USC Marshall
School of Business

Periodic Investment with Moving Average Strategy

FBE 551: Quantitative Investing Group Project

Group Members: Yingying Qian, Xuan Yang, Sunjie Wang

Table of Contents

Background

In recent years, the continuous bull markets, despite the downturn in 2020 because of the Covid-19, attracts more and more investors into the financial market. Robinhood, a stockbroker app that is popular among young people, recorded all-time high monthly active users of 18.9 million in the third quarter of 2021. With more investors, especially younger investors who are in the early stage of their career, we see their needs for a more passive investment strategy. They will not have the time to monitor the market every day in the post-quarantine era, but they still want to invest in the market. As a result, we want to evaluate different kinds of **Periodic Investment Plans (PIPs)** and come up with one that will help younger investors to get stable and prosperous returns while not monitoring the stock market on a daily basis.

Who is the plan designed for?

According to the US News survey, in 2019, more than half of the American families, 53%, were invested in the stock market. The percentage sharply rose again in 2021 because of the COVID-19 pandemic and more people were staying at home and having time to monitor the stock market. **The Periodic Investment Plan (PIP)** is not suitable for all the investors, it is designed for a specific crowd.

First of all, a periodic investment plan is only suitable for incremental funds, for example, wages, which generate regular and continuous cash flows. The cash flow should be predictable and with no emergency demand of usage. Some funds are not suitable for a periodic investment plan, such as $100,000 deposits in the bank or some short-term money that is scheduled to be paid out in several months.

This is determined by the nature of the frequent switch of bull and bear market of the US stock market. Only a steady stream of incremental funds can provide financial protection for the bear market fixed investment. As for incremental funds, no one can predict when the bear market will end and when the bull market will come. Therefore, there is no way to determine how many shares should be divided into fixed investment. If shares are less, the bear market like 2008 or 2020 will fall by half of the value, and the funds will be used up and investors have to bear the loss. A large book loss, or continuous small loss for several straight periods is still a lot of psychological pressure. If shares are too much, there will be no funds left until the bull market comes. Therefore, fixed amounts of deposits are more suitable for asset allocation. Incremental funds will be more suitable for periodic investment plans.

We can now get a result that the periodic investment plan is for people who have a job and with stable income. Part of the salary can be invested in a bear market, fortunately, and sold in a bull market. It should be treated as another way of saving accounts while obtaining higher returns than fixed-term saving deposits. At the same time, it is especially important to note that the money invested must be spare money which means that it will not be used within at least three to five or even more years. Periodic investment plans must be set with the determination to invest in the long-term and even insist on investing in the bull market. If people need the money in the short term, the periodic investment plans are not the way they should choose. It is easy to suffer a loss if people withdraw halfway.

We summarized the most appropriate segments to go for the periodic investment plan:

- Full-time workers with a working fixed time schedule

People with stable careers that demand them to either sit in the office or be somewhere else that cannot monitor the stock market. The plan is designed for people who don't have time to manage their investments. They only need to set it up once, and then sit tight, keep investing every term and enjoy the income. The periodic investment plan could also help people who just started their careers to learn to save money, control expenses and think of ways to make more money.

- Investors with low-risk tolerance

Many investors wanted to invest, but think that investing in stocks is too risky, investing in currency funds or bank fixed-term deposits has too low returns and is boring. Then automatic investment in an index fund is a perfect choice. First, it avoids the trouble of choosing fund managers. The index covers multiple underlying constituent stocks which reduces the risk of investing in a single stock. Index funds have been rising if you view it in the long-term and still pay out much higher returns compared to currency funds and bank deposits.

- People with long-term PIPs and who tend to spend all their money once they receive it

The former is planning to buy houses in the short-term, saving for education for children, parents, and their own pensions. All these plans need money and if they insist on investing for more than 10 years, they will enjoy the power of compounding interest.
The latter could view a periodic investment plan as a means of compulsory savings. As long as the periodic investment plan is turned on, it forces people to develop the habits of saving. The money is deducted once they receive the paycheck to make sure they will not waste it.

- Non-professional investors

Many people might be trying to pick their own stocks and time to invest for a long period of time. However, the result is not so good. When we are talking about the result, it means at least a 10% return every single year. If the return is less than the number, it means they do not know how to invest. It is ok to leave the investment to professional investors to handle.

Why should I use a periodic investment plan?

The periodic investment plan is a passive investment choice. Because of the high inflation or other reasons, people need high returns to ensure that the assets will not depreciate quickly. It is difficult to grasp the exact timing of a one-time investment. More importantly, this strategy is passive and automatic, it will make you persist until the day when you make money. Here are a few reasons why people should invest in a periodic investment plan.

- Inflation

The US inflation over the past decade has been high since the 2008 economic crisis and peaked last year due to the COVID-19 pandemic. According to the study of the real return in the US market in the past 200 years of Professor Schiller from Yale University, the value of a dollar in cash has depreciated from 1 to 0.052, and the annual real return is -1.4%; the annual real return of gold is 0.6%, basically maintaining its value; Long-term bonds are 3.5%; stocks are 6.7%.

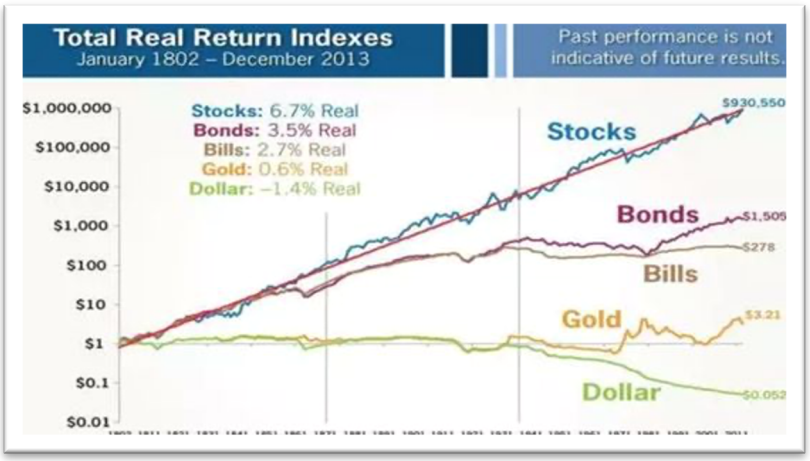

Figure 1: Total Real Return Indexes from 1802 to 2013

The above picture shows the real return from investing in stocks, bonds, bills, gold, and dollars. Although the picture is a little bit old and ends in 2013, the overall trend is set. After the Covid-19 pandemic, the dollars depreciated more because of the quantitative easing strategy. The return of investing in stocks has increased even more since then. Even on a global scale, stocks are an essential tool, sometimes the only tool, to counter inflation. Taking higher risks and obtaining higher returns, which is fair and reasonable.

- Non-professionals don't have the ability to choose the right market timing

When non-professionals want to avoid a potential downturn and play with market timing, one of the biggest costs is missing out on potentially the most lucrative moments. For example, an investor thinks the market is going to go down and sells stocks and invests in more conservative investments. The market is experiencing high returns while money is out of stock. Therefore, the investor miscalculated the time to market and missed the best month.

Here is a table made by Merrill Edge that presents the risk of missing out.

The Risk of Missing Out

	1991-2020	2001-2020	2011-2020
[1] Untouched	$21,115	$4,222	$3,671
[2] Miss 10 top-performing months	$8,117	$1,699	$1,648
[3] Miss 20 top-performing months	$3,855	$866	$1,043

Perhaps the most significant risk of market timing is missing out on the market's best-performing cycles. The three columns represent the growth of a $1,000 investment beginning in 1991, 2001, and 2011 and ending December 31, 2020.

Row 1 shows the investment if left untouched for the entire period shown above; Row 2 shows the investment if it was pulled out during the 10 top-performing months; and Row 3 shows the investment if it was pulled out during the 20 top-performing months.

Source: ChartSource®, DST Retirement Solutions, LLC, an SS&C company. Stocks are represented by Standard & Poor's Composite Index of 500 Stocks, an unmanaged index that is generally considered representative of the U.S. stock market. It is not possible to invest directly in an index. Past performance is not a guarantee of future results.

Not responsible for any errors or omissions. (CS000078)

Past performance is no guarantee of future results.

Figure 2: Risk of Missing Out

- Avoid fully buying at a high price at once

When people think about investments, the most terrifying thing is buying at a high price, but no one is sure if it is at a high price when buying. So simply pass the long-term periodic investment method to amortize the cost. As long as the market is rising in the long run, making money from periodic investment is at a high probability. However, a one-time investment requires professional ability or extraordinary luck, which is a small probability factor for most people.

- Emotion management

In investment, emotional management is more important than technical analysis most of the time. Some people tend to sell the floating profits on the account and keep the unrealized loss, hoping for it to bounce back.

The core is emotional management. In particular, the longer the time in the bottom area of the market, the more loss the investors are experiencing, and psychological pressure will increase accordingly. Periodic investment is a long-term investment strategy of regular fixed amount. The longer the market bottoms, the loss and the loss ratio will continue to decrease. The mental suffering will be relatively easy to overcome, and the main difficulty lies in persistence. Supported by a large amount of data, for the vast majority of unprofessional investors who do not have the ability to capture the market timing, it is still possible to obtain real excess returns through periodic investment.

Periodic Investing with Moving Average Strategy

Asset Traded

In this report, we choose to trade SPDR S&P 500 ETF (SPY) because it is an index with a wide market breadth of the large-cap companies included. For investors who are target users of this strategy, they have little experience in trading stocks so that trading SPY is a less risky option.

In addition to its broad scope, another advantage of the S&P 500 is that components of the index are updated on a quarterly basis. A committee determines which companies to include in the index. The factors considered include a market capitalization in excess of $6.1 billion, a public float of at least 50 percent, headquarters in the U.S., adequate liquidity, and financial viability. Therefore, investors do not need to actively select what to include in the index. Instead, there are experts chosen for us.

Moreover, SPY is also a constantly growing index. We could notice from the belove figure that despite several down due to the 2018 financial crisis and 2020 covid, the index is constantly increasing. From the start with a price less than $30 to a price above $450, the SPY index grows more than 15 times within 28 years.

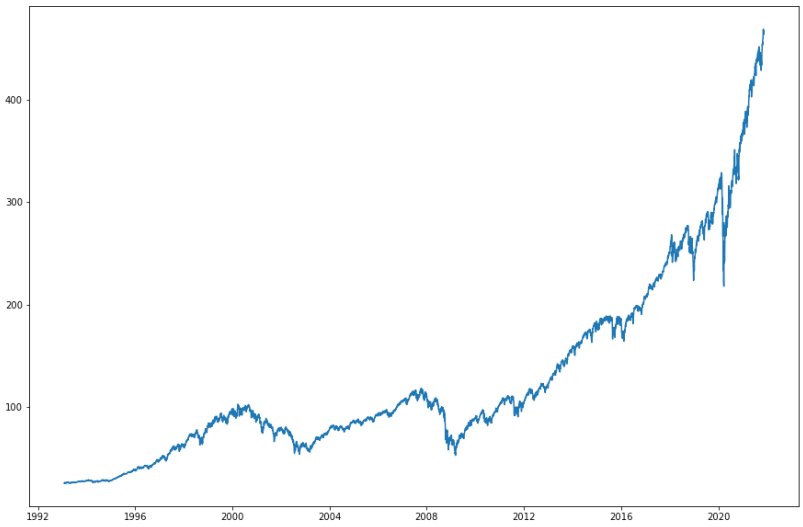

Figure 3: SPY Adjusted Closing Price Over Time

Moving Average Strategy

The moving average (MA) is a simple technical analysis tool that smooths out price data by creating a constantly updated average price. The average is taken over a specific period, for example, 10 days, 20 minutes, 30 weeks or any time period the trader chooses.

Crossovers are one of the main moving average strategies. The first type is a price crossover, which is when the price crosses above or below a moving average to signal a potential change in trend. Another strategy is to apply two moving averages to a chart: one longer and one shorter. When the shorter-term MA crosses above the longer-term MA, it's a buy signal, as it indicates that the trend is shifting up. This is known as a "golden cross." Meanwhile, when the shorter-term MA crosses below the longer-term MA, it's a sell signal, as it indicates that the trend is shifting down. This is known as a "dead/death cross."

The Strategy discussed in this report was inspired by the latter strategy but with some alterations. We no longer use two moving averages. Instead, we will only employ a long MA and the shorter one we choose to use the t-1 closing price, which technically can also be viewed as a one-day moving average. Strategic decision-making is also different. We think when the t-1 closing price is higher than the long-term moving average, it is an “overvalued” signal, and we should buy less. When the t-1 closing price is lower than the long-term moving average, it is an “undervalued” signal, and we should buy more.

Moving Average Strategy used in Periodic Investment

Periodic investing is a strategy of making regular investments on a regular basis, such as monthly, quarterly, or yearly. There are several forms of periodic investing, but they all have the same goals:

- To make investing automatic and eliminate the need to "time" the market
- To smooth out fluctuations in market prices, and

- To reduce the risk and impact of a loss associated with a sudden downturn in the market.

Periodic investing can work well over time. It is particularly useful for investors with small streams of discretionary savings who want long-term investments.

The simplest periodic investment is to invest a fixed amount each month. Our team has determined this as our benchmark strategy with its returns as benchmark return. This report aims to explore how to employ the moving average strategy in periodic investment in order to get a higher return than the simplest periodic investment. As mentioned in the previous section, when comparing t-1 closing price and the long-term moving average, there will be an "overvalued" or "undervalued" signal. Therefore, we can adjust the amount invested according to the signal given by the moving average. For example, the simplest periodic investment plan will invest $100 every month, but with the moving average strategy, we noticed this month's t-1 closing price is lower than the long-term moving average. Thus, we decided to invest $20 more dollars in addition to the $100 this month.

Based on this rule, we conceived five parameters we need to consider for this strategy:

1. **Moving average period**: what are the optimal days used for calculating the long-term moving average? 10 days, 28 days, 100 days, 500 days or more?
2. **Investment period**: the benchmark, simplest periodic investment, is to invest monthly. However, people may also consider investing weekly, bi-weekly, etc.
3. **Function**: are we always investing in a fixed amount, or can we adjust the amount based on a function taking moving average price into account. If we want to adjust the amount, what rules or functions we can follow, for example, linear, log, exponential, etc.
4. **Minimum percentage to invest**: if we decide to invest less, what is the minimum amount we want to invest compared to our benchmark amount?
5. **Maximum percentage to invest**: if we decide to invest more, what is the maximum amount we want to invest compared to our benchmark amount?

In addition, this strategy will always start investing at the beginning of a year. With the above five factors waiting for exploration, we used the SPY dataset to backtest the strategy in order to find the best combination of parameters investors can use in the future.

Return Calculation

To estimate the return of a periodic investment strategy, we used the following formula. The return of the strategy after investing n months is calculated by:

$$\hat{R} = \frac{P_n \times \sum_{i=0}^{n-1} w_i}{\sum_{i=0}^{n-1} M_i} - 1$$

where
- P_n: stock price at the nth month
- w_i: portfolio share we traded at period i. It could also be calculated by $w_i = M_i/P_i$, where the M is the amount of money we invested and P is the portfolio value at period i
- M_i: the amount of money we invested at period i.

Benchmark Returns

We want to generate a table showing returns from the benchmark strategy, so we can compare it with our improved strategy later. The benchmark strategy is investing in S&P 500 monthly with a fixed amount of money. The table below shows the backtest results of return for 3, 5, 10 years of periodic investment into S&P500 at a start date in the first trading day of 1995, 2000, 2015 respectively. We generated 14 different combination results in total.

	1995-01-01	*2000-01-01*	*2005-01-01*	*2010-01-01*	*2015-01-01*
3 years	0.565	-0.258	0.09	0.255	0.309
5 years	0.870	0.107	-0.062	0.517	0.421
10 years	0.390	0.016	0.743	1.044	N/A

Table 1: Benchmark returns

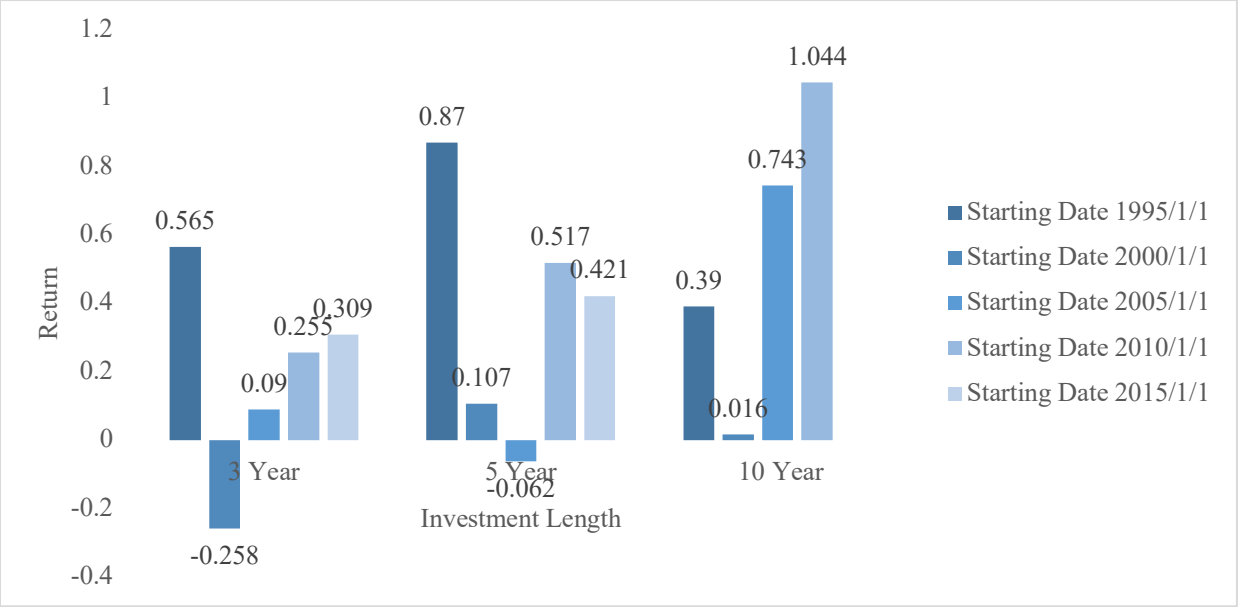

Figure 4: Benchmark returns

From the benchmark strategy, we can clearly find out the following observations:

- Investing in a longer period tends to generate a higher return, whereas if start investing in 2000, the return will be as low as 0.016.
- Two negative returns were generated from investing 3 years beginning 2000/1/1 and investing 5 years beginning 2005/1/1.
- Previously mentioned cases can be attributed to two main causes in general: the 2008 financial crisis and the long correction from 2000-2003 which includes black swan events like the burst of the dotcom bubble and the outbreak of accounting scandals.
- One thing we can learn from examining the benchmarks is that although investing in a longer period tends to generate higher returns, bad years and inappropriate exiting time will largely affect the return.

Exploring Parameters

After gaining the benchmark returns, in this step, we adjust each parameter in order to find the best combination that can outperform the benchmark returns. Specifically, we are going to do grid search on five parameters:

- ma_period: [28, 50, 100, 200, 500, 800, 1200]
- inv_period: ['weekly','monthly']
- function： ['linear','log','exp','ali','null']
- min_rate: np.arange(0.5,0.9,0.1)
- max_rate: np.arange(1.5,2.3,0.1)

We use different functions to determine the amount of investment based on t-1 price and the previous moving average price. In general, the amount to invest in month i should be denoted as:

$$w_i = f\left(\frac{P_i - MA_i}{MA_i}\right)$$

where the MA_i is the moving average price in the given number of days

Let $K_i = -\frac{P_i - MA_i}{MA_i}$, then each function could be explained as follows:

Function	***Mathematical formula***
linear	$w_i = 1 + K_i$
log	$w_i = max(\log_e K_i + 1, 0)$
exp	$w_i = e^{K_i - 1}$
ali	$w_i = \begin{cases} 0.6 & -K_i > 1 \\ 0.7 & -K_i > 0.5 \\ 0.8 & -K_i > 0.15 \\ 0.9 & -K_i > 0 \\ 1 & -K_i \le 0 \end{cases}$

Table 2: Function Formula

The ali function is originated from Alibaba's Alipay. In the fund investment services Alipay provides, investors can select a smart periodic investment option to invest in funds. The rule of this smart option Alipay uses is provided above as the ali formula.

Grid Search Results

Average Performance

The following table shows the description of the returns from the grid search result. Returns are not averaged or annualized according to the investment length.

start date	*investment length*	*mean*	*std*	*min*	*25%*	*50%*	*75%*	*max*
1995-01-01	***3***	0.507	0.054	0.432	0.457	0.498	0.565	0.569
	5	0.877	0.015	0.831	0.870	0.873	0.891	0.898
	10	0.392	0.005	0.384	0.389	0.392	0.397	0.403
2000-01-01	***3***	-0.232	0.035	-0.288	-0.261	-0.242	-0.194	-0.190
	5	0.101	0.010	0.070	0.097	0.106	0.108	0.111
	10	0.048	0.043	-0.026	0.013	0.035	0.095	0.099
2005-01-01	***3***	0.096	0.006	0.089	0.090	0.095	0.102	0.102
	5	-0.036	0.040	-0.106	-0.066	-0.048	0.009	0.013
	10	0.726	0.022	0.672	0.712	0.736	0.743	0.747
2010-01-01	***3***	0.233	0.019	0.206	0.215	0.230	0.253	0.255
	5	0.517	0.008	0.490	0.513	0.517	0.524	0.527
	10	1.049	0.011	1.017	1.042	1.046	1.058	1.064
2015-01-01	***3***	0.306	0.002	0.302	0.305	0.306	0.307	0.309
	5	0.428	0.008	0.417	0.420	0.427	0.436	0.438

Table 3: Grid Search Returns for Different Start Date and Investment Length

From the gird search result table, one notable thing is that some returns are negative. Based on intuition, even though may yield low returns, periodic investing is not supposed to lose money. However, as suggested by the result table, if we invest on 2000-01-01 for three years or invest on 2015-01-01 for 5 years, we will have negative returns (-0.232 and - 0.036 respectively). These two negative returns are likely due to events like 9·11 and financial crisis.

Another observation is that returns may not always be higher when investing in a longer period, because both the start date and investment length matter. We could notice the 5-year investment has the highest return compared to 3-year and 10-year when we start investing on 1995-01-01. Similar patterns can also be found in investments starting 2000-01-01. The reason behind this observation is that due to events like 9·11 and financial crisis, the price dropped a lot. If our investment period covers these events, the return might be low or negative. However, the return of investments starting after 2005 follows the pattern that the longer the investment, the higher the return you will get.

Highest Return

Next, for each combination of the start time and investment period, the table below shows the highest return with strategy parameters resulting in these returns. The numbers in the table represent the highest return we could get from the grid search of parameters. Parameters descriptions under the return are the parameters we used to get these highest returns. Using this table, we aim to find the best parameter sets that yield the highest return for different investment lengths.

	1995-01-01	*2000-01-01*	*2005-01-01*	*2010-01-01*	*2015-01-01*
3-year	**0.569**	**-0.190**	**0.102**	**0.255**	**0.309**
	Monthly 28 days MA ali	Weekly 28 days MA null	Weekly 500 days MA ali	Monthly 28 days MA null	Monthly 28 days MA null
5-year	**0.898**	**0.111**	**0.013**	**0.527**	**0.438**
	Weekly 28 days ma ali	Weekly, 28 days MA null	Weekly, 28 days MA null	Monthly 28 days MA null	Weekly, 28 days MA null
10-year	**0.403**	**0.099**	**0.747**	**1.064**	
	Monthly 800 days MA exp	Weekly 28 days MA null	Weekly 28 days MA null	Weekly 500 days MA exp	N/A

Table 4: Grid Search Highest Returns of Different Start Date and Investment Length

First of all, it is notable that parameters in the table do not include maximum and minimum rate of investing although we did a grid search on them. The reason is that we found max and min rates do not influence the highest return at all. Fixing the investment length, moving average, and function used, we could get the same return by using any combination of min rate from 0.5-0.8 and max rate from 1.5-2.2. Therefore, we concluded that setting the min and max rates is not very helpful in this strategy.

For a 3-year investment, both monthly and weekly investment frequency can get the highest result. Moving average period of 28 days appears the most time. Both null function (means no adjustment of investing amount) and ali function can result of the highest return.

For a 5-year investment, the weekly investment seems more likely to get the highest result. Moving average period of 28 days appears all the time. Null functions also appear most of the time, meaning investing in a fixed amount each period.

For a 10-year investment, weekly investment also beats monthly investing in the likelihood of getting the highest return. However, 10-year investment looks at different moving averages, including 28 days, 800 days, and 500 days, which all possibly result from the highest moving average. The functions used also include two kinds: exponential and null.

Below is the graph showing the highest return of different year. Despite the period including 9·11 and financial crisis, we could find the general trend that the higher the investment length, the higher the return.

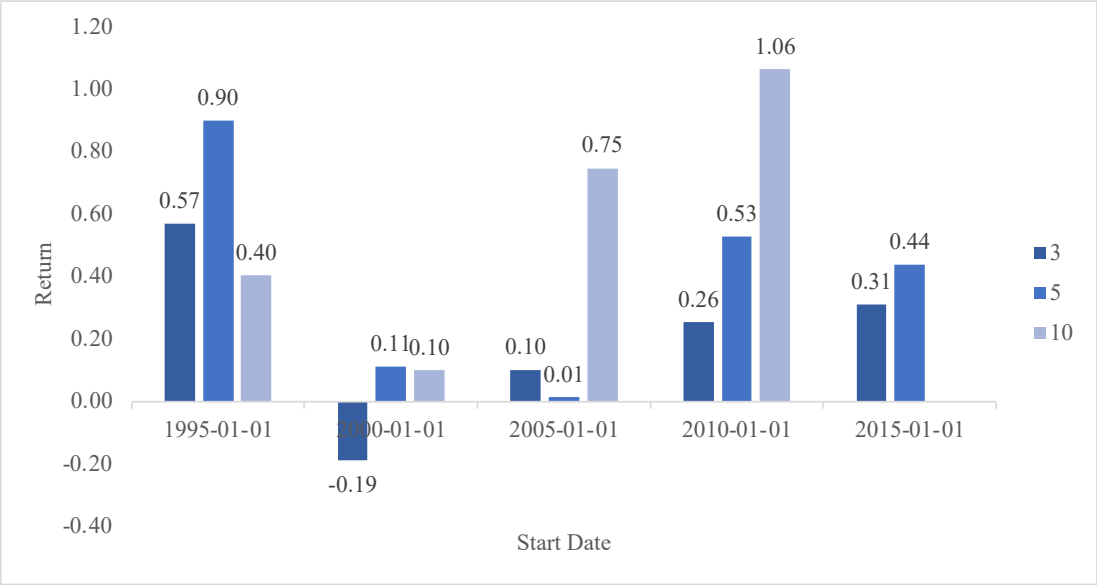

Figure 5: Compare Highest Return of Different Investment Length

From the above highest return table, we could get a general impression of what kind of parameter settings are more likely to result in a higher return. Nevertheless, this observation might be too qualitative, so next, we will analyze the impact of each parameter more quantitatively by controlling other parameters.

Explore Individual Parameters

Explore Investment Period

Investment Length	*Investment Frequency*	*mean*	*std*	*min*	*25%*	*50%*	*75%*	*max*	*max-mean*
3 years	**monthly**	**0.1889**	0.2730	-0.2878	0.0895	0.2533	0.3081	**0.5687**	**0.3798**
	weekly	0.1752	0.2196	-0.2252	0.1009	0.2153	0.3067	0.4594	0.2842
5 years	**monthly**	0.3646	0.3284	-0.1061	0.0946	0.4198	0.5155	0.8792	**0.5146**
	weekly	**0.3899**	0.3166	-0.0343	0.0983	0.4360	0.5254	**0.8978**	0.5079
10 years	**monthly**	0.5415	0.3842	-0.0257	0.2964	0.5374	0.8112	1.0485	**0.5070**
	weekly	**0.5655**	0.3630	0.0529	0.3125	0.5366	0.8190	**1.0641**	0.4986

Table 5: Return for Different Investment Length and Investment Frequency (Unannualized)

The table above shows the level of returns given the investment length and frequency by retrieving all kinds of parameters we set.

The return varies on investment frequency. When we only invest for 3-year, we could get a higher return if reinvest monthly, while weekly investment performs better in 5-year and 10-year investments. While we get a higher return, the standard deviation also increases.

Such a relationship is also similar when we consider their max returns. In other words, the global optimal return (when we prune our other parameters very carefully) for monthly investment is higher than a weekly investment when we only spend 3 years on our PIP, while we get reverse results when we spend 5 or 10 years on PIP.

Explore Moving Average Period

Investment Length	*Moving Average*	*mean*	*std*	*min*	*25%*	*50%*	*75%*	*max*	*Max-mean*
3 years	**28**	**0.1849**	0.247	-0.2602	0.0903	0.235	0.3079	**0.5688**	0.3839
	50	0.1847	0.247	-0.2601	0.0903	0.2346	0.3074	0.568	0.3833
	100	0.1841	0.2471	-0.2614	0.0901	0.2346	0.3067	0.5654	0.3813
	200	0.1831	0.2476	-0.265	0.09	0.2341	0.3062	0.5654	0.3823
	500	0.1814	0.2483	-0.2722	0.0891	0.2354	0.3063	0.5654	0.3840
	800	0.1794	0.2492	-0.2815	0.0895	0.2334	0.3073	0.5654	**0.3860**
	1200	0.1771	0.2493	-0.2879	0.0895	0.2298	0.3074	0.5654	0.3883
5 years	**28**	**0.3828**	0.321	-0.0641	0.1061	0.4286	0.5255	0.8974	0.5146
	50	0.3825	0.3212	-0.0647	0.1057	0.4287	0.5247	**0.8979**	0.5154
	100	0.3817	0.3214	-0.0671	0.1048	0.4287	0.5234	0.8956	0.5139
	200	0.3802	0.3225	-0.0742	0.1028	0.4283	0.5216	0.8944	0.5142
	500	0.3759	0.3252	-0.0964	0.0954	0.4274	0.5224	0.8918	0.5159
	800	0.3714	0.3256	-0.1062	0.0858	0.4272	0.5168	0.8914	0.5200
	1200	0.3664	0.3233	-0.1061	0.0745	0.428	0.5168	0.8914	**0.5250**
10 years	**28**	**0.5598**	0.3741	0.0143	0.3126	0.5672	0.8213	**1.0604**	0.5006
	50	0.5594	0.3739	0.0146	0.3126	0.5666	0.8212	1.0604	0.5010
	100	0.5585	0.3735	0.0129	0.3126	0.564	0.821	1.0604	0.5019
	200	0.5567	0.3733	0.0084	0.3126	0.5602	0.8204	1.0604	0.5037
	500	0.5525	0.3758	-0.0063	0.3126	0.5473	0.8216	1.0641	0.5116
	800	0.5473	0.3754	-0.0174	0.3126	0.5396	0.8205	1.0604	0.5131
	1200	0.5408	0.3724	-0.0258	0.3126	0.5351	0.8149	1.0604	**0.5196**

Table 6: Return for Different Investment Length and Moving Average

Above all the options of strategy, it always gets the highest mean returns when considering the 28-days moving average. As the time length of the moving average increases, the returns decrease gradually. This could be interpreted that the return of price might have more significantly correlated with short-term average price rather than the long-term price. When we consider moving average price as our benchmark to decide the investment weight. Short-term MA price may give us a higher expected return.

However, values in the max columns do have not much different from each other when we consider different MA benchmarks. This could be drawn that if we could prune suitable

parameters for adjusting our investment weight given the length of moving average we want to look backward, we could still get similar results with different MA options.

Explore Functions

Investment Length	*function*	*mean*	*std*	*min*	*25%*	*50%*	*75%*	*max*	*Max-mean*
3 years	**ali**	0.1836	0.2485	-0.268	0.0904	0.2334	0.3069	**0.5688**	**0.3852**
	exp	0.1805	0.2475	-0.282	0.0899	0.2297	0.3067	0.5649	0.3844
	linear	0.1806	0.2481	-0.284	0.0899	0.2300	0.3067	0.5649	0.3843
	log	0.1805	0.2488	-0.287	0.0900	0.2303	0.3067	0.5650	0.3845
	null	**0.1854**	0.2467	-0.258	0.0904	0.2358	0.3086	0.5650	0.3796
5 years	**ali**	0.3802	0.3237	-0.077	0.0985	0.4286	0.5235	**0.8979**	0.5177
	exp	0.3748	0.3220	-0.090	0.0907	0.4270	0.5220	0.8924	0.5176
	linear	0.3745	0.3234	-0.096	0.0888	0.4272	0.5222	0.8926	0.5181
	log	0.3736	0.3253	-0.106	0.0858	0.4273	0.5224	0.8927	**0.5191**
	null	**0.3832**	0.3202	-0.062	0.1067	0.4297	0.5269	0.8914	0.5082
10 years	**ali**	0.5560	0.3737	0.002	0.3160	0.5579	0.8172	1.0597	0.5037
	exp	0.5520	0.3736	-0.015	0.3131	0.5452	0.8131	**1.0641**	0.5121
	linear	0.5508	0.3740	-0.019	0.3131	0.5423	0.8134	1.0637	0.5129
	log	0.5487	0.3745	-0.026	0.3131	0.5372	0.8136	1.0633	**0.5146**
	null	**0.5604**	0.3745	0.016	0.3126	0.5662	0.8216	1.0604	0.5000

Table 8: Return for Different Investment Length and Function

In the above table, we explored for different investment length what is the best performing function. The means in this table means for each investment length if we only look at the function and do not care other parameters, what would the mean return be. For a 3-year length investment, using null function yield the highest mean return, and using ali yield the second highest mean return if we do not care other parameters. This pattern happens for 5- and 10-year length investment as well.

The maximum column in this table means the highest return this function will bring when considering its combination with other parameters. In this case, for a 3-year length investment, using ali function yield the highest return, while null performed poorly. This result can also be found for a 5-year length investment as well. For a 10-year length investment, the exponential function works the best when it forms a good combination with other parameters.

The max-mean column represents the premium brought by adjusting other parameters. The higher the value, the higher the return of the strategy also incorporates other parameters besides considering function. It is interesting to find that log will have the highest parameter premium for 5- and 10-year investments.

To conclude the effect of functions in the periodic investing strategy, we would note that if the function is the only factor considered in this strategy, then fixing the periodic investing amount is likely to yield higher returns than adjusting investing amount each period according to moving average. However, if investors also want to consider other parameters such as moving average

period and investment frequency, then we would recommend using ali function for 3- and 5-year investments and using exp function for 10-year investments.

Suggested Parameters

Based on all analytical results above, our final suggestion for investors is summarized in the following table. This suggestion is based on both the highest return table as a result of grid search and finding when exploring the individual and combined effect of investment frequency, moving average period, and functions.

Investment Length	*Investment Frequency*	*MA period*	*Function*
3 years	monthly	28 days	ali
5 years	weekly	28 days	ali
10 years	weekly	28 days	exp

Table 9: Final Suggested Parameters Given Investment Length

As suggested in above table, if investors plan to invest periodically for three years, then investing monthly, comparing 28 days SPY moving average with t-1 SPY price, and adjustment investing amount using ali function is likely to generate higher returns than using other parameters. For investors who plan to do a 5-year periodic investment, a better practice is to invest weekly using 28 days moving average and ali function. For 10-year investors, a better practice is to invest weekly using 28 days moving average and exponential function.

Backtest using Suggested Parameters

Three-year Periodic Investment

To backtest the suggested parameter, we used the suggested parameters with start date being the first trading day on each of the year from 1995 to 2018.

Figure 6: return vs. benchmark return

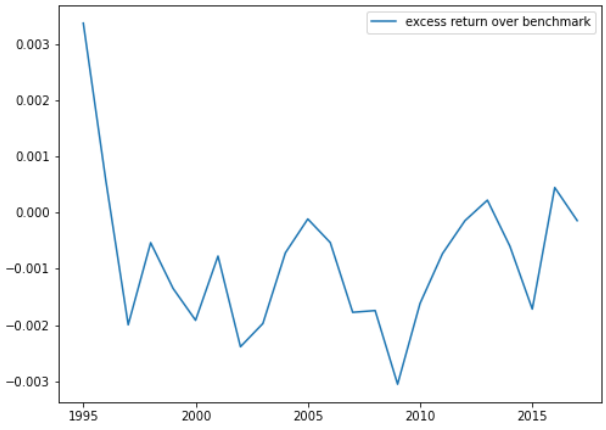

Figure 7: excess return over benchmark

Returns using suggested parameters and benchmark returns using the simplest monthly fixed amount investment (our benchmark return) are plotted in the left graph. We could find these two returns overlapped for all start years. The right graph shows the excess return brought by suggested parameters over the benchmark return. Most excess returns fall below 0, meaning the suggested parameters do not work well and could not beat the benchmark. However, we think this result is normal as three years investment are still so short that may introduce a lot of volatility. With the backtest result, investors invest periodically with both sets of parameters are fine:

- Suggested: monthly investment using 28 days moving average and ali function
- Benchmark: monthly fixed amount investment

Five-year Periodic Investment

To backtest the suggested parameter for five-year investments, we used start date being the first trading day on each of the year from 1995 to 2015.

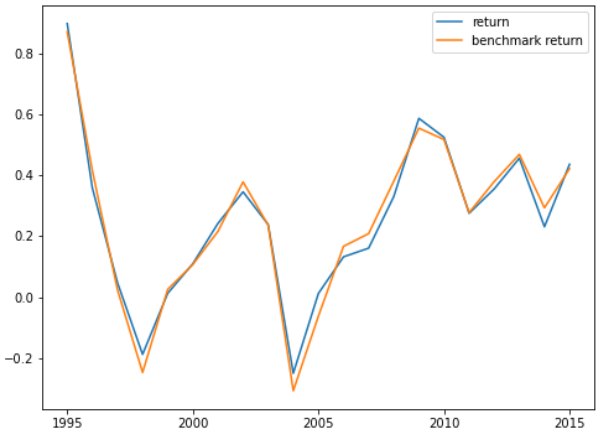

Figure 8: return vs. benchmark return

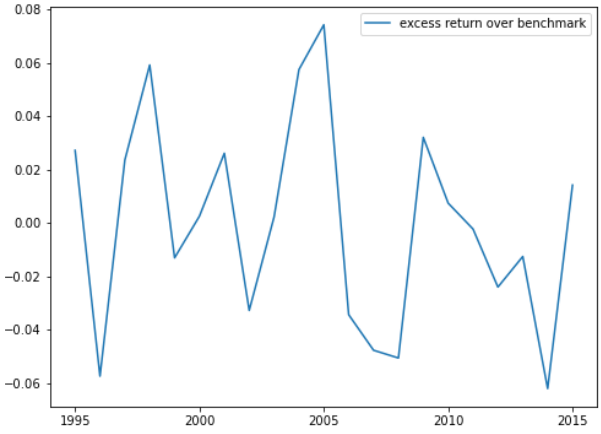

Figure 9: excess return over benchmark

Five-year results are better than the results of three-year investments. From the above two graphs, we could roughly estimate that about half of the time, return from our suggested parameters can beat the benchmark return. The right graph shows excess returns can be as high as 0.07 and as low as -0.06. Once we discover patterns underlying the change in excess return, we may adjust our parameters year over year to obtain higher returns.

Ten-year Periodic Investment

To backtest the suggested parameter for ten-year investments, we used start date being the first trading day on each of the year from 1995 to 2010.

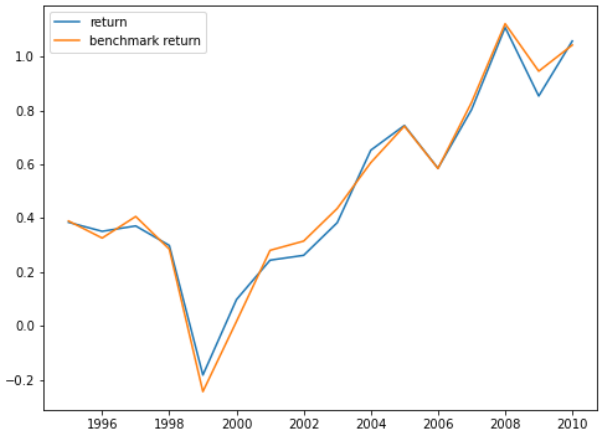

Figure 10: return vs. benchmark return

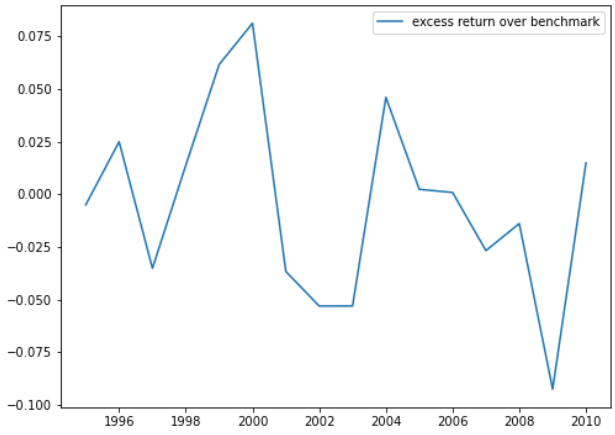

Figure 11: excess return over benchmark

Before comparing the return with benchmark returns, we can observe from the left graph that 10-year investment returns are constantly growing since 1999. Ten-year results are also better than the results of three-year investments. Similarly, return from our suggested parameters can beat the benchmark return about half of the time. The highest excess return is achieved in year 2000 with a value above 0.075, and the lowest value is -0.09 observed for year 2009. Again, once we discover patterns underlying the change in excess return, we may adjust our parameters year over year to obtain higher returns.

Evaluation and Future Actions

Limitations and Potential Improvements

- Limited number of parameter values used in the grid search

In the grid search, due to the computational power of our laptops, we only choose a limited number of values for each parameter. However, the resulting highest returns coming from these limited values may not be the actual highest returns we are able to achieve. For example, for the moving average period, we only tried eight values: [28, 50, 100, 200, 500, 800, 1200]. However, the highest return may be achieved by using 56 days moving average. So, with this limited number of parameter values, we tried to impose the risk of missing out on the actual best parameters. If having more time, we would try more values.

- Fixed the start date at the first trading day of a year

The periodic investment plan using the moving average strategy discussed in this report allows starting trading only on the first trading day of a year. However, this time might not be a good time to start the periodic investment plan. In real practice, people may want to start trading at any time. Therefore, to improve on this issue, we may randomly select 30-50 start times from a period of time and calculate the average return, which the result is more likely to be generalized to other start dates.

- Only tried 3, 5, 10 years investment length

In this report, we only discussed parameters for 3, 5, 10 years investment length. However, in real practice, people may invest in any length of time, or people may not decide the exact length of investment when they started. To improve, we may include more values of investment length ranging from months to 30 years. Also, we should suggest a more generalized parameter setting regardless of the investment length.

Future Actions

- Locate the best exit timing or time to secure profits

There has never been an omnipotent investment method in the world, and no investment method that is sure to make money, any method will have its shortcomings. One shortcoming of periodic investment is that the cost rises too fast in the bull market. When the bull market turns into a bear market, the fixed investment portfolio is prone to large losses, that is, the "inverted smile curve cycle". This means that the periodic investment fund still needs to be managed to reach better profits. the Periodic investment plan should be reduced, stopped, or even sold at the right time to secure profits.

- Portfolio building across different asset categories

According to the different risk tolerance levels of each investor, the portfolio building could be varying. S&P 500 will not be the only investing target to focus on. Configuring a portfolio with an index fund and funds from different sectors might generate better results.

References

Who owns stocks in America? mostly, it's the wealthy and ... (n.d.). Retrieved November 25, 2021, from https://www.usnews.com/news/national-news/articles/2021-03-15/who-owns-stocks-in-america-mostly-its-the-wealthy-and-white

Edge, M. (2021). Here's why time in market is more important than market timing. Merrill Edge. Retrieved December 3, 2021, from https://www.merrilledge.com/article/focus-on-time-in-market-not-market-timing

Tomwfranck. (2020, August 5). The ballooning money supply may be the key to unlocking inflation in the U.S. CNBC. Retrieved December 3, 2021, from https://www.cnbc.com/2020/08/05/the-ballooning-money-supply-may-be-the-key-to-unlocking-inflation-in-the-us.html.