_prob_timid[1, "8"], 3), "and the probab
```

```
## [1] "The probability of Smith reaching $8 is 0.02 and the probability of Smith reaching $0 is 0.98"
```

Bold strategy

This problem is basically a binomial probability proble. In order for Smith to win, he needs to win 3 times in a row. Each with a probability of 0.4. We can calculate that as 0.4^3.

```r
0.4 ** 3
```

```
## [1] 0.064
```

We can also solve this using Markov Chains as before to get the same result:

```r
# Define the transition matrix for the bold strategy
bold_mat <- matrix(c(1, 0, 0, 0, 0,
                     0.6, 0, 0.4, 0, 0,
                     0.6, 0, 0, 0.4, 0,
                     0.6, 0, 0, 0, 0.4,
                     0, 0, 0, 0, 1), byrow=TRUE, nrow=5)

states <- as.character(0:4)

bold_chain <- new("markovchain", states = states, transitionMatrix = bold_mat)

abs_prob_bold <- absorptionProbabilities(bold_chain)

print(paste("The probability of Smith reaching $8 is", round(abs_prob_bold[1, "4"], 3), "and the probabi
```

```
## [1] "The probability of Smith reaching $8 is 0.064 and the probability of Smith reaching $0 is 0.936
```

Better strategy results

As we can see from the above responses, Smith has around three times the chance of successfully achieving $8 by going with the bold strategy.