

## PPL – Assignment 5

### Question 1

**b.** Proving `append$` is CPS-equivalent to `append`:

We will prove:  $(\text{append\$ } l1 \ l2 \ c) = (c \ (\text{append } l1 \ l2))$ .

We will prove by induction on the length of `l1`.

**מקרה בסיס** עבור  $|l1| = 0$ :

$a-e \ [ \ (\text{append\$ } l1 \ l2 \ c) \ ] \rightarrow a-e \ [ \ (c \ l2) \ ] \rightarrow a-e \ [ \ (c \ (\text{append } l1 \ l2)) \ ]$   
as needed.

**הנחת האינדוקציה:** נניח שעבור  $|l1| = n$  כש- $n$  טבעי הטענה מתקיימת, כלומר:

$(\text{append\$ } l1 \ l2 \ c) = (c \ (\text{append } l1 \ l2))$

**צעד האינדוקציה:** נוכיח שהטענה מתקיימת עבור  $|l1| = n+1$ : (נסמן את הרשימה `l1` באורך  $n$  ב-`l1'` לנוחות)

$a-e \ [ \ (\text{append\$ } l1 \ l2 \ c) \ ] \rightarrow a-e \ [ \ (\text{append\$ } (\text{cdr } l1) \ l2 \ (\text{lambda } (res) \ (c \ (\text{cons } (\text{car } l1) \ res)))) \ ]$

נשתמש בהנחת האינדוקציה (אורך `(cdr l1)` הוא  $n$  ולכן ניתן להשתמש) ונקבל:

$a-e \ [ \ ((\text{lambda } (res) \ (c \ (\text{cons } (\text{car } l1) \ res)))) \ (\text{append } l1' \ l2)) \ ] \rightarrow$

$\rightarrow a-e \ [ \ (c \ (\text{cons } (\text{car } l1) \ (\text{append } l1' \ l2))) \ ] \rightarrow$

$\rightarrow a-e \ [ \ (c \ (\text{append } l1 \ l2)) \ ]$

מ.ש.ל.

### Question 2

**d.** We will use `reduce1-lz` when we know the lists are finite, so we know the operation will end. We will use `reduce2-lz` when we only want the operation applied to a finite number of elements in the lists, but the lists may be infinite. We will use `reduce3-lz` when the lists are possibly infinite, and we want to apply the procedure a non-fixed number of times, possibly infinitely.

**g.** The advantage of `generate-pi-approximations` implementation comparing to `pi-sum` implementation is that we get to see every step in the approximation process, thus having a compact view of the recursive calls.

The disadvantage is that it is difficult to manipulate the outcome of the calculation, you have to build another method to simplify it, if needed.

### Question 3.1

a.  $\text{unify}[t(s(s), G, s, p, t(K), s), t(s(G), G, s, p, t(K), U)]$

$s = \{ \}$

$A \circ s = t(s(s), G, s, p, t(K), s)$

$B \circ s = t(s(G), G, s, p, t(K), U)$

$s = s \circ \{ G = s \} = \{ G = s \}$

$A \circ s = t(s(s), s, s, p, t(K), s)$

$B \circ s = t(s(s), s, s, p, t(K), U)$

$s = s \circ \{ U = s \} = \{ G = s, U = s \}$

$A \circ s = t(s(s), s, s, p, t(K), s)$

$B \circ s = t(s(s), s, s, p, t(K), s)$

**Unification success**  $\rightarrow s = \{ G = s, U = s \}$

b.  $\text{unify}[p([v \mid [V \mid W]]), p([[v \mid V] \mid W])]$

$s = \{ \}$

$A \circ s = p([v \mid [V \mid W]])$

$B \circ s = p([[v \mid V] \mid W])$

FAIL :  $v \neq [v \mid V]$  not the same structure.

Proof Tree is in the next page:

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\*\*For clarification reasons, I just mentioned the substitution but didn't always the variables in the text box for easier understanding of the process.

