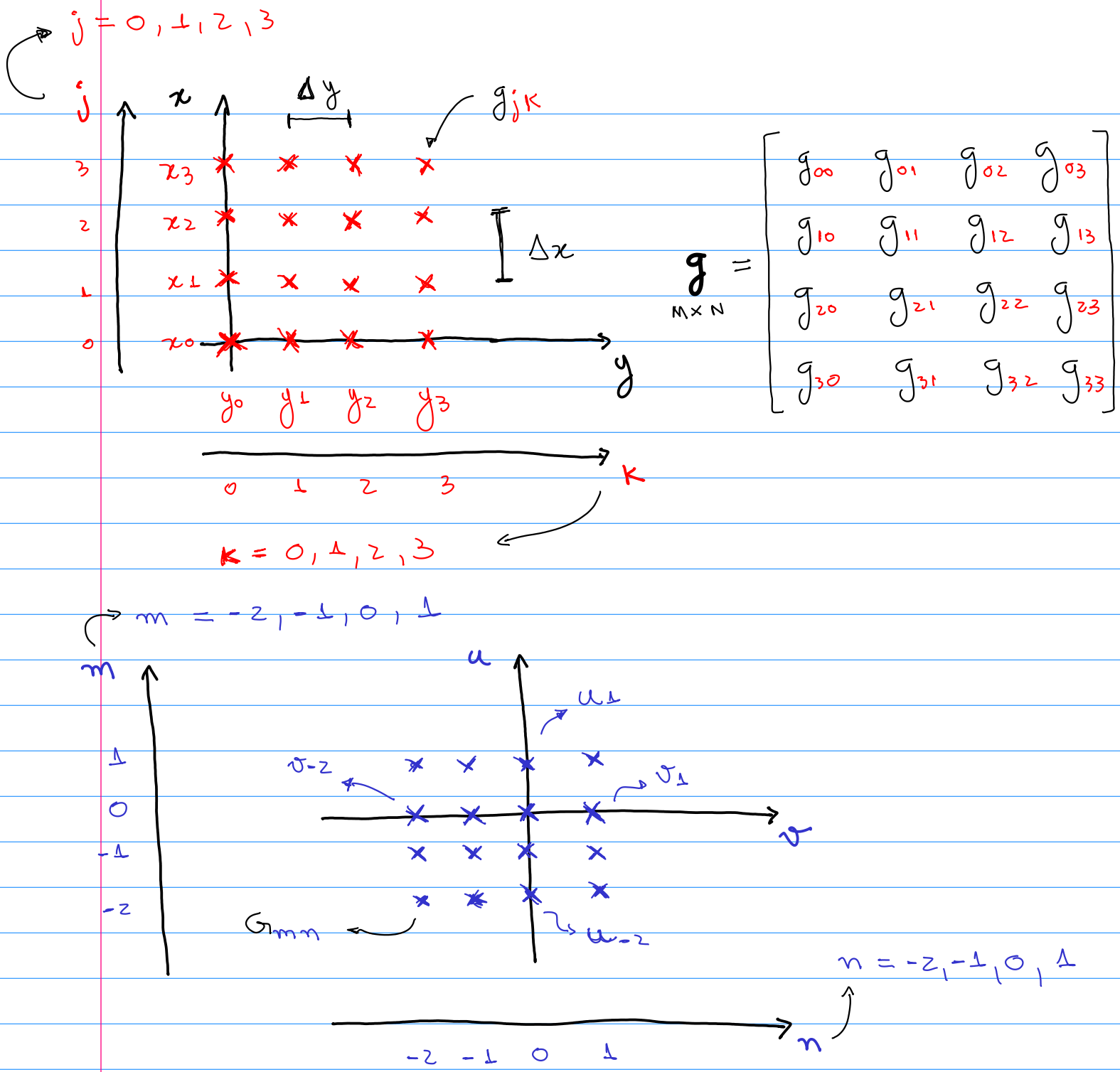
j = 0, 1, 2, 3
j
x
Δy
g_{jk}
3
2
1
0
x_3
x_2
x_1
x_0
Δx
y
y_0 y_1 y_2 y_3
k
0 1 2 3
k = 0, 1, 2, 3
$g_{M \times N}$ =
g_{00} g_{01} g_{02} g_{03}
g_{10} g_{11} g_{12} g_{13}
g_{20} g_{21} g_{22} g_{23}
g_{30} g_{31} g_{32} g_{33}
m = -2, -1, 0, 1
m
1
0
-1
-2
u
u_1
u_{-2}
v_{-2}
v_1
v
G_{mn}
n = -2, -1, 0, 1
n
-2 -1 0 1

eq 76

$$G_{mn} = \sum_{j=0}^{M-1} \left(\sum_{k=0}^{N-1} g_{jk}\, \omega_N^{(kn)} \right) \omega_M^{(mj)}$$

$$= \sum_{j=0}^{M-1} \left(\begin{bmatrix} g_{j0} & g_{j1} & g_{j2} & g_{j3} \end{bmatrix} \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix} \right) \omega_M^{(mj)}$$

$$= \begin{bmatrix} \omega_M^{(m0)} & \omega_M^{(m1)} & \omega_M^{(m2)} & \omega_M^{(m3)} \end{bmatrix} \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} \omega_N^{(0n)} \\ \omega_N^{(1n)} \\ \omega_N^{(2n)} \\ \omega_N^{(3n)} \end{bmatrix}$$

$G_{-2\,-1}$ (m, n)

$$= \begin{bmatrix} \omega_M^{(-20)} & \omega_M^{(-21)} & \omega_M^{(-22)} & \omega_M^{(-23)} \end{bmatrix} \underbrace{\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}}_{g} \begin{bmatrix} \omega_N^{(0-1)} \\ \omega_N^{(1-1)} \\ \omega_N^{(2-1)} \\ \omega_N^{(3-1)} \end{bmatrix}$$

$$\omega_M^{(m+M)j} = \omega_M^{(mj)}$$
$$\omega_N^{k(n+N)} = \omega_N^{(kn)}$$

M=4, N=4

$$G_{23} = \begin{bmatrix} \omega_M^{(20)} & \omega_M^{(21)} & \omega_M^{(22)} & \omega_M^{(23)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(63)} \\ \omega_N^{(13)} \\ \omega_N^{(23)} \\ \omega_N^{(33)} \end{bmatrix}$$

$-2+4$, $-1+4$

$$\begin{bmatrix} G_{-2-2} & G_{-2-1} & G_{-20} & G_{-21} \\ G_{-1-2} & G_{-1-1} & G_{-10} & G_{-11} \\ G_{0-2} & G_{0-1} & G_{00} & G_{01} \\ G_{1-2} & G_{1-1} & G_{10} & G_{11} \end{bmatrix} = \begin{bmatrix} \omega_M^{(-2\,0)} & \omega_M^{(-2\,1)} & \omega_M^{(-2\,2)} & \omega_M^{(-2\,3)} \\ \omega_M^{(-1\,0)} & \omega_M^{(-1\,1)} & \omega_M^{(-1\,2)} & \omega_M^{(-1\,3)} \\ \omega_M^{(0\,0)} & \omega_M^{(0\,1)} & \omega_M^{(0\,2)} & \omega_M^{(0\,3)} \\ \omega_M^{(1\,0)} & \omega_M^{(1\,1)} & \omega_M^{(1\,2)} & \omega_M^{(1\,3)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(0\,-2)} & \omega_N^{(0\,-1)} & \omega_N^{(0\,0)} & \omega_N^{(0\,1)} \\ \omega_N^{(1\,-2)} & \omega_N^{(1\,-1)} & \omega_N^{(1\,0)} & \omega_N^{(1\,1)} \\ \omega_N^{(2\,-2)} & \omega_N^{(2\,-1)} & \omega_N^{(2\,0)} & \omega_N^{(2\,1)} \\ \omega_N^{(3\,-2)} & \omega_N^{(3\,-1)} & \omega_N^{(3\,0)} & \omega_N^{(3\,1)} \end{bmatrix}$$

$$\begin{bmatrix} G_{22} & G_{23} & G_{20} & G_{21} \\ G_{32} & G_{33} & G_{30} & G_{31} \\ G_{02} & G_{03} & G_{00} & G_{01} \\ G_{12} & G_{13} & G_{10} & G_{11} \end{bmatrix} = \begin{bmatrix} \omega_M^{(2\,0)} & \omega_M^{(2\,1)} & \omega_M^{(2\,2)} & \omega_M^{(2\,3)} \\ \omega_M^{(3\,0)} & \omega_M^{(3\,1)} & \omega_M^{(3\,2)} & \omega_M^{(3\,3)} \\ \omega_M^{(0\,0)} & \omega_M^{(0\,1)} & \omega_M^{(0\,2)} & \omega_M^{(0\,3)} \\ \omega_M^{(1\,0)} & \omega_M^{(1\,1)} & \omega_M^{(1\,2)} & \omega_M^{(1\,3)} \end{bmatrix} g \begin{bmatrix} \omega_N^{(0\,2)} & \omega_N^{(0\,3)} & \omega_N^{(0\,0)} & \omega_N^{(0\,1)} \\ \omega_N^{(1\,2)} & \omega_N^{(1\,3)} & \omega_N^{(1\,0)} & \omega_N^{(1\,1)} \\ \omega_N^{(2\,2)} & \omega_N^{(2\,3)} & \omega_N^{(2\,0)} & \omega_N^{(2\,1)} \\ \omega_N^{(3\,2)} & \omega_N^{(3\,3)} & \omega_N^{(3\,0)} & \omega_N^{(3\,1)} \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Permutation matrix used to shift negative frequencies

$$P \begin{bmatrix} (11) & (10) \\ (01) & (00) \end{bmatrix} P = P \begin{bmatrix} (10) & (11) \\ (00) & (01) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P$$

$$\begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P = \begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix} g \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P$$

$$\underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{G} = \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{F_M} g \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{F_N}$$

eq 9

$$g = \frac{1}{MN}$$

$$\begin{bmatrix} \tilde{\omega}_m^{(0,-2)} & \tilde{\omega}_m^{(0,-1)} & \tilde{\omega}_m^{(0,0)} & \tilde{\omega}_m^{(0,1)} \\ \tilde{\omega}_m^{(1,-2)} & \tilde{\omega}_m^{(1,-1)} & \tilde{\omega}_m^{(1,0)} & \tilde{\omega}_m^{(1,1)} \\ \tilde{\omega}_m^{(2,-2)} & \tilde{\omega}_m^{(2,-1)} & \tilde{\omega}_m^{(2,0)} & \tilde{\omega}_m^{(2,1)} \\ \tilde{\omega}_m^{(3,-2)} & \tilde{\omega}_m^{(3,-1)} & \tilde{\omega}_m^{(3,0)} & \tilde{\omega}_m^{(3,1)} \end{bmatrix} \begin{bmatrix} G_{-2,-2} & G_{-2,-1} & G_{-2,0} & G_{-2,1} \\ G_{-1,-2} & G_{-1,-1} & G_{-1,0} & G_{-1,1} \\ G_{0,-2} & G_{0,-1} & G_{0,0} & G_{0,1} \\ G_{1,-2} & G_{1,-1} & G_{1,0} & G_{1,1} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_n^{(-2,0)} & \tilde{\omega}_n^{(-2,1)} & \tilde{\omega}_n^{(-2,2)} & \tilde{\omega}_n^{(-2,3)} \\ \tilde{\omega}_n^{(-1,0)} & \tilde{\omega}_n^{(-1,1)} & \tilde{\omega}_n^{(-1,2)} & \tilde{\omega}_n^{(-1,3)} \\ \tilde{\omega}_n^{(0,0)} & \tilde{\omega}_n^{(0,1)} & \tilde{\omega}_n^{(0,2)} & \tilde{\omega}_n^{(0,3)} \\ \tilde{\omega}_n^{(1,0)} & \tilde{\omega}_n^{(1,1)} & \tilde{\omega}_n^{(1,2)} & \tilde{\omega}_n^{(1,3)} \end{bmatrix}$$

$$g = \frac{1}{MN}$$

$$\begin{bmatrix} \tilde{\omega}_m^{(0,2)} & \tilde{\omega}_m^{(0,3)} & \tilde{\omega}_m^{(0,0)} & \tilde{\omega}_m^{(0,1)} \\ \tilde{\omega}_m^{(1,2)} & \tilde{\omega}_m^{(1,3)} & \tilde{\omega}_m^{(1,0)} & \tilde{\omega}_m^{(1,1)} \\ \tilde{\omega}_m^{(2,2)} & \tilde{\omega}_m^{(2,3)} & \tilde{\omega}_m^{(2,0)} & \tilde{\omega}_m^{(2,1)} \\ \tilde{\omega}_m^{(3,2)} & \tilde{\omega}_m^{(3,3)} & \tilde{\omega}_m^{(3,0)} & \tilde{\omega}_m^{(3,1)} \end{bmatrix} \begin{bmatrix} G_{22} & G_{23} & G_{20} & G_{21} \\ G_{32} & G_{33} & G_{30} & G_{31} \\ G_{02} & G_{03} & G_{00} & G_{01} \\ G_{12} & G_{13} & G_{10} & G_{11} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_n^{(2,0)} & \tilde{\omega}_n^{(2,1)} & \tilde{\omega}_n^{(2,2)} & \tilde{\omega}_n^{(2,3)} \\ \tilde{\omega}_n^{(3,0)} & \tilde{\omega}_n^{(3,1)} & \tilde{\omega}_n^{(3,2)} & \tilde{\omega}_n^{(3,3)} \\ \tilde{\omega}_n^{(0,0)} & \tilde{\omega}_n^{(0,1)} & \tilde{\omega}_n^{(0,2)} & \tilde{\omega}_n^{(0,3)} \\ \tilde{\omega}_n^{(1,0)} & \tilde{\omega}_n^{(1,1)} & \tilde{\omega}_n^{(1,2)} & \tilde{\omega}_n^{(1,3)} \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ (01) & (00) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ (00) & (01) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (11) & (10) \\ (01) & (00) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T P \begin{bmatrix} (10) & (11) \\ (00) & (01) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \begin{bmatrix} (01) & (00) \\ (11) & (10) \end{bmatrix} P^T \begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix} \begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}$$

$$g = \frac{1}{MN} \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{F_m^*} \underbrace{\begin{bmatrix} (00) & (01) \\ (10) & (11) \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} (00) & (01) \\ (01) & (11) \end{bmatrix}}_{F_n^*}$$

eq 10