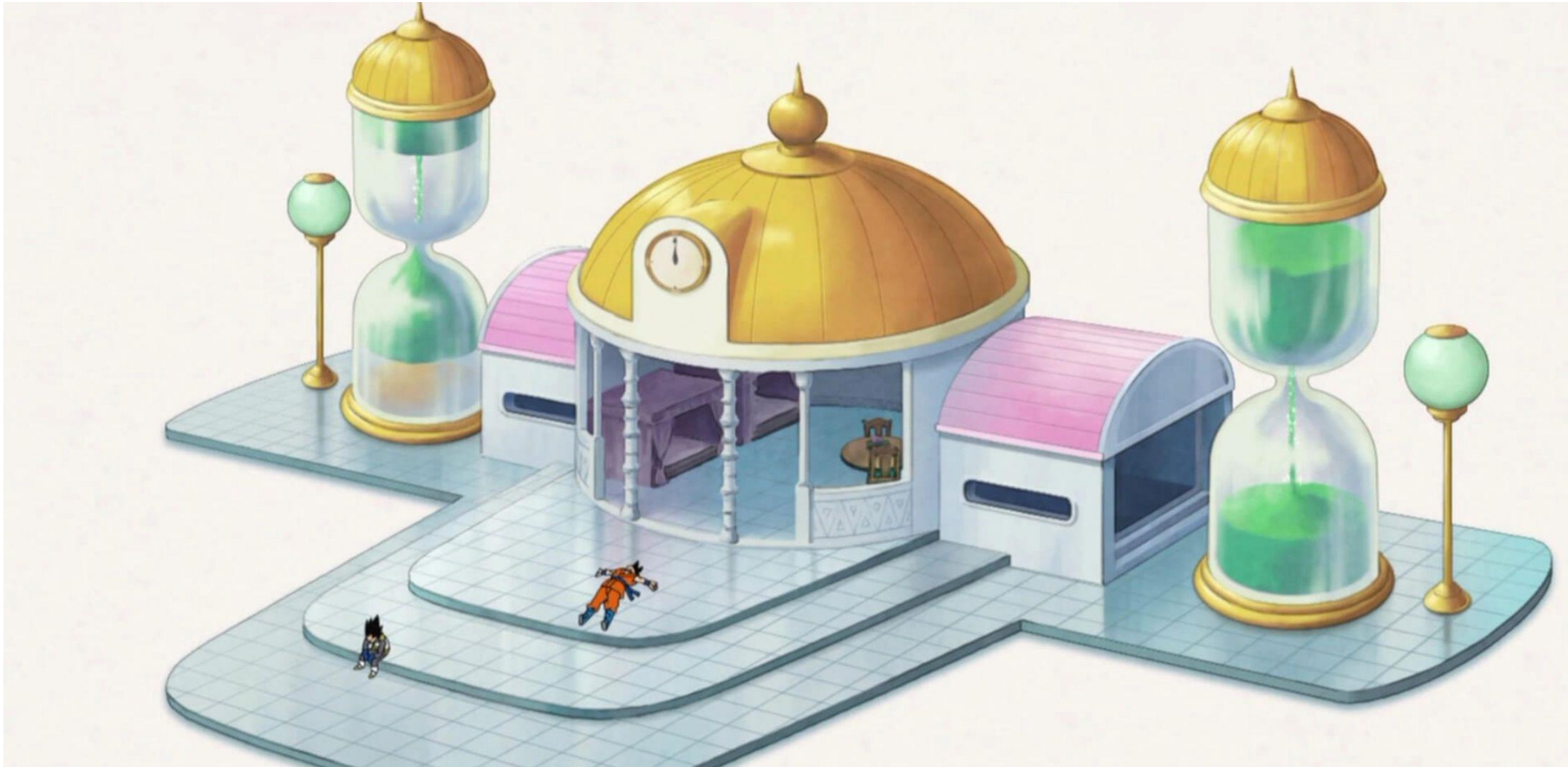
Learn enumerative combinatorics in hyperbolic time chamber

Andersen Ang



What is the foundation of complete search

- IF ignore efficiency (speed, time, storage),
  THEN Complete search = brute force

- Backbone of brute force is "list all possible cases"

- "list all possible case" in expert language is "enumerative combinatorics"

Enumerative combinatorics

- Enumeration := list all possible configurations

- Permutation ∈ configurations ok you learnt this
- Combination ∈ configurations ok you learnt this

- Permutation and Combination ARE NOT the full story
- Partition
- De-rangement
- 12-fold way

Partition

- Ground set: G, a given set we working on
- A k-partition P of G is a collection of non-empty subsets
  P = { A_1, A_2, …, A_k}) such that
  - Complete $A_1 \cup A_2 \cup \ldots \cup A_k = A$
  - no sharing $A_i \cap A_j = \emptyset \quad \forall i \neq j$
  - Nonempty $A_i \neq \emptyset$ for all (i)
- G = {1,2,3}
  1. { {1}, {2,3} }
  2. { {2}, {1,3} }
  3. { {3}, {1,2} }
  4. { {1}, {2},{3} }
  5. { {1, 2,3} }

Num of partition of n-set: Bell's number

1,1,2,5,15,52,203

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k .$$

De-rangement

- Permutation map π(i) = j means put *i* in *j*
- De-rangement := a π(G) such that π(i) ≠ i

- De-rangement {1,2,3}

1. { 2,3,1 }
2. { 3,1,2 }

Num of partition of de-rangement

1,0,1,2,9,...

$$!n = (n-1)\left(!(n-1) + !(n-2)\right)$$

for $n \geq 2$, where $!0 = 1$ and $!1 = 0$.[6]

12-fold way

- Distribute n balls into k boxes
- Think of arranging things as a function f

	No restriction	Injective f 1-to-1 All balls distributed	Surjective f Onto All box filled
Distinct balls Distinct boxes	k^n	k-permute-n k! / (k-n)!	k! S(n,k) 2nd Stirling number
InDistinct balls Distinct boxes	(n+k-1)-choose-(k-1) (n+k-1)-choose-(n)	k-choose-n (Combination)	(n-1)-choose-(k-1)
Distinct balls InDistinct boxes	S(n,1) + … + S(n,k)	1 if n <= k 0 if n > k	S(n,k)
InDistinct balls InDistinct boxes	n-partition-k	1 if n <= k 0 if n > k	n-partition-k - n-partition-(k-1)

WTF are these, how da f**k to use: learn by yourself, practices & practise !!