$j = 0, 1, 2, 3$

j x Δy g_{jk}

3   x_3 ✗ ✗ ✗ ✗

2   x_2 ✗ ✗ ✗ ✗ Δx

1   x_1 ✗ ✗ ✗ ✗

0   x_0 ✗ ✗ ✗ ✗ y

   y_0 y_1 y_2 y_3
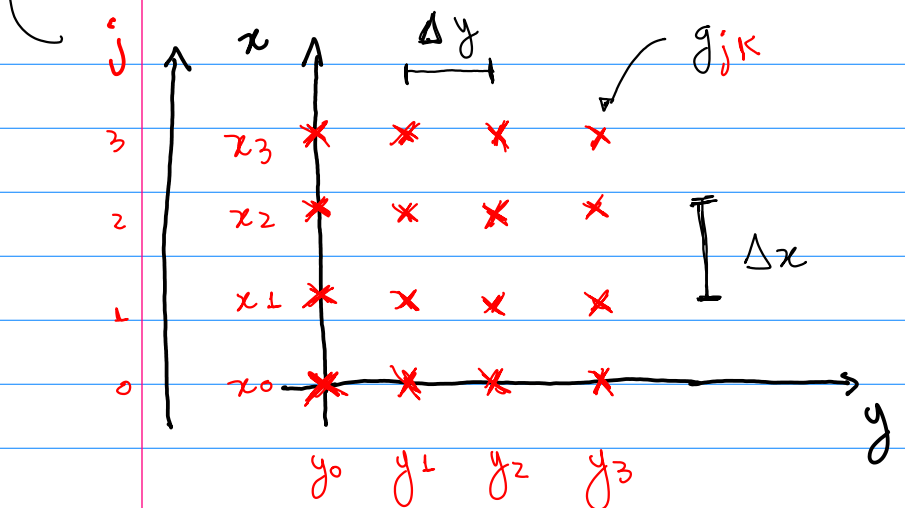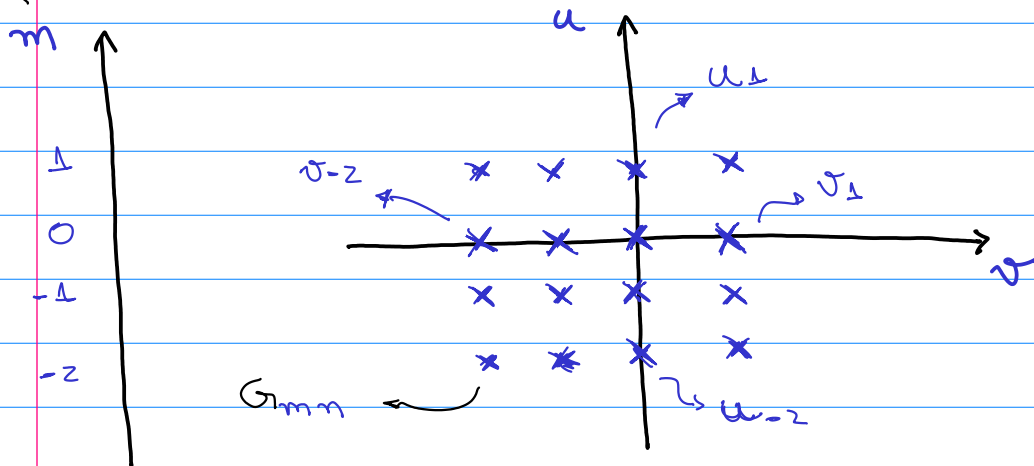
$$g_{M \times N} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}$$

   0   1   2   3 k

$k = 0, 1, 2, 3$

$m = -2, -1, 0, 1$

m u u_1

1   v_{-2} ✗ ✗ ✗ ✗ v_1

0   ✗ ✗ ✗ ✗ v

-1   ✗ ✗ ✗ ✗

-2   ✗ ✗ ✗ ✗ u_{-2}

G_{mn}

$n = -2, -1, 0, 1$

   -2   -1   0   1 n

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$$G_{mn} = \sum_{j=0}^{M-1} \left(\sum_{k=0}^{N-1} g_{jk} \, \omega_N^{(kn)} \right) \omega_M^{(mj)}$$

$$= \sum_{j=0}^{M-1} \left(\begin{bmatrix} g_{j0} & g_{j1} & g_{j2} & g_{j3} \end{bmatrix} \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix} \right) \omega_M^{(mj)}$$

$$= \begin{bmatrix} \omega_M^{(m0)} & \omega_M^{(m1)} & \omega_M^{(m2)} & \omega_M^{(m3)} \end{bmatrix} \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix}$$

$$G_{\underset{\substack{\uparrow \\ m}}{-2} \underset{\substack{\uparrow \\ n}}{-1}} = \begin{bmatrix} \omega_M^{(-20)} & \omega_M^{(-21)} & \omega_M^{(-22)} & \omega_M^{(-23)} \end{bmatrix} \underbrace{\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}}_{g} \begin{bmatrix} \omega_N^{(0-1)} \\ \omega_N^{(1-1)} \\ \omega_N^{(2-1)} \\ \omega_N^{(3-1)} \end{bmatrix}$$

$$\begin{cases} \omega_M^{(m+M)j} = \omega_M^{(mj)} \\ \omega_N^{k(n+N)} = \omega_N^{(kn)} \end{cases}$$

$$\underset{\substack{M=4 \\ -2+4}}{\overbrace{}} \quad \underset{\substack{N=4 \\ -1+4}}{\overbrace{}}$$

$$G_{23} = \begin{bmatrix} \omega_M^{(20)} & \omega_M^{(21)} & \omega_M^{(22)} & \omega_M^{(23)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(03)} \\ \omega_N^{(13)} \\ \omega_N^{(23)} \\ \omega_N^{(33)} \end{bmatrix}$$

$$\begin{array}{cc} \ominus\ominus & \ominus\oplus \end{array}$$

$$\begin{bmatrix} G_{-2-2} & G_{-2-1} & G_{-20} & G_{-21} \\ G_{-1-2} & G_{-1-1} & G_{-10} & G_{-11} \\ \hline G_{0-2} & G_{0-1} & G_{00} & G_{01} \\ G_{1-2} & G_{1-1} & G_{10} & G_{11} \end{bmatrix} = \begin{bmatrix} \omega_M^{(-20)} & \omega_M^{(-21)} & \omega_M^{(-22)} & \omega_M^{(-23)} \\ \omega_M^{(-10)} & \omega_M^{(-11)} & \omega_M^{(-12)} & \omega_M^{(-13)} \\ \hline \omega_M^{(00)} & \omega_M^{(01)} & \omega_M^{(02)} & \omega_M^{(03)} \\ \omega_M^{(10)} & \omega_M^{(11)} & \omega_M^{(12)} & \omega_M^{(13)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(0-2)} & \omega_N^{(0-1)} & \omega_N^{(00)} & \omega_N^{(01)} \\ \omega_N^{(1-2)} & \omega_N^{(1-1)} & \omega_N^{(10)} & \omega_N^{(11)} \\ \hline \omega_N^{(2-2)} & \omega_N^{(2-1)} & \omega_N^{(20)} & \omega_N^{(21)} \\ \omega_N^{(3-2)} & \omega_N^{(3-1)} & \omega_N^{(30)} & \omega_N^{(31)} \end{bmatrix}$$

$$\begin{array}{cc} \oplus\ominus & \oplus\oplus \end{array}$$

$$\begin{array}{cccc} (11) & (10) & (01) & (00) \end{array}$$

$$\begin{bmatrix} (11)\; & (10) \\ G_{22} & G_{23} & G_{20} & G_{21} \\ G_{32} & G_{33} & G_{30} & G_{31} \\ \hline G_{02} & G_{03} & G_{00} & G_{01} \\ G_{12} & G_{13} & G_{10} & G_{11} \\ (01)\; & (00) \end{bmatrix} = \begin{bmatrix} \omega_M^{(20)} & \omega_M^{(21)} & \omega_M^{(22)} & \omega_M^{(23)} \\ \omega_M^{(30)} & \omega_M^{(31)} & \omega_M^{(32)} & \omega_M^{(33)} \\ \hline \omega_M^{(00)} & \omega_M^{(01)} & \omega_M^{(02)} & \omega_M^{(03)} \\ \omega_M^{(10)} & \omega_M^{(11)} & \omega_M^{(12)} & \omega_M^{(13)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(02)} & \omega_N^{(03)} & \omega_N^{(00)} & \omega_N^{(01)} \\ \omega_N^{(12)} & \omega_N^{(13)} & \omega_N^{(10)} & \omega_N^{(11)} \\ \hline \omega_N^{(22)} & \omega_N^{(23)} & \omega_N^{(20)} & \omega_N^{(21)} \\ \omega_N^{(32)} & \omega_N^{(33)} & \omega_N^{(30)} & \omega_N^{(31)} \end{bmatrix}$$

$$\begin{array}{cccc} (00) & (01) & (11) & (10) \end{array}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Permutation matrix used to shift negative frequencies

$$P \begin{bmatrix} (11) & (10) \\ \hline (01) & (00) \end{bmatrix} P = P \begin{bmatrix} (10) & (11) \\ \hline (00) & (01) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P$$

$$\begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P = \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P$$

$$\underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{G} = \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_M} g \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_N}$$

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$$g = \frac{1}{MN} \begin{bmatrix} \tilde{\omega}_m^{(0-2)} & \tilde{\omega}_m^{(0-1)} & \tilde{\omega}_m^{(00)} & \tilde{\omega}_m^{(01)} \\ \tilde{\omega}_m^{(1-2)} & \tilde{\omega}_m^{(1-1)} & \tilde{\omega}_m^{(10)} & \tilde{\omega}_m^{(11)} \\ \tilde{\omega}_m^{(2-2)} & \tilde{\omega}_m^{(2-1)} & \tilde{\omega}_m^{(20)} & \tilde{\omega}_m^{(21)} \\ \tilde{\omega}_m^{(3-2)} & \tilde{\omega}_m^{(3-1)} & \tilde{\omega}_m^{(30)} & \tilde{\omega}_m^{(31)} \end{bmatrix} \begin{bmatrix} \overset{\ominus\ominus}{} & & \overset{\ominus\oplus}{} & \\ G_{-2-2} & G_{-2-1} & G_{-20} & G_{-21} \\ G_{-1-2} & G_{-1-1} & G_{-10} & G_{-11} \\ G_{0-2} & G_{0-1} & G_{00} & G_{01} \\ G_{1-2} & G_{1-1} & G_{10} & G_{11} \\ \underset{\oplus\ominus}{} & & \underset{\oplus\oplus}{} & \end{bmatrix} \begin{bmatrix} \tilde{\omega}_N^{(-20)} & \tilde{\omega}_N^{(-21)} & \tilde{\omega}_N^{(-22)} & \tilde{\omega}_N^{(-23)} \\ \tilde{\omega}_N^{(-10)} & \tilde{\omega}_N^{(-11)} & \tilde{\omega}_N^{(-12)} & \tilde{\omega}_N^{(-13)} \\ \tilde{\omega}_N^{(00)} & \tilde{\omega}_N^{(01)} & \tilde{\omega}_N^{(02)} & \tilde{\omega}_N^{(03)} \\ \tilde{\omega}_N^{(10)} & \tilde{\omega}_N^{(11)} & \tilde{\omega}_N^{(12)} & \tilde{\omega}_N^{(13)} \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} \tilde{\omega}_m^{(02)} & \tilde{\omega}_m^{(03)} & \tilde{\omega}_m^{(00)} & \tilde{\omega}_m^{(01)} \\ \tilde{\omega}_m^{(12)} & \tilde{\omega}_m^{(13)} & \tilde{\omega}_m^{(10)} & \tilde{\omega}_m^{(11)} \\ \tilde{\omega}_m^{(22)} & \tilde{\omega}_m^{(23)} & \tilde{\omega}_m^{(20)} & \tilde{\omega}_m^{(21)} \\ \tilde{\omega}_m^{(32)} & \tilde{\omega}_m^{(33)} & \tilde{\omega}_m^{(30)} & \tilde{\omega}_m^{(31)} \end{bmatrix} \begin{bmatrix} G_{22} & G_{23} & G_{20} & G_{21} \\ G_{32} & G_{33} & G_{30} & G_{31} \\ G_{02} & G_{03} & G_{00} & G_{01} \\ G_{12} & G_{13} & G_{10} & G_{11} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_N^{(20)} & \tilde{\omega}_N^{(21)} & \tilde{\omega}_N^{(22)} & \tilde{\omega}_N^{(23)} \\ \tilde{\omega}_N^{(30)} & \tilde{\omega}_N^{(31)} & \tilde{\omega}_N^{(32)} & \tilde{\omega}_N^{(33)} \\ \tilde{\omega}_N^{(00)} & \tilde{\omega}_N^{(01)} & \tilde{\omega}_N^{(02)} & \tilde{\omega}_N^{(03)} \\ \tilde{\omega}_N^{(10)} & \tilde{\omega}_N^{(11)} & \tilde{\omega}_N^{(12)} & \tilde{\omega}_N^{(13)} \end{bmatrix}$$

(top labels: (01) (00) ; (11) (10) ; (10) (11))
(bottom labels: (11) (10) ; (01) (00) ; (00) (01))

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ (01) & (00) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ (00) & (01) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ (01) & (00) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ (00) & (01) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{F_m^*} \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}}_{F_N^*}$$

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