UNIVERSITY OF LUCKNOW

"ENGINEERING MATHEMATICS"

" ASSIGNMENT - 04 "

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CLASS - CSE(AI) - II (P1)

ROLL NO - 115

Submitted to - Dr. Khusboo Verma.

1. Expand e^x and a^x in power of x by Maclaurin's series.

Soln:-

$f(x) = e^x$

$f'(x) = e^x$

$f''(x) = e^x$

$x - a = x$

$a = x - x$

$a = 0$

$$f(a+(x-a)) = f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a)$$

$$\cdots\cdots \frac{(x-a)^n}{n!} f^n(a)$$

By Taylor's theorem and Maclaurin Theorem.

$f(x) = e^x \quad , \quad a = 0$

$$\Rightarrow f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \cdots\cdots$$

$$e^x \Rightarrow 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots\cdots \frac{x^n}{n!}$$

$f(x) = a^x$

$f'(x) = a^x \log a$

$f''(x) = a^x (\log a)^2$

$f^n(x) = a^x (\log a)^n$

$x - a = x \quad \Rightarrow \quad a = 0$

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots\cdots \frac{(x-a)^n}{n!} f^n(a)$$

$$f(x) = f(0) + \frac{x}{1!} \log a + \frac{(x-a)^2}{2!} (\log a)^2 \cdots\cdots \frac{(x)^n}{n!} (\log a)^n$$

$$a^x = 1 + \frac{x}{1!} \log a + \frac{x^2}{2!} (\log a)^2 \cdots\cdots \frac{x^n}{n!} (\log a)^n$$

$^{n}2 \quad ^{n}2 \to n \qquad y^3 = a^3$

2). Prove by Maclaurin's Theorem that

$$e^{\sin x} = 1 + x + \frac{x^2}{1.2} - \frac{3.x^4}{1.2.3.4}$$

Soln:-

$f(x) = e^{\sin x}$

$f'(x) = \cos x \, e^{\sin x}$

$f''(x) = e^{\sin x} \cdot \cos^2 x - \sin x \cdot e^{\sin x}$

$= e^{\sin x}(\cos^2 x - \sin^2 x)$

$f'''(x) = e^{\sin x}[2\cos x(-\sin x) - \cos x] + e^{\sin x}\cos x \cdot [\cos^2 x - \sin x]$

$= e^{\sin x}\cos x[-2\sin x - 1 + \cos^2 x - \sin x]$

$= e^{\sin x}\cos x[-3\sin x + \sin^2 x]$

$f''''(x) = -e^{\sin x}\cos x[3\cos x + 2\sin x\cos x] + e^{\sin x} \cdot \sin x$
$[3\sin x + \sin^2 x] - [3\sin x + \sin^2 x]\cos^2 x \cdot e^{\sin x} \cdot \cos x$

$f(0) = 1 \quad f'(0) = 1 \quad f''(0) = 1 \quad f'''(0) = 0 \quad f^{IV}(0) = -3$

Taylor's Theorem,

$$f(x) = f(x) + x f'(x) + \frac{x^2}{2!} f''(x) + \frac{x^3}{3!} f'''(x) + f^{IV}(x) \cdot \frac{x^4}{4!}$$

$$e^{\sin x} = 1 + \frac{x}{1!}(1) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-3)$$

$$e^{\sin x} \Rightarrow 1 + x + \frac{x^2}{1.2} + \frac{-3x^4}{1.2.3.4} + \cdots$$

3) Expand $\sin(x+y)$ in power of 'y'

Soln:- Put $f(x+y) = \sin(x+y)$

let $y = 0$

$f(x) = \sin x$

$f'(x) = \cos x \qquad f''(x) = -\sin x$

$f'''(x) = -\cos x \qquad f^{IV}(x) = \sin x$

Using Taylor's Theorem,

$$f(x+y) = f(x) + \frac{y f'(x)}{1!} + \frac{y^2}{2!} f''(x) + \frac{y^3}{3!} f'''(x) + \cdots$$

$$\sin(x+y) = \sin x + y\cos x + \frac{y^2}{2!}(-\sin x) + \frac{y^3}{3!}(-\cos x) + \frac{y^4}{4!}\sin x$$

$$\sin(x+y) = \sin x + y\cos x - \frac{y^2}{2!}\sin x - \frac{y^3}{3!}\cos x + \frac{y^4}{4!}\sin x.$$

4) Expand $\log x$ in power of $(x-1)$ by Taylor's Theorem.

Soln:-

$f(x) = \log x$ $\qquad$ $x-a = x-1$

$\boxed{a=1}$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{+2}{x^3}$

$f^{iv}(x) = \frac{-6}{x^4}$

$x = a = 1$

$f'(a) = \log 1 = 0$ $\qquad$ $f'(0) = 1$

$f''(a) = -1$

$f'''(a) = 2$

$f^{iv}(a) = -6$

Using Taylor's Theorem;

$\log x = f(x) + (x-1) f'(x) + \frac{(x-1)^2}{2!} f''(x) + \frac{(x-1)^3}{3!} f'''(x)$

$\log x = f(1) + (x-1) f'(1) + \frac{(x-1)^2}{2!} f''(1) + \frac{(x-1)^3}{3!} f''''(1) \ldots$

$= 0 + (x-1) + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{3!}(+2) + \frac{(x-1)^4}{4!}(-6)$

$\log x = (x-1) \mp \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3} \mp \frac{1}{4}(x-1)^4 + \ldots$

5) Expand $e^x \sin y$ in powers of x & y as far as terms of the third degree.

Soln: let $f(x,y) = e^x \sin y$

$f_x(x,y) = e^x \sin y$ $\qquad$ $f(0,0) = 0$

$f_y(x,y) = e^x \cos y$ $\qquad$ $f_x(0,0) = 0$

$f_{xx}(x,y) = e^x \sin y$ $\qquad$ $f_y(0,0) = 1$

$f_{yy}(x,y) = -e^x \sin y$ $\qquad$ $f_{xx}(0,0) = 0$

$f_{xy}(x,y) = +e^x \cos y$ $\qquad$ $f_{yy}(0,0) = 0$

$f_{xy}(0,0) = 1$

$f_{xxx}(x,y) = e^x \sin y \qquad f_{xxx}(0,0) = 0$

$f_{yyy}(x,y) = \cancel{-e^x \sin} \; -e^x \cos y \qquad f_{yyy}(0,0) = -1$

$f_{xxy}(x,y) = e^x \cos y \qquad f_{xxy}(0,0) = 1$

$f_{xyy}(x,y) = -e^x \sin y \qquad f_{xyy}(0,0) = 0$

$\therefore f(x,y) = f(0,0) + [x f_x(0,0) + y f_y(0,0)] + \frac{1}{2!}[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)] + \frac{1}{3!}[x^3 f_{xxx}(0,0) + 3x^2y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0) + y^3 f_{yyy}(0,0)]$ ----

$\Rightarrow 0 + [x(0) + y(1)] + \frac{1}{2!}(x^2(0) + 2xy(1) + y^2(0)) + \frac{1}{3!}[x^3(0) + 3x^2y(1) + 3xy^2(0) + y^3(-1)]$

$\Rightarrow y + \frac{1}{2!}(2xy) + \frac{1}{3!}(3x^2y - y^3)$

6) Find the first 6 terms of the expansions of the function $e^x \log(1+y)$ in a Taylor series in the neighbourhood of the point $(0,0)$.

Soln: Given, $f(x,y) = e^x \log(1+y)$

$f_x(x,y) = e^x \log(1+x) \cdot \quad f_x(0,0) = 0$

$f_y(x,y) = e^x \cdot \frac{1}{1+y} \qquad f_y(0,0) = 1$

$f_{xx}(x,y) = e^x \log(1+y) \qquad f_{xx}(0,0) = 0$

$f_{yy}(x,y) = \frac{-e^x}{(1+y)^2} \qquad f_{yy}(0,0) = -1$

$f_{xy}(x,y) = e^x (1+y)^{-1} \qquad f_{xy}(0,0) = 1$

$f_{xxx}(x,y) = e^x \log(1+y) \qquad f_{xxx}(0,0) = 0$

$f_{yyy}(x,y) = \frac{2e^x}{(1+y)^3} \qquad f_{yyy}(0,0) = 2$

$f_{xxy}(x,y) = \frac{e^x}{1+y} \qquad f_{xxy}(0,0) = 1$

$$f_{xyy}(x,y) = \frac{-2e^x}{(1+y)^2} \quad . \quad f_{xyy}(0,0) = -1$$

$$f(x,y) = f(0,0) + [x f_x(0,0) + y f_y(0,0)] + \frac{1}{2!}[x^2 f_{xx}(0,0)$$
$$+ y^2 f_{yy}(0,0) + 2xy f_{xy}(0,0)] + \frac{1}{3!}[x^3 f_{xxx}(0,0) +$$
$$y^3 f_{yyy}(0,0) + 3x^2y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0)]$$

$$e^x \log(1+y) = 0 + [x(0) + y(1)] + \frac{1}{2!}[x^2(0) + y^2(-1) + 2xy(1)]$$
$$+ \frac{1}{3!}[x^3(0) + y^3(2) + 3x^2y(1) - 3xy^2(+1)$$

$$e^x \log(1+y) = y + \left[\frac{-y^2}{2} + xy\right] + \left[\frac{y^3}{3} + \frac{x^2y}{2} - \frac{xy^2}{2}\right] - - -$$

7) Expand $x^2y + 3y - 2$ is powers of $(x-1)$ and $(y+2)$ using Taylor's Theorem.

Soln: $f(x,y) = f(a,b) + [(x-a) f_x(a,b) + (y-b) f_y(a,b)] + \frac{1}{2!}[(x-a)^2$
$f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)]$
$+ \frac{1}{3!}[(x-a)^3 f_{xxx}(a,b) + 3(x-a)^2(y-b) \cdot f_{xxy}(a,b) + 3(x-a)$
$(y-b)^2 f_{xyy}(a,b) + (y-b)^3 f_{yyy}(a,b)] + \_ \_ \_$

$$f(x,y) = x^2y + 3y - 2$$

$x - a = x - 1$, $y - b = y + 2$

$\boxed{a = 1}$ $\boxed{b = -2}$

$$f(1,-2) = (1)^2(-2) + 3(-2) - 2$$
$$= -2 - 6 - 2$$
$$= -10$$

$f_x = 2xy$ $\quad f_x(1,-2) = 2(1)(-2) = -4$

$f_y = x^2 + 3$ $\quad f_y(1,-2) = (1)^2 + 3 = 4$

$f_{xx} = 2y$ $\quad f_{xx}(1,-2) = 2(-2) = -4$

$f_{yy} = 0$ $\quad f_{yy}(1,-2) = 0$

$f_{xy} = 2x \qquad f_{xy}(1,-2) = 2(1) = 2$

$f_{xxx} = 0 \qquad f_{xxx}(1,-2) = 0$

$f_{yyy} = 0 \qquad f_{yyy}(1,-2) = 0$

$f_{xxy} = 2 \qquad f_{xxy}(1,-2) = 2$

$f_{xyy} = 0 \qquad f_{xyy}(1,-2) = 0$

$f(x,y) = f(-a,-b)$

$= -10 + [(x-a)(-4) + (y+2)(4)] + \frac{1}{2!}[(x-1)^2(-4) +$
$2(x-1)(y+2)2 + (y+2)^2(0) + \frac{1}{3!}[(x-1)^3(0) + 3(x-1)^2(y+2)0$
$+ 3(x-1)(y+2)^2 0 + (y+2)^3 \cdot (0)]$

$\Rightarrow -10 + [(x-1)(-4) + 4(y+2)] + \frac{1}{2}[-4(x-1)^2 + 4(x-1)^2(y+2)$
$+ 0 + (3(x-1)^2(y+2)2)\frac{1}{3!}$

$= -10 + [-4(x-1) + 4(y+2)] + [-2(x-1)^2 + 2(x-1)(y+2)]$
$+ \frac{3 \times 2}{3 \times 2 \times 1}(x-a)^2(y+2)$

$\Rightarrow -10 - 4(x-1) + 4(y+2) - 2(x-1)^2 + 2(x-1)(y+2) +$
$(x-1)^2(y+2$

8) Find the extreme values of (i) x^3+y^3-3axy
(ii) $x^3y^2(6-x-y)$

Sol^n:- (i) x^3+y^3-3axy

let $z = x^3+y^3-3axy$

$\frac{\partial z}{\partial x} = 3x^2 - 3ay \qquad \frac{\partial z}{\partial y} = 3y^2 - 3ax$

$\frac{\partial z}{\partial x} = 0 \qquad \frac{\partial z}{\partial y} = 0$

$3x^2 - 3ay = 0 \qquad 3y^2 - 3ax = 0$

$3x^2 = 3ay \qquad y^2 = ax$

$x^2 = ay \qquad \frac{y^3}{a^2} = ay$

$y^2 = ax$

$y^3 - a^3 = 0$ $\qquad\qquad$ $y^3 = a^3$

$\boxed{y = 0, a}$

at $y = 0$, $x = 0$ $\qquad$ point $A(0,0)$ & $B(0,a)$

$y = 0$, $x = a$

$\frac{\partial^2 z}{\partial x^2} = 6x - r$ $\quad$ $\frac{\partial^2 z}{\partial y^2} = 6y = t$ $\quad$ $\frac{\partial^2 z}{\partial y \partial x} = -3a = s$

if $(rt - s^2) > 0$, $r > 0$ at any point (a,b)
then the points are minima.

$\Rightarrow 6x \cdot 6y - (-3a)^2 \Rightarrow 36xy - 9a^2$ $\qquad$ $\therefore$ Not an extreme
at $(0,0)$ $\qquad -9a^2 < 0$ $\qquad$ point $A(0,0)$

At $B(a,a)$

$rt - s^2 = 6x \cdot 6y - 9a^2$ $\qquad$ $r = 6x = 6a$
at B $\qquad\qquad$ $= 6a > 0$ if $a = +ve$

$= 36xy - 9a^2$

$= 36a^2 - 9a^2$

$= 27a^2 > 0$

$\therefore$ $B(a,a)$ is the minima point.

$x^3 + y^3 - 3axy$

At (a,a),

$\Rightarrow a^3 + a^3 - 3a^3$

$2a^3 - 3a^3$

$= -a^3$

$-a^3$ is the Minimum value.

(ii) let $z = 6x^3y^2 - x^4y^2 - x^3y^3$

$\frac{\partial z}{\partial x} = 18x^2y^2 - 4x^3y^2 - 3x^2y^3$ $\quad$ $\frac{\partial z}{\partial y} = 12x^3y - 2x^4y - 3y^2x^3$

$\frac{\partial z}{\partial x} = 0$ $\qquad\qquad$ $\frac{\partial z}{\partial y} = 0$

$18x^2y^2 - 4x^3y^2 - 3x^2y^3 = 0$ $\qquad$ $x^3y(12 - 2x - 3y) = 0$

$x^2y^2(18-4x-3y)=0$ $\qquad$ $12-2x-3y=0$ — (ii)

$18-4x-3y=0$ — (i)

Adding eq (i) & (ii)

$$\begin{array}{r} 18-4x-3y=0 \\ \underset{(-)}{}\ 12\underset{(+)}{-}2x\underset{(+)}{-}3y=0 \\ \hline 6-2x=0 \end{array}$$

$\boxed{x=3}$

$12-6-3y=0$

$6=3y$ $\qquad$ $\boxed{y=2}$

Point A (3,2)

$\frac{\partial^2 z}{\partial x^2}=36xy^2-12x^2y^2-6xy^3$ $\quad$ $\frac{\partial^2 z}{\partial y^2}=12x^3-2x^4-6yx^3$

$\frac{\partial^2 z}{\partial x\partial y}=36x^2y-8x^3y-9x^2y^2$

$r=6xy^2(6-2x-y)$. $\qquad$ $t=2x^3(6-x-3y)$

$s=3x^2y(36-8x-9y)$

$rt-s^2=6x^4y^2(6-2x-y)(12-2x-6y)-x^4y^2(36-8x-9y)^2$

$=23328-324(36)$

$=23328-11664$

$=11664$

$rt-s^2>0$

$r=6xy^2(6-2x-y)$

$=-144<0$

$\therefore$ (3,2) is a maxima point.

9) Find the minimum distance from the point (1,2,0) to the cone $z^2=x^2+y^2$

Soln:- By using distance formula.

$$D=\sqrt{(x-1)^2+(y-2)^2+z^2}$$

$$4=D^2=(x-1)^2+(y-2)^2+z^2$$

~~ϕ~~ $\phi = x^2+y^2-z^2=0$

Lagrange's function.

$$F(x,y,z,\lambda) = f + \lambda\phi$$
$$= (x-1)^2+(y-2)^2+z^2+\lambda(x^2+y^2-z^2)$$
$$dF=0$$

$$dF = \{2(x-1)+\lambda 2(x)\}\,dx + (2(y-2)+\lambda 2y)\cdot dy + (2z-\lambda 2z)\,dz$$

$dF=0$

$\cancel{2(x-1)+\lambda 2(x)\,dx}$

$\cancel{2}(x-1)+\lambda(\cancel{2}x)=0$

$(x-1)+(x\lambda)=0$

$x\cdot(1+\lambda)-1=0$

$x=\dfrac{1}{1+\lambda}$

$y-2+\lambda y=0$

$\lambda(1+y)=2$

$\lambda=\dfrac{2}{1+y}$

$2z-\lambda 2z=0$

$2z(1-\lambda)=0$

$1-\lambda=0$

$\boxed{\lambda=1}$

$x=\frac{1}{2}\quad y=1$

$\phi=x^2+y^2-z^2=0$

$\left(\frac{1}{2}\right)^2+(1)^2-z^2=0$

$\frac{1}{4}+1=z^2$

$\frac{5}{4}=z^2$

$z=\sqrt{\frac{5}{4}}=\pm\frac{\sqrt{5}}{2}$

Hence, minimum distance from the point $(1,2,0)$ is

$$u=(x-1)^2+(y-2)^2+z^2$$
$$=\left(\tfrac{1}{2}-1\right)^2+(1-2)^2+\tfrac{5}{4}$$
$$=\left(\tfrac{-1}{2}\right)^2+(-1)^2+\tfrac{5}{4}$$
$$=\tfrac{1}{4}+1+\tfrac{5}{4}=\tfrac{6}{4}+1=\tfrac{3}{2}+1=\tfrac{5}{2}$$

$D^2=\frac{5}{2}\qquad D=\sqrt{\frac{5}{2}}$

10) Use lagrange's method of indetermined multipliers to find the minimum value of $x^2+y^2+z^2$ subject to the conditions $x+y+z=1$, $xyz+1=0$ (10)

Soln:- $F(x,y,z,\lambda;u) = x^2+y^2+z^2+\lambda(x+y+z-1)+$
$u(xyz+1)$

$dF=0$

$$\left.\begin{aligned} 2x+\lambda+uyz&=0 \\ 2y+\lambda+uxz&=0 \\ 2z+\lambda+uxy&=0 \end{aligned}\right\} \begin{aligned} &\times x \text{ --- (i)} \\ &\times y \text{ --- (ii)} \\ &\times z \text{ --- (iii)} \end{aligned}$$

$2x^2+x\lambda+uxyz+2y^2+\lambda y+uxyz+2z^2+\lambda z+uzxy=0$

$2(x^2+y^2+z^2)+\lambda(x+y+z)+u(3xyz)=0$

$2(x^2+y^2+z^2)+\lambda+u(3(-1))=0$

$2u+\lambda-3u=0$

$\boxed{2u+\lambda=3u}$

$\lambda=3u-2u$

$2x+3u-u+uyz=0$

$(x-u)+u(3+yz)=0$

$y+3u-u+uxz=0$

$(y-u)+(3+xz)=0$

$z+3u-u+uxy=0$

$(z-u)+u(3-xy)=0$

Subtracting eq (ii) from eq (i)

$2x+\cancel{\lambda}+uyz-2y-\cancel{\lambda}-uxz=0$

$2(x-y)+xz(y-x)=0$

$x-y(1-uz)=0$

$x=y$ & $1-uz=0$

$y=z$ $\quad u=\frac{1}{z}$

Multiplying eq (i) & eq (i)

$\cancel{x^2+y}$ $x^2-y^2+\lambda(x-y)=0$

$(x-y)(x+y+\lambda)=0$

$x = y \cdot \qquad \lambda = y - x$

In eq (i)

$(y+2) + \frac{1}{x}(yz) = 0$

$x - (1-x) + \frac{1}{x}\left(\frac{-1}{x}\right) = 0$

$x - 1 + x - \frac{1}{x^2} = 0$

$2x - 1 - \frac{1}{x^2} = 0$

$2x^3 - x^2 - 1 = 0$

$(-x+1)(2x^2+x+1) = 0$

$\boxed{x = 1}$, $y = x - 1$, $z = \frac{xyz}{xy} = -1$

The minimum $u = x^2 + y^2 + z^2$

$= (1)^2 + (1)^2 + (-1)^2$

$= 1 + 1 + 1$

$= 3$

11) Find the maxima & minima of $u = x^2 + y^2 + z^2$ subject to the condition $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$

Soln: $u = x^2 + y^2 + z^2$

$F(x, y, z, \lambda, \mu) = x^2 + y^2 + z^2 + \lambda(ax^2 + by^2 + cz^2) + \mu(lx + \cancel{ly}\, my + nz)$

$dF = 0$

$2x + 2ax\lambda + \mu l = 0$ ——— (i) $\times x$

$2y + 2by\lambda + \mu m = 0$ ——— (ii) $\times y$

$2z + 2cz\lambda + \mu n = 0$ ——— (iii) $\times z$

$dF = (2x^2 + 2ax^2\lambda + \mu lx + 2y^2 + 2by^2\lambda + \mu my + 2z^2 + \cancel{c}z^2\lambda + \mu nz) = 0$

$0 = 2(x^2 + y^2 + z^2) + 2\lambda(ax^2 + by^2 + cz^2) + \mu(\cancel{1}lx + my + nz)$

$0 = 2u + 2\lambda(1) + 0$

$$2\lambda + 2u = 0$$
$$\boxed{\lambda = -u}$$

$$2x + 2ax(-u) + ul = 0$$
$$2x(1-au) + ul = 0$$
$$x = \frac{-ul}{2(1-au)}$$

Similarly:-

$$y = \frac{-um}{2(1-bu)} \quad z = \frac{-un}{2(1-cu}$$

∴ Putting x, y, z value in eq $(lx+my+nz)=0$

$$\frac{-ul}{2(1-au)} - \frac{um}{2(1-bu)} - \frac{um}{2(1-cu)} = 0$$

$$= -\frac{u}{2}\left(\frac{l}{(1-au)} + \frac{m}{(l-bu)} + \frac{n}{(l-cu)}\right) = 0 \,//$$

12) Find the volume of ~~of~~ the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Soln:- $F(x, y, z, \lambda) = 8xyz + \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)$

$$dF = 0$$

$$8yz + \frac{2x\lambda}{a^2} = 0 \quad \times x \text{ — (i)}$$

$$8xz + \frac{2y\lambda}{b^2} = 0 \quad \times y \text{ — (ii)}$$

$$8xy + \frac{2z\lambda}{c^2} = 0 \quad \times z \text{ — (iii)}$$

$$2\lambda + 2u = 0$$

$$\boxed{\lambda = -u}$$

$$2x + 2ax(-u) + wl = 0$$

$$2x(1-au) + wl = 0$$

$$x = \frac{-wl}{2(1-au)}$$

Similarly:-

$$y = \frac{-wm}{2(1-bu)} \quad z = \frac{-wn}{2(1-cu}$$

∴ Putting x, y, z value in eq $(lx + my + nz) = 0$

$$\frac{-wl}{2(1-au)} - \frac{wm}{2(1-bu)} - \frac{wm}{2(1-cu)} = 0$$

$$= -\frac{w}{2}\left(\frac{l}{(1-au)} + \frac{m}{(l-bu)} + \frac{n}{(l-cu)}\right) = 0 \text{ //}$$

12) Find the volume of of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Soln:- $F(x, y, z, \lambda) = 8xyz + \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)$

$$dF = 0$$

$$\left.\begin{aligned} 8yz + \frac{2x\lambda}{a^2} &= 0 \\ 8xz + \frac{2y\lambda}{b^2} &= 0 \\ 8xy + \frac{2z\lambda}{c^2} &= 0 \end{aligned}\right\} \begin{aligned} &\times x \text{ — (i)} \\ &\times y \text{ — (ii)} \\ &\times z \text{ — (iii)} \end{aligned}$$

$$8xyz + 8xyz + 8xyz + \frac{2x^2\lambda}{a^2} + \frac{2y^2\lambda}{b^2} + \frac{2z^2\lambda}{c^2} = 0$$

$$24xyz + 2\lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 0$$

$$24xyz + 2\lambda = 0$$

$$12xyz + \lambda = 0$$

$$\lambda = -12xyz$$

Putting $\lambda = -12xyz$ in eq (i)

$$8yz + (-12xyz)\cdot\frac{2x}{a^2} \Rightarrow$$

$$8yz\left(1 - \frac{3x^2}{a^2}\right) = 0$$

$$1 - \frac{3x^2}{a^2} = 0$$

$$\frac{3x^2}{a^2} = 1$$

$$3x^2 = a^2$$

$$x^2 = \frac{a^2}{3}$$

$$x = \sqrt{\frac{a^2}{3}} \Rightarrow \frac{a}{\sqrt{3}}$$

Similarly, $y = \frac{b}{\sqrt{3}}$, $z = \frac{c}{\sqrt{3}}$

$\therefore$ Volume of the largest rectangular parallelopiped:

$$= 8xyz = 8 \times \frac{a}{\sqrt{3}} \times \frac{b}{\sqrt{3}} \times \frac{c}{\sqrt{3}}$$

$$= \frac{8abc}{3\sqrt{3}} \;//$$