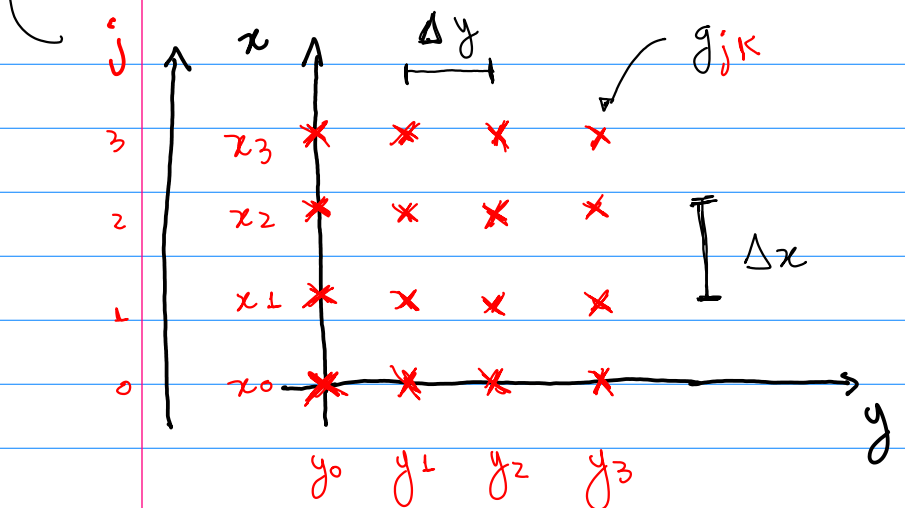
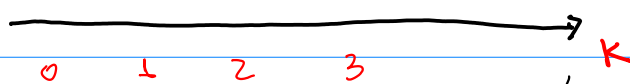


$$j = 0, 1, 2, 3$$

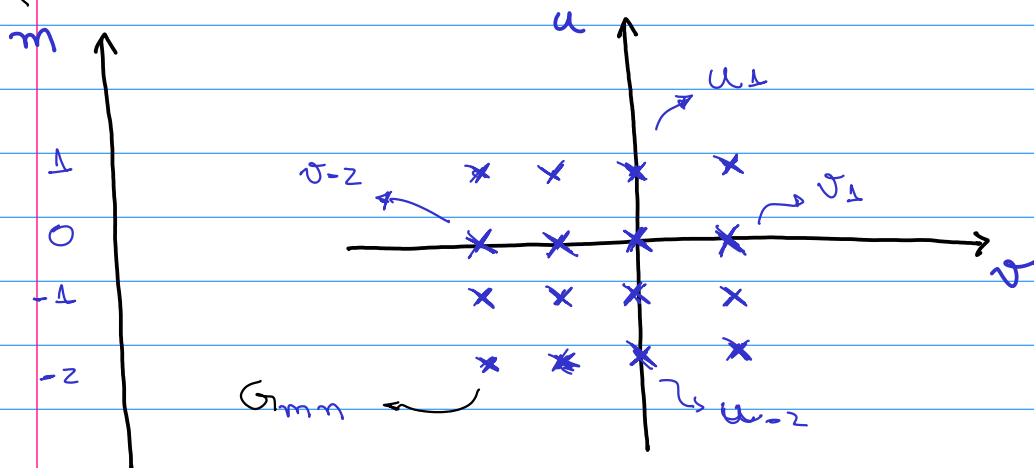


$$g_{M \times N} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}$$

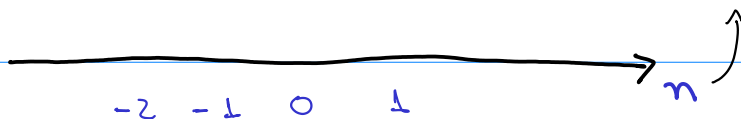


$$k = 0, 1, 2, 3$$

$$m = -2, -1, 0, 1$$



$$n = -2, -1, 0, 1, 2$$



eq 7b

$$G_{mn} = \sum_{j=0}^{M-1} \left(\sum_{k=0}^{N-1} g_{jk} \omega_N^{(kn)} \right) \omega_m^{(mj)}$$

$$= \sum_{j=0}^{M-1} \left([g_{j0} \ g_{j1} \ g_{j2} \ g_{j3}] \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix} \right) \omega_m^{(mj)}$$

$$= [\omega_m^{(m0)} \ \omega_m^{(m1)} \ \omega_m^{(m2)} \ \omega_m^{(m3)}] \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix}$$

$$G_{\substack{-2-1 \\ m \quad n}} = [\omega_m^{(-20)} \ \omega_m^{(-21)} \ \omega_m^{(-22)} \ \omega_m^{(-23)}] \underbrace{\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}}_g \begin{bmatrix} \omega_N^{(0-1)} \\ \omega_N^{(1-1)} \\ \omega_N^{(2-1)} \\ \omega_N^{(3-1)} \end{bmatrix}$$

$$\boxed{\begin{aligned} \omega_m^{(m+m)} j &= \omega_m^{(mj)} \\ \omega_N^{(k(m+N))} &= \omega_N^{(kn)} \end{aligned}}$$

$$G_{\substack{23 \\ m \quad n}} = [\omega_m^{(20)} \ \omega_m^{(21)} \ \omega_m^{(22)} \ \omega_m^{(23)}] g \begin{bmatrix} \omega_N^{(03)} \\ \omega_N^{(13)} \\ \omega_N^{(23)} \\ \omega_N^{(33)} \end{bmatrix}$$

$$\begin{array}{cc} \ominus \ominus & \ominus \oplus \\ \begin{bmatrix} G_{-2-2} & G_{-2-1} & G_{-20} & G_{-21} \\ G_{-1-2} & G_{-1-1} & G_{-10} & G_{-11} \\ \hline G_{0-2} & G_{0-1} & G_{00} & G_{01} \\ G_{1-2} & G_{1-1} & G_{10} & G_{11} \end{bmatrix} & = \begin{bmatrix} \omega_m^{(-20)} & \omega_m^{(-21)} & \omega_m^{(-22)} & \omega_m^{(-23)} \\ \omega_m^{(-10)} & \omega_m^{(-11)} & \omega_m^{(-12)} & \omega_m^{(-13)} \\ \hline \omega_m^{(00)} & \omega_m^{(01)} & \omega_m^{(02)} & \omega_m^{(03)} \\ \omega_m^{(10)} & \omega_m^{(11)} & \omega_m^{(12)} & \omega_m^{(13)} \end{bmatrix} \\ \oplus \ominus & \oplus \oplus \end{array} \quad g \quad \begin{bmatrix} \omega_n^{(0-2)} & \omega_n^{(0-1)} & \omega_n^{(00)} & \omega_n^{(01)} \\ \omega_n^{(1-2)} & \omega_n^{(1-1)} & \omega_n^{(10)} & \omega_n^{(11)} \\ \hline \omega_n^{(2-2)} & \omega_n^{(2-1)} & \omega_n^{(20)} & \omega_n^{(21)} \\ \omega_n^{(3-2)} & \omega_n^{(3-1)} & \omega_n^{(30)} & \omega_n^{(31)} \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{cc} (10) & (11) \end{array} \\ \begin{bmatrix} G_{22} & G_{23} & G_{20} & G_{21} \\ G_{32} & G_{33} & G_{30} & G_{31} \\ \hline G_{02} & G_{03} & G_{00} & G_{01} \\ G_{12} & G_{13} & G_{10} & G_{11} \end{bmatrix} & = \begin{bmatrix} \omega_m^{(20)} & \omega_m^{(21)} & \omega_m^{(22)} & \omega_m^{(23)} \\ \omega_m^{(30)} & \omega_m^{(31)} & \omega_m^{(32)} & \omega_m^{(33)} \\ \hline \omega_m^{(00)} & \omega_m^{(01)} & \omega_m^{(02)} & \omega_m^{(03)} \\ \omega_m^{(10)} & \omega_m^{(11)} & \omega_m^{(12)} & \omega_m^{(13)} \end{bmatrix} \\ \begin{array}{cc} (01) & (00) \end{array} & \begin{array}{cc} (00) & (01) \end{array} \end{array} \quad g \quad \begin{bmatrix} \omega_n^{(02)} & \omega_n^{(03)} & \omega_n^{(00)} & \omega_n^{(01)} \\ \omega_n^{(12)} & \omega_n^{(13)} & \omega_n^{(10)} & \omega_n^{(11)} \\ \hline \omega_n^{(22)} & \omega_n^{(23)} & \omega_n^{(20)} & \omega_n^{(21)} \\ \omega_n^{(32)} & \omega_n^{(33)} & \omega_n^{(30)} & \omega_n^{(31)} \end{bmatrix} \\ & \begin{array}{cc} (11) & (10) \end{array} \end{array}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Permutation matrix used to shift negative frequencies

$$P \begin{bmatrix} (11) & (10) \\ \hline (01) & (00) \end{bmatrix} P = P \begin{bmatrix} (10) & (11) \\ \hline (00) & (01) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P$$

$$\begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P = \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P$$

$$\underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_G = \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_m} g \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_n}$$

(eq 9)

$$g = \frac{1}{M_N} \begin{bmatrix} \tilde{\omega}_m^{(0-2)} & \tilde{\omega}_m^{(0-1)} & \tilde{\omega}_m^{(00)} & \tilde{\omega}_m^{(01)} \\ \tilde{\omega}_m^{(1-2)} & \tilde{\omega}_m^{(1-1)} & \tilde{\omega}_m^{(10)} & \tilde{\omega}_m^{(11)} \\ \tilde{\omega}_m^{(2-2)} & \tilde{\omega}_m^{(2-1)} & \tilde{\omega}_m^{(20)} & \tilde{\omega}_m^{(21)} \\ \tilde{\omega}_m^{(3-2)} & \tilde{\omega}_m^{(3-1)} & \tilde{\omega}_m^{(30)} & \tilde{\omega}_m^{(31)} \end{bmatrix} \begin{bmatrix} \ominus \ominus & \ominus \oplus \\ G_{-2-2} & G_{-2-1} & G_{-20} & G_{-21} \\ \hline G_{-1-2} & G_{-1-1} & G_{-10} & G_{-11} \\ \hline G_{0-2} & G_{0-1} & G_{00} & G_{01} \\ \hline G_{1-2} & G_{1-1} & G_{10} & G_{11} \\ \oplus \ominus & \oplus \oplus \end{bmatrix} \begin{bmatrix} \tilde{\omega}_n^{(-20)} & \tilde{\omega}_n^{(-21)} & \tilde{\omega}_n^{(-22)} & \tilde{\omega}_n^{(-23)} \\ \tilde{\omega}_n^{(-10)} & \tilde{\omega}_n^{(-11)} & \tilde{\omega}_n^{(-12)} & \tilde{\omega}_n^{(-13)} \\ \tilde{\omega}_n^{(00)} & \tilde{\omega}_n^{(01)} & \tilde{\omega}_n^{(02)} & \tilde{\omega}_n^{(03)} \\ \tilde{\omega}_n^{(10)} & \tilde{\omega}_n^{(11)} & \tilde{\omega}_n^{(12)} & \tilde{\omega}_n^{(13)} \end{bmatrix}$$

$$g = \frac{1}{M_N} \begin{bmatrix} \tilde{\omega}_m^{(01)} & \tilde{\omega}_m^{(00)} & \tilde{\omega}_m^{(00)} & \tilde{\omega}_m^{(01)} \\ \tilde{\omega}_m^{(12)} & \tilde{\omega}_m^{(13)} & \tilde{\omega}_m^{(10)} & \tilde{\omega}_m^{(11)} \\ \tilde{\omega}_m^{(22)} & \tilde{\omega}_m^{(23)} & \tilde{\omega}_m^{(20)} & \tilde{\omega}_m^{(21)} \\ \tilde{\omega}_m^{(32)} & \tilde{\omega}_m^{(33)} & \tilde{\omega}_m^{(30)} & \tilde{\omega}_m^{(31)} \end{bmatrix} \begin{bmatrix} (11) & (10) \\ G_{22} & G_{23} & G_{20} & G_{21} \\ \hline G_{32} & G_{33} & G_{30} & G_{31} \\ \hline G_{02} & G_{03} & G_{00} & G_{01} \\ \hline G_{12} & G_{13} & G_{10} & G_{11} \\ (01) & (00) \end{bmatrix} \begin{bmatrix} (10) & (11) \\ \tilde{\omega}_n^{(20)} & \tilde{\omega}_n^{(21)} & \tilde{\omega}_n^{(22)} & \tilde{\omega}_n^{(23)} \\ \tilde{\omega}_n^{(30)} & \tilde{\omega}_n^{(31)} & \tilde{\omega}_n^{(32)} & \tilde{\omega}_n^{(33)} \\ \tilde{\omega}_n^{(00)} & \tilde{\omega}_n^{(01)} & \tilde{\omega}_n^{(02)} & \tilde{\omega}_n^{(03)} \\ \tilde{\omega}_n^{(10)} & \tilde{\omega}_n^{(11)} & \tilde{\omega}_n^{(12)} & \tilde{\omega}_n^{(13)} \end{bmatrix}$$

(11) (10) (01) (00)

$$g = \frac{1}{M_N} \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ \hline (01) & (00) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ \hline (00) & (01) \end{bmatrix}$$

$$g = \frac{1}{M_N} \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ \hline (01) & (00) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix} \curvearrowright$$

$$g = \frac{1}{M_N} \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ \hline (00) & (01) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}$$

$$g = \frac{1}{M_N} \begin{bmatrix} (01) & (00) \\ \hline (11) & (10) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix} \curvearrowright$$

$$g = \frac{1}{M_N} \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_m^*} \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_G \underbrace{\begin{bmatrix} (00) & (01) \\ \hline (10) & (11) \end{bmatrix}}_{F_n^*}$$