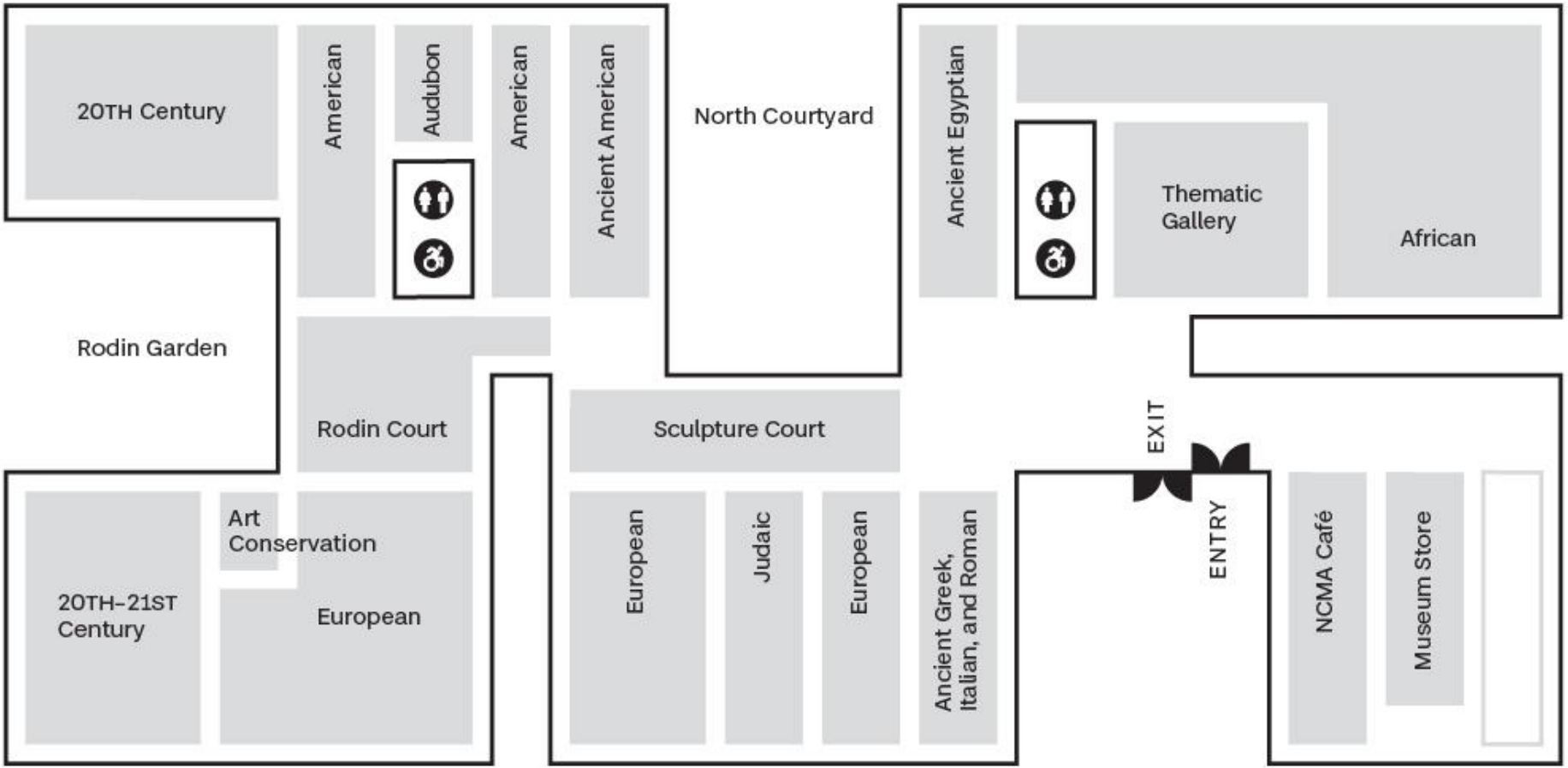
Who's Guarding the Louvre?! The Art Gallery Problem Explained

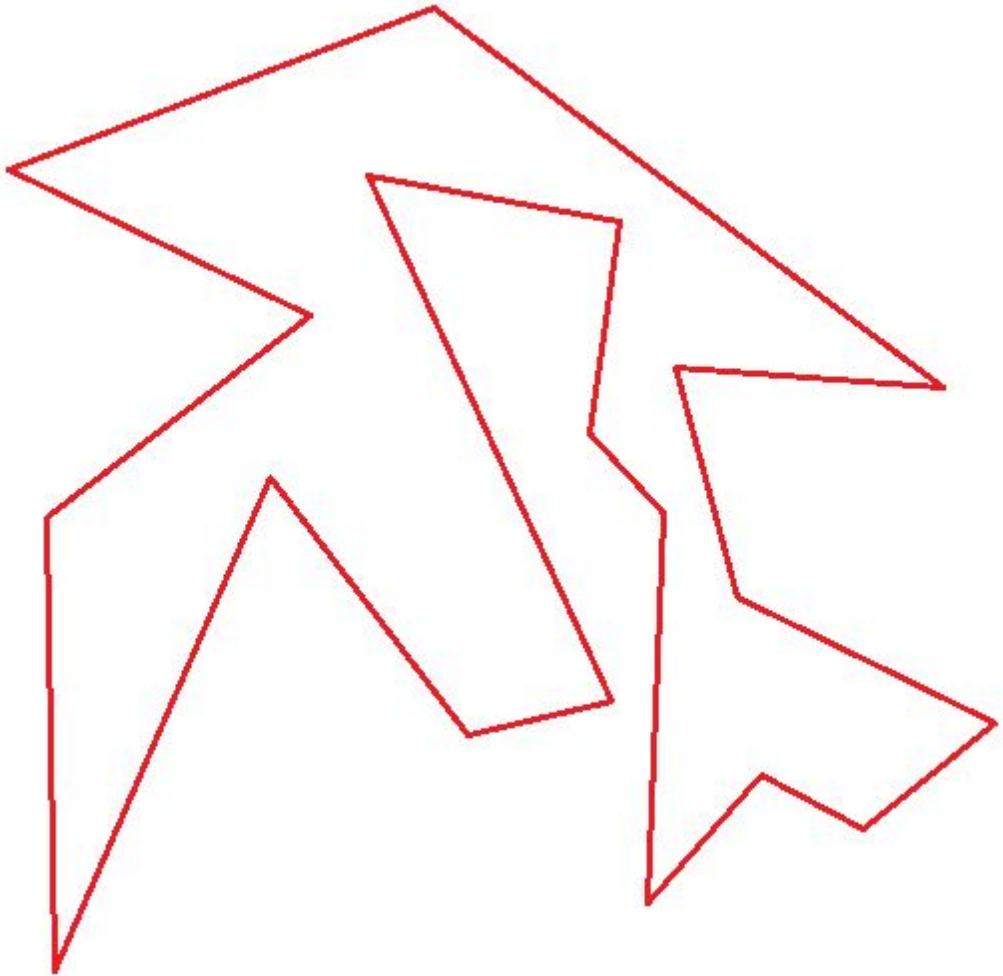
Shayan Shahrabi – Fall 2025
shayanshahrabi.github.io

Problem Statement

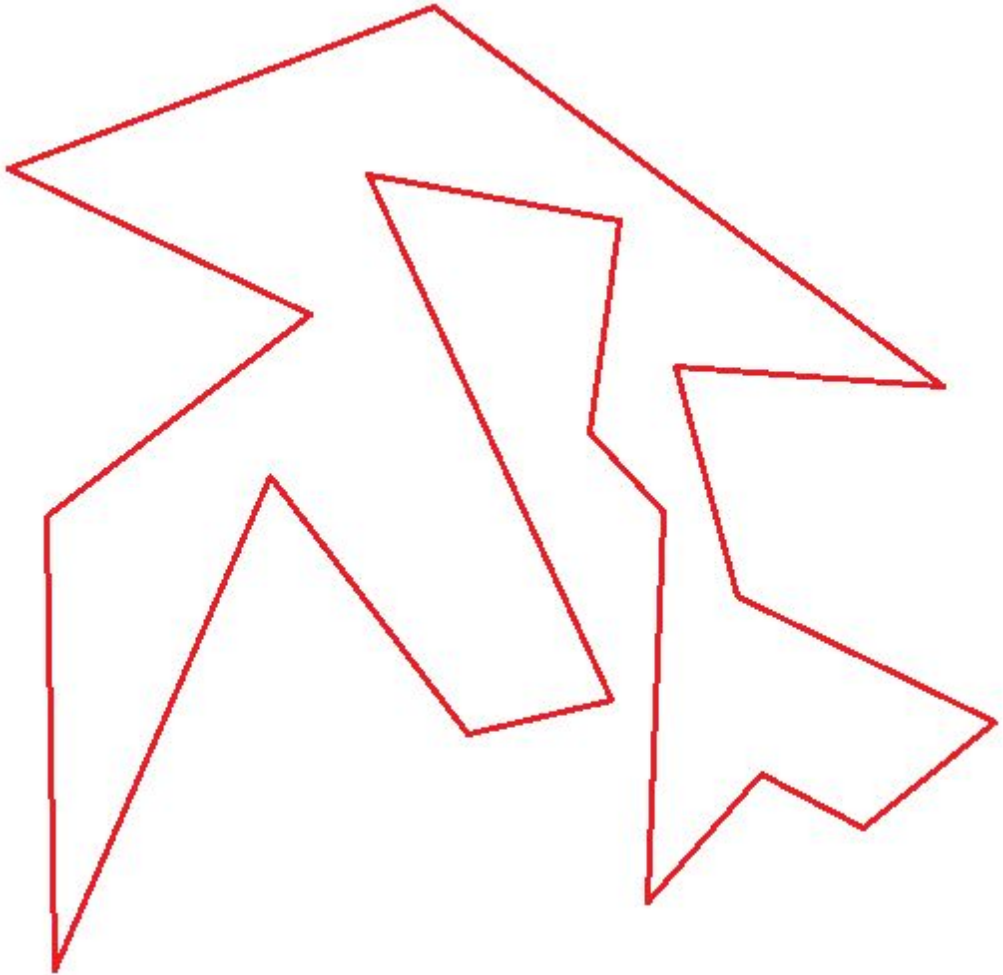

20TH Century
American
Audubon
American
Ancient American
North Courtyard
Ancient Egyptian
Thematic Gallery
African
Rodin Garden
Rodin Court
Sculpture Court
EXIT
ENTRY
Art Conservation
European
European
Judaic
European
Ancient Greek, Italian, and Roman
20TH–21ST Century
NCMA Café
Museum Store

Assumptions

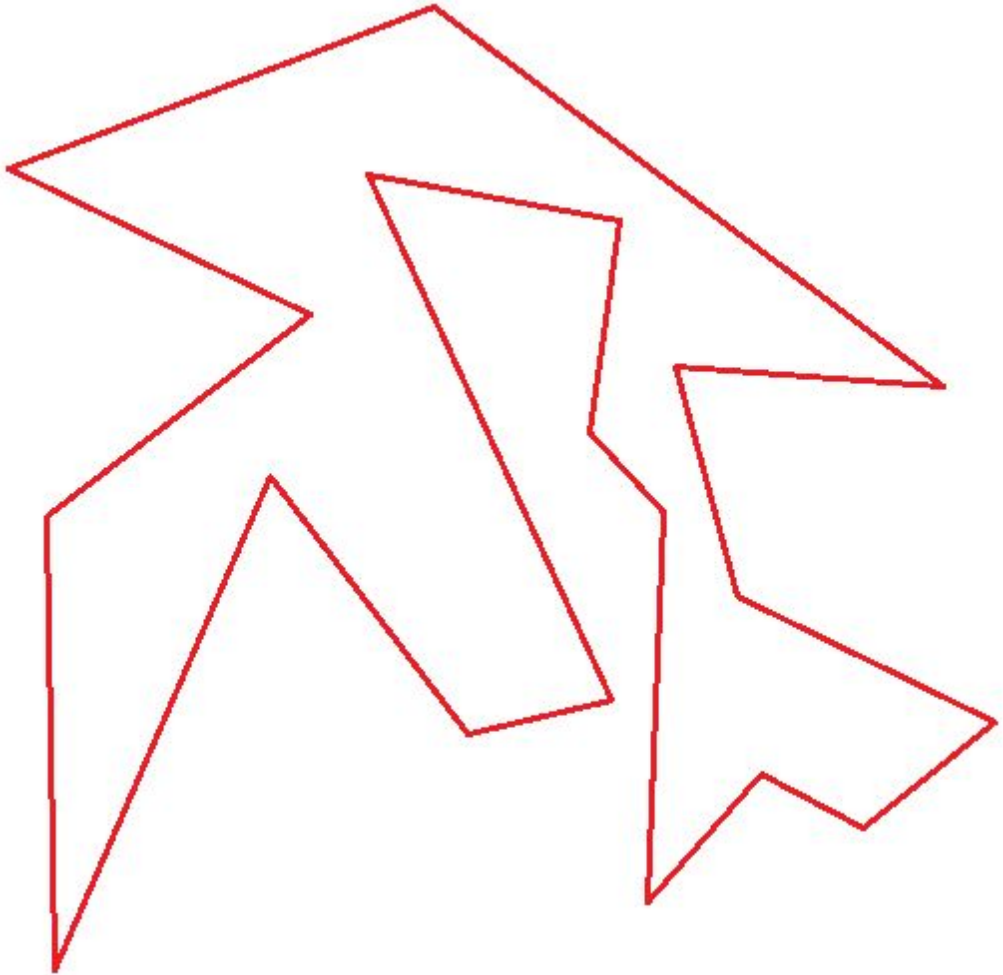
1. 360-degree FOV



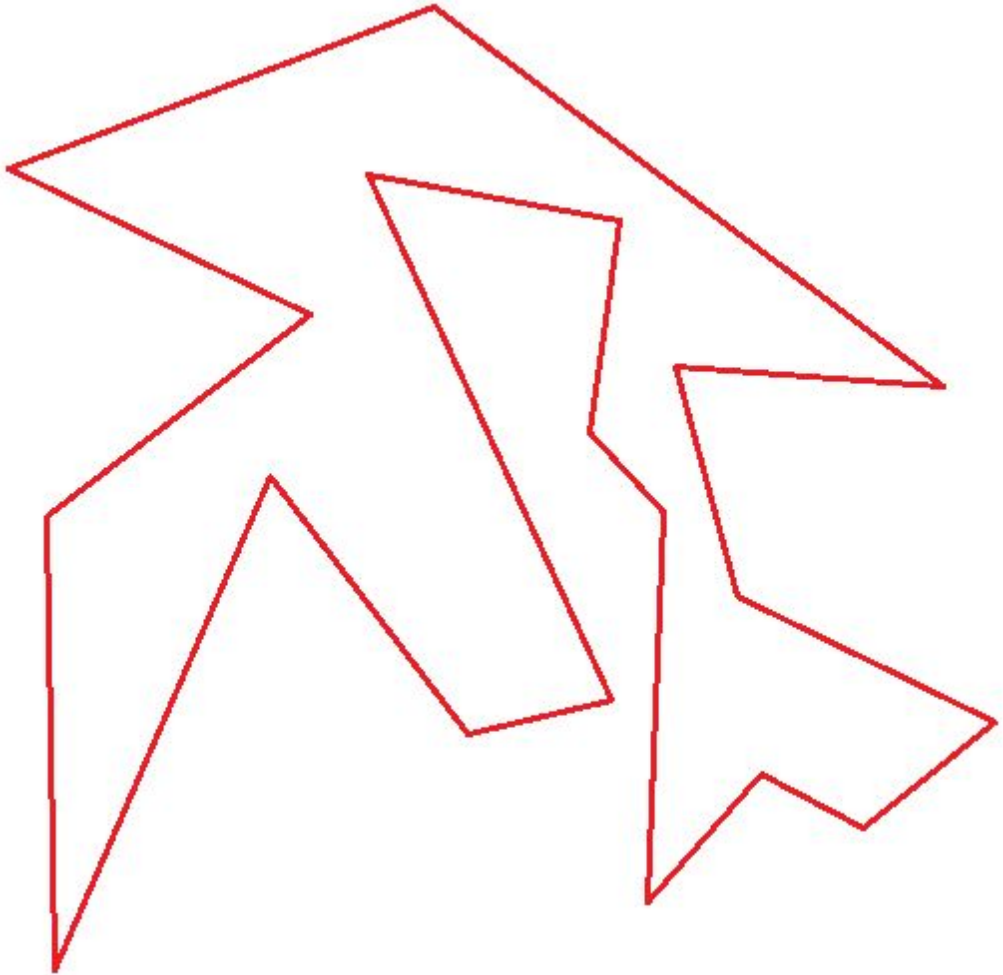
2. Unbounded visibility dist.



3. For guard G & point P …

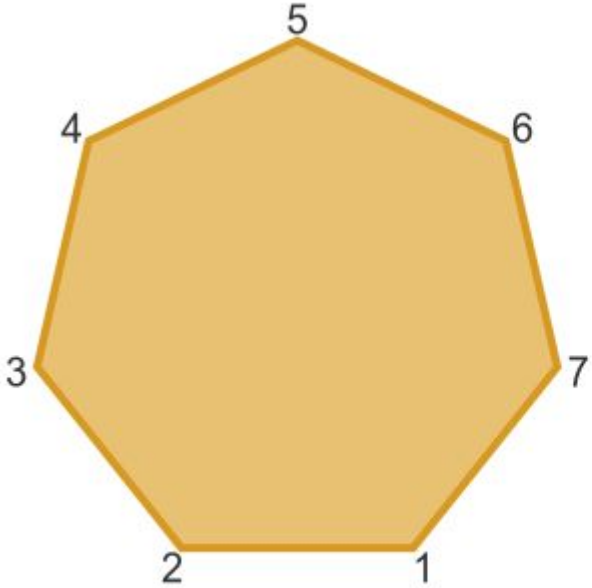


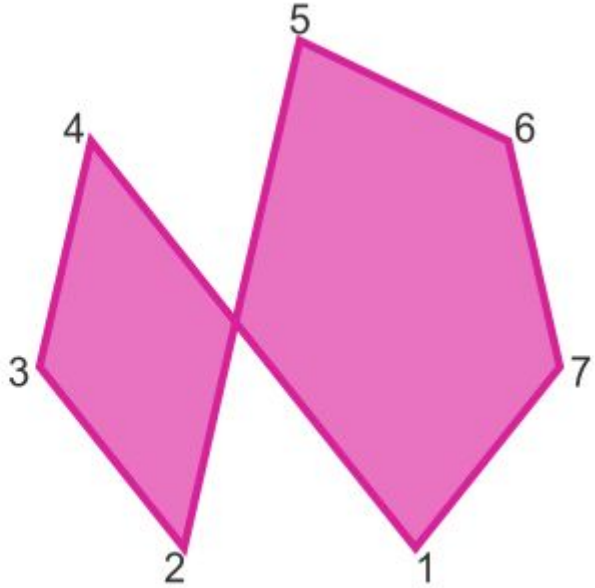
Problem Statement?!



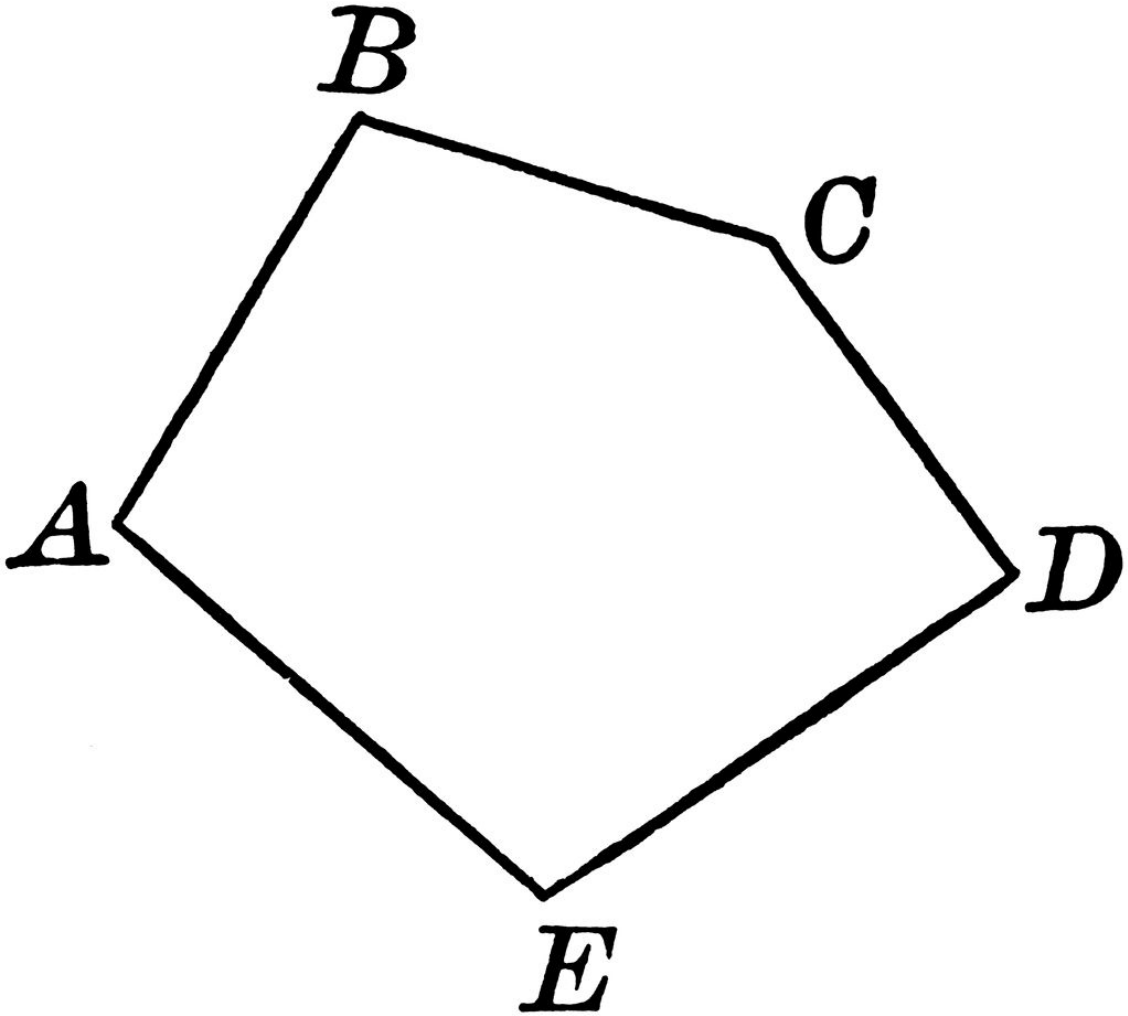
Some Examples

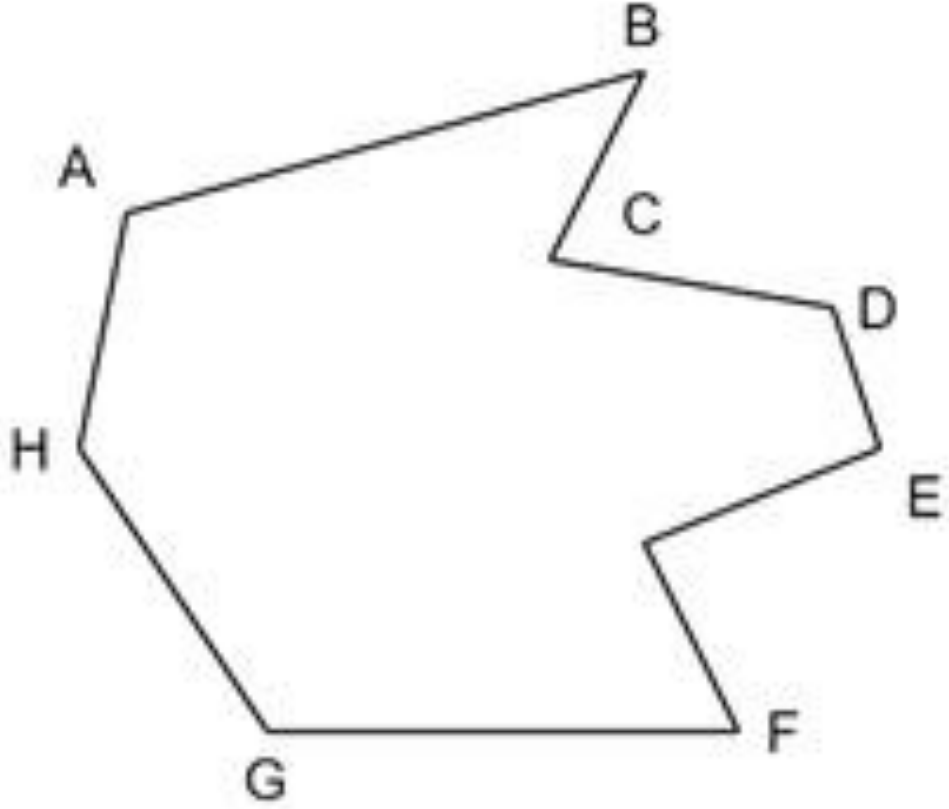
But Before That…

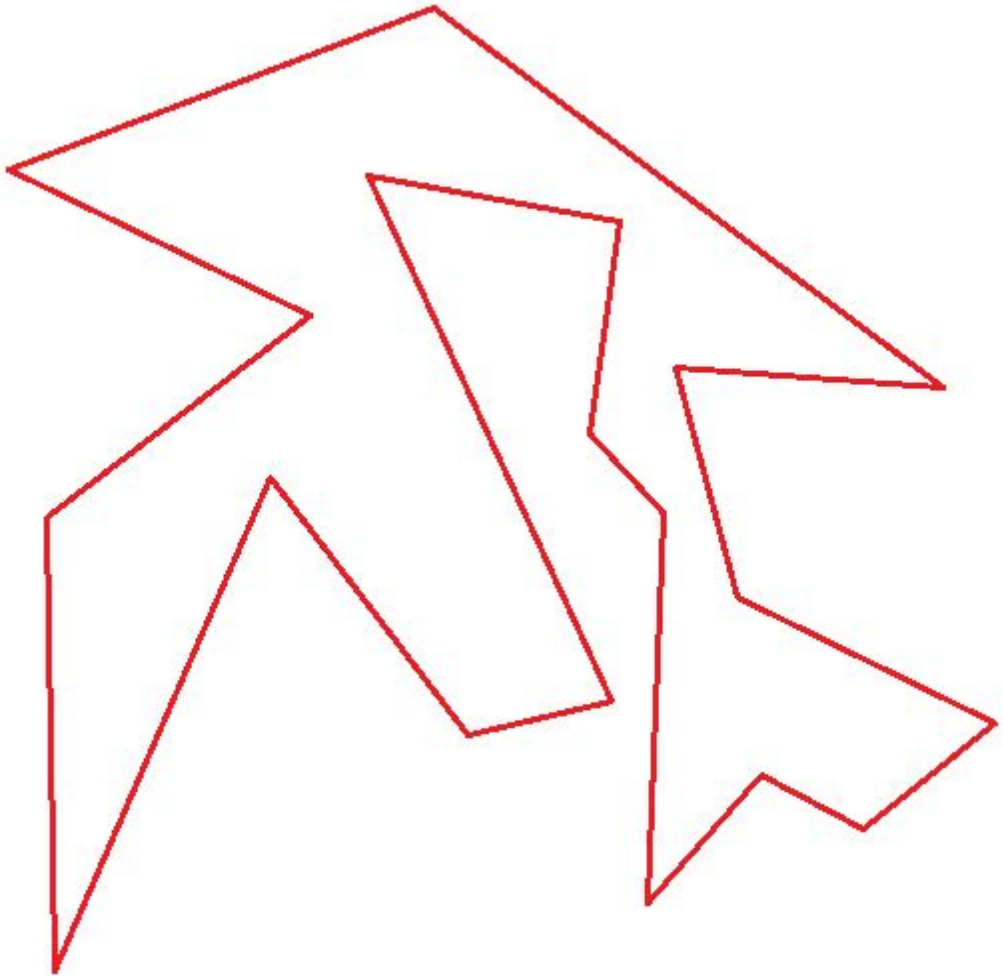

5
4
6
3
7
2
1


5
4
6
3
7
2
1

Now Some Examples!


B
C
A
D
E


B
A
C
D
H
E
F
G



There are 2 parts to the problem

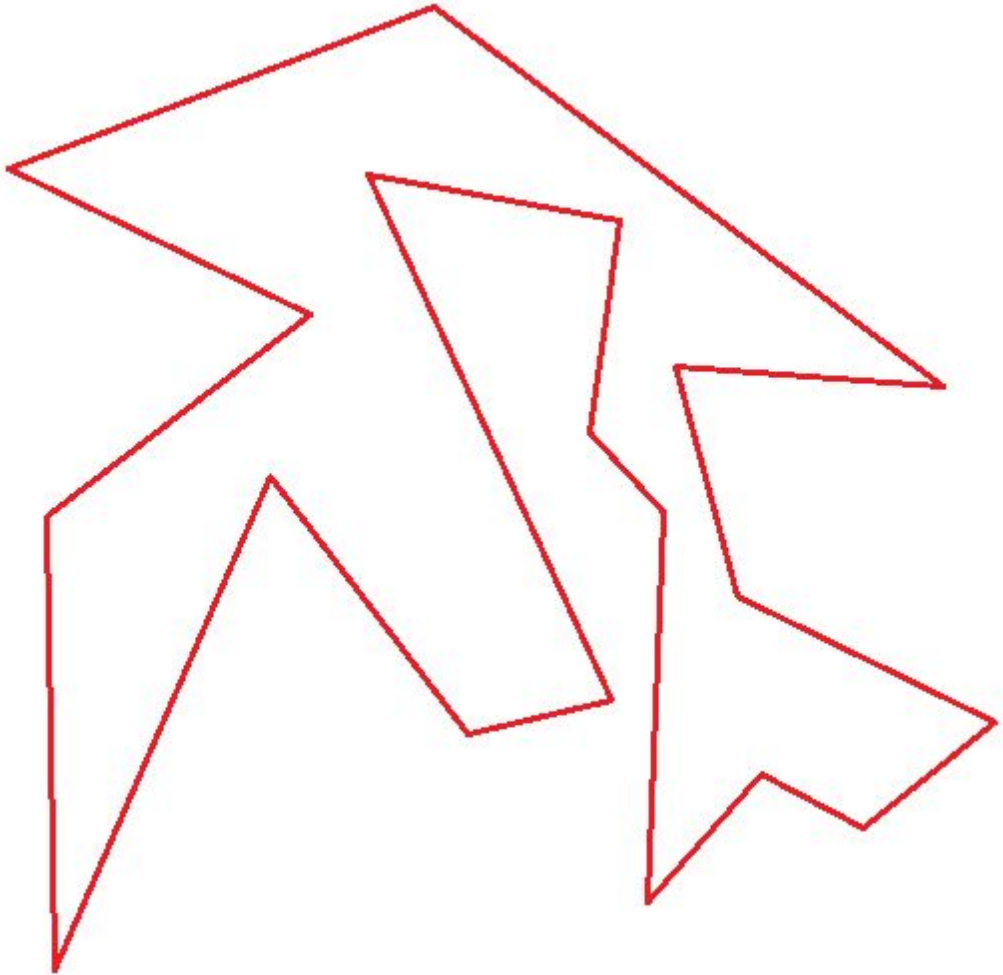
Part I : # Sufficient Guards

Part II : Minimal # Guards

Part I Vs. Part II ?

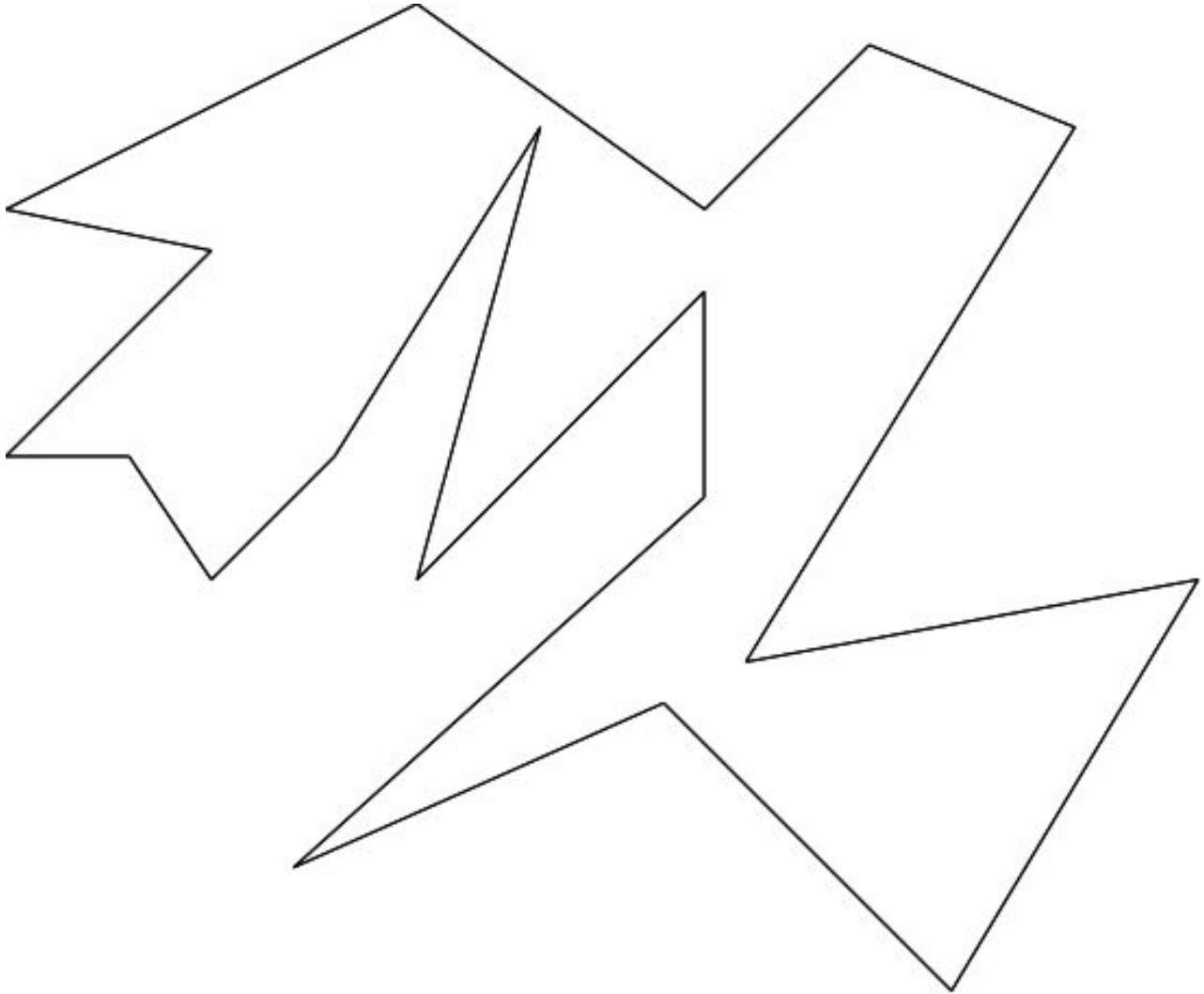
Note: We do NOT Consider Polygons with Holes

Guards Can Be Placed ANYWHERE!



Our Thinking Procedure?

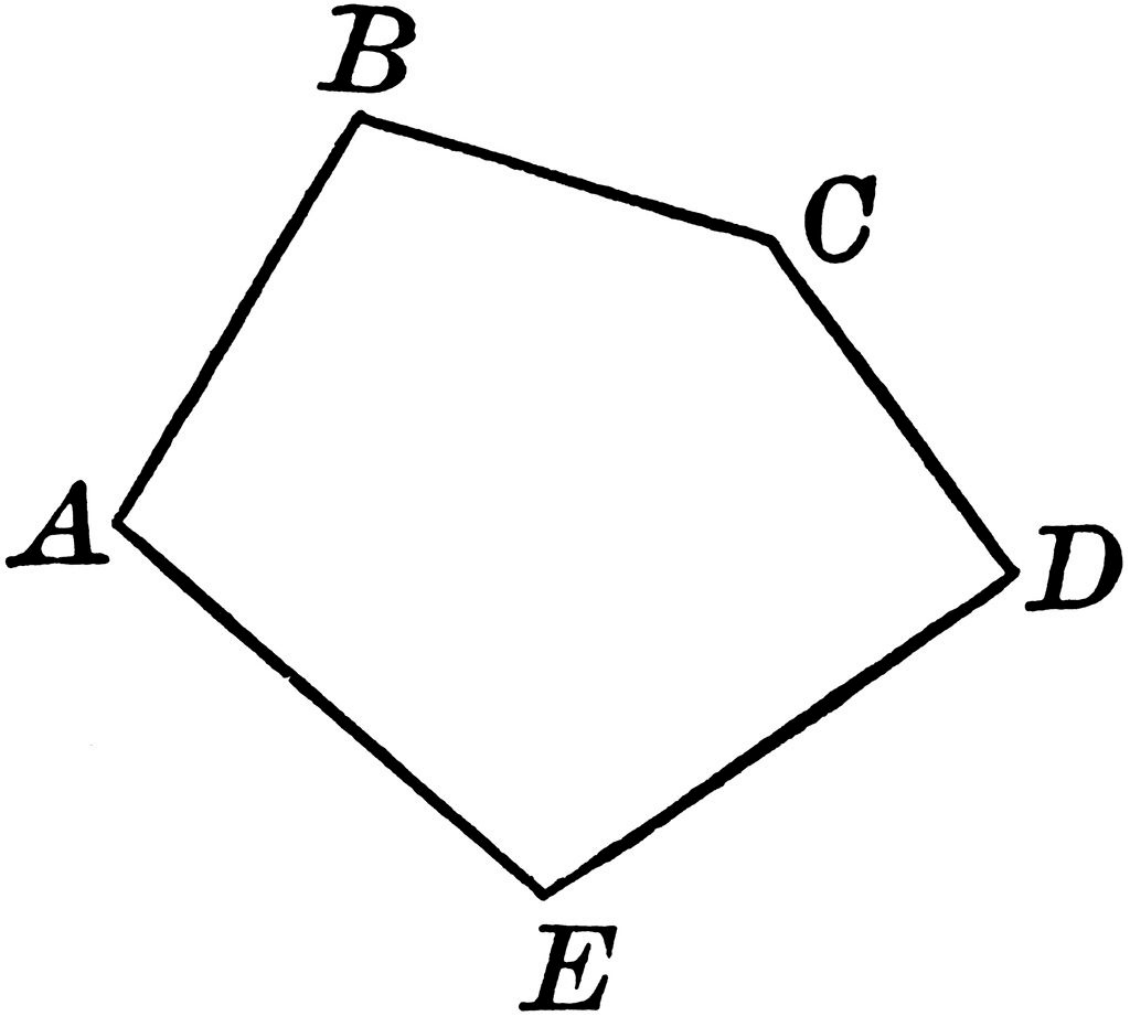
A Naive Approach (G1 Func.)

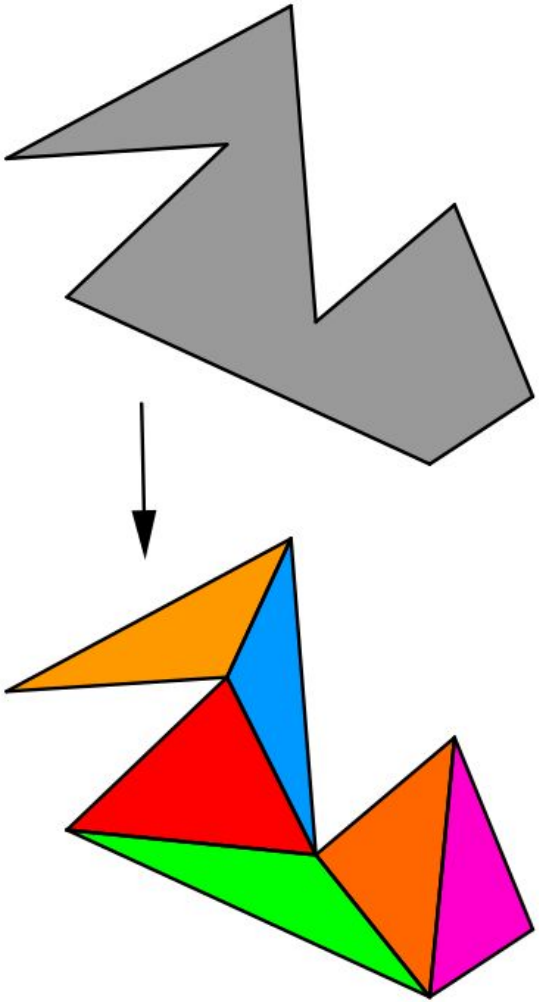


The Tightest Upper Bound?

The Concept of Triangulation

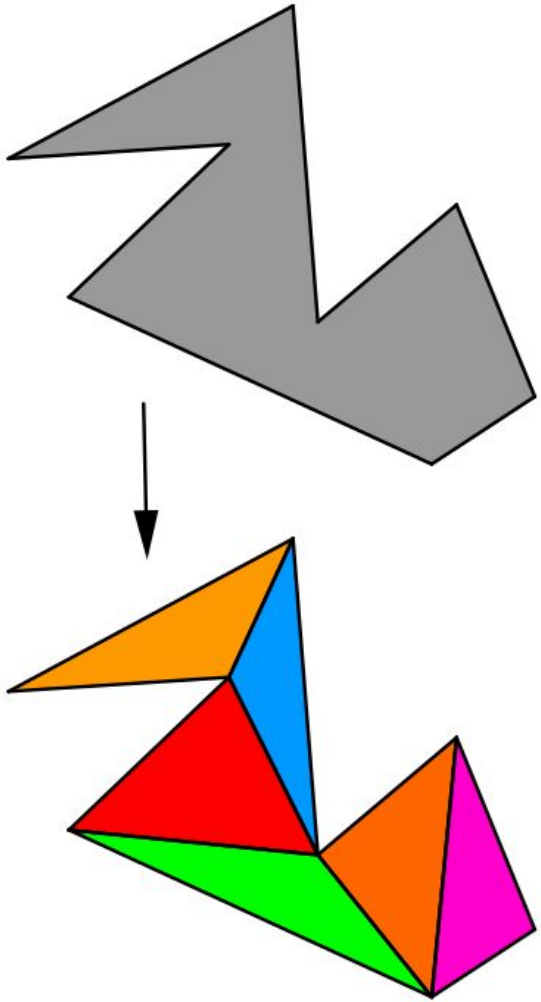
Definition


B
C
A
D
E



Is It Unique?

NO!



Always Feasible?

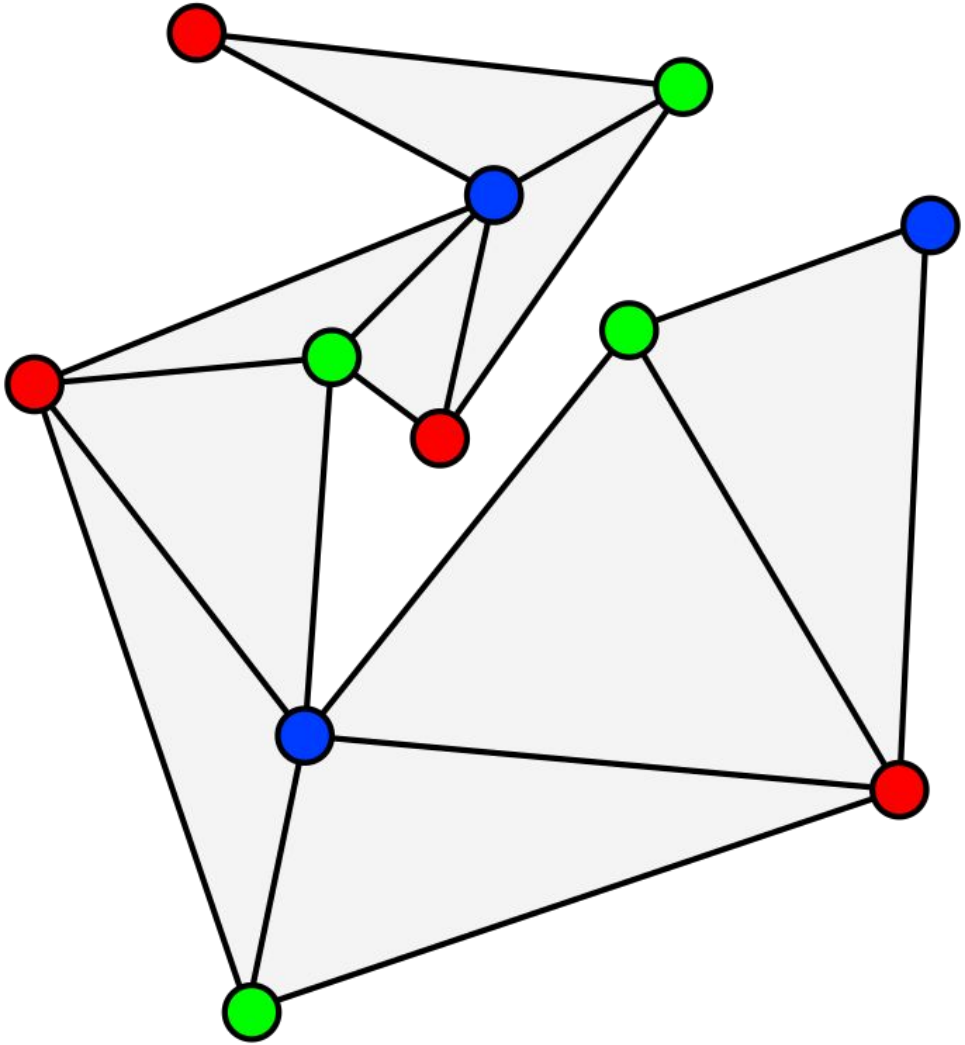
YES!
(Triangulation Theorem)

HOW?

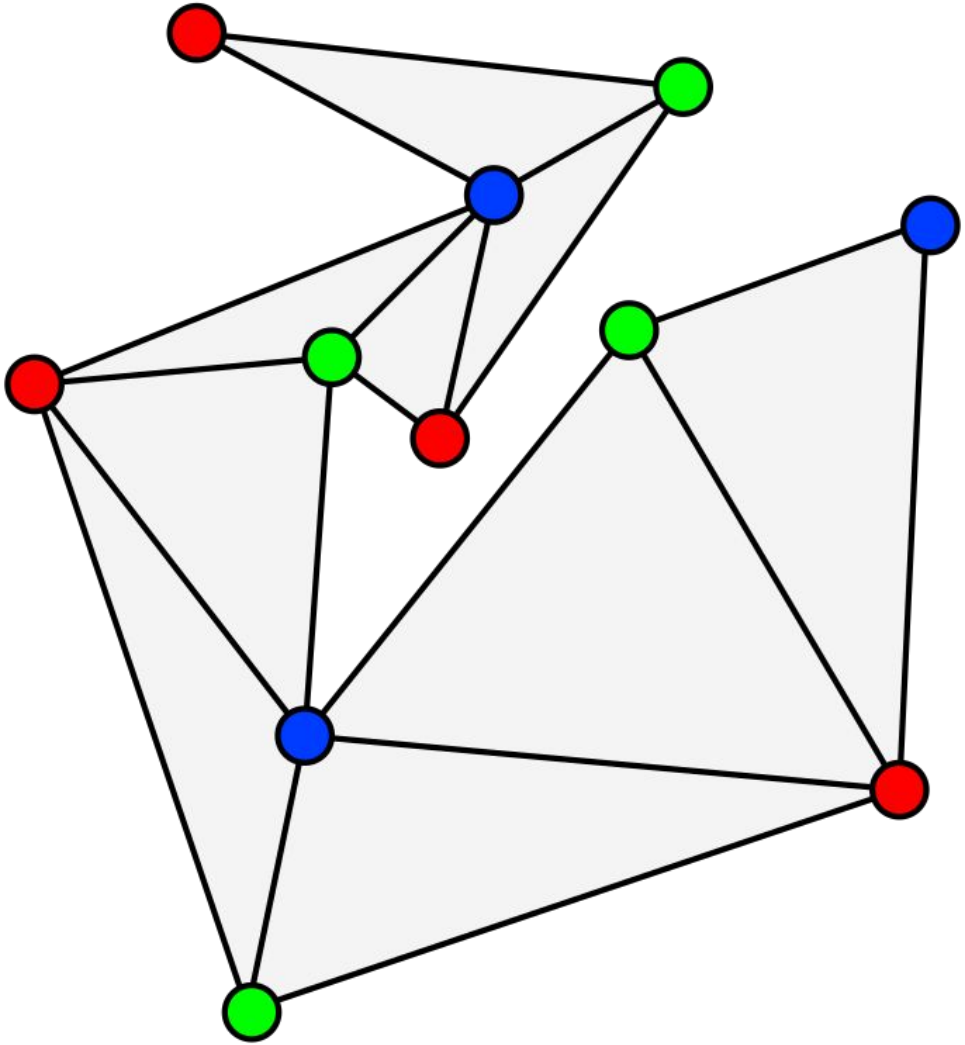
The Ear Clipping Algorithm

Triangulation by Ear Clipping, David Eberly

Polygon Triangulation Wikipedia Page



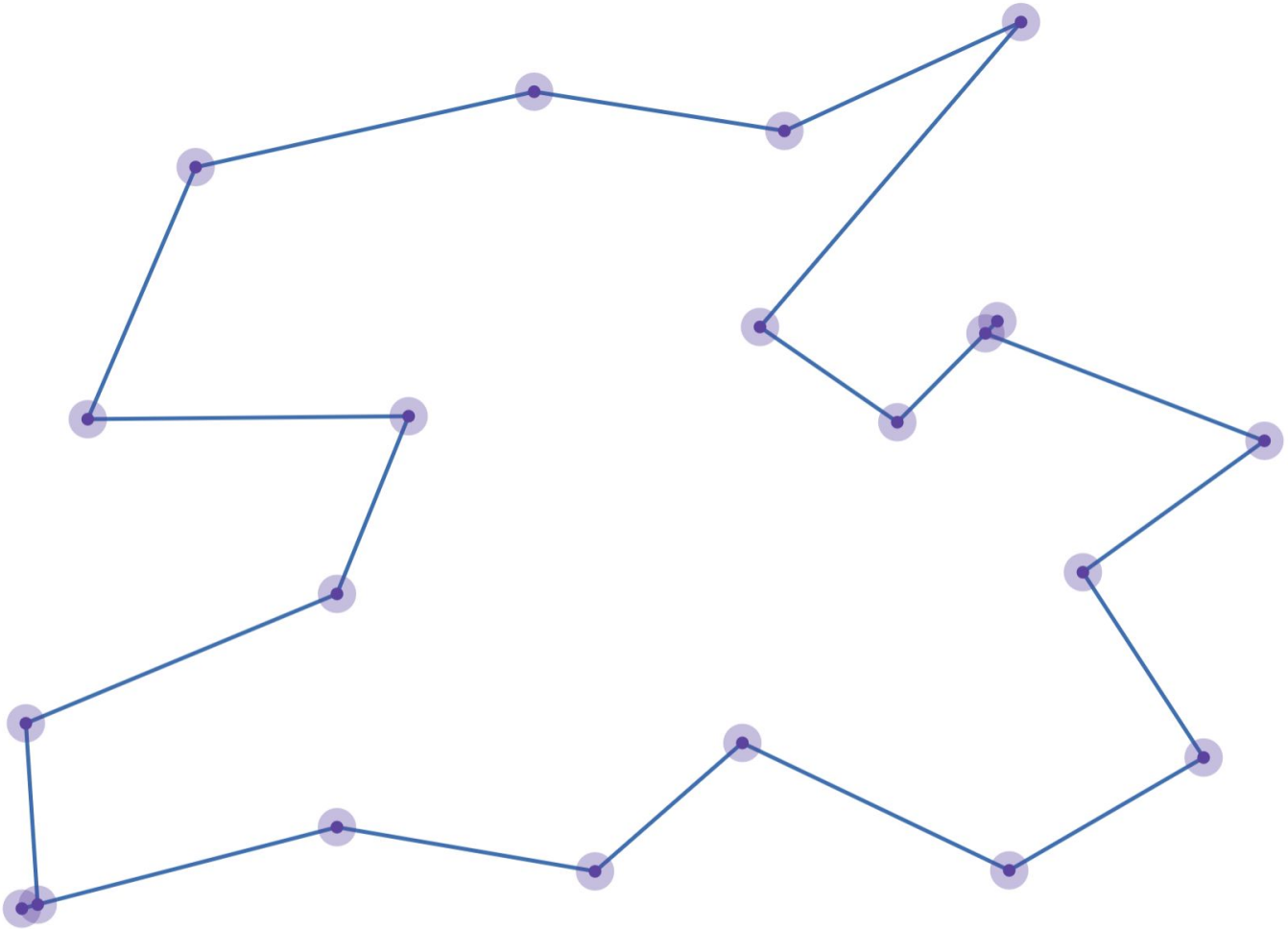
Maybe a Better Bound? (G_2 Func.)

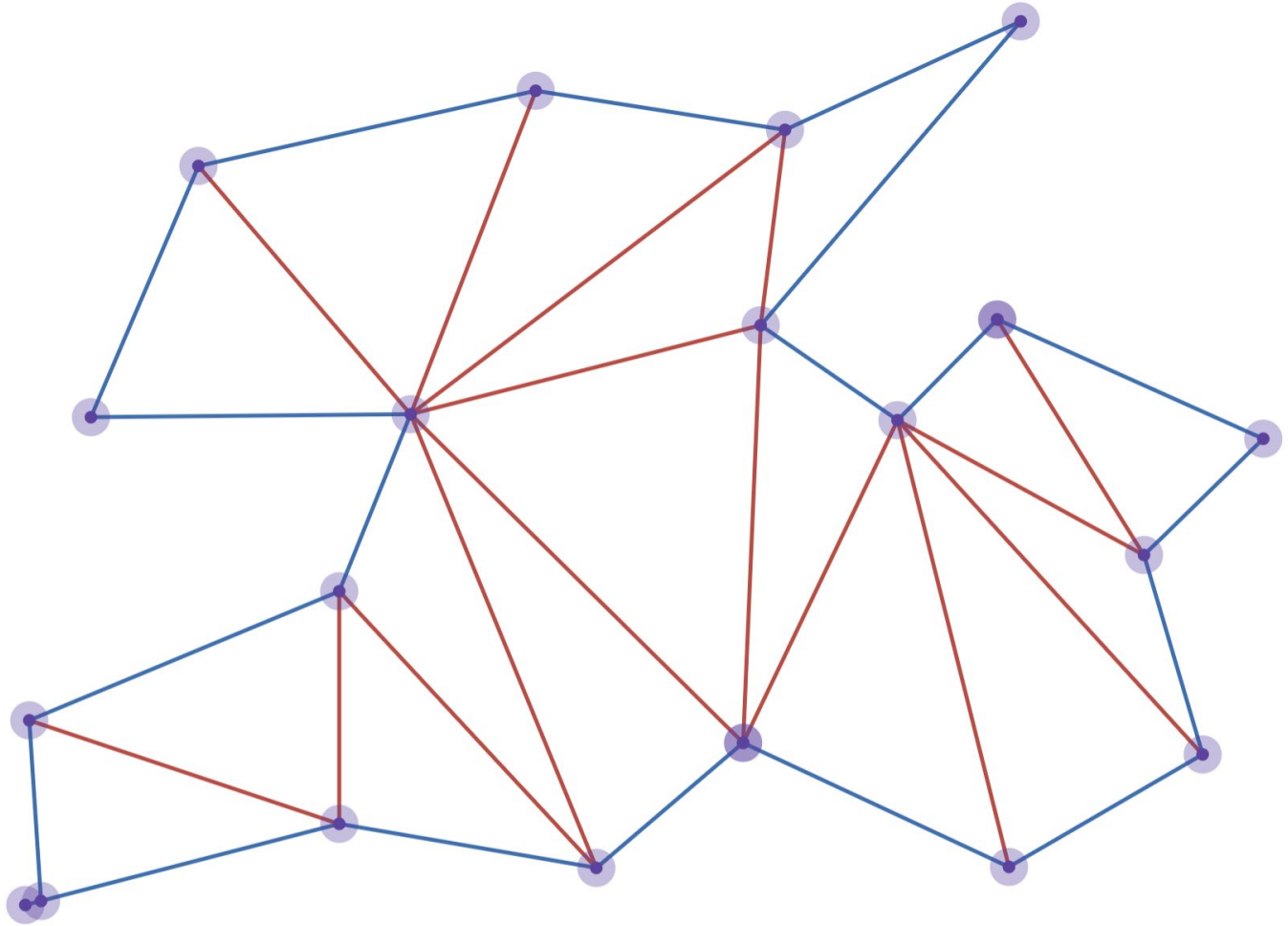


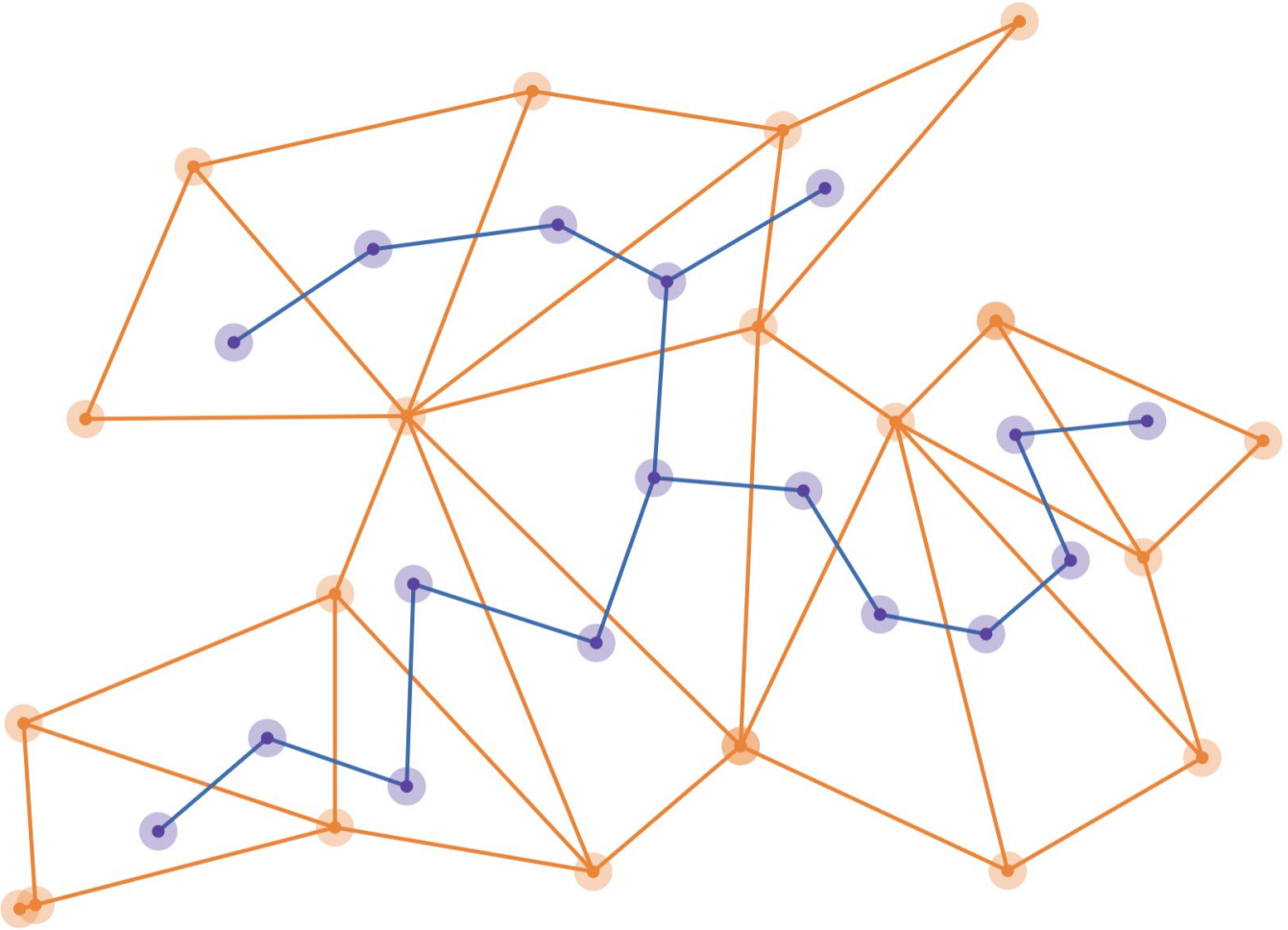
Are We Close to the Answer?

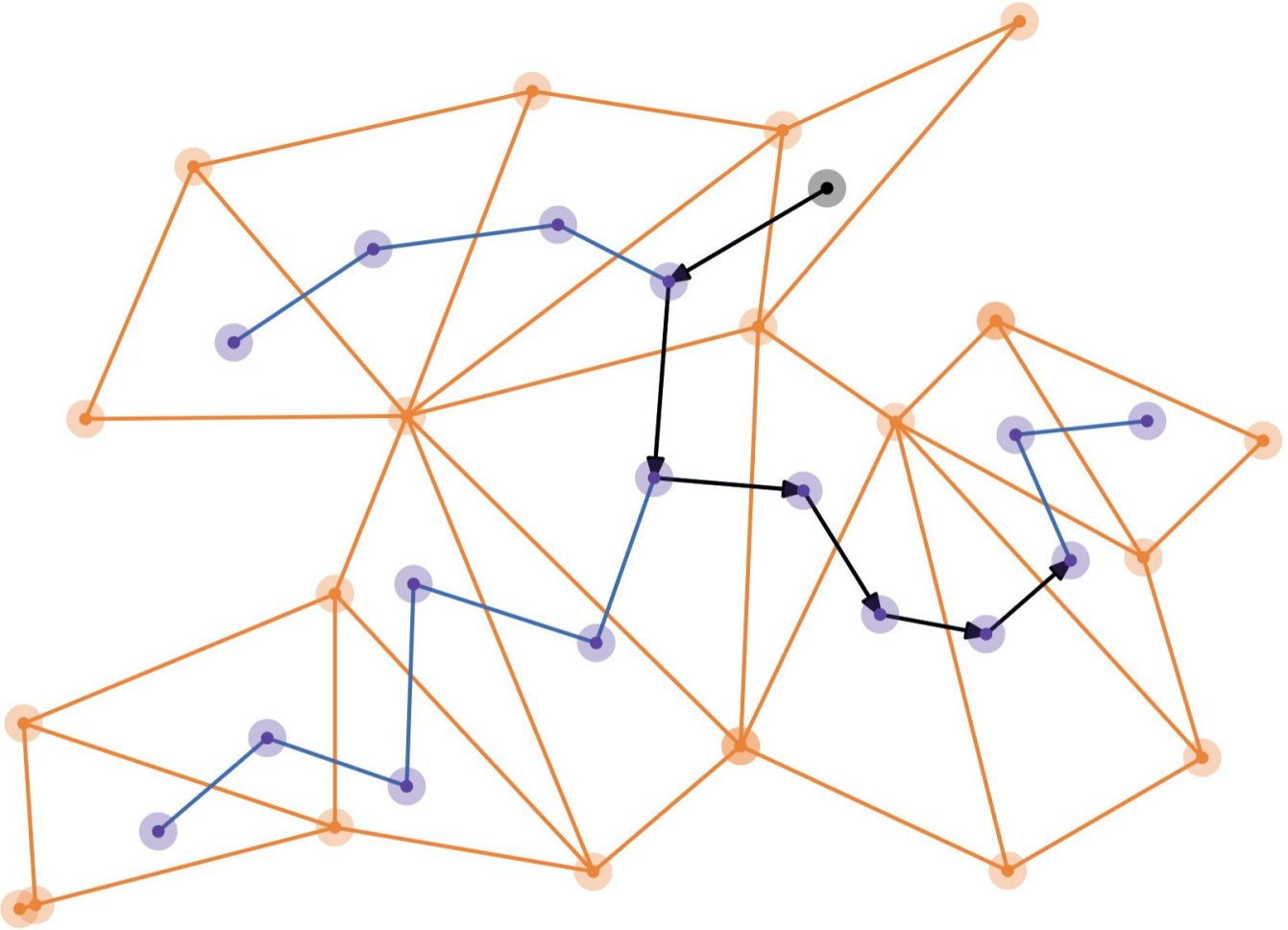

Well yes, but actually no

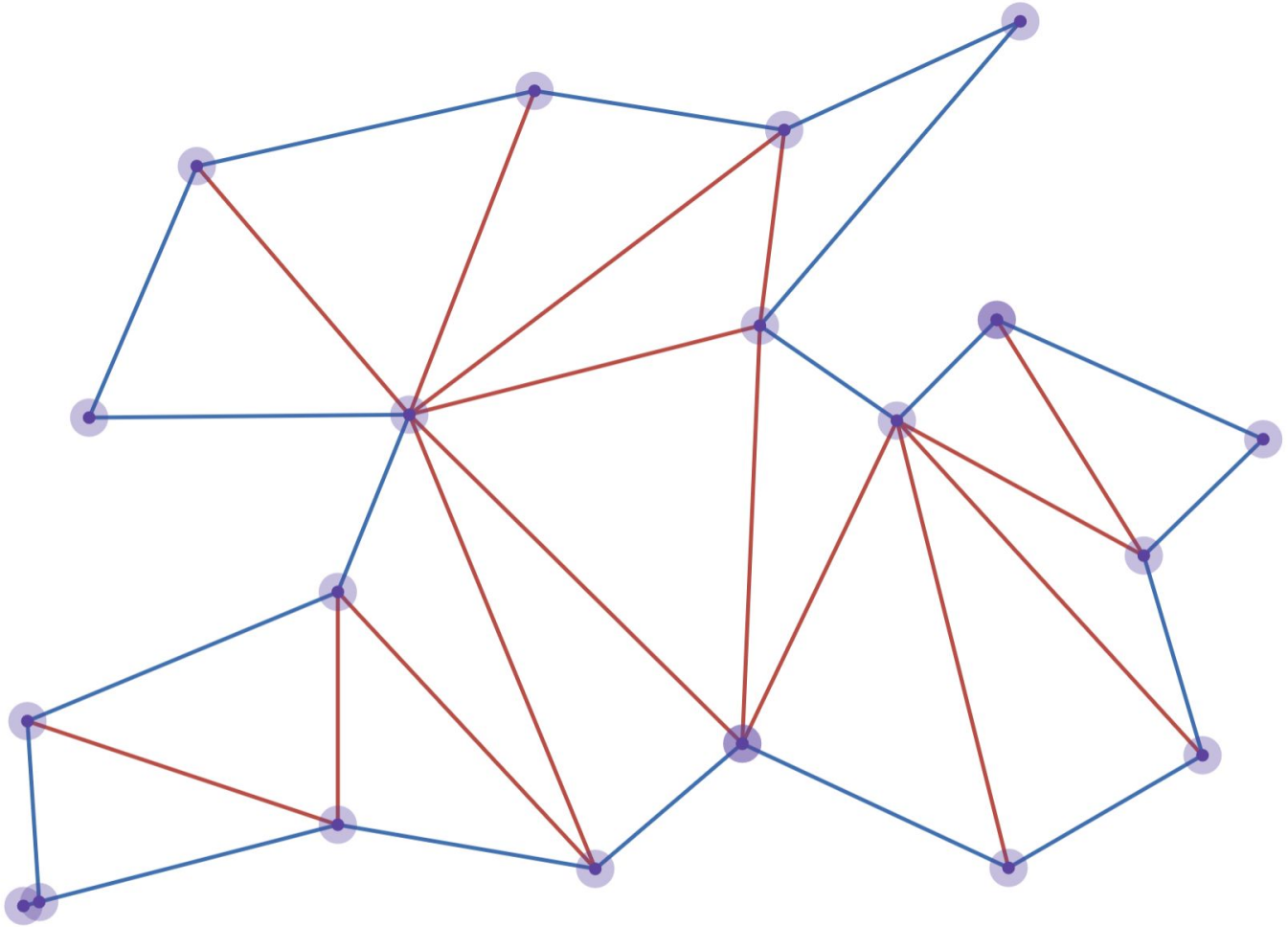
Chvátal's Art Gallery Theorem (G_3 Func.)



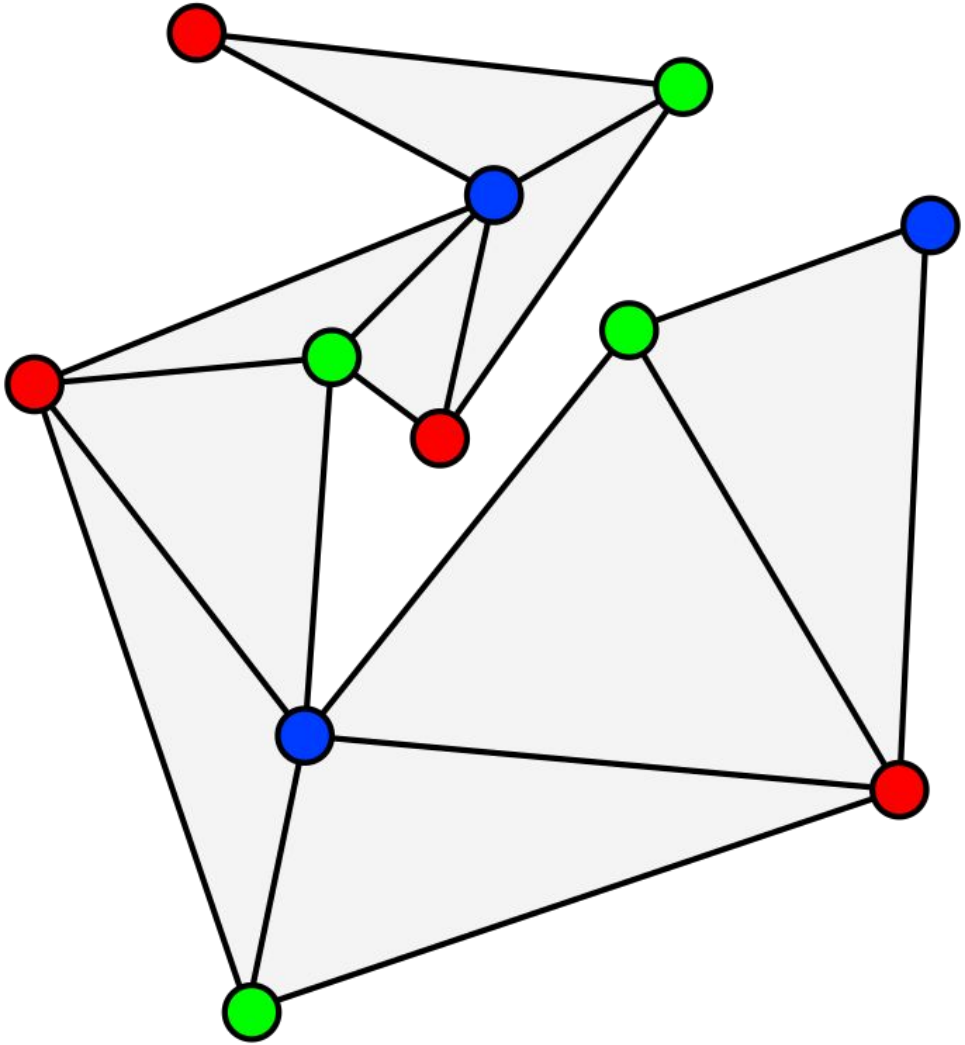




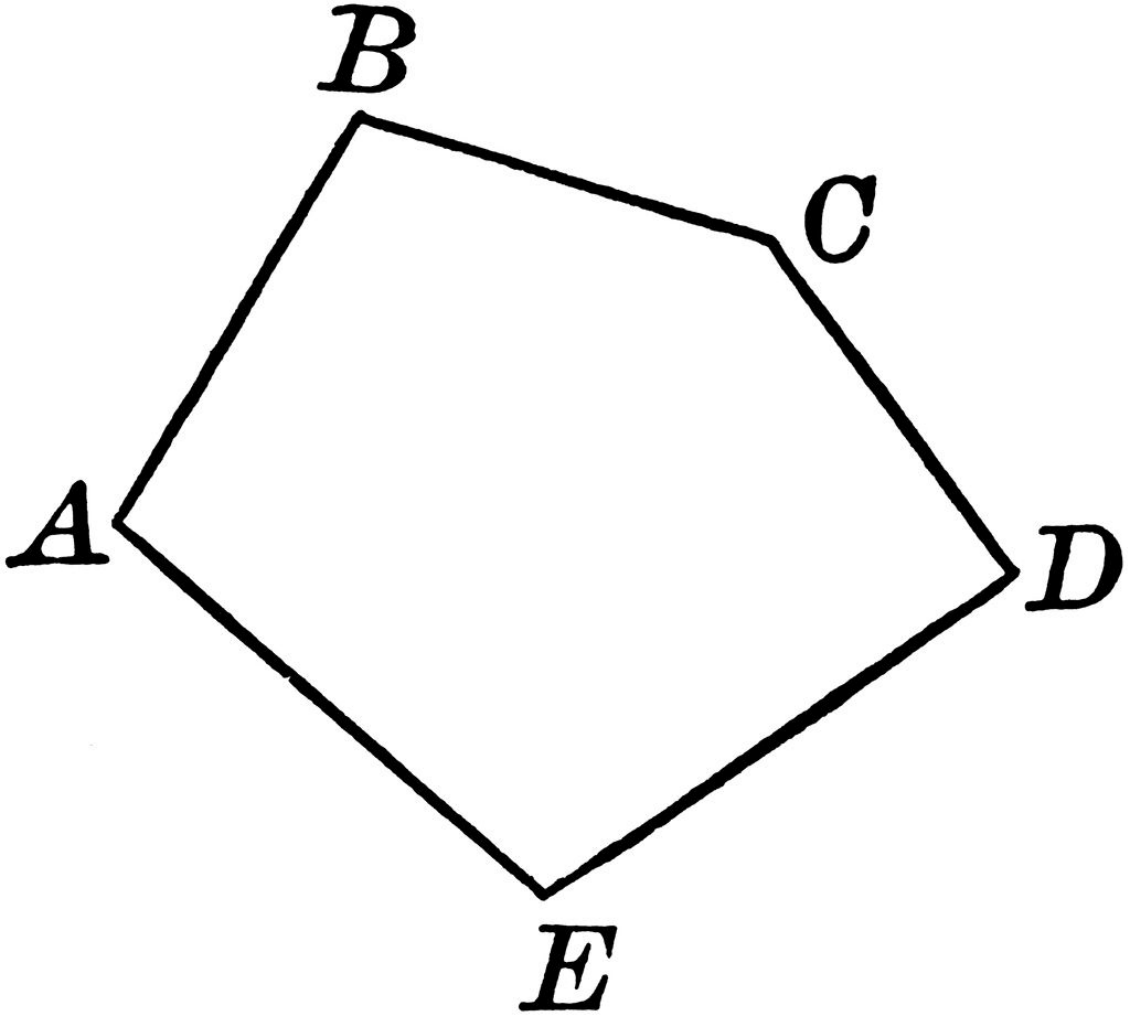




Placement of Guards?



Are As Many Guards Needed?


B
C
A
D
E

Can We Do Any Better?


Well yes, but actually no

The Set Cover Problem

Optimization Online
(supported by the Mathematical Optimization Society)

A Practical Iterative Algorithm for the Art Gallery Problem using Integer Linear Programming

Davi C. Tozoni · Pedro J. de Rezende · Cid C. de Souza

Resources

- A Combinatorial Problem in Geometry by V.Chvatal
- Solving the Art Gallery Problem by CC ACADEMY
- A Practical Iterative Algorithm for the Art Gallery Problem using Integer Linear Programming by Tozoni
- Could a 50-year-old math problem have saved the Louvre from robbery?
- A Constant-Factor Approximation Algorithm for Point Guarding an Art Gallery