$N = 6$

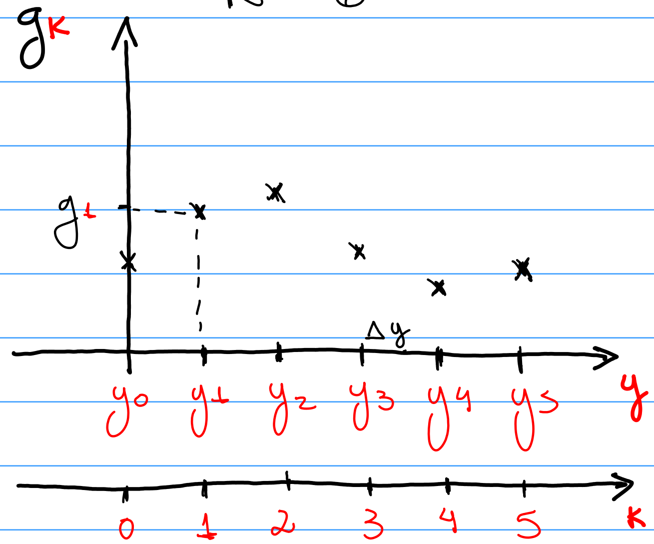
(eq41a)

$g_k \equiv g(y_k)$

(eq41b)

$y_k = k\Delta y$

$k = 0, \dots, N-1$



$$\mathbf{g} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_5 \end{bmatrix}_{6\times 1}$$

$$G_n \approx \frac{G(f_n)}{\Delta y}$$

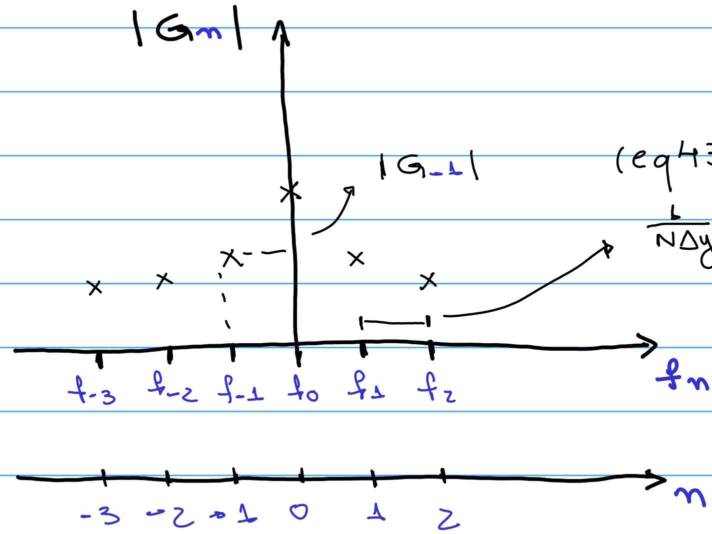
(eq43)

$f_n = \frac{n}{N\Delta y}$

$n = -\frac{N}{2}, \dots, \frac{N}{2} - 1$

(eq43)

$\frac{1}{N\Delta y}$



$$G_n = g_0\, \omega_N^{(n0)} + g_1\, \omega_N^{(n1)} + g_2\, \omega_N^{(n2)} + \dots + g_5\, \omega_N^{(n5)} \quad \text{(eq 45)}$$

$$g_k = \frac{1}{N}\left[G_{-3}\, \tilde{\omega}_N^{(-3k)} + G_{-2}\, \tilde{\omega}_N^{(-2k)} + G_{-1}\, \tilde{\omega}_N^{(-1k)} + \dots + G_2\, \omega_N^{(2k)} \right] \quad \text{(eq 47)}$$

From eq. 45:

$$G_{-3} = g_0\,\omega_6^{(-3\cdot 0)} + g_1\,\omega_6^{(-3\cdot 1)} + \cdots + g_5\,\omega_6^{(-3\cdot 5)} \qquad \boxed{\omega_6 = e^{-i2\pi/6}}$$

$$G_{-2} = g_0\,\omega_6^{(-2\cdot 0)} + g_1\,\omega_6^{(-2\cdot 1)} + \cdots + g_5\,\omega_6^{(-2\cdot 5)}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$G_{2} = g_0\,\omega_6^{(2\cdot 0)} + g_1\,\omega_6^{(2\cdot 1)} + \cdots + g_5\,\omega_6^{(2\cdot 5)}$$

$$\begin{array}{c} f_{-3}\\ f_{-2}\\ f_{-1}\\ f_0\\ f_1\\ f_2 \end{array}
\underbrace{\begin{bmatrix} G_{-3}\\ G_{-2}\\ G_{-1}\\ G_0\\ G_1\\ G_2 \end{bmatrix}}_{6\times 1}
=
\underbrace{\begin{bmatrix}
\omega_6^{(-3\cdot 0)} & \omega_6^{(-3\cdot 1)} & \cdots & \omega_6^{(-3\cdot 5)}\\
\omega_6^{(-2\cdot 0)} & \omega_6^{(-2\cdot 1)} & \cdots & \omega_6^{(-2\cdot 5)}\\
\omega_6^{(-1\cdot 0)} & \omega_6^{(-1\cdot 1)} & \cdots & \omega_6^{(-1\cdot 5)}\\
\omega_6^{(0\cdot 0)} & \omega_6^{(0\cdot 1)} & \cdots & \omega_6^{(0\cdot 5)}\\
\omega_6^{(1\cdot 0)} & \omega_6^{(1\cdot 1)} & \cdots & \omega_6^{(1\cdot 5)}\\
\omega_6^{(2\cdot 0)} & \omega_6^{(2\cdot 1)} & \cdots & \omega_6^{(2\cdot 5)}
\end{bmatrix}}_{6\times 6}
\underbrace{\overset{\mathbf{g}}{\begin{bmatrix} g_0\\ g_1\\ g_2\\ g_3\\ g_4\\ g_5 \end{bmatrix}}}_{6\times 1}
\begin{array}{c} y_0\\ y_1\\ y_2\\ y_3\\ y_4\\ y_5 \end{array}$$

$$\omega_N^{(n+N)k} = \omega_N^{(nk)} \longrightarrow N = 6, \quad
\begin{aligned}
\omega_6^{(-3+6)k} &= \omega_6^{(3\cdot k)}\\
\omega_6^{(-2+6)k} &= \omega_6^{(4\cdot k)}\\
\omega_6^{(-1+6)k} &= \omega_6^{(5\cdot k)}
\end{aligned}$$

$$G_{(n+N)} = g_0\,\omega_N^{(n+N)0} + g_1\,\omega_N^{(n+N)1} + \cdots + g_5\,\omega_N^{(n+N)5} =$$

$$= g_0\,\omega_N^{(n\cdot 0)} + g_1\,\omega_N^{(n\cdot 1)} + \cdots + g_5\,\omega_N^{(n\cdot 5)} = G_n$$

$$\begin{array}{c} f_{3}\\ f_{4}\\ f_{5}\\ f_0\\ f_1\\ f_2 \end{array}
\underbrace{\begin{bmatrix} G_{3}\\ G_{4}\\ G_{5}\\ G_0\\ G_1\\ G_2 \end{bmatrix}}_{6\times 1}
=
\underbrace{\begin{bmatrix}
\omega_6^{(3\cdot 0)} & \omega_6^{(3\cdot 1)} & \cdots & \omega_6^{(3\cdot 5)}\\
\omega_6^{(4\cdot 0)} & \omega_6^{(4\cdot 1)} & \cdots & \omega_6^{(4\cdot 5)}\\
\omega_6^{(5\cdot 0)} & \omega_6^{(5\cdot 1)} & \cdots & \omega_6^{(5\cdot 5)}\\
\omega_6^{(0\cdot 0)} & \omega_6^{(0\cdot 1)} & \cdots & \omega_6^{(0\cdot 5)}\\
\omega_6^{(1\cdot 0)} & \omega_6^{(1\cdot 1)} & \cdots & \omega_6^{(1\cdot 5)}\\
\omega_6^{(2\cdot 0)} & \omega_6^{(2\cdot 1)} & \cdots & \omega_6^{(2\cdot 5)}
\end{bmatrix}}_{6\times 6}
\underbrace{\overset{\mathbf{g}}{\begin{bmatrix} g_0\\ g_1\\ g_2\\ g_3\\ g_4\\ g_5 \end{bmatrix}}}_{6\times 1}
\begin{array}{c} y_0\\ y_1\\ y_2\\ y_3\\ y_4\\ y_5 \end{array}$$

$$P\,\mathbf{G} = P\,\mathbf{F}_6\,\mathbf{g}\;, \qquad P = \begin{bmatrix} 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0 \end{bmatrix}$$

Permutation matrix used to shift negative frequencies

$$\begin{array}{r} f_0\\ f_1\\ f_2\\ f_{-3}\to f_3\\ f_{-2}\to f_4\\ f_{-1}\to f_5 \end{array}\; \underbrace{\begin{bmatrix} G_0\\ G_1\\ G_2\\ G_3\\ G_4\\ G_5 \end{bmatrix}_{6\times1}}_{\mathbf{G}} = \underbrace{\begin{bmatrix} \omega_6^{(0\cdot0)} & \omega_6^{(0\cdot1)} & \cdots & \omega_6^{(0\cdot5)}\\ \omega_6^{(1\cdot0)} & \omega_6^{(1\cdot1)} & \cdots & \omega_6^{(1\cdot5)}\\ \omega_6^{(2\cdot0)} & \omega_6^{(2\cdot1)} & \cdots & \omega_6^{(2\cdot5)}\\ \omega_6^{(3\cdot0)} & \omega_6^{(3\cdot1)} & \cdots & \omega_6^{(3\cdot5)}\\ \omega_6^{(4\cdot0)} & \omega_6^{(4\cdot1)} & \cdots & \omega_6^{(4\cdot5)}\\ \omega_6^{(5\cdot0)} & \omega_6^{(5\cdot1)} & \cdots & \omega_6^{(5\cdot5)} \end{bmatrix}_{6\times6}}_{\mathbf{F}_6} \underbrace{\begin{bmatrix} g_0\\ g_1\\ g_2\\ g_3\\ g_4\\ g_5 \end{bmatrix}_{6\times1}}_{\mathbf{g}} \; \begin{array}{l} y_0\\ y_1\\ y_2\\ y_3\\ y_4\\ y_5 \end{array}$$

$\mathbf{F}_6$ ← DFT matrix for N = 6

$$\boxed{\mathbf{G} = \mathbf{F}_N\,\mathbf{g} \quad (\text{eq. } 49)}$$

From eq 47:

$$g_0 = \frac{1}{6}\left[G_{-3}\,\tilde{\omega}_6^{(-3\cdot0)} + G_{-2}\,\tilde{\omega}_6^{(-2\cdot0)} + \cdots\; G_2\,\tilde{\omega}_6^{(2\cdot0)} \right]$$

$$g_1 = \frac{1}{6}\left[G_{-3}\,\tilde{\omega}_6^{(-3\cdot1)} + G_{-2}\,\tilde{\omega}_6^{(-2\cdot1)} + \cdots\; G_2\,\tilde{\omega}_6^{(2\cdot1)} \right]$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$g_5 = \frac{1}{6}\left[G_{-3}\,\tilde{\omega}_6^{(-3\cdot5)} + G_{-2}\,\tilde{\omega}_6^{(-2\cdot5)} + \cdots\; G_2\,\tilde{\omega}_6^{(2\cdot5)} \right]$$

$$\boxed{\tilde{\omega}_6 = e^{i2\pi/6}}$$

$$\begin{array}{l} y_0\\ y_1\\ y_2\\ y_3\\ y_4\\ y_5 \end{array}\; \overbrace{\begin{bmatrix} g_0\\ g_1\\ g_2\\ g_3\\ g_4\\ g_5 \end{bmatrix}_{6\times1}}^{\mathbf{g}} = \frac{1}{6} \begin{bmatrix} \tilde{\omega}_6^{(-3\cdot0)} & \tilde{\omega}_6^{(-2\cdot0)} & \cdots & \tilde{\omega}_6^{(2\cdot0)}\\ \tilde{\omega}_6^{(-3\cdot1)} & \tilde{\omega}_6^{(-2\cdot1)} & \cdots & \tilde{\omega}_6^{(2\cdot1)}\\ \tilde{\omega}_6^{(-3\cdot2)} & \tilde{\omega}_6^{(-2\cdot2)} & \cdots & \tilde{\omega}_6^{(2\cdot2)}\\ \tilde{\omega}_6^{(-3\cdot3)} & \tilde{\omega}_6^{(-2\cdot3)} & \cdots & \tilde{\omega}_6^{(2\cdot3)}\\ \tilde{\omega}_6^{(-3\cdot4)} & \tilde{\omega}_6^{(-2\cdot4)} & \cdots & \tilde{\omega}_6^{(2\cdot4)}\\ \tilde{\omega}_6^{(-3\cdot5)} & \tilde{\omega}_6^{(-2\cdot5)} & \cdots & \tilde{\omega}_6^{(2\cdot5)} \end{bmatrix}_{6\times6} \begin{bmatrix} G_{-3}\\ G_{-2}\\ G_{-1}\\ G_0\\ G_1\\ G_2 \end{bmatrix}_{6\times1} \; \begin{array}{l} f_{-3}\\ f_{-2}\\ f_{-1}\\ f_0\\ f_1\\ f_2 \end{array}$$

$$\tilde{\omega}_N^{(n+N)k} = \tilde{\omega}_N^{(nk)} \longrightarrow N = 6 \; , \quad \begin{aligned} \tilde{\omega}_6^{(-3+6)k} &= \tilde{\omega}_6^{(3\cdot k)} \\ \tilde{\omega}_6^{(-2+6)k} &= \tilde{\omega}_6^{(4\cdot k)} \\ \tilde{\omega}_6^{(-1+6)k} &= \tilde{\omega}_6^{(5\cdot k)} \end{aligned}$$

$$g_k = \frac{1}{6}\left[G_{(-3+6)}\,\tilde{\omega}_6^{(-3+6)k} + G_{(-2+6)}\,\tilde{\omega}_6^{(-2+6)k} + G_{(-1+6)}\,\tilde{\omega}_6^{(-1+6)k} + \cdots \right] =$$

$$= \frac{1}{6}\left[G_3\,\tilde{\omega}_6^{(3\cdot k)} + G_4\,\tilde{\omega}_6^{(4\cdot k)} + G_5\,\tilde{\omega}_6^{(5\cdot k)} + \cdots \right]$$

$$\begin{array}{c} \mathbf{g} \\ y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array}
\underbrace{\begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix}}_{6\times 1}
= \frac{1}{6}
\underbrace{\begin{bmatrix}
\tilde{\omega}_6^{(3\cdot 0)} & \tilde{\omega}_6^{(4\cdot 0)} & \cdots & \tilde{\omega}_6^{(2\cdot 0)} \\
\tilde{\omega}_6^{(3\cdot 1)} & \tilde{\omega}_6^{(4\cdot 1)} & \cdots & \tilde{\omega}_6^{(2\cdot 1)} \\
\tilde{\omega}_6^{(3\cdot 2)} & \tilde{\omega}_6^{(4\cdot 2)} & \cdots & \tilde{\omega}_6^{(2\cdot 2)} \\
\tilde{\omega}_6^{(3\cdot 3)} & \tilde{\omega}_6^{(4\cdot 3)} & \cdots & \tilde{\omega}_6^{(2\cdot 3)} \\
\tilde{\omega}_6^{(3\cdot 4)} & \tilde{\omega}_6^{(4\cdot 4)} & \cdots & \tilde{\omega}_6^{(2\cdot 4)} \\
\tilde{\omega}_6^{(3\cdot 5)} & \tilde{\omega}_6^{(4\cdot 5)} & \cdots & \tilde{\omega}_6^{(2\cdot 5)}
\end{bmatrix}}_{6\times 6}
\underbrace{\begin{bmatrix} G_3 \\ G_4 \\ G_5 \\ G_0 \\ G_1 \\ G_2 \end{bmatrix}}_{6\times 1}
\begin{array}{l} f_3 \leftarrow f_{-3} \\ f_4 \leftarrow f_{-2} \\ f_5 \leftarrow f_{-1} \\ f_0 \\ f_1 \\ f_2 \end{array}$$

$$[\;] = [\qquad] \, P^T P \, [\;]$$

$$\begin{array}{c} \mathbf{g} \\ y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array}
\underbrace{\begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix}}_{6\times 1}
= \frac{1}{6}
\underbrace{\underbrace{\begin{bmatrix}
\tilde{\omega}_6^{(0\cdot 0)} & \tilde{\omega}_6^{(1\cdot 0)} & \cdots & \tilde{\omega}_6^{(5\cdot 0)} \\
\tilde{\omega}_6^{(0\cdot 1)} & \tilde{\omega}_6^{(1\cdot 1)} & \cdots & \tilde{\omega}_6^{(5\cdot 1)} \\
\tilde{\omega}_6^{(0\cdot 2)} & \tilde{\omega}_6^{(1\cdot 2)} & \cdots & \tilde{\omega}_6^{(5\cdot 2)} \\
\tilde{\omega}_6^{(0\cdot 3)} & \tilde{\omega}_6^{(1\cdot 3)} & \cdots & \tilde{\omega}_6^{(5\cdot 3)} \\
\tilde{\omega}_6^{(0\cdot 4)} & \tilde{\omega}_6^{(1\cdot 4)} & \cdots & \tilde{\omega}_6^{(5\cdot 4)} \\
\tilde{\omega}_6^{(0\cdot 5)} & \tilde{\omega}_6^{(1\cdot 5)} & \cdots & \tilde{\omega}_6^{(5\cdot 5)}
\end{bmatrix}}_{6\times 6}}_{\mathbf{F}_6^*}
\underbrace{\underbrace{\begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \end{bmatrix}}_{6\times 1}}_{\mathbf{G}}
\begin{array}{l} f_0 \\ f_1 \\ f_2 \\ f_3 \leftarrow f_{-3} \\ f_4 \leftarrow f_{-2} \\ f_5 \leftarrow f_{-1} \end{array}$$

$$\boxed{\mathbf{g} = \frac{1}{N}\mathbf{F}_N^* \mathbf{G} \quad (\text{eq } 50)}$$