MATHS ASSIGNMENT - 2

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BRANCH : CSE(AL&ML)

1.

i. From the partial differential equation by eliminating arbitrary function from $f(x+y+z, x^2+y^2+z^2)=0$

Ans)

By Applying Jacobi method .

$$\varphi(u, v) = 0$$

$$u = x + y + z \qquad v = x^2 + y^2 + z^2$$

Partially differentiate each of them separately .

$$\frac{\partial u}{\partial x} = 1 + \frac{\partial z}{\partial x} \qquad \frac{\partial u}{\partial y} = 1 + \frac{\partial z}{\partial y} = 1 + q$$

$$= 1 + p$$

$$\frac{\partial u}{\partial x} = 2x + 2z\frac{\partial z}{\partial x} = 2(x + zp)$$

$$\frac{\partial v}{\partial y} = 2y + 2z\frac{\partial z}{\partial y} = 2(y + zq)$$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 + p & 1 + q \\ 2(x + zp) & 2(y + zq) \end{vmatrix} = 0$$

$$2(1+p)(y+zq)-2(1+q)(x+zp)=0$$

$$(1+q)(y+zq)-(1+q)(x+zp)=0$$

ii. **Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$**

Ans)

$P=(x^2-yz),\quad Q=(y^2-zx),\ R=(z^2-xy)$ --------------(1)

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

$$\frac{dx}{x^2-y^2}=\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}=\frac{d(x-y)}{(x-y)(y+x+z)}=\frac{d(y-z)}{(y-z)(x+y+z)}$$

$choose$ $(1,-1,0)$ are multipliers, then the equivalent subsidiary equation becomes

$$\frac{dx-dy}{x^2-yz-y^2+xz}=\frac{d(x-y)}{x^2-y^2+z(x-y)}$$

$$=\frac{d(x-y)}{(x-y)(x+y+z)}$$

$at(0,1,-1)$

$$=\frac{dy-dz}{y^2-zx-z^2+xy}$$

$$=\frac{d(y-z)}{y^2-z^2+x(y-z)}$$

$$=\frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\frac{d(x-y)}{(x-y)(x+y+z)}=\frac{d(y-z)}{(y-z)(x+y+z)}$$

Integrating on both sides

$$\int\frac{1}{x-y}d(x-y)=\int\frac{1}{y-z}d(y-z)$$

$$\log(x-y)=\log(y-z)+logc1$$

$\frac{x-y}{y-z}=c1$ -------------------------(2)

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - xz} = \frac{dz}{z^2 - xy}$$

Multiply x,y,z on each side

$$= \frac{xdx + ydy + zdz}{x^3 + y^3 + z^3 - 3xyz}$$

$$= \frac{xdx + ydy + zdz}{(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)}$$

At(1,1,1)

$$= \frac{dx + dy + dz}{x^2 + y^2 + z^2 - xy - yz - zx}$$

Now, $\frac{xdx+ydy+zdz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)} = \frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}$

$$\frac{xdx + ydy + zdz}{(x + y + z)} = d(x + y + z)$$

Integrating on both sides

$$\int x\,dx + \int y\,dy + \int z\,dz = \int (x + y + z)\ d(x + y + z)$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{(x + y + z)^2}{2} + c2$$

$x^2 + y^2 + z^2 - (x + y + z)^2 = c2$ ------------------------(3)

$$\emptyset(c1, c2) = 0$$

2.

i. **Find the angle between the normal to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3)**

Ans)

Given

$$f = xy - z^2$$

$$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$
$$= i(y) + j(x) + k(-2z)$$
$$\nabla f_{(4,1,2)} = i + 4j - 4k$$
$$\nabla f_{(3,3,-3)} = 3i + 3j + 6k$$
$$\cos\theta = \frac{(1 + 4j - 4k)(3i + 3j + 6k)}{\sqrt{1 + 16 + 16}\ \ \sqrt{9 + 9 + 36}}$$
$$= \frac{3 + 12 - 24}{\sqrt{33}\sqrt{54}}$$
$$= \frac{-9}{\sqrt{33}\,3\sqrt{6}}$$
$$\cos\theta = \frac{-3}{\sqrt{198}} = \frac{-3}{3\sqrt{22}} = \frac{-1}{\sqrt{22}}$$
$$\theta = \cos^{-1}(\frac{-1}{\sqrt{22}})$$

ii. **If $\boldsymbol{R = xi + yj + zk}$,show that (i)$\boldsymbol{\nabla \cdot R = 3}$,(ii) $\boldsymbol{\nabla * R = 0}$**

Ans)

Given

$$R = x\vec{i} + y\vec{j} + z\vec{k}$$
$$div\ R = \nabla \cdot R$$
$$= (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}) \cdot (x\vec{i} + y\vec{j} + z\vec{k})$$
$$= 1 + 1 + 1$$
$$= 3$$
$$curl\ R = \nabla * R$$

$$\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{bmatrix} = \vec{i}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \vec{j}\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + \vec{k}(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y})$$

=0

3. find the work done in moving particle in the force field $\boldsymbol{F = 3x^2 i + (2xz - y) + zk}$ along the straight line from (0,0,0) to (2,1,3)

Ans)

Given

$F = 3x^2\vec{\imath} + (2xz - y)\vec{\jmath} + z\vec{k}$ at (0,0,0) to (2,1,3)

The equation of a straight line is

$$\frac{x - x1}{x2 - x1} = \frac{y - y1}{y2 - y1} = \frac{z - z1}{z2 - z1}$$

$$\frac{x - 0}{2 - 0} = \frac{y - 0}{1 - 0} = \frac{z - 0}{3 - 0}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{3} = t$$

x=2t , y=t , z=3t

$$R = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

$dR = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$

l.l t=0(0,0,0)

u.l t=1(2,1,3)

Now $\int_c F \cdot R = \int_c (3x^2\vec{\imath} + (2xz - y)\vec{\jmath} + 3\vec{k}) \cdot (dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k})$

$$= \int_c [3x^2 dx + (2xz - y)dy + zdz]$$

$$= \int_c [3(2t)^2 + (2 * 2t * 3t - t)dt + 3t * 3dt]$$

$$= \int_c [24t^2 + 12t^2 - t + 9t]dt$$

$$= \int_0^1 (36t^2 + 8t)dt$$

$$= [36\frac{t^3}{3} + 8\frac{t^2}{2}]_0^1$$

=12+4

=16