

# 2D convolution

## Circular convolution

$$M_a = 4$$

$$N_a = 4$$

$$A = \begin{bmatrix} 00 & 01 & 02 & 03 \\ 10 & 11 & 12 & 13 \\ 20 & 21 & 22 & 23 \\ 30 & 31 & 32 & 33 \end{bmatrix}$$

$M_a \times N_a$

$a_{31}$

$$B = \begin{bmatrix} 00 & 01 & 02 & 03 \\ 10 & 11 & 12 & 13 \\ 20 & 21 & 22 & 23 \\ 30 & 31 & 32 & 33 \end{bmatrix}$$

$M_b \times N_b$

$b_{32}$

$$M_b = 4$$

$$N_b = 4$$

Note that the elements of matrices A and B are represented by using only their indices. Otherwise I would get crazy.

The circular convolution of A and B produces a third matrix W.

The elements of W can be computed by following the steps below:

- ① Put matrix A in a fixed position

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33

- ② Create a temporary matrix by flipping the rows and columns of matrix B

33	32	31	30
23	22	21	20
13	12	11	10
03	02	01	00

- ③ Expand the temporary matrix by repeating the flipped matrix

33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00
33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00

- ④ Put the expanded matrix over matrix A

33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00
33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00

- ⑤ Compute the 00 element of matrix W by adding all the products of superposed elements

$$w_{00} = a_{00}b_{00} + a_{01}b_{03} + a_{02}b_{02} + a_{03}b_{01} +$$

$$+ a_{10}b_{30} + a_{11}b_{33} + a_{12}b_{32} + a_{13}b_{31} +$$

$$+ a_{20}b_{20} + a_{21}b_{23} + a_{22}b_{22} + a_{23}b_{21} +$$

$$+ a_{30}b_{10} + a_{31}b_{13} + a_{32}b_{12} + a_{33}b_{11}$$

- ⑥ The remaining elements of W are computed by sliding the temporary matrix and adding the products of superposed elements

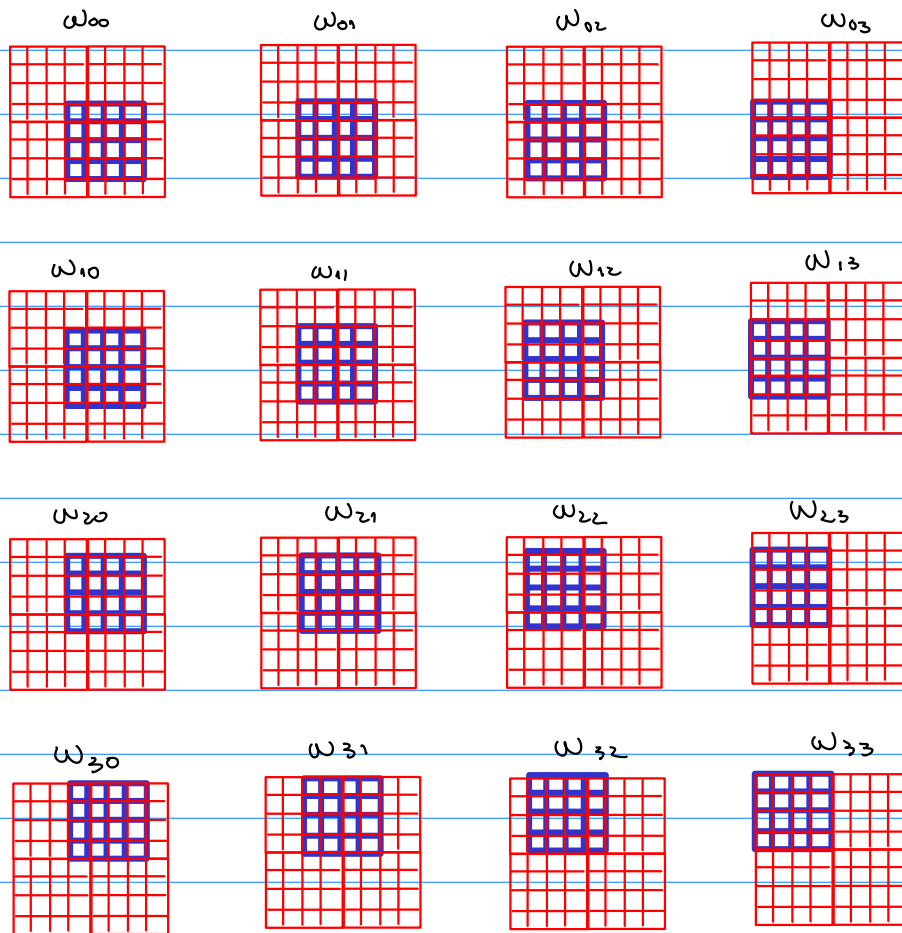
$w_{01}$

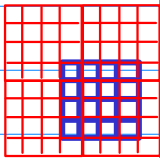
33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00
33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00

$w_{10}$

33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00
33	32	31	30	33	32	31	30
23	22	21	20	23	22	21	20
13	12	11	10	13	12	11	10
03	02	01	00	03	02	01	00

By repeating step 6, we obtain all elements of  $W$ :





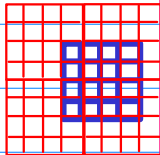
$$00 = \begin{matrix} 00 & 10 & 20 & 30 \\ 00 & 30 & 20 & 10 \end{matrix} \quad \begin{matrix} 01 & 11 & 21 & 31 \\ 03 & 33 & 23 & 13 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 02 & 32 & 22 & 12 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 01 & 31 & 21 & 11 \end{matrix}$$

Note that:

1) All elements are conveniently represented by using only their indices in order to make things as clean as possible;

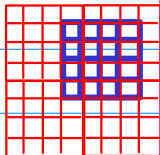
2) All plus "+" symbols are conveniently omitted in order to make things as clean as possible;

3) The elements of A are arranged by column.

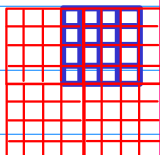


$$10 = \begin{matrix} 00 & 10 & 20 & 30 \\ 10 & 00 & 30 & 20 \end{matrix} \quad \begin{matrix} 01 & 11 & 21 & 31 \\ 13 & 03 & 33 & 23 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 12 & 02 & 32 & 22 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 11 & 01 & 31 & 21 \end{matrix}$$

$$C_k = \begin{bmatrix} b_{0k} & b_{3k} & b_{2k} & b_{1k} \\ b_{1k} & b_{0k} & b_{3k} & b_{2k} \\ b_{2k} & b_{1k} & b_{0k} & b_{3k} \\ b_{3k} & b_{2k} & b_{1k} & b_{0k} \end{bmatrix}$$



$$20 = \begin{matrix} 00 & 10 & 20 & 30 \\ 20 & 10 & 00 & 30 \end{matrix} \quad \begin{matrix} 01 & 11 & 21 & 31 \\ 23 & 13 & 03 & 33 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 22 & 12 & 02 & 32 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 21 & 11 & 01 & 31 \end{matrix}$$

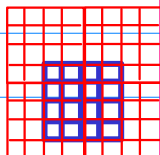


$$30 = \begin{matrix} 00 & 10 & 20 & 30 \\ 30 & 20 & 10 & 00 \end{matrix} \quad \begin{matrix} 01 & 11 & 21 & 31 \\ 33 & 23 & 13 & 03 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 32 & 22 & 12 & 02 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 31 & 21 & 11 & 01 \end{matrix}$$

$$w_k = \begin{bmatrix} w_{0k} \\ w_{1k} \\ w_{2k} \\ w_{3k} \end{bmatrix} \quad a_k = \begin{bmatrix} a_{0k} \\ a_{1k} \\ a_{2k} \\ a_{3k} \end{bmatrix}$$

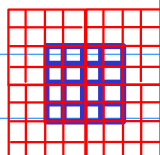
$$w_0 = C_0 a_0 + C_3 a_1 + C_2 a_2 + C_1 a_3$$

$$A = [a_0 a_1 a_2 a_3]$$

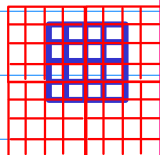


$$01 = \begin{matrix} 00 & 01 & 02 & 03 \\ 01 & 31 & 21 & 11 \end{matrix} \quad \begin{matrix} 10 & 11 & 21 & 31 \\ 00 & 30 & 20 & 10 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 03 & 33 & 23 & 13 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 02 & 32 & 22 & 12 \end{matrix}$$

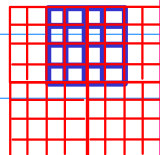
$$W = [w_0 w_1 w_2 w_3]$$



$$11 = \begin{matrix} 00 & 01 & 02 & 03 \\ 11 & 01 & 31 & 21 \end{matrix} \quad \begin{matrix} 10 & 11 & 21 & 31 \\ 10 & 00 & 30 & 20 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 13 & 03 & 33 & 23 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 12 & 02 & 32 & 22 \end{matrix}$$

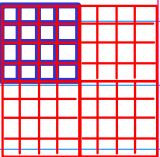
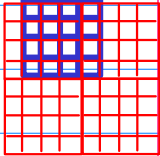


$$21 = \begin{matrix} 00 & 01 & 02 & 03 \\ 21 & 11 & 01 & 31 \end{matrix} \quad \begin{matrix} 10 & 11 & 21 & 31 \\ 20 & 10 & 00 & 30 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 23 & 13 & 03 & 33 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 22 & 12 & 02 & 32 \end{matrix}$$



$$31 = \begin{matrix} 00 & 01 & 02 & 03 \\ 31 & 21 & 11 & 01 \end{matrix} \quad \begin{matrix} 10 & 11 & 21 & 31 \\ 30 & 20 & 10 & 00 \end{matrix} \quad \begin{matrix} 02 & 12 & 22 & 32 \\ 33 & 23 & 13 & 03 \end{matrix} \quad \begin{matrix} 03 & 13 & 23 & 33 \\ 32 & 22 & 12 & 02 \end{matrix}$$

$$w_1 = C_1 a_0 + C_0 a_1 + C_3 a_2 + C_2 a_3$$



$$\omega_3 = c_3 a_0 + c_2 a_1 + c_1 a_2 + c_0 a_3$$

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_0 & c_3 & c_2 & c_1 \\ c_1 & c_0 & c_3 & c_2 \\ c_2 & c_1 & c_0 & c_3 \\ c_3 & c_2 & c_1 & c_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$