Smart Sensing Systems
Pushing Technology Frontiers

Activity Report

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Report Title: *A Modified analysis of the radar bistatic scenario*

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Description: The previous report's diagram depicting the antenna beam ranges as a function of target's height have been modified. The analysis of the new diagram is described herein.

Objectives:

- Conduct a MATLAB simulation describing the receive antenna beam range as a function of the target's height for various elevation angles.
- Conduct a MATLAB simulation examining the duration of time at which the target remains in sight as a function of its speed for various elevation angles.

1. Design Basis and parameters

The diagram describing the passive radar bistatic scenario is shown in figure 1. The target detection zone is a sector of a circle of a radius equal to the maximum detectable range($R_{r,max}$), and the angle of the sector is the half power angle of the receiving antenna. The maximum detectable range equation is given by Eq.1.

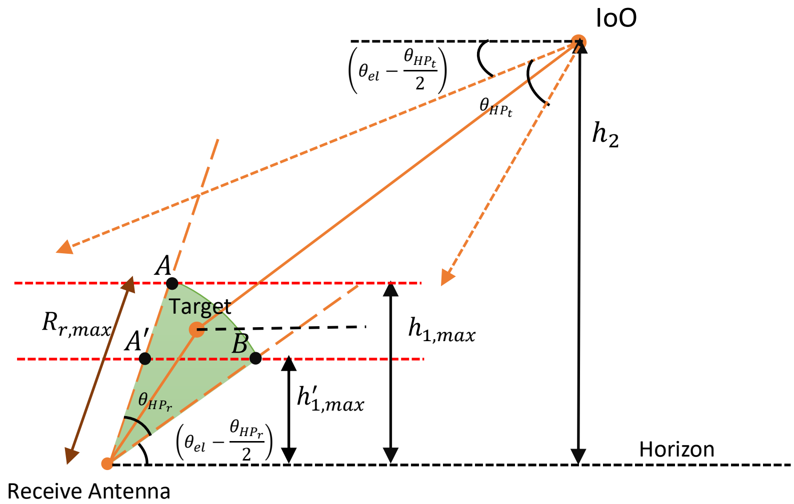


Figure 1. Passive radar bistatic scenario. The shaded green region is the target detection zone region.

$$R_{r,max} = \left[\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R_t^2 P_{r,min}}\right]^{\frac{1}{2}} \tag{1}$$

The coordinates of points A, A' and B defining the detection zone with respect to the receive antenna are given below.

$$A(x,y) = \left(R_{r,max} \times \cos\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right), R_{r,max} \times \sin\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)\right)$$

$$A'(x,y) = \left(\frac{h'_{1,max}}{tan\left(\theta_{el}+\frac{\theta_{HP_r}}{2}\right)}, h'_{1,max}\right) = \left(\frac{R_{r,max}\times \sin\left(\theta_{el}-\frac{\theta_{HP_r}}{2}\right)}{tan\left(\theta_{el}+\frac{\theta_{HP_r}}{2}\right)}, R_{r,max} \times \sin\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right)\right)$$

$$B(x,y) = \left(R_{r,max} \times \cos\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right), R_{r,max} \times \sin\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right)\right)$$

The equation describing the receive antenna beam range as a function of the target's height d(h) for ($h < h'_{1,max}$) is derived below using figure 2. The final equation is given by Eq.2. The equation is a function of the receive antenna elevation angle.

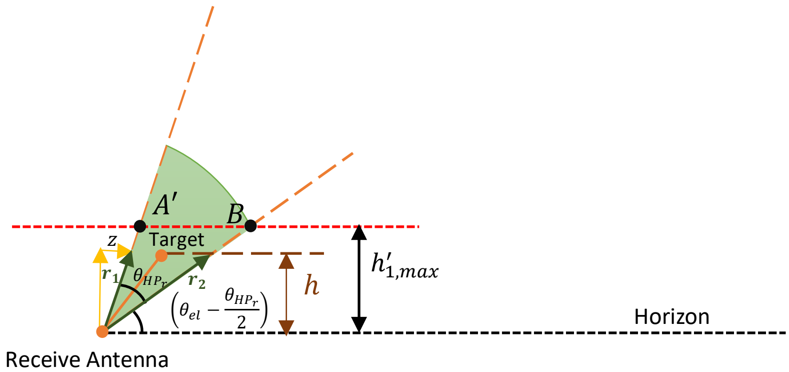


Figure 2. Calculating antenna beam range in the case when the target's height is below $h'_{1,max}$.

$$z = h \times \tan\left(90 - \theta_{el} - \frac{\theta_{HP_r}}{2}\right) = h \times \cot\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)$$

$$r_2 = h \times \csc\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right)$$

$$(d+z)^2 = (h)^2 + (r_2)^2$$

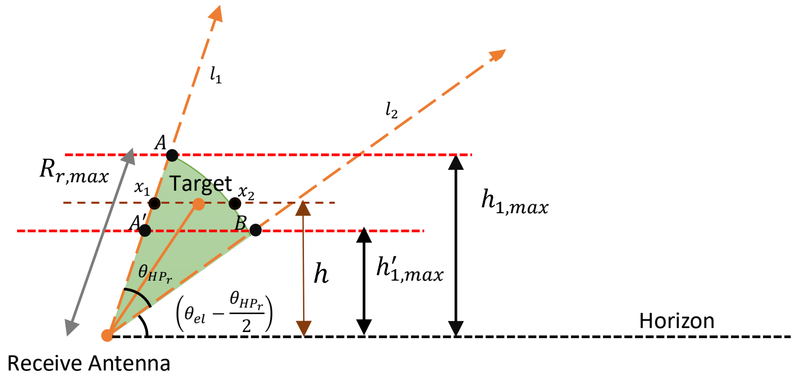
$$d(h) = \sqrt{(h)^2 + (r_2)^2} - z$$

$$d(h) = \sqrt{(h)^2 + h^2 \times csc^2\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right)} - h \times \cot\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)$$

$$d(h) = h\sqrt{1 + csc^2\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right)} - h \times \cot\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)$$

$$d(h) = h\left[\cot\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right) - \cot\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)\right], \text{ for } (h < h'_{1,max}) \qquad (2)$$

The receive antenna beam range for ($h > h'_{1,max}$) is derived below using figure 3. The final equation is given by Eq. 3. Moreover, the piecewise function of the receive antenna beam range is given by Eq.4.



$$d(h) = x_2 - x_1$$

$$l_1: y = mx, m = tan\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)$$

x_1 occurs at the intersection of the lines $y = h$ and $y = x \times tan\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)$

$$x_1 = \frac{h}{tan\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)}$$

x_2 occurs at the intersection of the lines $y = h$ and $y = \sqrt{{R_{r,max}}^2 - x^2}$

$x_2 = \sqrt{{R_{r,max}}^2 - h^2}$, therefore, d(h):

$$d(h) = \sqrt{{R_{r,max}}^2 - h^2} - \frac{h}{tan\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)}, \text{ for } (h > h'_{1,max}) \quad (3)$$

$$d(h) = \begin{cases} h\left[\cot\left(\theta_{el} - \frac{\theta_{HP_r}}{2}\right) - \cot\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)\right], & for\ 0 \le h \le h'_{1,max} \\ \sqrt{{R_{r,max}}^2 - h^2} - \frac{h}{tan\left(\theta_{el} + \frac{\theta_{HP_r}}{2}\right)}, & for\ h'_{1,max} \le h \le h_{1,max} \\ The\ target\ is\ undetectable, & for\ h_{1,max} < h \end{cases} \quad (4)$$

The duration of time for which the target remains in sight is the receive antenna beam range divided by the target's speed, as given by Eq. 5.

$$T = \frac{d(h)}{Speed} \tag{5}$$

2. Simulation Code

2.1 The receive antenna beam ranges against the target's height for various elevations angles.

The code given below simulates equation 4, producing multiple curves for elevation angles of $(1°, 5°, 10°, 15°, 25°, 35°, 45°, 55°, 65°, 75°, 85°, 90°)$; this is to study the effect of the elevation angle on the receive antenna beam range and the maximum detectable height of a given target. The code generates 11 subplots to visualize these relationships.

```
clc;clear;
Rr_max = 20e3; %Assumed value, Rr_max =
% ((P_t*G_t*G_r*(lambda^2)*RCS)/(((4*pi)^3)*)*Pr_min*(R_t^2))^(1/2)
theta_Elv =  [1 10 15 25 35 45 55 65 75 85 90]   ;  % Elevation angle
theta_HP = 1; % Half power angle

for m = 1:numel(theta_Elv)
h1_max_a(m) = Rr_max*sind(theta_Elv(m)-theta_HP/2); % Calculates the maximum heights
h1_max_b(m) = Rr_max*sind(theta_Elv(m)+theta_HP/2); %corresponding to each elevation
angle
h1 = linspace(0,h1_max_a(m),350);
d1(:,m) = h1*(cotd(theta_Elv(m)-theta_HP/2)-cotd(theta_Elv(m)+theta_HP/2)); %
Expression of d for h<h1_max_a
h2 = linspace(h1_max_a(m),h1_max_b(m),350);
d2(:,m) = -(h2)/tand(theta_Elv(m)+theta_HP/2)...
+ sqrt((Rr_max)^2-(h2).^2); % Expression of d for h>h1_max_a
d(:,m) = [d1(:,m)'  d2(:,m)']; %adds the two expressions
end
for j = 1:numel(theta_Elv)
h = linspace(0,h1_max_b(j),700);
subplot(numel(theta_Elv),1,j)
plot(h, d(:,j))
grid on;
xlabel('Height of the Target in m')
ylabel('The Receive Antenna Beam Range in m')
title('Comparing The Range of The Receive Antenna Beam Against The Target''s Height
for Various Elevation, \n Elevation angle = %d degrees', theta_Elv(j))
end
```

2.2 The duration of time for which the target with a given height remains in sight against its speed

The program plots the time for which the signal from the target remains available against its speed. For each target's height, multiple T vs speed curves are produced for elevations angles of $(5°, 18°, 36°, 54°, 72°, 85°)$. As mentioned above, the maximum detectable height is a function of the elevation angle, and hence the program is made to state when a target is undetectable for a given elevation angle and target's height.

```
Rr_max = 20e3; %Assumed value, Rr_max =
% ((P_t*G_t*G_r*(lambda^2)*RCS)/(((4*pi)^3)*)*Pr_min*(R_t^2))^(1/2)
theta_Elv = [ 5 18 36 54 72 85];theta_HP = 1;
S_target = linspace((15e3/60),(45e3/60),1500);h = [ 500 1e3  5e3 12e3 ];
for j = 1:numel(h)
for k = 1:numel(theta_Elv)
h1_max_a(k) = Rr_max*sind(theta_Elv(k)-theta_HP/2); % Calculates the maximum heights
h1_max_b(k) = Rr_max*sind(theta_Elv(k)+theta_HP/2); %coressponding to each elevation
angle
if (h(j) <=h1_max_a(k))
T(:,k) = (h(j)*(cotd(theta_Elv(k)-theta_HP/2)-
cotd(theta_Elv(k)+theta_HP/2)))./(S_target);
elseif (h(j)>=h1_max_a(k) && h(j)<=h1_max_b(k))
T(:,k) = (-(h(j))/tand(theta_Elv(m)+theta_HP/2)+ sqrt((Rr_max)^2-
(h(j)).^2))./(S_target); % Expression of d for h>h1_max_a
else
    fprintf('When the elevation angle is %d, a target with a height of %d is
undetectable\n',theta_Elv(k),h(j))
    T(:,k) = zeros(numel(S_target), 1);
end
end
if (j == 1)
 subplot(4,1,1)
semilogy(S_target,T(:,1),S_target,T(:,2),S_target,T(:,3),S_target,T(:,4))
set(gca,'ColorOrderIndex',1); % reset color index for better comparison
legend('Elevation angle (in degrees) = 5', '18','36','72','85')
grid on;
ylim([0 5])
title('Comparing the time taken for a target with a given height to exit the receive
antenna beam range against its speed, h = 500 m ');
end
if (j == 2)
     subplot(4,1,2)
semilogy(S_target,T(:,1),S_target,T(:,2),S_target,T(:,3),S_target,T(:,4))
set(gca,'ColorOrderIndex',1); % reset color index for better comparison
grid on;
ylim([0 10])
title('h = 1 km ');
end
if ( j == 3)
subplot(4,1,3)
semilogy(S_target,T(:,1),S_target,T(:,2),S_target,T(:,3),S_target,T(:,4))
set(gca,'ColorOrderIndex',1); % reset color index for better comparison
grid on;
ylabel('Vertical distance from the target to the receiver in m' );
title('h = 5 km ');
end
if (j == 4)
subplot(4,1,4)
semilogy(S_target,T(:,1),S_target,T(:,2),S_target,T(:,3),S_target,T(:,4))
set(gca,'ColorOrderIndex',1); % reset color index for better comparison
grid on;
ylim([0 2])
xlabel('Speed of the target' );title('h = 12 km ');
end
end
```

3. Simulation Results and Conclusions

3.1 The receive antenna beam ranges against the target's height for various elevations angles.

Figures 4-14 shows the MATLAB simulation results of the receive antenna beam range versus the height of the target for elevation angles of $(1°, 5°, 10°, 15°, 25°, 35°, 45°, 55°, 65°, 75°, 85°, 90°)$.

It should be noted that as the elevation angle increases, the maximum detectable height increases, however, the antenna beam range decreases.

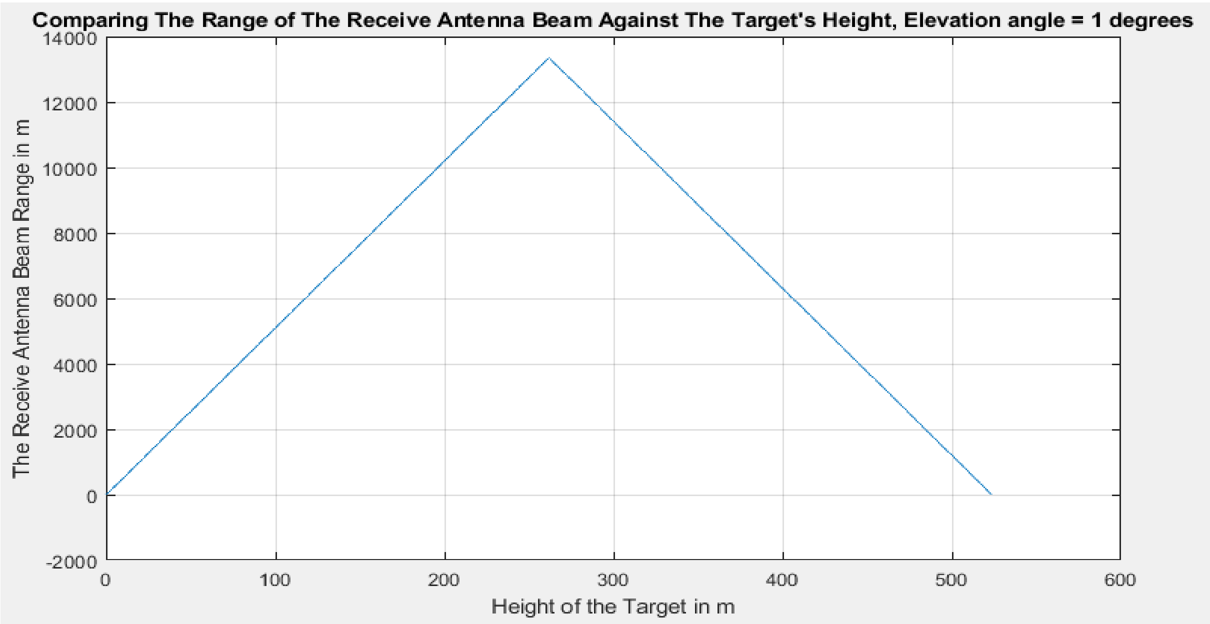


Figure 4. Receive antenna beam range versus the target's height, with the elevation angle equal to 1 degree.

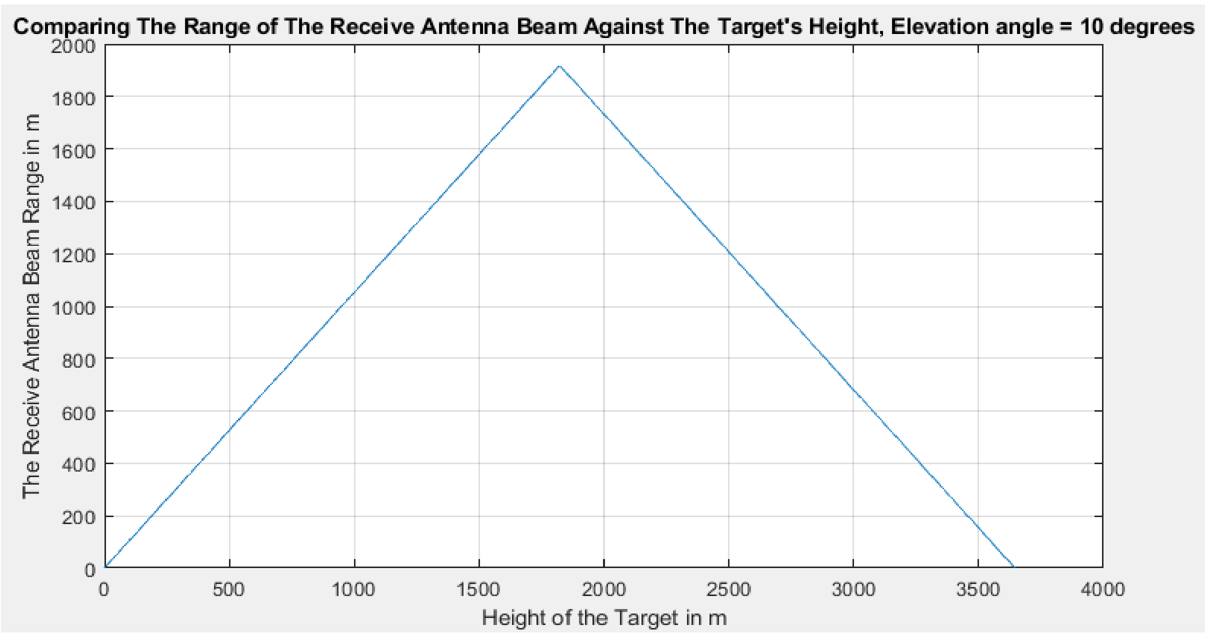


Figure 5. Receive antenna beam range versus the target's height, with the elevation angle equal to 10 degrees.

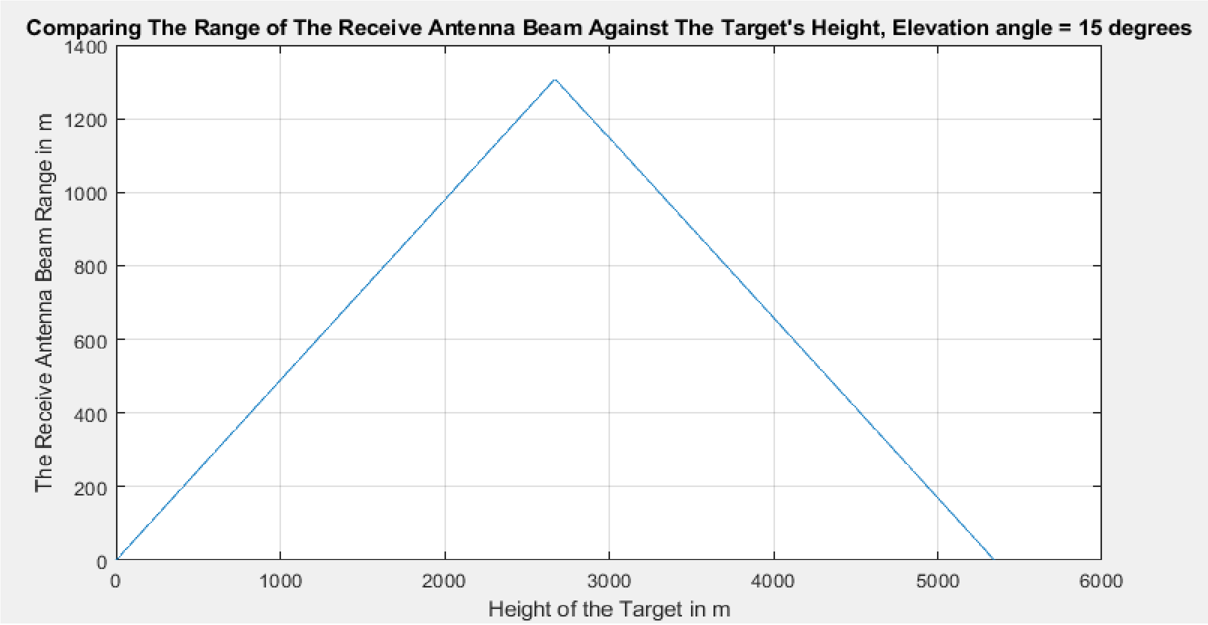


Figure 6. Receive antenna beam range versus the target's height, with the elevation angle equal to 15 degrees.

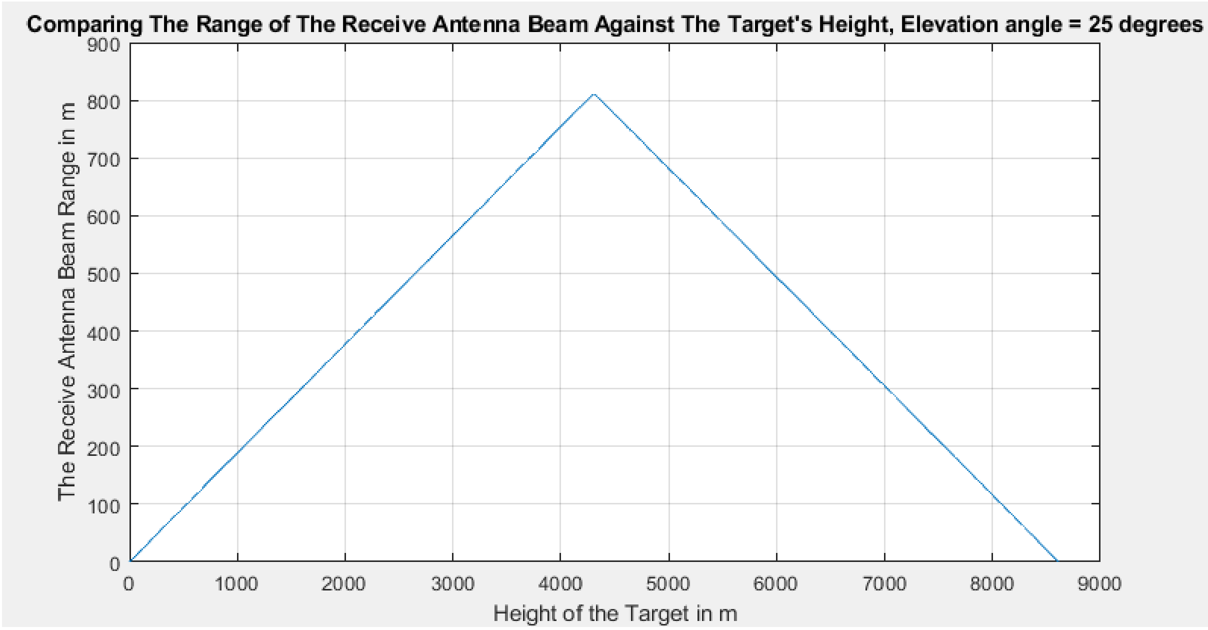


Figure 7. Receive antenna beam range versus the target's height, with the elevation angle equal to 25 degrees.

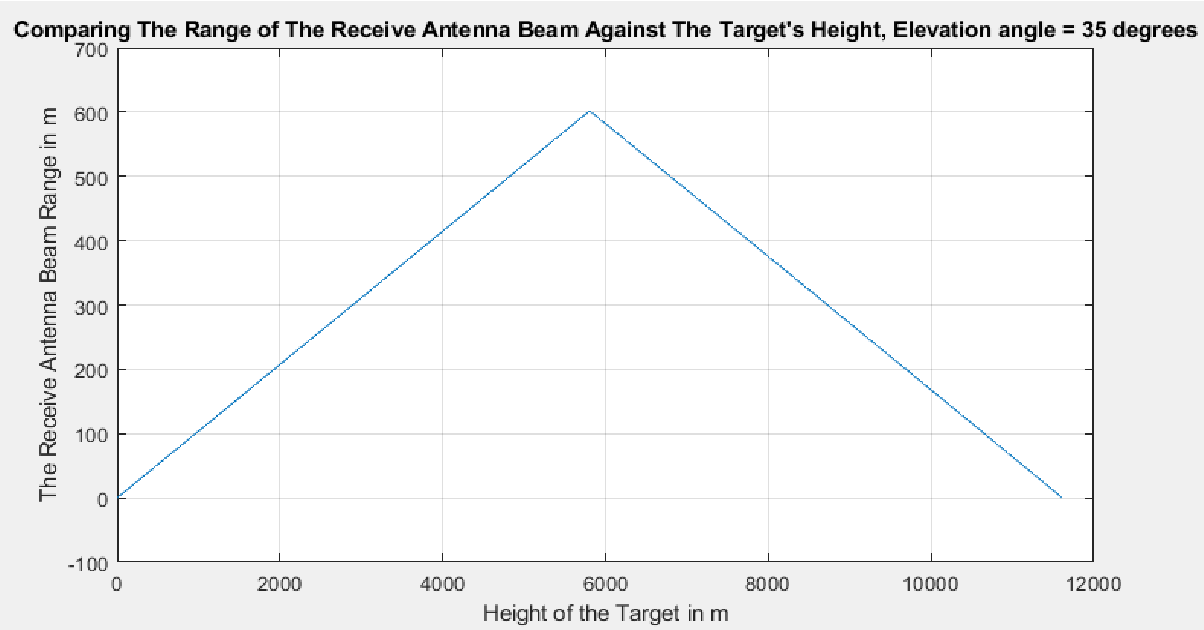


Figure 8. Receive antenna beam range versus the target's height, with the elevation angle equal to 35 degrees.

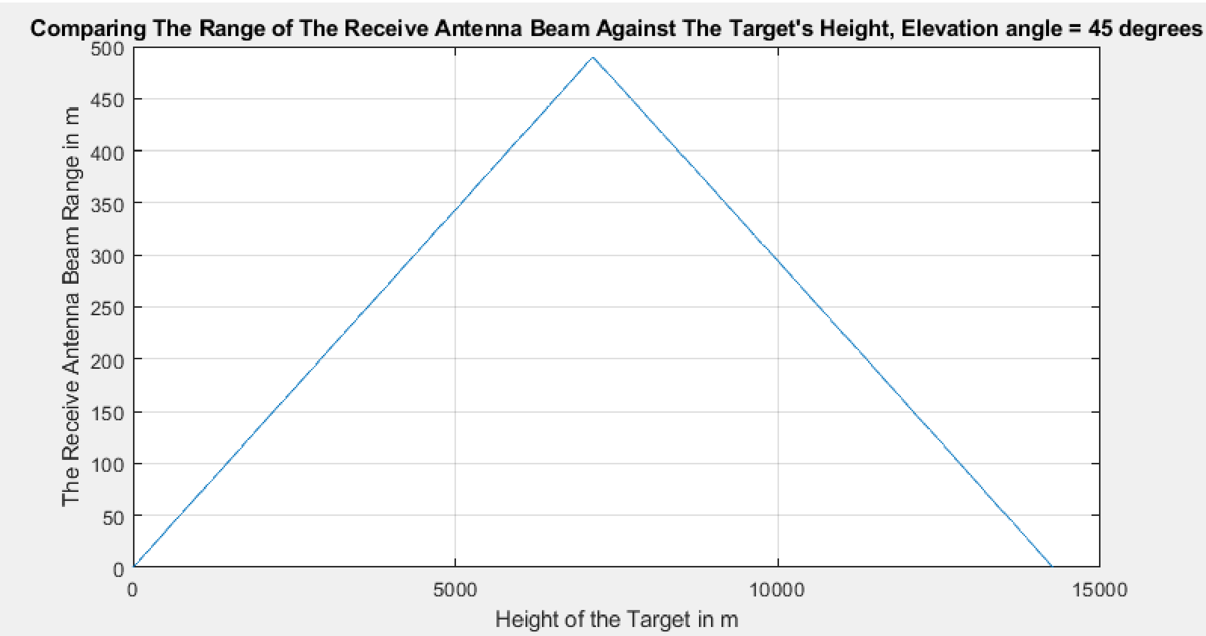


Figure 9. Receive antenna beam range versus the target's height, with the elevation angle equal to 45 degrees.

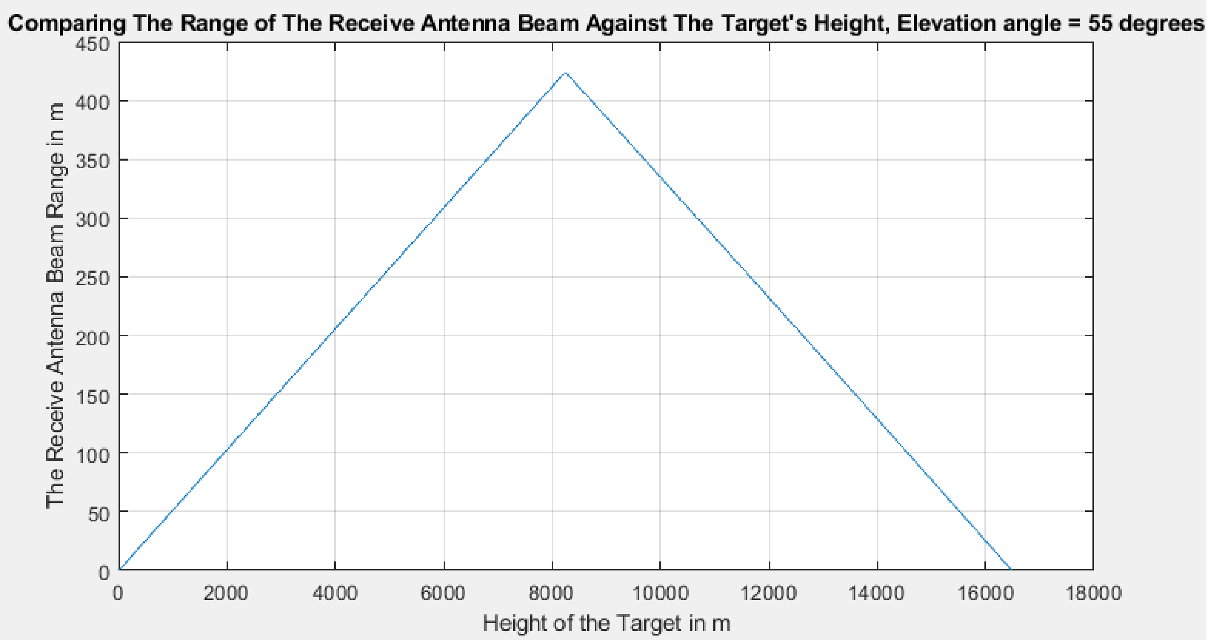


Figure 10. Receive antenna beam range versus the target's height, with the elevation angle equal to 55 degrees.

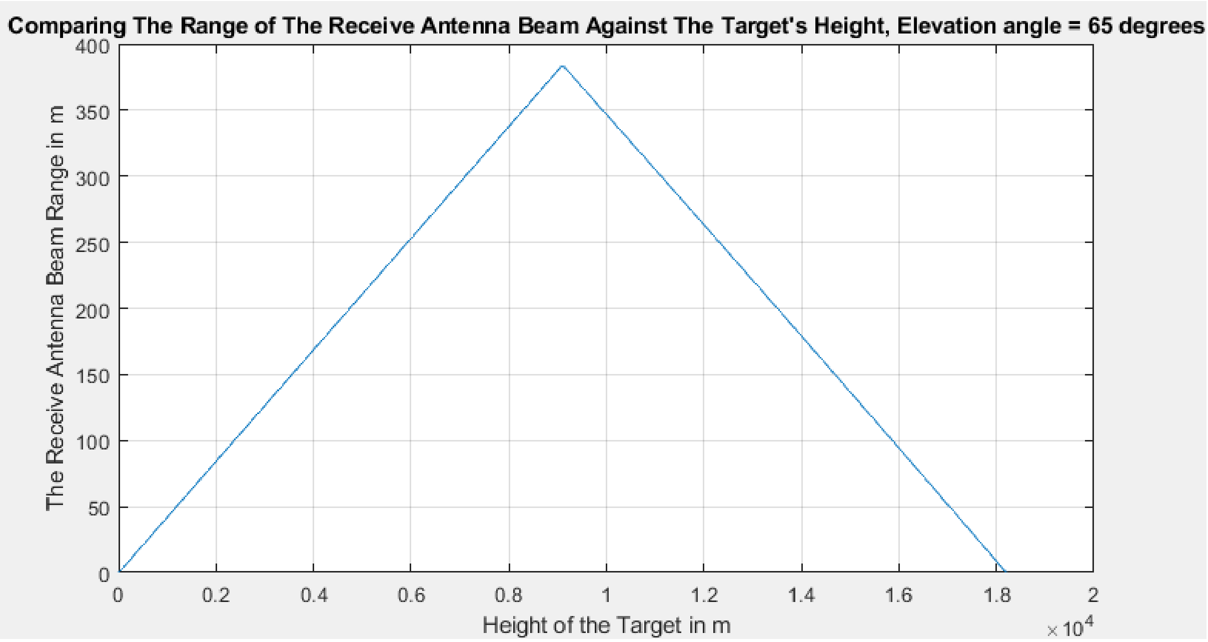


Figure 11. Receive antenna beam range versus the target's height, with the elevation angle equal to 65 degrees.

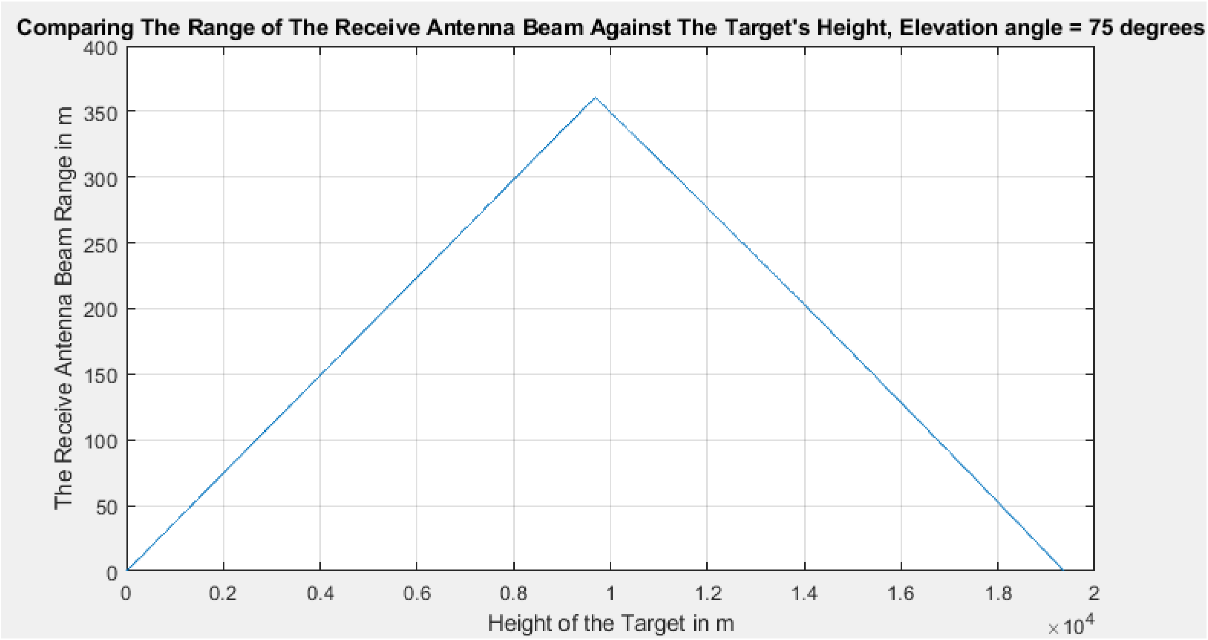


Figure 12. Receive antenna beam range versus the target's height, with the elevation angle equal to 75 degrees.

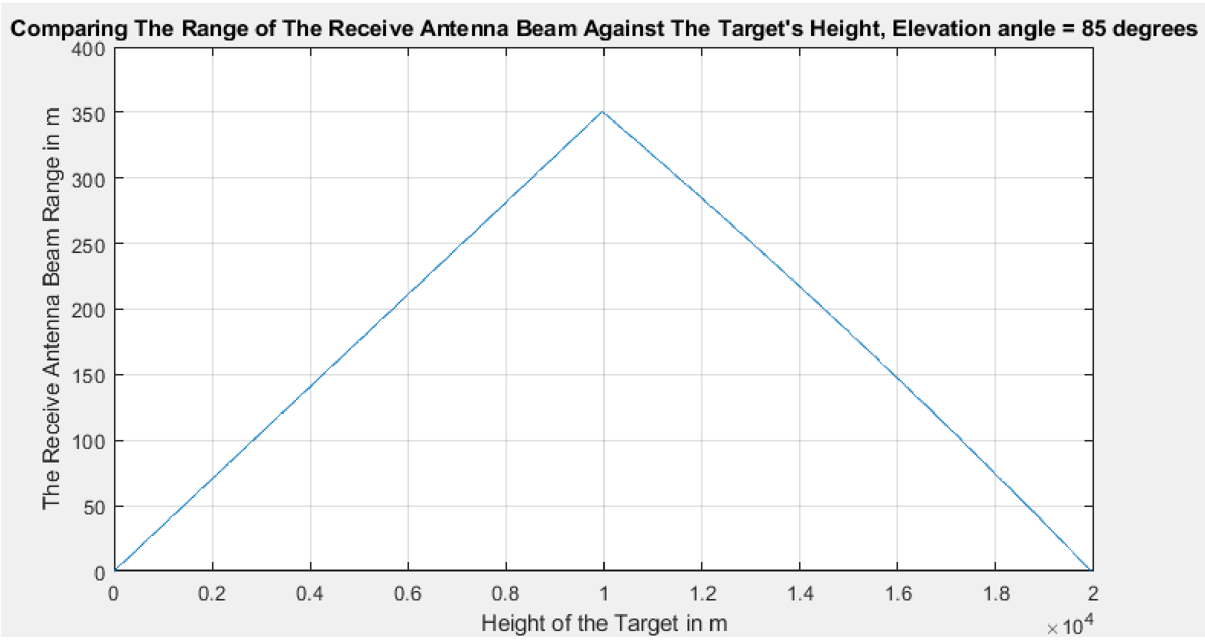


Figure 13. Receive antenna beam range versus the target's height, with the elevation angle equal to 85 degrees.

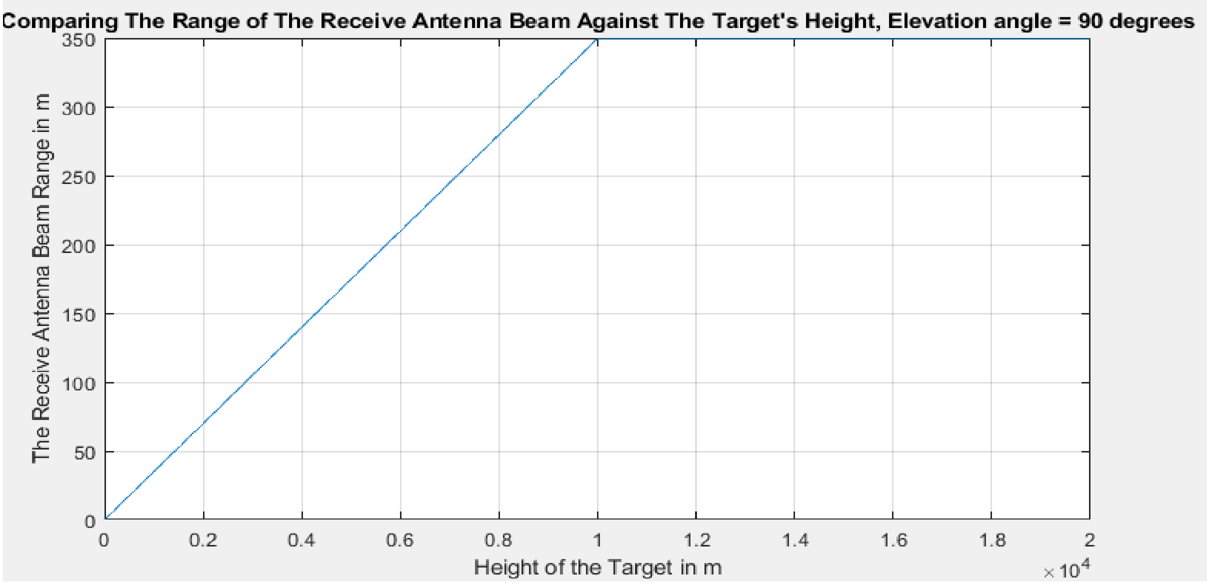


Figure 14. Receive antenna beam range versus the target's height, with the elevation angle equal to 90 degrees.

3.2 The duration of time for which the target with a given height remains in sight against its speed

Figure 15 shows the MATLAB simulation plot of the antenna beam range for h = 500 m, 1 km, 5 km, and 15 km divided by the target's speed against the target's speed for elevation angles of (5°, 18°, 36°, 54°, 72°, 85°). The curves for some elevation angles for a target with a given height is zero, indicating that the target cannot be detected; this is also indicated by the program result shown in figure 16.

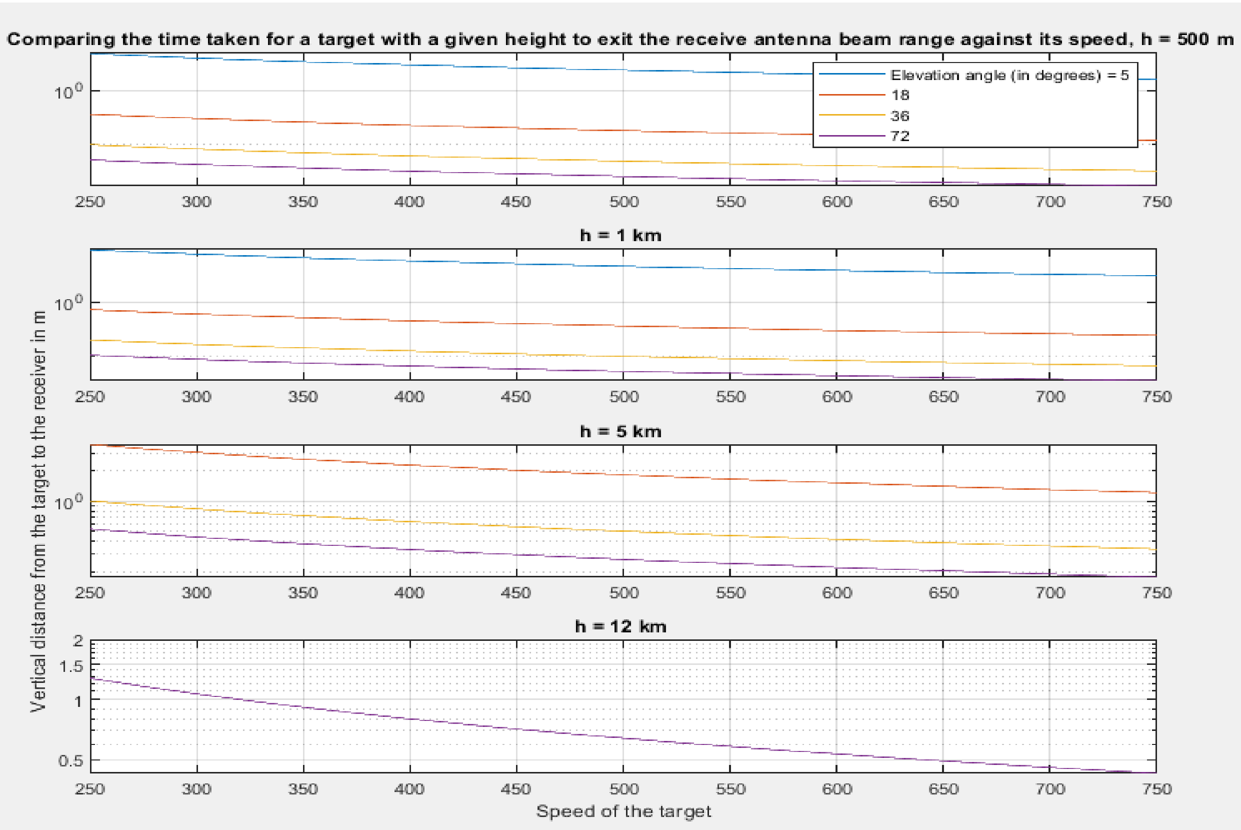


Figure 15. Duration of time for which the signal from a target remains available versus the target's speed for various target's height and elevation angles.

```
When the elevation angle is 5, a target with a height of 5000 is undetectable
When the elevation angle is 5, a target with a height of 12000 is undetectable
When the elevation angle is 18, a target with a height of 12000 is undetectable
When the elevation angle is 36, a target with a height of 12000 is undetectable
```

Figure 16. MATLAB simulation results indicating when a target with a given height is undetectable for a receive antenna with a given elevation angle.

4. Future Steps

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