

Genetic Algorithms with Python



Clinton Sheppard

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Preface

Genetic Algorithms with Python distills more than 5 years of experience using genetic algorithms and helping others learn how to apply Genetic algorithms, into a graduated series of lessons that will impart to you a powerful life-long skill.

This book is available in the following formats:

- Paperback - <http://amzn.to/2gJGUin>,
- Kindle Edition - <http://amzn.to/2fPKunf>, and
- PDF - <http://bit.ly/GAPython>

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A brief introduction to genetic algorithms

Genetic algorithms are one of the tools we can use to apply machine learning to finding good, sometimes even optimal, solutions to problems that have billions of potential solutions. They use biological processes in software to find answers to problems that have really large search spaces by continuously generating candidate solutions, evaluating how well the solutions fit the desired outcome, and refining the best solutions.

When solving a problem with a genetic algorithm, instead of asking for a specific solution, you provide characteristics that the solution must have or rules its solution must pass to be accepted. For example, when filling a moving truck you provide a set of rules like: load big things first, distribute the weight to both sides, put light things on top but not loose, interlock things with odd shapes so they don't move around. The more constraints you add the more potential solutions are blocked. Suppose you say: put the refrigerator, washer and dryer along the front left wall, load boxes of books along the front right wall, mattresses down the middle, and clothes on top of the mattresses. These more specific rules do not work if you are loading packages, or Christmas trees, but the previous goal oriented ones still do.

Goal oriented problem solving

Imagine you are given 10 chances to guess a number between 1 and 1000 and the only feedback you get is whether your guess is right or wrong. Could you reliably guess the number? With only *right* or *wrong* as feedback, you have no way to improve your guesses so you have at best a 1 in 100 chance of guessing the number. A fundamental aspect of solving problems using genetic algorithms is that they must provide feedback that helps the engine select the better of two guesses. That feedback is called the fitness, for how closely the guess fits the desired result. More importantly it implies a general progression.

If instead of *right* or *wrong* as feedback you receive *higher* or *lower* indicating that the number is higher or lower than your guess, you can always find the number because 10 guesses are sufficient to binary search your way to any number in the 1 to 1000 range.

Now imagine multiplying the size of this problem so that instead of trying to find 1 number you are simultaneously trying to find a set of 100 numbers, all in the range 1 to 1000, your only receive back a fitness value indicating how closely that set of numbers matches the desired outcome. Your goal would be to maximize or minimize that fitness. Could you find the right set of 100 numbers? You might be able to do better than random guessing if you have problem-specific knowledge that helps you eliminate certain number combinations. Using problem-specific knowledge to guide the genetic algorithm's creation and modification of potential solutions can help them find a solution orders of magnitude faster.

Genetic algorithms and genetic programming are very good at finding solutions to very large problems. They do it by taking millions of samples from the search space, making small changes, possibly recombining parts of the best solutions, comparing the resultant fitness against that of the current best solution, and keeping the better of the two. This process repeats until a stop condition like one of the following occurs: the known solution is found, a solution meeting all requirements is found, a certain number of generations has passed, a specific amount of time has passed, etc.

First project

Imagine you are asked to guess a 3-letter password; what kind of feedback would you want? If the password is 'aaa' and you guess 'abc' what should the fitness value be? Would something simple like how many of the letters in your guess are correct be sufficient? Should 'bab', which has one correct letter, get a better fitness value than 'zap', also one correct letter but the wrong letters are alphabetically farther away, or should the fitness be the same? These are some of the first decisions you have to make when planning to implement a genetic algorithm to find a solution to your problem. Genetic algorithms are good at finding good solutions to problems with large search spaces because they can quickly find the parts of the guesses that improve fitness values or lead to better solutions.

In the project above, let's say the fitness function returns a count of the number of letters that match the password. That means 'abc', 'bab' and 'zba' all get a fitness value of one, because they each have one letter correct. The genetic algorithm might then combine the first two letters of 'abc' with the last letter of 'zba' through crossover, to create the guess 'aba'. The

fitness for that guess would be two because two letters match the password. The algorithm might also mutate the last letter of 'zba' to get 'zbc' and a fitness value of zero. Depending on how the engine is configured it might throw out 'zbc' immediately, it might keep it in order to maintain genetic diversity, or perhaps it would only keep it if it is better than some cutoff fitness value when compared with all the other guesses tried by the engine.

We will look more at the password project in the first chapter, and go on to explore a variety of projects to learn different ways of solving problems with genetic algorithms. However, this book is not about showing you a hand-picked set of problems you can solve with genetic algorithms. It is about giving you experience making genetic algorithms work for you using sample projects that you understand and can fall back upon when learning to use other machine learning tools and techniques, or applying genetic algorithms in your own field of expertise.

Genetic programming with Python

This book uses the Python programming language to explore genetic algorithms. Why Python? Because Python is a low ceremony, powerful and easy-to-read language whose code can be understood by entry-level programmers. I explain the occasional Python feature but should you encounter a programming construct you've never seen before and can't intuit, Python.org and StackOverflow.com are great places to find explanations. If you have experience with another programming language then you should have no difficulty learning Python by induction while also exploring genetic algorithms.

example Python syntax

```
# this is a comment
import math # imports make code from other modules available

# code blocks are initiated by a trailing colon followed by indented lines
class Circle: # define a class
    def __init__(self, radius): # constructor with parameter radius
        self.radius = radius # store the parameter in a class variable

    def get_area(self): # define a function that belongs to the class
        return math.pi \
            * self.radius \
            * self.radius # trailing \ continues the expression on the next line

# code that is not in a class is executed immediately
for i in range(1, 10):
    if (i & 1) == 0:
        continue
    circle = Circle(i) # create an instance
    print("A circle with radius {0} has area {1:0.2f}".format(
        i, circle.get_area() # 'print' writes output to the console
    ))
```

You can run the code above in your browser at <https://repl.it/C0uK>.

Like blacksmiths, programmers create their own tools. We frequently prototype a solution by using tools we already have available, not unlike using a pair of pliers to pull a nail. Once we get a good understanding of the problem, however, we usually restart with a better combination of tools or build a problem-specific one. In this book we will co-evolve a genetic engine while examining increasingly difficult projects with the engine. Why not just use one of the genetic programming packages already available for Python like Pyvolution, DEAP, Pyevolve, pySTEP, etc? Because they all have different

interfaces and options, some of which may not be applicable to a problem, and we're trying to learn about genetic algorithms not specific engines. By co-evolving the engine you'll know exactly how it works so you'll be able to use its features effectively to solve the next problem with a genetic algorithm of your own design. The engine will be a by-product of applying genetic algorithms to the different projects in this book. If you were to co-evolve an engine with a different set of projects, or even the projects in this book in a different order, you would end up with a different engine. But, by co-evolving an engine you will gain experience with some of the features available in commonly used packages, and see how they can affect the performance of your code.

About the author

I am a polyglot programmer with more than 15 years of professional programming experience. Occasionally I step out of my comfort zone and learn a new language to see what that development experience is like and to keep my skills sharp. This book grew out of my experiences while learning Python, but it isn't about Python.

When learning a new programming language, I start with a familiar project and try to learn enough of the new language to solve it. For me, writing a genetic engine is that familiar project. Why a genetic engine? For one thing, it is a project where I can explore interesting puzzles, and where even a child's game like Tic-Tac-Toe can be viewed on a whole new level. Also, I can select increasingly complex puzzles to drive evolution in the capabilities of the engine. This allows me to discover the expressiveness of the language, the power of its tool chain, and the size of its development community as I work through the idiosyncrasies of the language.

About the text

The Python 3.5 code snippets in this book were programmatically extracted from **working code files** using the [tags feature](#) of AsciiDoc's include directive.

Chapter 1. Hello World!

1.1. Guess my number

Let's begin by learning a little bit about genetic algorithms. Reach way back in your memories to a game we played as kids. It is a simple game for two people where one picks a secret number between 1 and 10 and the other has to guess that number.

```
Is it 2? No
Is it 3? No
Is it 7? No
Is it 1? Yes
```

That works reasonably well for 1..10 but quickly becomes frustrating or boring as we increase the range to 1..100 or 1..1000. Why? Because we have no way to improve our guesses. There's no challenge. The guess is either right or wrong, so it quickly becomes a mechanical process.

```
Is it 1? No
Is it 2? No
Is it 3? No
Is it 4? No
Is it 5? No
...
```

So, to make it more interesting, instead of *no* let's say *higher* or *lower*.

```
1? Higher
7? Lower
6? Lower
5? Lower
4? Correct
```

That might be reasonably interesting for a while for 1..10 but soon you'll increase the range to 1..100. Because people are competitive, the next revision is to see who is a better guesser by trying to find the number in the fewest guesses. At this point the person who evolves the most efficient guessing strategy wins.

However, one thing we automatically do when playing the game is make use of domain knowledge. For example, after this sequence:

1? Higher
7? Lower

Why wouldn't we guess 8, 9, or 10? The reason is, of course, because we know that those numbers are not *lower* than 7. Why wouldn't we guess 1? Because we already tried it. We use our memory of what we've tried, our successes and failures, and our *knowledge of the domain*, number relationships, to improve our guesses.



When playing a card game inexperienced players build a mental map using the cards in their hand and those on the table. More experienced players also take advantage of their knowledge of the problem space, the entire set of cards in the deck. This means they may also keep track of cards that have not yet been played, and may know they can win the rest of the rounds without having to play them out. Highly experienced card players also know the probabilities of various winning combinations. Professionals, who earn their living playing the game, also pay attention to the way their competitors play... whether they bluff in certain situations, play with their chips when they think they have a good hand, etc.

A genetic algorithm does not know what *lower* means. It has no intelligence. It does not learn. It will make the same mistakes every time. It will only be as good at solving a problem as the person who writes the code. And yet, it can be used to find solutions to problems that humans would struggle to solve or could not solve at all. How is that possible?

Genetic algorithms use random exploration of the problem space combined with evolutionary processes like mutation and crossover (exchange of genetic information) to improve guesses. But also, because they have no experience in the problem domain, they *try things a human would never think to try*. Thus, a person using a genetic algorithm may learn more about the problem space and potential solutions. This gives them the ability to make improvements to the algorithm, in a virtuous cycle.

What can we learn from this?

Technique

The genetic algorithm should make informed guesses.

1.2. Guess the Password

Now let's see how this applies to guessing a password. Start with a randomly generated initial sequence of letters, then mutate/change one random letter at a time until the sequence of letters is "Hello World!". Conceptually:

pseudo code

```
_letters = [a..zA..Z !]  
target = "Hello World!"  
guess = get 12 random letters from _letters  
while guess != target:  
    index = get random value from [0..length of target]  
    guess[index] = get 1 random value from _letters
```

If you try this in your favorite programming language you'll find that it performs worse than playing the number guessing game with only *yes* and *no* answers because it cannot tell when one guess is better than another.

One solution is to help it make an informed guess by telling it how many of the letters from the guess are in the correct locations. For example "World!Hello?" would get 2 because only the 4th letter of each word is correct. The 2 indicates how close the answer is to correct. This is called the fitness value. "hello world?" would get a fitness value of 9 because 9 letters are correct. Only the h, w, and question mark are wrong.

1.3. First Program

It is time to start writing Python. By the way, if you do not already have a favorite Python development environment, I highly recommend [JetBrains' PyCharm IDE](#).

1.3.1. Genes

To begin with, the genetic algorithm needs a gene set to use for building guesses. For this project that will be a generic set of letters. It also needs a target password to guess:

guessPassword.py

```
geneSet = " abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ!."
target = "Hello World!"
```



You can run the code for this section in your browser at <https://repl.it/CZL1/1>

1.3.2. Generate a guess

Next the algorithm needs a way to generate a random string from the gene set.

```
import random

def generate_parent(length):
    genes = []
    while len(genes) < length:
        sampleSize = min(length - len(genes), len(geneSet))
        genes.extend(random.sample(geneSet, sampleSize))
    return ''.join(genes)
```



`random.sample` takes `sampleSize` values from the input without replacement. This means there will be no duplicates in the generated parent unless `geneSet` contains duplicates, or `length` is greater than `len(geneSet)`. The implementation above can generate a long string with a small set of genes and uses as many unique genes as possible.

1.3.3. Fitness

The **fitness** value the genetic algorithm provides is the **only** feedback the engine gets to guide it toward a solution. In this project the **fitness** value is the total number of letters in the guess that match the letter in the same position of the password.

```
def get_fitness(guess):
    return sum(1 for expected, actual in zip(target, guess)
               if expected == actual)
```

1.3.4. Mutate

Next, the engine needs a way to produce a new guess by mutating the current one. The following implementation converts the parent string to an array with `list(parent)`, then replaces 1 letter in the array with a randomly selected one from `geneSet`, and finally recombines the result into a string with `''.join(childGenes)`.

```
def mutate(parent):
    index = random.randrange(0, len(parent))
    childGenes = list(parent)
    newGene, alternate = random.sample(geneSet, 2)
    childGenes[index] = alternate \
        if newGene == childGenes[index] \
        else newGene
    return ''.join(childGenes)
```

This implementation uses an alternate replacement if the randomly selected `newGene` is the same as the one it is supposed to replace, which can prevent a significant number of wasted guesses.

1.3.5. Display

Next, it is important to monitor what is happening so that the engine can be stopped if it gets stuck. Having a visual representation of the gene sequence, which may not be the literal gene sequence, is often critical to identifying what works and what does not so that the algorithm can be improved.

Normally the display function also outputs the fitness value and how much time has elapsed.

```
import datetime
...
def display(guess):
    timeDiff = datetime.datetime.now() - startTime
    fitness = get_fitness(guess)
    print("{0}\t{1}\t{2}".format(guess, fitness, str(timeDiff)))
```

1.3.6. Main

The main program begins by initializing `bestParent` to a random sequence of letters and calling the display function.

```
random.seed()
startTime = datetime.datetime.now()
bestParent = generate_parent(len(target))
bestFitness = get_fitness(bestParent)
display(bestParent)
```

The final piece is the heart of the genetic engine. It is a loop that:

- generates a guess,
- requests the `fitness` for that guess, then
- compares the `fitness` to that of the previous best guess, and
- keeps the guess with the better fitness.

This cycle repeats until a stop condition occurs, in this case when all the letters in the guess match those in the target.

```
while True:
    child = mutate(bestParent)
    childFitness = get_fitness(child)
    if bestFitness >= childFitness:
        continue
    display(child)
    if childFitness >= len(bestParent):
        break
    bestFitness = childFitness
    bestParent = child
```

Run the code and you'll see output similar to the following. Success!

```
ftljCDPvhasn    1    0:00:00
ftljC Pvhasn    2    0:00:00
ftljC Pohasn    3    0:00:00.001000
HtljC Pohasn    4    0:00:00.002000
HtljC Wohasn    5    0:00:00.004000
Htljo Wohasn    6    0:00:00.005000
Htljo Wohas!    7    0:00:00.008000
Htljo Wohls!    8    0:00:00.010000
Heljo Wohls!    9    0:00:00.013000
Hello Wohls!   10    0:00:00.013000
Hello Wohld!   11    0:00:00.013000
Hello World!   12    0:00:00.015000
```

1.4. Extract a reusable engine

We have a working engine but it is currently tightly coupled to the Password project, so the next task is to extract the genetic engine code from that specific to guessing the password so it can be reused for other projects. Start by creating a new file named `genetic.py`.

Next move the `mutate` and `generate_parent` functions to the new file and rename them to `_mutate` and `_generate_parent`. This is how protected functions are named in Python. Protected functions are only accessible to other functions in the same module.

1.4.1. Generate and Mutate

Future projects will need to be able to customize the gene set, so that needs to become a parameter to `_generate_parent` and `_mutate`.

```
import random

def _generate_parent(length, geneSet):
    genes = []
    while len(genes) < length:
        sampleSize = min(length - len(genes), len(geneSet))
        genes.extend(random.sample(geneSet, sampleSize))
    return ''.join(genes)
```

```
def _mutate(parent, geneSet):
    index = random.randrange(0, len(parent))
    childGenes = list(parent)
    newGene, alternate = random.sample(geneSet, 2)
    childGenes[index] = alternate \
        if newGene == childGenes[index] \
        else newGene
    return ''.join(childGenes)
```


1.4.2. `get_best`

The next step is to move the main loop into a new public function named `get_best` in the `genetic` module. Its parameters are:

- the function it calls to request the fitness for a guess,
- the number of genes to use when creating a new gene sequence,
- the optimal fitness value,
- the set of genes to use for creating and mutating gene sequences, and
- the function it should call to display, or report, each improvement found.

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet, display):
    random.seed()
    bestParent = _generate_parent(targetLen, geneSet)
    bestFitness = get_fitness(bestParent)
    display(bestParent)
    if bestFitness >= optimalFitness:
        return bestParent

    while True:
        child = _mutate(bestParent, geneSet)
        childFitness = get_fitness(child)

        if bestFitness >= childFitness:
            continue
        display(child)
        if childFitness >= optimalFitness:
            return child
        bestFitness = childFitness
        bestParent = child
```

Notice that `display` and `get_fitness` are called with only one parameter - the child gene sequence. This is because a generic engine does not need access to the target value, and it does not care about how much time has passed, so those are not passed to it.

The result is a reusable module named `genetic` that can be used in other programs via `import genetic`.

1.4.3. Use the `genetic` module

The code remaining in `guessPassword.py` is specific to the password guessing project. To get it working again first import the `genetic` module.

guessPassword.py

```
import datetime
import genetic
```

Next, helper functions that take only one parameter must be defined so they are compatible with what the **genetic** engine expects. Each helper function will take the candidate gene sequence it receives and call the local functions with additional required parameters as necessary.

```
def test_Hello_World():
    target = "Hello World!"
    guess_password(target)

def guess_password(target):
    geneset = " abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ!."
    startTime = datetime.datetime.now()

    def fnGetFitness(genes):
        return get_fitness(genes, target)

    def fnDisplay(genes):
        display(genes, target, startTime)

    optimalFitness = len(target)
    genetic.get_best(fnGetFitness, len(target), optimalFitness, geneset, fnDisplay)
```

1.4.4. Display

Now change **display** to take the target password as a parameter. It could remain a global variable in the algorithm file but this change facilitates trying different passwords without side effects.

```
def display(genes, target, startTime):
    timeDiff = datetime.datetime.now() - startTime
    fitness = get_fitness(genes, target)
    print("{0}\t{1}\t{2}".format(genes, fitness, str(timeDiff)))
```

1.4.5. Fitness

The fitness function also needs to receive the target password as a parameter.

```
def get_fitness(genes, target):
    return sum(1 for expected, actual in zip(target, genes)
               if expected == actual)
```

1.4.6. Main

There are many ways to structure the main code, the most flexible is as a unit test. To start that transition first rename `guessPassword.py` to `guessPasswordTests.py`. Next, to make it possible to execute the code from the command line add:

guessPasswordTests.py

```
if __name__ == '__main__':  
    test_Hello_World()
```

If you are following along in an editor be sure to run your code to verify it works at this point.

1.5. Use Python's `unittest` framework

The next step is to make the code work with Python's built in test framework.

```
import unittest
```

To do that the main test function must be moved into a class that inherits from `unittest.TestCase`. The other functions can be moved into the class as well if you want, but if they are then `self` must be added as the first parameter to each because they will then belong to the test class.

```
class GuessPasswordTests(unittest.TestCase):  
    geneset = " abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ!.,"  
  
    def test_Hello_World(self):  
        target = "Hello World!"  
        self.guess_password(target)  
  
    def guess_password(self, target):  
    ...  
        optimalFitness = len(target)  
        best = genetic.get_best(fnGetFitness, len(target),  
                                optimalFitness, self.geneset,  
                                fnDisplay)  
        self.assertEqual(best, target)
```

When the `unittest` module's `main` function is called, it automatically executes each function whose name starts with "test".

```
if __name__ == '__main__':  
    unittest.main()
```

This allows the test to be run from the command line and, incidentally, without its `display` function output.

```
python -m unittest -b guessPasswordTests
.
-----
Ran 1 test in 0.020s

OK
```



If you get an error like `'module' object has no attribute 'py'` then you used the filename `guessPasswordTests.py` instead of the module name `guessPasswordTests`.

1.6. A longer password

"Hello World!" doesn't sufficiently demonstrate the power of the genetic engine so try a longer password:

```
def test_For_I_am_fearfully_and_wonderfully_made(self):
    target = "For I am fearfully and wonderfully made."
    self.guess_password(target)
```

1.6.1. Run

```
...
ForMI am feabaully and wWndNyfulll made.    33  0:00:00.047094
For I am feabaully and wWndNyfulll made.    34  0:00:00.047094
For I am feabfully and wWndNyfulll made.    35  0:00:00.053111
For I am feabfully and wondNyfulll made.    36  0:00:00.064140
For I am feabfully and wondNyfully made.    37  0:00:00.067148
For I am feabfully and wondeyfully made.    38  0:00:00.095228
For I am feabfully and wonderfully made.    39  0:00:00.100236
For I am fearfully and wonderfully made.    40  0:00:00.195524
```

Nice!

1.7. Introduce a `Chromosome` object

The next change is to introduce a `Chromosome` object that has `Genes` and `Fitness` attributes. This will make the genetic engine more flexible by making it possible to pass those values around as a unit.

```

class Chromosome:
    Genes = None
    Fitness = None

    def __init__(self, genes, fitness):
        self.Genes = genes
        self.Fitness = fitness

```

```

def _mutate(parent, geneSet, get_fitness):
    index = random.randrange(0, len(parent.Genes))
    childGenes = list(parent.Genes)
    ...
    genes = ''.join(childGenes)
    fitness = get_fitness(genes)
    return Chromosome(genes, fitness)

```

```

def _generate_parent(length, geneSet, get_fitness):
    ...
    genes = ''.join(genes)
    fitness = get_fitness(genes)
    return Chromosome(genes, fitness)

```

```

def get_best(get_fitness, targetLen, optimalFitness, geneSet, display):
    random.seed()
    bestParent = _generate_parent(targetLen, geneSet, get_fitness)
    display(bestParent)
    if bestParent.Fitness >= optimalFitness:
        return bestParent

    while True:
        child = _mutate(bestParent, geneSet, get_fitness)

        if bestParent.Fitness >= child.Fitness:
            continue
        display(child)
        if child.Fitness >= optimalFitness:
            return child
        bestParent = child

```


It does require compensating changes to the algorithm file functions but those changes remove some double work.

guessPasswordTests.py

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        candidate.Genes, candidate.Fitness, str(timeDiff)))
```

```
class GuessPasswordTests(unittest.TestCase):
    ...
    def guess_password(self, target):
    ...
        def fnDisplay(candidate):
            display(candidate, startTime) ①

        optimalFitness = len(target)
        best = genetic.get_best(fnGetFitness, len(target),
                                optimalFitness, self.geneset,
                                fnDisplay)
        self.assertEqual(best.Genes, target) ②
```

1.8. Benchmarking

The next improvement is to add support for benchmarking to *genetic* because it is useful to know how long the engine takes to find a solution on average and the standard deviation. That can be done with another class as follows:

genetic.py

```
class Benchmark:
    @staticmethod
    def run(function):
        timings = []
        for i in range(100):
            startTime = time.time()
            function()
            seconds = time.time() - startTime
            timings.append(seconds)
        mean = statistics.mean(timings)
        print("{0} {1:3.2f} {2:3.2f}".format(
            1 + i, mean,
            statistics.stdev(timings, mean)
            if i > 1 else 0))
```

That requires the following imports:

genetic.py

```
import statistics
import time
```



You may need to install the `statistics` module on your system. This can be accomplished from the command line with `python -m pip install statistics`

Now, to use the benchmarking capability simply add a test and pass the function to be benchmarked.

guessPasswordTests.py

```
def test_benchmark(self):
    genetic.Benchmark.run(self.test_For_I_am_fearfully_and_wonderfully_made)
```

When run, this function works great but is a bit chatty because it also shows the `display` output for all 100 runs. That can be fixed in the `run` function by temporarily redirecting standard output to nowhere while running the function being benchmarked.

genetic.py

```
import sys
...
class Benchmark:
    @staticmethod
    def run(function):
    ...
        timings = []
        stdout = sys.stdout          ①
        for i in range(100):
            sys.stdout = None        ②
            startTime = time.time()
            function()
            seconds = time.time() - startTime
            sys.stdout = stdout      ③
            timings.append(seconds)
    ...
```



If you get an error like the following when you run the benchmark test:

```
AttributeError: 'NoneType' object has no attribute 'write'
```

Then you are probably using Python 2.7. It does not support redirecting `stdout` to `None`. It must have someplace to write the data. One solution to that problem is to add the following class:

genetic.py

```
class NullWriter():
    def write(self, s):
        pass
```

Then replace the following in `Benchmark.run`:

```
for i in range(100):
    sys.stdout = None
```

with:

```
for i in range(100):
    sys.stdout = NullWriter()
```

That change allows you work around the difference between Python 2.7 and 3.5 this time. However, the code in this book uses other features of Python 3.5 so I suggest switching to Python 3.5 so that you can focus on learning about genetic algorithms without these additional issues. If you want to use machine learning tools that are tied to Python 2.7, wait until you have a solid understanding of genetic algorithms then switch to Python 2.7.

The output can also be improved by only displaying statistics for the first ten runs and then every 10th run after that.

genetic.py

```
...
    timings.append(seconds)
    mean = statistics.mean(timings)
    if i < 10 or i % 10 == 9:
        print("{0} {1:3.2f} {2:3.2f}".format(
            1 + i, mean,
            statistics.stdev(timings, mean)
            if i > 1 else 0))
```

Now the benchmark test output looks like the following.

sample output

```
1 0.19 0.00
2 0.17 0.00
3 0.18 0.02
...
9 0.17 0.03
10 0.17 0.03
20 0.18 0.04
...
90 0.16 0.05
100 0.16 0.05
```

This means that, averaging 100 runs, it takes .16 seconds to guess the password, and 68 percent of the time (one standard deviation) it takes between .11 (.16 - .05) and .21 (.16 + .05) seconds. Unfortunately that is probably too fast to determine if a change is due to a code improvement or due to something else running on the computer at the same time. That problem can be solved by making the genetic algorithm guess a random sequence that takes 1-2 seconds to run. Your processor likely is different from mine so adjust the length as necessary.

guessPasswordTests.py

```
import random
...
def test_Random(self):
    length = 150
    target = ''.join(random.choice(self.geneset) for _ in range(length))
    self.guess_password(target)

def test_benchmark(self):
    genetic.Benchmark.run(self.test_Random)
```

On my system that results in:

Benchmarks

average (seconds)	standard deviation
1.46	0.35

1.9. Summary

In this chapter we built a simple genetic engine that makes use of random mutation to produce better results. This engine was able to guess a secret password given only its length, a set of characters that might be in the password, and a fitness function that returns a count of the number characters in the guess that match the secret. This is a good benchmark project for the engine because as the target string gets longer the engine wastes more and more guesses trying to change positions that are already correct. As the engine evolves in later projects, we'll try to keep this benchmark fast. Also, as you work your way through this book you'll learn ways to improve the performance of the code in this project.

1.10. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch01>

Chapter 2. One Max Problem

Our second project involves maximizing the number of 1's in an array of 100 1-bit numbers. For this project the gene set will be only 0 or 1.

2.1. Test class

oneMaxTests.py

```
import unittest
import datetime
import genetic

class OneMaxTests(unittest.TestCase):
    def test(self, length=100):
        geneset = [0, 1]
```

2.2. Change *genetic* to work with lists

The *genetic* module is currently hard coded to work with strings, so we need to modify it to work with lists instead. We can start by using Python's array slicing feature to copy the genes in *_mutate* instead of using the list constructor.

```
def _mutate(parent, geneSet, get_fitness):
    childGenes = parent.Genes[:]
    ...
```

Then remove the following line from *_mutate* and *_generate_parent* since we no longer need to convert the list back to a string:

```
genes = ''.join(childGenes)
```

Next, update those functions to use the list, as follows:

genetic.py

```
def _mutate(parent, geneSet, get_fitness):
    childGenes = parent.Genes[:]
    index = random.randrange(0, len(parent.Genes))
    newGene, alternate = random.sample(geneSet, 2)
    childGenes[index] = alternate \           ①
        if newGene == childGenes[index] \    ②
        else newGene
    fitness = get_fitness(childGenes)         ③
    return Chromosome(childGenes, fitness)    ④
```

```
def _generate_parent(length, geneSet, get_fitness):
    ...
    fitness = get_fitness(genes) ①
    return Chromosome(genes, fitness) ②
```

To keep `guessPasswordTests.py` working, we must recombine the genes into a string in its `display` function

guessPasswordTests.py

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        ''.join(candidate.Genes), ①
        candidate.Fitness,
        str(timeDiff)))
```

and the assertion in `guess_password`.

```
class GuessPasswordTests(unittest.TestCase):
    def guess_password(self, target):
    ...
        self.assertEqual(''.join(best.Genes), target)
```

2.3. Genes

OK, back to the One Max problem. The fitness will be the number of 1's in the array.

oneMaxTests.py

```
def get_fitness(genes):
    return genes.count(1)
```

2.4. Display

Since 100 numbers would be a lot to display, we'll just show the first and last 15 along with the fitness and elapsed time.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}...{1}\t{2:3.2f}\t{3}".format(
        ''.join(map(str, candidate.Genes[:15])),
        ''.join(map(str, candidate.Genes[-15:])),
        candidate.Fitness,
        str(timeDiff)))
```

This uses `str` to convert the integers in `candidate.Genes` to strings. Without `map` the code would need a loop like the following to convert the candidate genes to strings:

```
result = []
for i in candidate.Genes:
    result += str(candidate[i])
return result
```

2.5. Test

And here's the full test harness.

```
def test(self, length=100):
    geneset = [0, 1]
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    optimalFitness = length
    best = genetic.get_best(fnGetFitness, length, optimalFitness,
                           geneset, fnDisplay)
    self.assertEqual(best.Fitness, optimalFitness)
```

2.6. Run

Now it can find the solution very quickly.

sample output

```
010101101010010...101010110100101  50.00  0:00:00.001000
010101101010010...101010110100101  51.00  0:00:00.001000
...
110111101111111...111111110111111  95.00  0:00:00.008000
110111111111111...111111110111111  96.00  0:00:00.008000
110111111111111...111111110111111  97.00  0:00:00.009000
110111111111111...111111111111111  98.00  0:00:00.009000
111111111111111...111111111111111  99.00  0:00:00.010000
111111111111111...111111111111111  100.00 0:00:00.011000
```


2.7. Benchmarks

Since it runs so fast we'll benchmark this project with a longer array. As with the Guess Password benchmark I'm choosing an array length that takes between 1 and 2 seconds on average on my box. You may want to select a different length.

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test(4000))
```

We can see in the updated benchmarks that eliminating the string conversion may also have given us a tiny performance improvement in Guess Password.

Updated Benchmarks

project	average (seconds)	standard deviation
Guess Password	1.21	0.25
One Max	1.25	0.17

2.8. Aside

In this project the genetic engine randomly chooses an array index to change, even if the array already contains a 1 at that index, because the engine doesn't know the change it is making is useless until after it has called the fitness function. A physical equivalent is to take 100 coins and put green sticky dots on one side and yellow sticky dots on the other. Then have a partner blindfold you and drop the coins on a table top. Your goal is to turn all the coins yellow-side up. If you turn one yellow-side up they tell you it was a success. Otherwise, they undo the change and tell you it was a failure. To keep you from building a mental map they could optionally move the coin somewhere else afterward. Tough game right?

Now think about possible changes to the coin game that would make it solvable. For example, what if they were to remove the coin from the table if you turn it yellow-side up. That would be extremely useful right? Assuming every coin started yellow-side up, you would at most turn each coin twice, meaning you could turn all coins yellow-side up in at most 200 turns no matter which way they started. If the genetic algorithm, instead of the engine, had control of choosing the next index, then the equivalent would be for it to record the index it changes. Then, if `display` is called next by the engine instead of `get_index`, the algorithm would know that the array contains a 1 at that index. I could then simply add the index to a list of indexes it ignores when guessing. A side benefit of this solution is that, like the human player in the coin game, the genetic algorithm does not need to know the state of the entire array. It can begin with zero knowledge and start tracking indexes that improve the fitness, just like [the classic Battleship game](#).

2.9. Summary

In this chapter the flexibility of the `genetic` module was increased by allowing the genes to be any type instead of only characters in a string. This is a critical feature, as before long we will want to use something other than just keyboard characters for genes.

2.10. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch02>

Chapter 3. Sorted Numbers

In this chapter we will make a genetic algorithm that produces a sorted list of numbers. This project is slightly more difficult than the previous two because even though there are many potential solutions instead of just one, now each of the genes has a constraint based on another gene's value; specifically, it must have a value greater than the gene to its left in the array.

3.1. Test class

sortedNumbersTests.py

```
import unittest

class SortedNumbersTests(unittest.TestCase):
    def test_sort_10_numbers(self):
        self.sort_numbers(10)

    def sort_numbers(self, totalNumbers):
```

3.2. Genes

For this project the gene set will be integers in the range 0 to 99.

```
def sort_numbers(self, totalNumbers):
    geneset = [i for i in range(100)]
```

List comprehensions

`[i for i in range(100)]` is an example of a list comprehension, a powerful Python feature that enables us to build a list by saying what we want instead of how to get it. This is equivalent to:

```
geneset = []
for i in range(100):
    geneset.append(i)
```

The Python compiler may be able to write faster code for list comprehensions in some cases. You may recall that we sometimes use a variant of this in `display` to convert a list of numbers to strings using `map`.

3.3. Fitness

The fitness function will return a count of the adjacent numbers that are in ascending order with 1 freebie because the initial position doesn't have a gene to its left.

```
def get_fitness(genes):
    fitness = 1
    for i in range(1, len(genes)):
        if genes[i] > genes[i - 1]:
            fitness += 1
    return fitness
```

3.4. Display

The `display` function will output the array values separated by a comma.

```
import datetime
...
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t=> {1}\t{2}".format(
        ', '.join(map(str, candidate.Genes)),
        candidate.Fitness,
        str(timeDiff)))
```

3.5. Test

First we'll try to get the engine to produce 10 sorted numbers.

```
import genetic
...
def test_sort_10_numbers(self):
    self.sort_numbers(10)
```

```

def sort_numbers(self, totalNumbers):
    geneset = [i for i in range(100)]
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    optimalFitness = totalNumbers
    best = genetic.get_best(fnGetFitness, totalNumbers,
                           optimalFitness, geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)

```

3.6. Run

Now run the test and...

```

54, 59, 7, 69, 73, 76, 42, 46, 39, 87  => 7 0:00:00.001000
54, 59, 7, 69, 73, 76, 42, 46, 63, 87  => 8 0:00:00.001000
54, 59, 66, 69, 73, 76, 42, 46, 63, 87 => 9 0:00:00.003000

```

it stalls.

3.7. Our first stall

When a genetic algorithm stalls it is often a clue to a pattern in the result or that there is something you know that the algorithm does not. For example, in the final output above the numbers form a pattern of runs that our current fitness function did not anticipate. The easiest way for the algorithm to make progress would be to change the values of the 7th-9th indexes from [42, 46, 63] to values in the range 77-86. The problem is it has no way to do that. So we will look at a simpler version of the issue to see why. For example, in the following sequence

```
3, 1, 6
```

the engine can change **1** to **4** or **5** to get a higher fitness. But in the following sequence

```
3, 1, 2, 6
```

the engine has to change 2 values to get a higher fitness. The problem is `get_best` is coded to only keep a change if the fitness is higher. So changing **2** to **5** or **1** to **4** in the above sequence does not improve the fitness and those changes will be abandoned. One solution would be to change the engine somehow, perhaps to keep changes that don't make the fitness worse, instead of keeping only those that increase it. We could do that but there's a better way that shows us something important.

3.8. Engineer a solution

Consider this sequence.

9, 30, 1, 20, 60

Would the following be in an improvement?

9, 30, 15, 20, 60

Or this?

9, 16, 1, 20, 60

Both changes would be improvements because they smooth the point where the two runs meet from a gap of 29 to a gap of 15. Either could then be followed by another similar change that further reduces the gap, or even converts the 2 sorted runs into a single one. The way to help the engine keep this kind of change is to give partial credit in the fitness.

Fitness Technique

Give partial credit in the fitness if possible.

3.9. Use a Fitness object

We need to somehow prefer sequences with small gaps between the runs to those with large gaps. There are a couple of ways to do that. One way that is often seen when the engine has constraints on the type of the fitness value is to scale up the fitness value, multiplying by a large value like 1000 for example, and then subtracting the amount of the gap. Another way would be to make the fitness score a floating point value with the gap value in the decimal portion and subtracted from 1. These would both work but we then have to decode them either mentally or actually if we want to break out each part of the fitness value for display.

Lucky for us our engine doesn't have a type limitation so we're going to keep the values separate but encapsulated in a problem-specific fitness object that can be compared to its own type. This allows us to determine the better of two gene sequences based on their fitness, while making the fitness object responsible for how it is displayed.

We start with the constructor.

sortedNumbersTests.py

```
class Fitness:
    NumbersInSequenceCount = None
    TotalGap = None

    def __init__(self, numbersInSequenceCount, totalGap):
        self.NumbersInSequenceCount = numbersInSequenceCount
        self.TotalGap = totalGap
    ...
```

Next we need to be able to compare two fitness values.

```
...
def __gt__(self, other):
    if self.NumbersInSequenceCount != other.NumbersInSequenceCount:
        return self.NumbersInSequenceCount > other.NumbersInSequenceCount
    return self.TotalGap < other.TotalGap
...
```



We could implement other comparisons, such as less-than, equal, greater-than-or-equal, etc., but we only need to know if the fitness of the new gene sequence is better than the previous one. If we're careful with the comparisons we make in `get_best` then we can save ourselves unnecessary work writing the other comparison functions.

Lastly, the `Fitness` object needs to know how to convert itself to a string for `display`:

```
...
def __str__(self):
    return "{0} Sequential, {1} Total Gap".format(
        self.NumbersInSequenceCount,
        self.TotalGap)
```

3.10. Use only > for fitness comparison

We need to be able to compare a **Chromosome**'s fitness with the optimal fitness in the engine and only use greater-than comparisons in the implementation. We're doing that so we can be lazy in implementing **Fitness** objects in the future.

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet, display):
    ...
    if not optimalFitness > bestParent.Fitness:
        return bestParent

    while True:
        child = _mutate(bestParent, geneSet, get_fitness)
        if not child.Fitness > bestParent.Fitness:
            continue
        display(child)
        if not optimalFitness > child.Fitness:
            return child
        bestParent = child
```

Then we need to pass an instance of **Fitness** to **get_best** as the optimal value

sortedNumbersTests.py

```
def sort_numbers(totalNumbers):
    ...
    optimalFitness = Fitness(totalNumbers, 0)
    best = genetic.get_best(fnGetFitness, totalNumbers,
                           optimalFitness, geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)
```

Finally, in **get_fitness** we need to return an instance of **Fitness**.

```
def get_fitness(genes):
    fitness = 1
    gap = 0
    for i in range(1, len(genes)):
        if genes[i] > genes[i - 1]:
            fitness += 1
        else:
            gap += genes[i - 1] - genes[i]

    return Fitness(fitness, gap)
```

3.11. Run 2

Now when we run the test it can find a solution about 70 percent of the time. When it can't the result looks like this:

```
...
10, 16, 18, 28, 30, 33, 7, 8, 9, 11 => 9 Sequential, 26 Total Gap    0:00:00.008001
10, 16, 18, 28, 30, 6, 7, 8, 9, 11  => 9 Sequential, 24 Total Gap    0:00:00.012001
10, 16, 18, 28, 29, 6, 7, 8, 9, 11  => 9 Sequential, 23 Total Gap    0:00:00.021002
```

3.12. Study the results

By studying the last line of output we see that we can get partial fitness improvements by moving **29** and **6** closer to each other. However, they are both prevented from moving by a neighbor - 6 can't be replaced by a higher number, and 29 can't be replaced by a lower number, without reducing the fitness. This means considering only the gap at the discontinuity is insufficient. What would let us move forward is if **28** was reduced or **9** was increased, or if one of the neighbors changed so that we could have room to reduce the gap.

3.13. Engineer a solution

Now we will implement a change mentioned earlier in the chapter. Instead of keeping only changes that have a better fitness than the parent, we're going to allow the engine to keep any change that results in a fitness equivalent to or better than the parent, like this:

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet, display):
    ...
    child = _mutate(bestParent, geneSet, get_fitness)
    if bestParent.Fitness > child.Fitness: ①
        continue
    if not child.Fitness > bestParent.Fitness:
        bestParent = child ②
        continue
    display(child)
    if not optimalFitness > child.Fitness:
    ...
```


3.14. Run 3

Now when we run the test it can find a sorted sequence every time.

```
...
11, 3, 9, 27, 55, 69, 71, 76, 89, 86    => 8 Sequential, 11 Total Gap    0:00:00.001000
11, 3, 9, 41, 45, 47, 71, 76, 83, 86    => 9 Sequential, 8 Total Gap 0:00:00.001000
9, 3, 6, 18, 45, 47, 52, 76, 83, 86 => 9 Sequential, 6 Total Gap 0:00:00.002000
2, 3, 6, 36, 45, 47, 52, 76, 83, 86 => 10 Sequential, 0 Total Gap    0:00:00.002000
```

We can tell that it kept equivalent genetic lines to get 9 items sorted and again to get 10 sorted because more than one number changed to get each of those improvements. We can also see it switching genetic lines when multiple characters change at the same time in [GuessPasswordTests](#).

```
C!pDrdy!QtDq    0    0:00:00
ULzEoMJmzF!a    1    0:00:00.001000
HXoSocjYQtZb    2    0:00:00.005000
HxiKoIWeVj!x    3    0:00:00.007000
HxiKoIWerj!x    4    0:00:00.007000
HxEKoIWYrl!x    5    0:00:00.007000
HovlobWJrlUX    6    0:00:00.010000
HovlobWJrldv    7    0:00:00.010000
HuTlo WRrldp    8    0:00:00.016001
HeUlo WirldK    9    0:00:00.017001
HeUlo WUrld!   10    0:00:00.017001
Hello WNrld!   11    0:00:00.024001
Hello World!   12    0:00:00.034002
```

3.15. Split `get_best`

The loop code in `get_best` is becoming complex. One way to fix that is to split the loop code into two parts. The first part will only be responsible for generating successively better gene sequences which it will send back to `get_best` via `yield`.

genetic.py

```
def _get_improvement(new_child, generate_parent):
    bestParent = generate_parent()
    yield bestParent
    while True:
        child = new_child(bestParent)
        if bestParent.Fitness > child.Fitness:
            continue
        if not child.Fitness > bestParent.Fitness:
            bestParent = child
            continue
        yield child
        bestParent = child
```

And `get_best` will be responsible for displaying improvements and breaking the loop.

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet, display):
    random.seed()

    def fnMutate(parent):
        return _mutate(parent, geneSet, get_fitness)

    def fnGenerateParent():
        return _generate_parent(targetLen, geneSet, get_fitness)

    for improvement in _get_improvement(fnMutate, fnGenerateParent):
        display(improvement)
        if not optimalFitness > improvement.Fitness:
            return improvement
```

3.16. Benchmarks

Benchmark this project by having it produce a sequence of sorted numbers that takes 1-2 seconds to run on average.

sortedNumbersTests.py

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.sort_numbers(40))
```

Since we modified the `genetic` module the previous benchmarks may have changed, so here are the updated benchmarks for all of the projects we've done so far:

project	average (seconds)	standard deviation
Guess Password	1.25	0.31
One Max	1.19	0.15
Sorted Numbers	1.27	0.75

3.17. Summary

In this chapter we considered a couple of ways to encode multiple goals into the fitness value, and how to use a comparable class instead of a number for fitness values. We also improved the engine by making it switch to new, equally good genetic lines should they come about. Also, we extracted a generator function from `get_best` to separate concerns.

3.18. Final Code

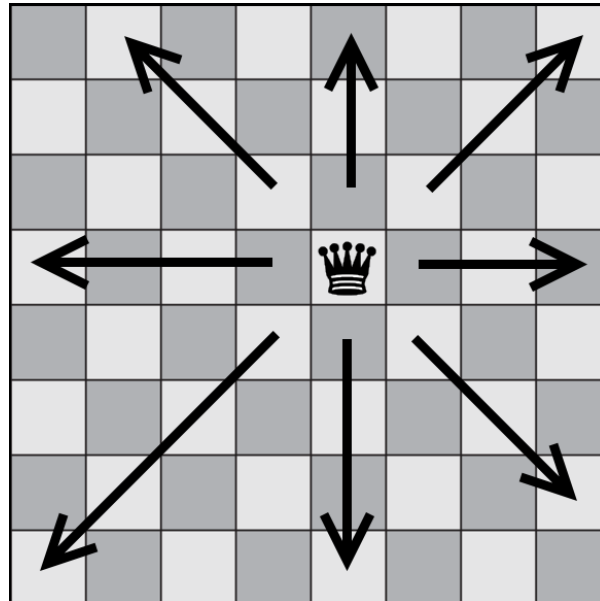
The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch03>

Chapter 4. The 8 Queens Puzzle

In this chapter we will solve the 8 Queens Puzzle.

In the game of chess, the queen can attack across any number of unoccupied squares on the board horizontally, vertically, or diagonally.



The 8 Queens Puzzle involves putting 8 queens on a standard chessboard such that none are under attack.

Take a couple of minutes to try to solve this with something physical like pennies on a paper chessboard to get a feel for how it might work.

It turns out that getting 7 queens into safe positions on a chessboard isn't too difficult.

```
Q . . . . . . .
. . . . Q . .
. . . . . Q .
. Q . . . . .
. . . Q . . .
. . . . . Q
. . . . . .
. . Q . . . .
```

Getting 8 takes a bit more work. [According to Wikipedia](#) there are only 92 solutions to this puzzle and once we remove mirrors and rotations there are only 12 unique solutions.

There are $64 \times 63 \times 62 \times 61 \times 60 \times 59 \times 58 \times 57$ potential locations for the queens assuming we don't apply some logic to reduce the search space. That's a very large number, so clearly a straight iterative method is impractical.

This puzzle is like the sorted numbers project in that there are constraints on the genes, but instead of one or two constraints per gene we now have many because of the relationships between the genes that the engine knows nothing about.

Also, in the projects we've done so far the genes **were** the solution, so we were able to display them without transformation and our fitness code could simply compare them to each other or the known answer.

At a high level, the community calls the genetic encoding the **genotype** and the genes' ultimate form or behavior in the solution the **phenotype**.

The **genotype** is the way the parts of the problem are encoded so they can be manipulated by the genetic algorithm and/or engine.

Example: potential genotypes for this project include:

- 64 bits, one for each of the 64 squares on the board
- 48 bits, 6 for each of the queen locations, because we can count to 64 with 6 bits
- 8 integers in the range 0..63 or 1..64
- 16 integers representing the row and column location of each queen

The **phenotype** is how the decoded genes are used in solving the problem. In each of the examples above the phenotype is locations of 8 queens on the board.

The fitness function then evaluates the phenotype in the context of the problem being solved to return a fitness value to the engine.

Also, like in the sorted numbers project, we have multiple potential solutions, and we're not going to hard-code them. So, we will have to calculate fitness based on characteristics.

4.1. Test class

```
import unittest
import datetime
import genetic

class EightQueensTests(unittest.TestCase):
    def test(self, size=8):
```

To start with we need to define the genotype. We will use two genes for the position of each queen – one each for the row and column. The chessboard conveniently has the same number of rows as columns (8) so we'll use the digits 0-7.

```
def test(self, size=8):
    geneset = [i for i in range(size)]
```

4.2. Board

We will use the genes as row and column indexes to plot queen locations on a board.

```
class Board:
    def __init__(self, genes, size):
        board = [['.' * size for _ in range(size)]
        for index in range(0, len(genes), 2):
            row = genes[index]
            column = genes[index + 1]
            board[column][row] = 'Q'
        self._board = board
```

We could have introduced a Location class to convert and encapsulate pairs of genes as Row and Column locations but since there is a direct correlation we don't need it. If we had chosen one of the other genotypes described above, it would have been an important step.

4.3. Display

The `display` function will let us visualize the queen locations

```
def display(candidate, startTime, size):
    timeDiff = datetime.datetime.now() - startTime
    board = Board(candidate.Genes, size)
    board.print()
    print("{0}\t- {1}\t{2}".format(
        ' '.join(map(str, candidate.Genes)),
        candidate.Fitness,
        str(timeDiff)))
```

but first we need to add a `print` function to `Board`:

```
class Board:
    ...
    def print(self):
        # 0,0 prints in bottom left corner
        for i in reversed(range(0, len(self._board))):
            print(' '.join(self._board[i]))
```

This produces output like the following:

```
Q . . . . .
. . . . Q . Q .
. . Q . . . .
. . . . . . Q
. . . . . . .
. Q . . . . .
. . . Q . . .
. . . . . Q .
1 2 7 4 6 6 4 6 3 1 6 0 2 5 0 7 - 3 0:00:00.005012
```



Printing comma separated values without a format string automatically separates them with a space.

The row of digits under the board is the set of genes that created the board layout. The number to the right is the fitness, and the elapsed time is on the end.

4.4. Fitness

To drive improvement we'll need to increase the fitness value whenever more queens can coexist on the board.

We'll start with considering the number of columns that do not have a queen. Here's a layout that gets an optimal score but is undesirable:

```
Q Q Q Q Q Q Q Q
. . . . .
. . . . .
. . . . .
. . . . .
. . . . .
. . . . .
. . . . .
```

We'll also consider the number of rows that do not have queens. Here's a revised board where both situations are optimal but the layout still allows queens to attack one another:

```
Q . . . . .
. Q . . . . .
. . Q . . . .
. . . Q . . .
. . . . Q . .
. . . . . Q .
. . . . . . Q
. . . . . . Q
```

To fix this problem we'll include the number of southeast diagonals that do not have a queen. Again we can find a corner case as follows:

```

. . . . . Q
. . . . . Q .
. . . . Q . .
. . . Q . . .
. . Q . . . .
. Q . . . . .
. Q . . . . .
Q . . . . . .

```

To resolve this final issue we'll include the number of northeast diagonals that do not have a queen.

We can calculate indexes for the northeast diagonals in Excel using the formula `=$A2+B$1` which results in a grid as follows

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	8
2	2	3	4	5	6	7	8	9
3	3	4	5	6	7	8	9	10
4	4	5	6	7	8	9	10	11
5	5	6	7	8	9	10	11	12
6	6	7	8	9	10	11	12	13
7	7	8	9	10	11	12	13	14

The indexes of the southeast diagonals can be calculated using `=(8-1-$A2)+B$1` which we can visualize as follows:

	0	1	2	3	4	5	6	7
0	7	8	9	10	11	12	13	14
1	6	7	8	9	10	11	12	13
2	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11
4	3	4	5	6	7	8	9	10
5	2	3	4	5	6	7	8	9
6	1	2	3	4	5	6	7	8
7	0	1	2	3	4	5	6	7

Using the above 2 formulas along with the row and column values we can write a fitness function that touches each board position exactly once, which makes it run fast.

Fitness Rule

The fitness function should run as fast as possible because we're going to call it potentially millions of times.

The fitness value will be the sum of those four counts, subtracted from the maximum value (8+8+8+8, or 32). This means the optimal value will be zero and higher values will be worse. In all previous project, higher fitnesses were better. How do we make this work? The same way we did in the Sorted Numbers problem. We add a problem-specific Fitness class where `__gt__` is wired to prefer fewer queens under attack, as follows:

```
class Fitness:
    Total = None

    def __init__(self, total):
        self.Total = total

    def __gt__(self, other):
        return self.Total < other.Total

    def __str__(self):
        return "{0}".format(self.Total)
```

Then we count the number of rows, columns, and diagonals that have queens to determine how many are under attack:

```
def get_fitness(genes, size):
    board = Board(genes, size)
    rowsWithQueens = set()
    colsWithQueens = set()
    northEastDiagonalsWithQueens = set()
    southEastDiagonalsWithQueens = set()
    for row in range(size):
        for col in range(size):
            if board.get(row, col) == 'Q':
                rowsWithQueens.add(row)
                colsWithQueens.add(col)
                northEastDiagonalsWithQueens.add(row + col)
                southEastDiagonalsWithQueens.add(size - 1 - row + col)

    total = size - len(rowsWithQueens) \
        + size - len(colsWithQueens) \
        + size - len(northEastDiagonalsWithQueens) \
        + size - len(southEastDiagonalsWithQueens)

    return Fitness(total)
```

This requires the addition of a `get` function to `Board`:

```
class Board:
    ...
    def get(self, row, column):
        return self._board[column][row]
```

4.5. Test

Finally our test harness brings all the parts together.

```
def test(self, size=8):
    geneset = [i for i in range(size)]
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime, size)

    def fnGetFitness(genes):
        return get_fitness(genes, size)

    optimalFitness = Fitness(0)
    best = genetic.get_best(fnGetFitness, 2 * size,
                           optimalFitness, geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)
```

4.6. Run

Now we can run the test to see if the engine can find a solution.

```
. . . . . Q . .
. . . Q Q . . .
. . . . Q . . .
. . . . . . . .
. . . . . . . .
. Q . . . . . .
. . Q . . . . .
. . . Q . . . Q
3 6 7 0 1 2 4 5 4 6 3 0 2 1 5 7 - 9 0:00:00
```

```

Q . . . . Q . .
. . Q . . . . .
. . . . Q . . .
. . . . . . . Q
. . . . . . . .
. . . . . . Q .
. . Q . . . . .
. . Q . . . . .
0 7 7 4 6 2 4 5 2 6 2 0 2 1 5 7 - 4 0:00:00.001003

```

```

. . . . Q . . .
. . Q . . . . .
Q . . . . . . .
. . . . . Q . .
. Q . . . . . .
. . . . . . Q
. . . . . Q . .
. . . Q . . . .
7 2 1 3 0 5 4 7 3 0 6 4 2 6 5 1 - 0 0:00:00.098260

```

Some generations are left out for brevity but you can see that the engine can easily find optimal solutions to this puzzle. The solution above is particularly pleasing. We'll see it again in another chapter.

4.7. Benchmarks

The cool thing about our implementation is it works for N queens on an NxN chessboard too, so we can benchmark it with a more difficult problem, like 20 queens.

```

def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test(20))

```

```

. . . . . Q . . . . .
. . . . . . . . . . Q
. Q . . . . . . . . .
. . . . . Q . . . . .
. . . . . . . . . Q .
. . . . Q . . . . .
. . . . . Q . . . . .
. . . . . . . . . Q .
Q . . . . . . . . . .
. . . . . . . . . Q .
. . . . . Q . . . . .
. . . . Q . . . . .
. . . . . . . . . Q .
. . . . . . . . . Q .
. . . . . . . . . Q .
. . . . . Q . . . . .
. . . . . . . . . Q .
. . . . . . . . . Q .
. . . . . Q . . . . .
. . . . . . . . . Q .
. . . Q . . . . . . .
10 4 11 19 5 14 6 2 17 10 14 1 8 9 18 15 3 6 4 8 13 3 2 0 1 17 15 7 9 16 7 13 12 12 19 18
0 11 16 5 - 0 0:00:00.639702

```

We didn't change any code in the `genetic` module, so we can just run the N queens benchmark.

Benchmark

average (seconds)	standard deviation
1.38	1.17

4.8. Summary

In this chapter we learned the difference between genotype and phenotype. This was the first project we've had where the genotype was different from the phenotype. We also learned that we can easily make the engine select for gene sequences with lower fitness values instead of higher ones, should that be useful in solving a problem.

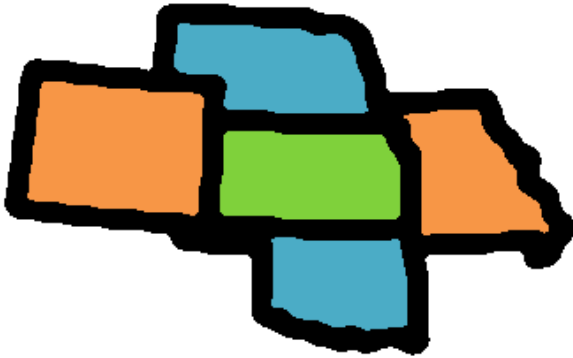
4.9. Final Code

The final code for this chapter is available from:

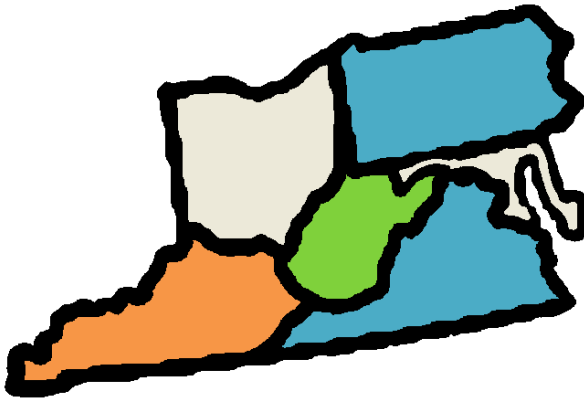
<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch04>

Chapter 5. Graph Coloring

In this chapter we will try a type of problem known as graph coloring. Variations involve using the fewest number of colors while making each node a unique color, trying to use an equal number of each color, etc. By experimenting we can see that a shape surrounded by an even number of neighbors can generally be filled with only 3 colors.



While a shape surrounded by an odd number of neighbors requires 4 colors.



We will try to use only 4 colors to color a map of the United States with the constraint that no adjacent states have the same color. To do this by hand, color any state then use alternating colors on its neighboring states, and only introduce a new color when necessary.

Now when we do this in code we don't care about the visual representation, just the physical relationships between the states, which we can encode as a graph, or more simply as a set of rules indicating which states cannot have the same color.

5.1. Data

We'll start off with a file containing a list of states and the set of states that are adjacent to each. Each row of the file contains the information for one state. There are 2 fields on each line, separated by a comma. The first field contains a unique code for the state. The second field contains a semicolon-separated list of the codes for its neighboring states. I used 2 columns in the table below to keep the list together.

AK,	MT,ID;ND;SD;WY
AL,FL;GA;MS;TN	NC,GA;SC;TN;VA
AR,LA;MO;MS;OK;TN;TX	ND,MN;MT;SD
AZ,CA;NM;NV;UT	NE,CO;IA;KS;MO;SD;WY
CA,AZ;NV;OR	NH,MA;ME;VT
CO,KS;NE;NM;OK;UT;WY	NJ,DE;NY;PA
CT,MA;NY;RI	NM,AZ;CO;OK;TX
DC,MD;VA	NV,AZ;CA;ID;OR;UT
DE,MD;NJ;PA	NY,CT;MA;NJ;PA;VT
FL,AL;GA	OH,IN;KY;MI;PA;WV
GA,AL;FL;NC;SC;TN	OK,AR;CO;KS;MO;NM;TX
HI,	OR,CA;ID;NV;WA
IA,IL;MN;MO;NE;SD;WI	PA,DE;MD;NJ;NY;OH;WV
ID,MT;NV;OR;UT;WA;WY	RI,CT;MA
IL,IA;IN;KY;MO;WI	SC,GA;NC
IN,IL;KY;MI;OH	SD,IA;MN;MT;ND;NE;WY
KS,CO;MO;NE;OK	TN,AL;AR;GA;KY;MO;MS;NC;VA
KY,IL;IN;MO;OH;TN;VA;WV	TX,AR;LA;NM;OK
LA,AR;MS;TX	UT,AZ;CO;ID;NV;WY
MA,CT;NH;NY;RI;VT	VA,DC;KY;MD;NC;TN;WV
MD,DC;DE;PA;VA;WV	VT,MA;NH;NY
ME,NH	WA,ID;OR
MI,IN;OH;WI	WI,IA;IL;MI;MN
MN,IA;ND;SD;WI	WV,KY;MD;OH;PA;VA
MO,AR;IA;IL;KS;KY;NE;OK;TN	WY,CO;ID;MT;NE;SD;UT
MS,AL;AR;LA;TN	

5.2. Reading the file

We'll use the `csv` module to read the file and split the lines on comma. Then we'll manually split the adjacent state list on semicolon and link the state and its list of adjacent states in a key-value table (`dict`).

graphColoringTests.py

```
import csv

def load_data(localFileName):
    """ expects: AA,BB;CC where BB and CC are the initial column values in other rows
    """
    with open(localFileName, mode='r') as infile:
        reader = csv.reader(infile)
        lookup = {row[0]: row[1].split(';') for row in reader if row}
    return lookup
```

5.3. Rule

Now that we've read the data we need to build the rules. A `Rule` connects two states indicating that they are adjacent. When we create the rule we always sort the `state` and `adjacent` codes alphabetically. This makes it possible to eliminate duplicates.

```
class Rule:
    Node = None
    Adjacent = None

    def __init__(self, node, adjacent):
        if node < adjacent:
            node, adjacent = adjacent, node
        self.Node = node
        self.Adjacent = adjacent
```

Hashing and Uniqueness

Since we want to be able to put rules in a key-value table and uniquely identify them in a `list` we need to define `__hash__` and `__eq__`. The important feature of the `__hash__` function is that no combination of state and adjacent codes get the same hash value. The implementation below multiplies the hash of the state code by a prime number then XORs it with the hash of the adjacent state code.

```
def __eq__(self, other):
    return self.Node == other.Node and \
           self.Adjacent == other.Adjacent

def __hash__(self):
    return hash(self.Node) * 397 ^ hash(self.Adjacent)
```

We may also want to be able to display a rule so we'll add a `__str__` implementation.

```
def __str__(self):
    return self.Node + " -> " + self.Adjacent
```

5.4. State adjacency Rules

Next we will build the set of `Rules`. While we're doing so we will perform a sanity check on the data as follows: Whenever a key state says it is adjacent to another state, the adjacent state's rules should also say the key state is adjacent to it. We do that by keeping a count of the number of times we've seen each state pair. We should see each twice, so if we see it only once we know we have a data problem.

```
def build_rules(items):
    rulesAdded = {}

    for state, adjacent in items.items():
        for adjacentState in adjacent:
            if adjacentState == '':
                continue
            rule = Rule(state, adjacentState)
            if rule in rulesAdded:
                rulesAdded[rule] += 1
            else:
                rulesAdded[rule] = 1

    for k, v in rulesAdded.items():
        if v != 2:
            print("rule {0} is not bidirectional".format(k))

    return rulesAdded.keys()
```

We now have the ability to convert a file of node relationships to a set of adjacency `Rules`.

5.5. Test class

Next we will build the code used by the genetic engine.

```
import unittest
import datetime
import genetic
...
class GraphColoringTests(unittest.TestCase):
    def test(self):
```

5.6. Test

We'll start by loading the states from the file and building the rules. Since the expected optimal situation will be that all adjacent states have different colors, we can set the optimal value to the number of rules. Our **Chromosome** will have 50 genes, one for each state in alphabetical order. This lets us use the index into its genes as an index into a list of sorted state codes.

```
def test(self):
    states = load_data("adjacent_states.csv")
    rules = build_rules(states)
    optimalValue = len(rules)
    stateIndexLookup = {key: index
                        for index, key in enumerate(sorted(states))}
    ...
```

5.7. Genes

Next, since we want to four-color the 50 states, our genotype can be four color codes (the first letter of each color name).

```
...
    colors = ["Orange", "Yellow", "Green", "Blue"]
    colorLookup = {color[0]: color for color in colors}
    geneset = list(colorLookup.keys())
    ...
```

Now add the usual function definitions and call `get_best`

```
...
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes, rules, stateIndexLookup)

    best = genetic.get_best(fnGetFitness, len(states),
                           optimalValue, geneset, fnDisplay)
    self.assertTrue(not optimalValue > best.Fitness)
...
```

And at the end we can write out the color of each state.

```
...
    keys = sorted(states.keys())
    for index in range(len(states)):
        print(keys[index] + " is " + colorLookup[best.Genes[index]])
```

5.8. Display

As for `display`, it should be sufficient to output the genes because they are also color codes.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        ''.join(map(str, candidate.Genes)),
        candidate.Fitness,
        str(timeDiff)))
```

This results in output like the following. The number to the right of the gene sequence will indicate how many rules this gene sequence satisfies.

```
YGGB00G00BBYGGYYYYGBGY00GBOYGG000YBOYBBGGGOBYOGOGOGG 74 0:00:00.001000
```

5.9. Fitness

Finally, we need a fitness function that counts how many constraints the gene sequence passed.

```
def get_fitness(genes, rules, stateIndexLookup):
    rulesThatPass = sum(1 for rule in rules
                        if rule.IsValid(genes, stateIndexLookup))
    return rulesThatPass
```

This uses a new function in **Rule**:

```
class Rule:
    ...
    def IsValid(self, genes, nodeIndexLookup):
        index = nodeIndexLookup[self.Node]
        adjacentStateIndex = nodeIndexLookup[self.Adjacent]
        return genes[index] != genes[adjacentStateIndex]
```

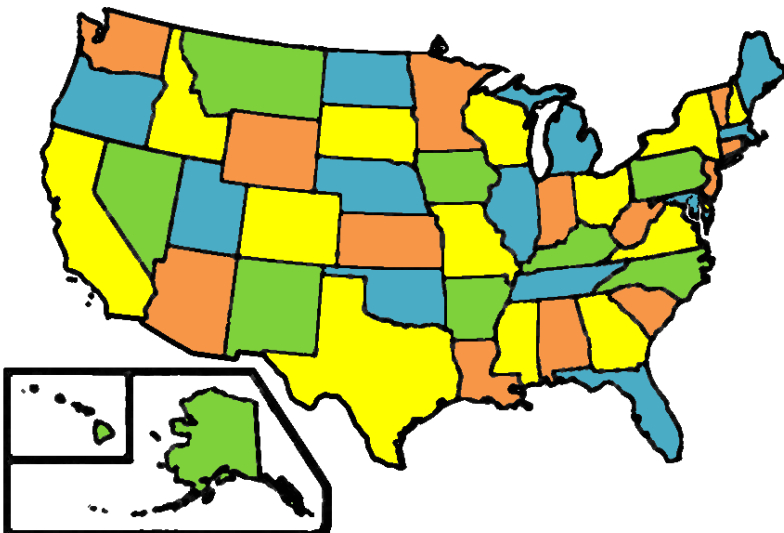
5.10. Run

Now when we run our main **test** function we get output like the following:

```
...
GOGYYGGBGYOYYOOOGOBYYGOYYGOBBYOGGYYBBGOBYBYBG00B00 103 0:00:00.014000
YOGYYGGBGYGYBOOGOBYYGOBBYOGGYYBBGOBYBYBY00B00 105 0:00:00.017001
YOGYYGGBGYGYBOOGOBYYGOBBYOGGYYBBGOBYBYBY00YB0 106 0:00:00.017001
GOGYYOGYBYGGYBOOGOB00BBBOYYGGBBYOGGYYBBGYOYBYBY00Y00 107 0:00:00.040002
AK is Green
AL is Orange
AR is Green
AZ is Orange
CA is Yellow
CO is Yellow
CT is Orange
DC is Green
DE is Yellow
FL is Blue
GA is Yellow
HI is Green
IA is Green
ID is Yellow
IL is Blue
IN is Orange
KS is Orange
KY is Green
LA is Orange
MA is Blue
MD is Blue
```

ME is Blue
MI is Blue
MN is Orange
MO is Yellow
MS is Yellow
MT is Green
NC is Green
ND is Blue
NE is Blue
NH is Yellow
NJ is Orange
NM is Green
NV is Green
NY is Yellow
OH is Yellow
OK is Blue
OR is Blue
PA is Green
RI is Yellow
SC is Orange
SD is Yellow
TN is Blue
TX is Yellow
UT is Blue
VA is Yellow
VT is Orange
WA is Orange
WI is Yellow
WV is Orange
WY is Orange

Which looks like this:



5.11. Benchmarking

To benchmark this project we're going to use R100_1gb, one of a set of graph coloring problem sets available from <https://turing.cs.hbg.psu.edu/txn131/graphcoloring.html>. The file format is line based, with each line prefixed with its type. The lines we're interested in start with **e**, for edge, followed by the IDs of the two nodes connected by that edge, as follows:

```
e 16 10
```

If we change the format of our adjacent state file to the same format, then we'll be able to reuse a lot of code. We'll start by adding a utility test-function that reads our current file and outputs the contents in the new format.

5.11.1. Convert the state file

```
@staticmethod
def test_convert_file():
    states = load_data("adjacent_states.csv")
    output = []
    nodeCount = edgeCount = 0
    for state, adjacents in states.items():
        nodeCount += 1
        for adjacent in adjacents:
            if adjacent == '':
                output.append("n {0} 0".format(state))
            else:
                output.append("e {0} {1}".format(state, adjacent))
                edgeCount += 1
    with open('./adjacent_states.col', mode='w+') as outfile:
        print("p edge {0} {1}".format(nodeCount, edgeCount), file=outfile)
        for line in sorted(output):
            print(line, file=outfile)
```

When we run this test we end up with a file whose content looks like the following:

```
p edge 51 214
e AL FL
e AL GA
...
e WY UT
n AK 0
n HI 0
```

Alaska and Hawaii have no edges, so the only way we know about them is from their (n)ode records.

The utility function and `import csv` can be removed after the file is converted.

5.11.2. Read the new file format

Next we need to update `load_data` to read the new file format.

```
def load_data(localFileName):
    """ expects: T D1 [D2 ... DN]
        where T is the record type
        and D1 .. DN are record-type appropriate data elements
    """
    rules = set()
    nodes = set()
    with open(localFileName, mode='r') as infile:
        content = infile.read().splitlines()
    for row in content:
        if row[0] == 'e': # e aa bb, aa and bb are node ids
            nodeIds = row.split(' ')[1:3]
            rules.add(Rule(nodeIds[0], nodeIds[1]))
            nodes.add(nodeIds[0])
            nodes.add(nodeIds[1])
            continue
        if row[0] == 'n': # n aa ww, aa is a node id, ww is a weight
            nodeIds = row.split(' ')
            nodes.add(nodeIds[1])
    return rules, nodes
```



Now we're returning a set of `Rules` and a set of node names. We'll capture those in variables in the test. Since we have `__hash__` and `__eq__` functions on `Rule`, putting the rules in a `set` automatically eliminates the bi-directional equivalents.

5.11.3. Extract parameters

We'll also take this opportunity to rename `test` to `color` and make the file name and color list into parameters. Plus we'll also add a `test_states` function that uses the `color` function.

```
def test_states(self):
    self.color("adjacent_states.col", ["Orange", "Yellow", "Green", "Blue"])

def color(self, file, colors):
    rules, nodes = load_data(file)
...

```

And remove this line from `color` since the colors are now a parameter.

```
colors = ["Orange", "Yellow", "Green", "Blue"]
```

5.11.4. Node indexes

The next change to the `color` function affects how we get the node indexes.

```
rules, nodes = load_data(file)
optimalValue = len(rules)
colorLookup = {color[0]: color for color in colors}
geneset = list(colorLookup.keys())
startTime = datetime.datetime.now()
nodeIndexLookup = {key: index
                    for index, key in enumerate(sorted(nodes))} ①

def fnDisplay(candidate):
    display(candidate, startTime)

def fnGetFitness(genes):
    return get_fitness(genes, rules, nodeIndexLookup) ②
```

5.11.5. Update the final output

Finally, we have to update the way we write out the results at the end of the `test` function:

```
best = genetic.get_best(fnGetFitness, len(nodes),
                       optimalValue, geneset, fnDisplay)
self.assertTrue(not optimalValue > best.Fitness)

keys = sorted(nodes) ③
for index in range(len(nodes)): ④
    print(keys[index] + " is " + colorLookup[best.Genes[index]])
```

Be sure to run the state test to verify that it still works.

5.11.6. Add the benchmark test

Next we can add a test for R100_1gb as follows:

```
def test_R100_1gb(self):
    self.color("R100_1gb.col",
               ["Red", "Orange", "Yellow", "Green", "Blue", "Indigo"])
```

5.12. Benchmarks

When we run the test it is able to find a way to color the graph using 6 colors relatively quickly. It can also find a solution using 5 colors but that takes longer than we want to spend on a benchmark, so we'll stay with 6 colors. We'll use this test as our benchmark for graph coloring.

```
def test_benchmark(self):  
    genetic.Benchmark.run(lambda: self.test_R100_1gb())
```

Benchmark

average (seconds)	standard deviation
0.74	0.42

5.13. Summary

In this chapter we read from a file and built constraints that can be applied to each candidate to determine its fitness. This was also the first project where we used test data from a standard set. Reading about others' solutions to the same standard problem is a good way to learn advanced techniques and to get inspiration for improving your fitness function and genotype.

5.14. Final Code

The state files and final code for this chapter are available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch05>

Chapter 6. Card Problem

In this chapter we will solve the Card Problem. We start off with the set of cards, Ace and 2-10, which we separate into 2 groups of 5 cards. The cards in each group have different constraints. The card values in one group must have a product of 360. The card values in the other group must sum to 36. We can only use each card once. First try it by hand with a deck of cards.



6.1. Test class and genes

For this project our genotype and phenotype can be the same, integers. Thus we can avoid an encoding/decoding step.

cardTests.py

```
import unittest
import datetime
import genetic

class CardTests(unittest.TestCase):
    def test(self):
        geneset = [i + 1 for i in range(10)]
```

6.2. Fitness

This keeps fitness calculation relatively easy because we can sum one range of genes and multiply the rest. In this project we have 3 values to include in the fitness. One is the sum of the numbers in the first group. Another is the product of the numbers in the second group. The last is a count of the duplicated numbers in the list; we don't want any duplicates.

```
import operator
import functools
...
def get_fitness(genes):
    group1Sum = sum(genes[0:5])
    group2Product = functools.reduce(operator.mul, genes[5:10])
    duplicateCount = (len(genes) - len(set(genes)))
    return Fitness(group1Sum, group2Product, duplicateCount)
```

Once again we'll use a `Fitness` class.

```
class Fitness:
    Group1Sum = None
    Group2Product = None
    TotalDifference = None
    DuplicateCount = None

    def __init__(self, group1Sum, group2Product, duplicateCount):
        self.Group1Sum = group1Sum
        self.Group2Product = group2Product
        sumDifference = abs(36 - group1Sum)
        productDifference = abs(360 - group2Product)
        self.TotalDifference = sumDifference + productDifference
        self.DuplicateCount = duplicateCount
```

When comparing two fitnesses, we'll prefer gene sequences with fewer duplicates, and when those are the same, we'll prefer the one whose sum and product values are closer to the optimal values.

```
class Fitness:
    ...
    def __gt__(self, other):
        if self.DuplicateCount != other.DuplicateCount:
            return self.DuplicateCount < other.DuplicateCount
        return self.TotalDifference < other.TotalDifference
```

6.3. Display

In `display` we'll separate the two groups visually with a dash.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0} - {1}\t{2}\t{3}".format(
        ', '.join(map(str, candidate.Genes[0:5])),
        ', '.join(map(str, candidate.Genes[5:10])),
        candidate.Fitness,
        str(timeDiff)))
```

We need to add a `__str__` function to `Fitness`:

```
class Fitness:
    ...
    def __str__(self):
        return "sum: {0} prod: {1} dups: {2}".format(
            self.Group1Sum,
            self.Group2Product,
            self.DuplicateCount)
```

6.4. Test

Here's the full test harness:

```
def test(self):
    geneset = [i + 1 for i in range(10)]
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    optimalFitness = Fitness(36, 360, 0)
    best = genetic.get_best(fnGetFitness, 10, optimalFitness,
                           geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)
```

6.5. Run

Now we're ready to try it.

sample output

```
2, 3, 1, 6, 4 - 8, 5, 9, 7, 10 sum: 16, prod: 25200, dups: 0 0:00:00
```

6.6. Study the result

We can see that `mutate` quickly eliminates the duplicate values, but the algorithm almost always gets stuck immediately thereafter. The reason is, in order to make progress it has to be able to change 2 numbers. But, it can only change 1 at a time and the priority of eliminating duplicates keeps it from moving forward.

6.7. Introducing custom mutation

We need to find a way to let the engine make two changes at once, ideally without introducing duplication in the process. This means changing the way mutation works. The most flexible way to do that is to introduce an optional parameter called `custom_mutate` into `get_best` that allows us to perform the mutation ourselves. Then we replace the current definition of `fnMutate` with a different one depending on whether or not `custom_mutate` is provided.

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
             display, custom_mutate=None):

    if custom_mutate is None:
        def fnMutate(parent):
            return _mutate(parent, geneSet, get_fitness)
    else:
        def fnMutate(parent):
            return _mutate_custom(parent, custom_mutate, get_fitness)
```

We'll use a variant of `_mutate` instead of the built-in implementation when `custom_mutate` is used.

```
def _mutate_custom(parent, custom_mutate, get_fitness):
    childGenes = parent.Genes[:]
    custom_mutate(childGenes)
    fitness = get_fitness(childGenes)
    return Chromosome(childGenes, fitness)
```

6.8. Mutate

Now back in our genetic algorithm we can add a `mutate` function that changes 1 random gene if there are duplicates, but otherwise swaps 2 genes.

cardTests.py

```
import random
...
def mutate(genes, geneset):
    if len(genes) == len(set(genes)):
        indexA, indexB = random.sample(range(len(genes)), 2)
        genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
    else:
        indexA = random.randrange(0, len(genes))
        indexB = random.randrange(0, len(geneset))
        genes[indexA] = geneset[indexB]
```

Then we pass that function to `get_best` in the test harness.

```
def test(self):
    ...
    def fnMutate(genes):
        mutate(genes, geneset)

    optimalFitness = Fitness(36, 360, 0)
    best = genetic.get_best(fnGetFitness, 10, optimalFitness,
                           geneset, fnDisplay,
                           custom_mutate=fnMutate)
    self.assertTrue(not optimalFitness > best.Fitness)
```

6.9. Run 2

Now when we run the test, it can find the solution about 20 percent of the time. The rest of the time it still gets stuck.

sample output

```
6, 8, 1, 10, 3 - 4, 2, 9, 7, 5 sum: 28, prod: 2520, dups: 0 0:00:00.001001
10, 8, 2, 6, 3 - 4, 1, 9, 7, 5 sum: 29, prod: 1260, dups: 0 0:00:00.001001
10, 8, 7, 6, 3 - 4, 1, 9, 2, 5 sum: 34, prod: 360, dups: 0 0:00:00.001001
```

6.10. Study the result

When we compare the last line in these results with the optimal solution, we can see that the test needs to be able to swap 2 elements on the left side with 2 on the right.

6.11. Engineer a solution

We can solve that by looping the swap portion of `mutate` a random number of times.

```
def mutate(genes, geneset):
    if len(genes) == len(set(genes)):
        count = random.randint(1, 4)
        while count > 0:
            count -= 1
            indexA, indexB = random.sample(range(len(genes)), 2)
            genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
    else:
        indexA = random.randrange(0, len(genes))
        indexB = random.randrange(0, len(geneset))
        genes[indexA] = geneset[indexB]
```

We'll see this pattern again in other chapters.

6.12. Run 3

Now, when we run the test it finds the optimal solution every time.

sample output

```
7, 4, 6, 10, 9 - 8, 3, 5, 1, 2 sum: 36, prod: 240, dups: 0 0:00:00
7, 4, 9, 8, 6 - 1, 3, 5, 2, 10 sum: 34, prod: 300, dups: 0 0:00:00
9, 8, 3, 7, 6 - 5, 4, 1, 2, 10 sum: 33, prod: 400, dups: 0 0:00:00.002000
9, 10, 3, 7, 6 - 5, 4, 1, 2, 8 sum: 35, prod: 320, dups: 0 0:00:00.002000
9, 7, 10, 5, 3 - 6, 1, 2, 8, 4 sum: 34, prod: 384, dups: 0 0:00:00.003000
5, 8, 10, 3, 9 - 4, 1, 7, 2, 6 sum: 35, prod: 336, dups: 0 0:00:00.003000
3, 6, 10, 7, 8 - 9, 2, 5, 1, 4 sum: 34, prod: 360, dups: 0 0:00:00.015001
7, 9, 2, 8, 10 - 6, 3, 1, 5, 4 sum: 36, prod: 360, dups: 0 0:00:00.023001
```

6.13. Retrospective

In all previous projects the only ability we've had to guide the algorithm towards a solution has been the fitness. We've seen that the fitness alone can usually be used to work around structural constraints in the genes. In this project we started using mutation to work around structural constraints as well, but custom mutation is much more powerful than that. It can also take advantage of problem-specific knowledge to narrow the search scope toward better solutions, away from unworkable ones, or both. When we start using mutation that way we're no longer building a simple genetic algorithm, we're building a memetic algorithm.

Memetic algorithms are capable of solving a wider range of problems than random-population-based genetic algorithms because they accelerate the search. Take some time and try to use custom mutation to improve the speed of previous project solutions.

Your mutate function for the 8 Queens Puzzle can now take advantage of problem-specific knowledge such as: no 2 queens will be in the same row or column. So, like this project's mutate function, if there is a duplicate row index, change a random row index, otherwise swap 2 row indexes. The same can be done for the column indexes.

In the Password and One Max projects you might try changing a random number of genes. Another option is to create an empty dictionary of index-to-known in the test harness and pass it to your mutate function. Then each time you make a random change, check the fitness and compare it with the initial fitness from when the mutate function was called. If the random change makes the fitness worse, then you'll know that the previous value at that index was the correct one. Add it to the dictionary and only change that location if it doesn't match what is in the dictionary. There's a lot you could do with this concept, so experiment.

The speed of the Sorted Numbers project could be improved in several ways. For example, we could increment all the values in the final run of the array by 1 if possible, or decrement all the values in the first run by 1 if possible, or change 1 value on both sides of a discontinuity to bring the lines together faster. You likely will think of more.

Also, in Graph Coloring you could take advantage of the adjacency rules to choose a compatible color for a particular gene index.

Memetic algorithm DOs and DON'Ts

It is important to balance keeping the algorithm fast with letting it take advantage of problem-specific knowledge. It is OKAY to check the fitness as long as that runs fast. DO NOT make a lot of changes without checking the fitness. If checking fitness, DO return the first improvement you find. DO NOT try to solve the entire problem, let the genetic engine and the random number generator do their jobs.

6.14. Benchmarks

```
def test_benchmark(self):  
    genetic.Benchmark.run(lambda: self.test())
```

Because we changed the `genetic` module we'll update all of the benchmarks.

Updated Benchmarks

project	average (seconds)	standard deviation
Guess Password	1.23	0.28
One Max	1.24	0.17
Sorted Numbers	1.15	0.66
Queens	1.45	1.07
Graph Coloring	0.79	0.41
Cards	0.01	0.01

6.15. Summary

In this chapter we added the very useful ability to take over mutation from the engine. We also learned about memetic algorithms and how the custom mutation function can be used to take advantage of problem-specific knowledge. Finally, we used the sum-of-difference technique in the fitness calculation. This is a technique commonly used in solving numerical problems.

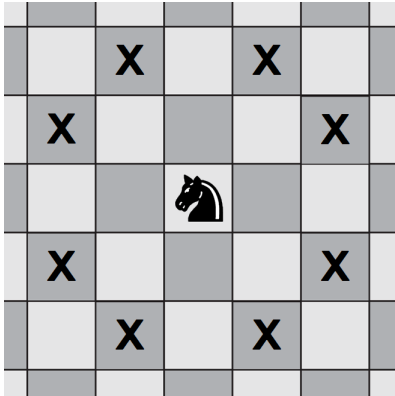
6.16. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch06>

Chapter 7. Knights Problem

The project for this chapter is to figure out the minimal number of chess knights necessary to attack every square on a chess board. This means our chessboard must be at least 3x4 for some number of knights to be able to attack all squares on the board because a knight can only attack certain squares relative to its own location:



The knight can attack, at most, 8 squares, fewer when positioned along an edge.

7.1. Genes

For our genotype we could use the same number as squares on our chessboard so that each represents a location. That has the advantage of preventing duplication. Its major disadvantage is that most of the positions will be empty. A better option is for each gene to be a knight location, just like we did in the 8 Queens chapter. This works as long as we know how many knights are required, which we do for square boards as you'll see later.

7.2. Position

This time we're going to use a `Position` class with X and Y coordinates.

knightsTests.py

```
class Position:
    X = None
    Y = None

    def __init__(self, x, y):
        self.X = x
        self.Y = y
```


7.3. Attacks

We're going to need a utility function that returns all the squares a knight could attack from a given location, respecting the bounds of the chessboard.

```
def get_attacks(location, boardWidth, boardHeight):
    return [i for i in set(
        Position(x + location.X, y + location.Y)
        for x in [-2, -1, 1, 2] if 0 <= x + location.X < boardWidth
        for y in [-2, -1, 1, 2] if 0 <= y + location.Y < boardHeight
        and abs(y) != abs(x))]
```

7.4. Introducing `custom_create`

We have a problem in that the engine doesn't know how to create a `Position` object. We could pass a list of all possible positions to the engine but there are a lot of them and there's a better way. We could also add an optional function that can be called to create an individual gene, but a more efficient and flexible solution is to provide a function for creating the entire set of genes for a `Chromosome`.

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
             display, custom_mutate=None, custom_create=None):
    ...
    return _mutate_custom(parent, custom_mutate, get_fitness)

    if custom_create is None: ①
        def fnGenerateParent():
            return _generate_parent(targetLen, geneSet, get_fitness)
    else:
        def fnGenerateParent(): ②
            genes = custom_create()
            return Chromosome(genes, get_fitness(genes))

    for improvement in _get_improvement(fnMutate, fnGenerateParent):
        ...
```

7.5. Create

Now we can add a `create` function to the algorithm file. It will assign a specific number of knights to unique board positions.

knightsTests.py

```
def create(fnGetRandomPosition, expectedKnights):
    genes = [fnGetRandomPosition() for _ in range(expectedKnights)]
    return genes
```

7.6. Mutate

We chose not to provide all possible genes to the engine, so we need to use a custom `mutate` function as well.

```
import random

def mutate(genes, fnGetRandomPosition):
    index = random.randrange(0, len(genes))
    genes[index] = fnGetRandomPosition()
```

7.7. Display

Since in `display` we want to show the board, we'll adapt the `Board` class from our 8 Queens Puzzle implementation

```
class Board:
    def __init__(self, positions, width, height):
        board = [['.' * width for _ in range(height)]

        for index in range(len(positions)):
            knightPosition = positions[index]
            board[knightPosition.Y][knightPosition.X] = 'N'
        self._board = board
        self._width = width
        self._height = height

    def print(self):
        # 0,0 prints in bottom left corner
        for i in reversed(range(self._height)):
            print(i, "\t", ' '.join(self._board[i]))
        print(" \t", ' '.join(map(str, range(self._width))))
```

and use it in `display`.

```
import datetime

def display(candidate, startTime, boardWidth, boardHeight):
    timeDiff = datetime.datetime.now() - startTime
    board = Board(candidate.Genes, boardWidth, boardHeight)
    board.print()

    print("{0}\n\t{1}\t{2}".format(
        ' '.join(map(str, candidate.Genes)),
        candidate.Fitness,
        str(timeDiff)))
```

We also want `display` to show us the positions of the knights as coordinates. Since we use `map` to convert the `Position` objects to strings, we have to implement `__str__`.

```
class Position:
    ...
    def __str__(self):
        return "{0},{1}".format(self.X, self.Y)
```

7.8. Fitness

For fitness we can just count the unique squares that can be attacked by the knights.

```
def get_fitness(genes, boardWidth, boardHeight):
    attacked = set(pos
                    for kn in genes
                    for pos in get_attacks(kn, boardWidth, boardHeight))
    return len(attacked)
```

In order to put the `Position` objects into a `set`, we also have to implement `__hash__` and `__eq__`.

```
class Position:
    ...
    def __eq__(self, other):
        return self.X == other.X and self.Y == other.Y

    def __hash__(self):
        return self.X * 1000 + self.Y
```

7.9. Test

Finally, we're going to try this out on a chessboard we can check by hand.

```
2   N N N .
1   . . . .
0   . N N N
    0 1 2 3
```

Note, indexes rise starting from the bottom left corner.

Here's the test harness:

```
import unittest
import genetic
...
class KnightsTests(unittest.TestCase):
    def test_3x4(self):
        width = 4
        height = 3
        # 1,0   2,0   3,0
        # 0,2   1,2   2,0
        # 2     N N N .
        # 1     . . . .
        # 0     . N N N
        #      0 1 2 3
        self.find_knight_positions(width, height, 6)
```

```
@staticmethod
def find_knight_positions(boardWidth, boardHeight, expectedKnights):
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime, boardWidth, boardHeight)

    def fnGetFitness(genes):
        return get_fitness(genes, boardWidth, boardHeight)

    def fnGetRandomPosition():
        return Position(random.randrange(0, boardWidth),
                        random.randrange(0, boardHeight))

    ...
```

```

def fnMutate(genes):
    mutate(genes, fnGetRandomPosition)

def fnCreate():
    return create(fnGetRandomPosition, expectedKnights)

optimalFitness = boardWidth * boardHeight
best = genetic.get_best(fnGetFitness, None, optimalFitness,
                        None, fnDisplay, fnMutate, fnCreate)
self.assertTrue(not optimalFitness > best.Fitness)

```

7.10. Run

When we run `test_3x4` we get output like the following:

sample output

```

2   N . . .
1   N N N .
0   . N . N
    0 1 2 3
0,1 2,1 3,0 0,2 1,0 1,1
10  0:00:00.001002

```

```

2   . . N N
1   . N . .
0   N N N .
    0 1 2 3
1,0 2,2 3,2 2,0 0,0 1,1
11  0:00:00.003007

```

```

2   . . N N
1   . . N .
0   N N N .
    0 1 2 3
3,2 2,2 2,0 1,0 2,1 0,0
12  0:00:00.006015

```

As expected it finds a solution equivalent to our hand coded one.

7.11. Test 8x8

We can use OEIS (<https://oeis.org/A261752>) to learn that 14 knights are required to attack all squares on a standard 8x8

chessboard. Let's see if we can verify it.

```
class KnightsTests(unittest.TestCase):
...
    def test_8x8(self):
        width = 8
        height = 8
        self.find_knight_positions(width, height, 14)
```

7.11.1. Run

sample output

```
...
7   . . . . . . . .
6   . . . . . . . .
5   . N N . . N N .
4   . . N N N N . .
3   . . . . . N . .
2   . . N N N N . .
1   . . N . . . . .
0   . . . . . . . .
    0 1 2 3 4 5 6 7
2,5 3,2 6,5 4,2 3,4 1,5 2,1 5,3 2,2 5,2 4,4 5,5 2,4 5,4
63  0:00:00.117311
```

```
7   . . . . . . . .
6   . . . . . . . .
5   . N N . . N N .
4   . . N N N N . .
3   . . . . . . . .
2   . . N N N N . .
1   . . N . . N . .
0   . . . . . . . .
    0 1 2 3 4 5 6 7
2,5 3,2 6,5 5,1 3,4 1,5 2,1 4,2 2,2 5,2 4,4 5,5 2,4 5,4
64  0:00:00.414100
```

It finds an optimal solution every time.

7.12. Try 10x10

Let's try it on a 10x10 board. OEIS says the optimal solution requires 22 knights.

```
class KnightsTests(unittest.TestCase):
    ...
    def test_10x10(self):
        width = 10
        height = 10
        self.find_knight_positions(width, height, 22)
```

7.12.1. Run

```
9  . . . . . . . . . .
8  . N . . . . . . . .
7  . N N . . N N N N .
6  . . N N . . . . . .
5  . . . N . . . N N .
4  . . . . . . . . . .
3  . . N N . N N . . .
2  . . N . . . N N . .
1  . . N . N . . N . .
0  . . . . . . . . . .
    0 1 2 3 4 5 6 7 8 9
8,7 7,2 2,1 7,5 3,6 3,5 2,2 2,7 3,3
6,3 2,3 8,5 5,7 2,6 5,3 4,1 6,7 1,7
6,2 7,1 1,8 7,7
    100 knights: 22 0:00:16.277931
```

It finds a solution, but it takes a while.

7.13. Performance

Let's see if we can improve the performance. First we'll benchmark it:

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test_10x10())
```



When you run this benchmark you might find that it cannot complete all 100 rounds. This can happen if it ends up with a knight configuration where moving any knight to any location results in a worse fitness. This is called a local minimum, or local maximum, depending on whether you are trying to find a lower or higher fitness respectively.

The change we are about to make is one way to work around the local maximum.

```
def mutate(genes, fnGetRandomPosition):
    count = 2 if random.randint(0, 10) == 0 else 1
    while count > 0:
        count -= 1
        index = random.randrange(0, len(genes))
        genes[index] = fnGetRandomPosition()
```

This does not fix the local maximum issue. It simply gives a 1/10 chance of making two changes instead of one, which means the engine now has a random chance of finding the right pair of moves to get past a local maximum. However, as the board size increases, the chance of making the right two moves in a row drops precipitously.

Here's my starting benchmark for the 10x10 board.

average (seconds)	standard deviation
15.49	15.52

7.13.1. Study the result

If you study the optimal 8x8 and 10x10 board layouts you may notice the following pattern: knights are never located on edge rows or columns. This makes sense because a knight can attack, at most, 4 squares from an edge whereas being located just one square away from any edge increases that to 6 squares.



When we exploit that pattern our genetic algorithm becomes a memetic algorithm because we're taking advantage of problem-specific knowledge.

7.13.2. Engineer a solution

Our first performance improvement will be to only use non-edge locations for knights in `create`.

```
@staticmethod
def find_knight_positions(boardWidth, boardHeight, expectedKnights):
    ...
    allPositions = [Position(x, y)
                     for y in range(boardHeight)
                     for x in range(boardWidth)]

    if boardWidth < 6 or boardHeight < 6:
        nonEdgePositions = allPositions
    else:
        nonEdgePositions = [i for i in allPositions
                             if 0 < i.X < boardWidth - 1 and
                                0 < i.Y < boardHeight - 1]

    def fnGetRandomPosition():
        return random.choice(nonEdgePositions)
```


7.13.3. Run

Updated Benchmark

average (seconds)	standard deviation
3.76	3.84

The pattern we discovered netted us a nice performance improvement. Not only did we improve the knights' chances of hitting, we also reduced the search space on a 10x10 board by removing more than 1/3 of the potential knight positions. That is the virtuous cycle I mentioned back in Chapter 1 in action.

7.13.4. Choose wisely

We can get another performance boost by moving knights whose attacks are all covered by other knights to a position where they can attack a square that is not currently under attack. There's a performance hit for checking all the squares and knight attacks but it is more than compensated for if it results in an improvement. Consider that for a 10x10 board the chances of picking the right knight (1 in 22) and moving it to bring a new square under attack (1 in 32 at worst because even the corner square can be attacked from 2 positions in the non-edge 8x8 center) is 1 in 704.

We start off as before but with some additional parameters.

```
def mutate(genes, boardWidth, boardHeight, allPositions, nonEdgePositions):
    count = 2 if random.randint(0, 10) == 0 else 1
    while count > 0:
        count -= 1
    ...
```

The parameters come from `find_knight_positions` as follows:

```
def find_knight_positions(boardWidth, boardHeight, expectedKnights):
    ...
    def fnMutate(genes):
        mutate(genes, boardWidth, boardHeight, allPositions, nonEdgePositions)
    ...
```

Now continuing `mutate`, the next thing we do is figure out which knights are attacking which squares. The array in the dictionary has each knight's gene index.

```
...
    positionToKnightIndexes = dict((p, []) for p in allPositions)
    for i, knight in enumerate(genes):
        for position in get_attacks(knight, boardWidth, boardHeight):
            positionToKnightIndexes[position].append(i)
    ...
```

Then we get a list of indexes of knights whose attacks are all covered by some other knight, and while we're at it we build a list of the squares that are not under attack.

```
...
knightIndexes = set(i for i in range(len(genes)))
unattacked = []
for kvp in positionToKnightIndexes.items():
    if len(kvp[1]) > 1:
        continue
    if len(kvp[1]) == 0:
        unattacked.append(kvp[0])
        continue
    for p in kvp[1]: # len == 1
        if p in knightIndexes:
            knightIndexes.remove(p)
...
```

Next we build the list of locations from which the **unattacked** squares can be attacked. We keep the duplicates because that makes them more likely to be selected, resulting in attacking multiple new squares at once.

```
...
potentialKnightPositions = \
    [p for positions in
     map(lambda x: get_attacks(x, boardWidth, boardHeight), unattacked)
     for p in positions if p in nonEdgePositions] \
     if len(unattacked) > 0 else nonEdgePositions
...
```

Then we choose a gene (knight) to replace.

```
...
geneIndex = random.randrange(0, len(genes)) \
    if len(knightIndexes) == 0 \
    else random.choice([i for i in knightIndexes])
...
```

Lastly, we replace that knight with one likely to improve the fitness.

```
...
position = random.choice(potentialKnightPositions)
genes[geneIndex] = position
```

7.13.5. Run 2

Updated Benchmark

average (seconds)	standard deviation
0.79	1.02

7.14. Retrospective

The ability to control Chromosome creation often goes hand-in-hand with a custom mutation function. With both tools at your disposal you can simplify mutation functions in previous problems, such as in the Card, Sorted Numbers and 8 Queens projects. Use `custom_create` to prevent the creation of invalid sequences so you don't have to correct for them during mutation or in the fitness value. As with custom mutation, don't try to force a solution in this function. Let the genetic engine do its job.

7.15. Benchmarks

We made another change to `genetic` so we'll update the benchmarks.

project	average (seconds)	standard deviation
Guess Password	1.21	0.31
One Max	1.23	0.16
Sorted Numbers	1.15	0.60
Queens	1.61	1.18
Graph Coloring	0.77	0.40
Cards	0.01	0.01
Knights	0.79	1.02

7.16. Summary

This was our first use of a custom object instead of a letter or number for our genotype. This led to the introduction of the ability to customize gene sequence creation in the `genetic` module. Custom creation and custom mutation are frequently used together. We'll use both of these techniques together again in future chapters.

7.17. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch07>

Chapter 8. Magic Squares

Magic squares are squares of numbers where each of the rows and columns and both major diagonals sum to the same value, and all of the numbers between 1 and n^2 are used only once.

example

```
6  7  2
1  5  9
8  3  4
```

8.1. Test class

We'll start with the test stub.

magicSquareTests.py

```
import unittest
import datetime
import genetic

class MagicSquareTests(unittest.TestCase):
    def test_size_3(self):
        self.generate(3)
```

8.2. Test harness

In the test harness we'll determine the range of numbers to use to fill the magic square from its diagonal size then calculate the expected sum. We then pass those to the fitness function.

```
def generate(self, diagonalSize):
    nSquared = diagonalSize * diagonalSize
    geneset = [i for i in range(1, nSquared + 1)]
    expectedSum = diagonalSize * (nSquared + 1) / 2

    def fnGetFitness(genes):
        return get_fitness(genes, diagonalSize, expectedSum)

    ...
```

8.3. Fitness

For fitness we can count the number of rows, columns, and diagonals where the sum is equal to `expectedSum`.

```
def get_fitness(genes, diagonalSize, expectedSum):
    rowSums = [0 for _ in range(diagonalSize)]
    columnSums = [0 for _ in range(diagonalSize)]
    southeastDiagonalSum = 0
    northeastDiagonalSum = 0

    for row in range(diagonalSize):
        for column in range(diagonalSize):
            value = genes[row * diagonalSize + column]
            rowSums[row] += value
            columnSums[column] += value
            southeastDiagonalSum += genes[row * diagonalSize + row]
            northeastDiagonalSum += genes[row * diagonalSize +
                                           (diagonalSize - 1 - row)]

    fitness = sum(1 for s in rowSums + columnSums +
                  [southeastDiagonalSum, northeastDiagonalSum]
                  if s == expectedSum)

    return fitness
```

Unfortunately, that means if we want to display the sums we have to recalculate them. So, let's extract a reusable function.

```
def get_sums(genes, diagonalSize):
    rows = [0 for _ in range(diagonalSize)]
    columns = [0 for _ in range(diagonalSize)]
    southeastDiagonalSum = 0
    northeastDiagonalSum = 0
    for row in range(diagonalSize):
        for column in range(diagonalSize):
            value = genes[row * diagonalSize + column]
            rows[row] += value
            columns[column] += value
            southeastDiagonalSum += genes[row * diagonalSize + row]
            northeastDiagonalSum += genes[row * diagonalSize +
                                           (diagonalSize - 1 - row)]
    return rows, columns, northeastDiagonalSum, southeastDiagonalSum
```

Then call it from the fitness function:

```
def get_fitness(genes, diagonalSize, expectedSum):
    rows, columns, northeastDiagonalSum, southeastDiagonalSum = \
        get_sums(genes, diagonalSize)

    fitness = sum(1 for s in rows + columns +
                  [southeastDiagonalSum, northeastDiagonalSum]
                  if s == expectedSum)

    return fitness
```

8.4. Display

We can call that function from `display` too.

```
def display(candidate, diagonalSize, startTime):
    timeDiff = datetime.datetime.now() - startTime

    rows, columns, northeastDiagonalSum, southeastDiagonalSum = \
        get_sums(candidate.Genes, diagonalSize)

    for rowNumber in range(diagonalSize):
        row = candidate.Genes[rowNumber * diagonalSize:(rowNumber + 1) * diagonalSize]
        print("\t ", row, "=", rows[rowNumber])
    print(northeastDiagonalSum, "\t", columns, "\t", southeastDiagonalSum)
    print(" - - - - - ", candidate.Fitness, str(timeDiff))
```

This will produce output like the following:

sample output

```
    [5, 1, 8] = 14
    [6, 4, 9] = 19
    [3, 7, 2] = 12
15  [14, 12, 19]  11
- - - - - 1 0:00:00.001000
```



We could use format strings to justify the numbers but that involves converting each number to a string, finding the longest one, and then dynamically building a format string. However, all we really want is an idea of what is happening; a comma in the output works fine for that.

Now, since one of the constraints is that we have to use all the numbers between 1 and n^2 , we'll pass a `custom_create` function that produces a random permutation of all values in that range

```
import random
...

def generate(self, diagonalSize):
...
    def fnCustomCreate():
        return random.sample(geneset, len(geneset))

    optimalValue = 2 + 2 * diagonalSize
    startTime = datetime.datetime.now()
    best = genetic.get_best(fnGetFitness, nSquared, optimalValue,
                           geneset, fnDisplay, fnMutate,
                           fnCustomCreate)
```

8.5. Mutate

We'll also pass a companion function for `custom_mutate` so we don't have to worry about duplication being introduced by the engine's built-in `mutate`. We can simply swap two genes but for performances we're going to pass in the index options. This can often provide a good performance boost versus generating a list of indexes each time we enter the function.

```
def mutate(genes, indexes):
    indexA, indexB = random.sample(indexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

Here's the full test harness.

```
def generate(self, diagonalSize):
    nSquared = diagonalSize * diagonalSize
    geneset = [i for i in range(1, nSquared + 1)]
    expectedSum = diagonalSize * (nSquared + 1) / 2

    def fnGetFitness(genes):
        return get_fitness(genes, diagonalSize, expectedSum)

    def fnDisplay(candidate):
        display(candidate, diagonalSize, startTime)
```

```

geneIndexes = [i for i in range(0, len(geneset))]

def fnMutate(genes):
    mutate(genes, geneIndexes)

def fnCustomCreate():
    return random.sample(geneset, len(geneset))

optimalValue = 2 + 2 * diagonalSize
startTime = datetime.datetime.now()
best = genetic.get_best(fnGetFitness, nSquared, optimalValue,
                        geneset, fnDisplay, fnMutate,
                        fnCustomCreate)
self.assertTrue(not optimalValue > best.Fitness)

```

8.6. Run

And, when we run it we get a result like the following:

sample result

```

...
    [8, 7, 4] = 19
    [9, 1, 5] = 15
    [2, 3, 6] = 11
7   [19, 11, 15]    15
- - - - - 3 0:00:00.001001
    [8, 3, 4] = 15
    [1, 9, 5] = 15
    [2, 7, 6] = 15
15  [11, 19, 15]    23
- - - - - 5 0:00:00.001001

```

We can run the test many times but it will rarely find a valid solution.

8.7. Use sum of differences

The problem is the fitness function is written exclusively rather than inclusively. It only gives credit when the sum exactly matches the expected sum, so there's no partial credit for the genetic engine to take advantage of. This is a problem we've encountered before. The solution is to take the sum of the differences between the actual sum and the expected sum for each row, column, and diagonal. That makes zero optimal, so we'll need a **Fitness** object to help reverse the greater-than logic used to compare fitnesses in the engine.


```

class Fitness:
    SumOfDifferences = None

    def __init__(self, sumOfDifferences):
        self.SumOfDifferences = sumOfDifferences

    def __gt__(self, other):
        return self.SumOfDifferences < other.SumOfDifferences

    def __str__(self):
        return "{0}".format(self.SumOfDifferences)

```

Next we update `get_fitness`:

```

def get_fitness(genes, diagonalSize, expectedSum):
    rows, columns, northeastDiagonalSum, southeastDiagonalSum = \
        get_sums(genes, diagonalSize)

    sumOfDifferences = sum(int(abs(s - expectedSum))
                           for s in rows + columns +
                           [southeastDiagonalSum, northeastDiagonalSum]
                           if s != expectedSum)

    return Fitness(sumOfDifferences)

```

And the optimal fitness value in our test.

```

def generate(self, diagonalSize):
    ...
    optimalValue = Fitness(0)
    startTime = datetime.datetime.now()

```

8.8. Run 2

Now when we run the test we get a valid solution about 60% of the time but only because it just happens to find a sequence of swaps that drive improvement. When it doesn't find a sequence

sample result

```

    [4, 8, 3] = 15
    [2, 6, 7] = 15
    [9, 1, 5] = 15
18  [15, 15, 15]  15
- - - - - 3 0:00:00.002005

```

it gets stuck at a position where it requires at least 2 swaps to make progress. We're hitting a local minimum.

8.9. Fixing the local minimum / maximum issue

To fix the local minimum/local maximum issue we're going to allow the current genetic line to die out. The first step in that is tracking how many generations have passed since the last improvement. We'll call this its **Age**.

genetic.py

```
class Chromosome:
    Genes = None
    Fitness = None
    Age = 0

    def __init__(self, genes, fitness):
        self.Genes = genes
        self.Fitness = fitness
```

Next, we'll add an optional parameter that allows us to set an upper limit on the age of a genetic line. That parameter will be passed through to **_get_improvement**.

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
            display, custom_mutate=None, custom_create=None,
            maxAge=None):
    ...
    for improvement in _get_improvement(fnMutate, fnGenerateParent, maxAge):
```

For the next part we need to import a couple of module functions.

```
from bisect import bisect_left
from math import exp
```

Then we need to separate the best parent from the current parent so that we still have something to compare to when a genetic line dies out. We're also going to keep a list of the fitnesses of the historical best parents.

```
def _get_improvement(new_child, generate_parent, maxAge):
    parent = bestParent = generate_parent()
    yield bestParent
    historicalFitnesses = [bestParent.Fitness]
    while True:
```

Next, we want to make sure we retain the current functionality if `maxAge` is not provided.

```
while True:
    child = new_child(parent)
    if parent.Fitness > child.Fitness:
        if maxAge is None:
            continue
```

However, when the child's fitness is worse than that of its parent, the most traveled path through the code, and `maxAge` is provided, then we need to check whether or not the genetic line's age has reached the maximum.

```
        continue
    parent.Age += 1
    if maxAge > parent.Age:
        continue
```

If so, we may allow the genetic line to die and replace it with something else. We're going to do that with simulated annealing.

Annealing is a method used to reduce internal stresses in materials like metal. At a high level, it works by heating the metal to a high temperature then allowing it to cool slowly. As you know, heat causes metal to expand. So the metal expands, loosening the bonds between materials in the metal, allowing them to move around. Then the metal is allowed to cool slowly. As the metal cools the bonds tighten again and the impurities get pushed along by the increasing pressure until they find something to bond with or until there is no more wiggle-room, thus reducing overall stress in the system.

Simulated annealing is a process used with genetic algorithms to break out of a local minimum or maximum. If the child gene sequence is **far away** from the current best solution then we give it a high probability of continuing to wiggle. If not, then we do something else. The something else is implementation dependent.

We need to determine how far away the child's fitness is from the best fitness. If we had a numeric fitness it would be a simple calculation. But our fitnesses aren't always numeric. No problem, we'll figure out where it would be in the realm of historical fitnesses.

```
        index = bisect_left(historicalFitnesses, child.Fitness, 0,
len(historicalFitnesses))
```

Then get its proximity to the best fitness.

```
        difference = len(historicalFitnesses) - index
```

Whether calculating the difference directly from the fitness, or by using its index into the historical fitnesses as we do here, we then convert it to a proportion by dividing it by the best value, in this case the highest index.

```
proportionSimilar = difference / len(historicalFitnesses)
```

Then we raise Euler's number, roughly 2.718, to the power of the proportion negated. The result is a floating point number that approaches 1 if the child's fitness is far away from the best fitness, but approaches ~0.36 when its fitness is very close to the best fitness. In the following sample chart assume the maximum current index is 50:

index	difference	proportion similar	e ^{-proportion}
0	50	0.0	1.00
5	45	0.1	0.90
10	40	0.2	0.82
40	10	0.8	0.45
45	5	0.9	0.41
50	0	1.0	0.37

A child whose fitness is close to the current best will have a high index (because the higher/better fitnesses are at the end of the array) and a low difference from the best fitness. As a result, it will have a lower chance of becoming the new parent. A child that has a fitness far from the current best will have a low index and high difference, and thus a high chance of becoming the new parent.

Next we pick a random number and if that random number is smaller than $e^{-\text{proportion}}$ then the child becomes the new parent.

```
if random.random() < exp(-proportionSimilar):  
    parent = child  
    continue
```

Otherwise we replace the parent with the best parent and reset its age to zero so it has time to anneal.

```
    continue  
    parent = bestParent  
    parent.Age = 0  
    continue
```

Next we consider what to do if the child's fitness is not lower than that of its parent.

```
        continue
    if not child.Fitness > parent.Fitness:
        # same fitness
        child.Age = parent.Age + 1
        parent = child
        continue
```

When the child has a better fitness than its parent, we reset its age to zero and make it the new parent.

```
        continue
    parent = child
    parent.Age = 0
```

Finally, when we find a child whose fitness is better than that of the best parent, we replace the best parent and append its fitness to the list of historical fitnesses.

```
    parent.Age = 0
    if child.Fitness > bestParent.Fitness:
        yield child
        bestParent = child
        historicalFitnesses.append(child.Fitness)
```

8.10. Set the max age

Now, to use it in our test harness we just need to set `maxAge`. But what value should we use? How do we pick a good maximum age? Let's sample the average run times for the projects where we've encountered local minimums or maximums.

	50	500	5000	no max
Knights	0.63 +/- 0.47	0.68 +/- 0.52	0.66 +/- 0.53	0.61 +/- 0.46
Magic Square	0.01 +/- 0.01	0.04 +/- 0.06	0.39 +/- 0.47	*

* *could not complete*

That's interesting. The Knights project doesn't appear to benefit from allowing genetic lines to age out. Magic squares, however, clearly benefits not only from having a maximum age but also from having a low one.

magicSquareTests.py

```
def test_size_3(self):
    self.generate(3, 50)
```

```
def generate(self, diagonalSize, maxAge):
...
    best = genetic.get_best(fnGetFitness, nSquared, optimalValue,
                           geneset, fnDisplay, fnMutate,
                           fnCustomCreate, maxAge)
```

8.11. Run 3

Now the test can quickly find a solution for a size 3 magic square every time.

```
    [6, 7, 2] = 15
    [1, 5, 9] = 15
    [8, 3, 4] = 15
15  [15, 15, 15]  15
- - - - - 0 0:00:00.008000
```

8.12. Size-5 Magic Squares

Size-5 magic squares need a higher maximum age.

```
def test_size_5(self):
    self.generate(5, 500)
```

8.12.1. Run

It can find a magic square every time but we're really giving the simulated annealing code a workout to get it.

sample result

```
    [25, 3, 10, 4, 23] = 65
    [9, 21, 8, 14, 13] = 65
    [22, 6, 5, 15, 17] = 65
    [2, 16, 24, 12, 11] = 65
    [7, 19, 18, 20, 1] = 65
65  [65, 65, 65, 65, 65]  64
- - - - - 1 0:00:00.285760
    [13, 16, 7, 4, 25] = 65
    [22, 19, 5, 9, 10] = 65
    [20, 3, 17, 14, 11] = 65
    [2, 6, 24, 15, 18] = 65
    [8, 21, 12, 23, 1] = 65
65  [65, 65, 65, 65, 65]  65
- - - - - 0 0:00:00.921482
```

8.13. Size 10 Magic Squares

```
def test_size_10(self):
    self.generate(10, 5000)
```

Given time, it can build larger squares too:

note: alignment manually adjusted for clarity

```

    [31,  9, 50, 21, 81, 92, 27, 34, 97, 63] = 505
    [ 8, 80, 86, 29, 40, 24, 100, 55, 72, 11] = 505
    [87, 53, 69, 95, 15, 67, 94,  6, 17,  2] = 505
    [79, 30, 37, 28, 38, 71, 42, 96, 62, 22] = 505
    [54, 46, 56, 93,  3, 74, 59, 52, 23, 45] = 505
    [35, 89, 98, 83, 32,  4, 10,  5, 58, 91] = 505
    [19, 75,  1, 41, 61, 70, 33, 76, 47, 82] = 505
    [20, 44, 78, 12, 85, 36, 65, 99, 18, 48] = 505
    [88, 13, 14, 64, 73, 60, 49, 25, 68, 51] = 505
    [84, 66, 16, 39, 77,  7, 26, 57, 43, 90] = 505
505  [505, 505, 505, 505, 505, 505, 505, 505, 505, 505] 505
- - - - - 0 0:04:24.477191
```

8.14. Retrospective

We saw that using various maximum ages on the Knight's project had no particular effect. What do you think will happen if you use a maximum age for the other projects? Try it.

8.15. Benchmarks

We'll benchmark this project with size-4 magic squares because size-5 runs a bit slower than I like in a benchmark.

```
def test_size_4(self):
    self.generate(4, 50)

def test_benchmark(self):
    genetic.Benchmark.run(self.test_size_4)
```

We rewrote a core function of the **genetic** module so we'll update the benchmarks to make sure there was no unintended slowdown in the ability to solve the previous projects.

Updated Benchmarks

project	average (seconds)	standard deviation
Guess Password	1.31	0.41
One Max	1.22	0.17
Sorted Numbers	1.23	0.83
Queens	1.39	1.03
Graph Coloring	0.78	0.34
Cards	0.01	0.01
Knights	0.61	0.46
Magic Square	0.33	0.56

8.16. Summary

In this chapter we encountered the first project where our forward progress was significantly impacted by a local minimum. We resolved this by adding an option to the engine that allows it to escape local minimums/maximums through the use of simulated annealing. This is also the first time we passed the gene indexes to the mutate function. We'll be using this again.

8.17. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch08>

Chapter 9. Knapsack Problem

The goal of this project is to put as much stuff into a container as it will hold while optimizing for constraints such as item weight, size, shape and value, and in variations of the problem, for the shape of the container. The bounded knapsack problem has a limitation on the number of each particular item available. In the 1/0 knapsack problem you can take no more than 1 of any given item. In the unbounded variant of the knapsack problem you are only limited by the container. We'll try a simple version of the unbounded knapsack problem so we can understand how it works.

9.1. Resources

We'll start by defining some resources:

Resources

Name	Value	Weight(kg)	Volume(L ³)
Flour	1680	0.265	0.41
Butter	1440	0.5	0.13
Sugar	1840	0.441	0.29

This implies a **Resource** class.

knapsackTests.py

```
class Resource:
    Name = None
    Value = None
    Weight = None
    Volume = None

    def __init__(self, name, value, weight, volume):
        self.Name = name
        self.Value = value
        self.Weight = weight
        self.Volume = volume
```

9.2. Test

We will hard code the resources into our test.

```
import unittest
import datetime
import genetic

class KnapsackTests(unittest.TestCase):
    def test_cookies(self):
        items = [
            Resource("Flour", 1680, 0.265, .41),
            Resource("Butter", 1440, 0.5, .13),
            Resource("Sugar", 1840, 0.441, .29)
        ]
```

For the container limits let's say the knapsack contents cannot weigh more than 10 kilograms and its maximum volume is 4 cubic liters.

```
maxWeight = 10
maxVolume = 4
```

Our goal is to maximize the value of the contents of the knapsack within those constraints.

Let's think about how we would accomplish that by hand. We want a high ratio of value to weight and value to volume so we can get the highest possible total value. When we can't stuff any more of the resource with the best ratio into the knapsack, we fill in the remaining space with the next most valuable resource, and so on. Once the knapsack is full then we have to consider whether we would be better off replacing one item type with a combination of others in order to increase the total value of the items in the knapsack.

9.3. ItemQuantity

This time we'll make the genes instances of an `ItemQuantity` class containing the resource and the quantity of that resource to take.

```
class ItemQuantity:
    Item = None
    Quantity = None

    def __init__(self, item, quantity):
        self.Item = item
        self.Quantity = quantity

    def __eq__(self, other):
        return self.Item == other.Item and self.Quantity == other.Quantity
```

9.4. Fitness

To calculate fitness we need to sum the weight, volume, and value of items in the knapsack.



We're going to be careful about how we select genes so that they will never exceed the weight or volume constraints. Therefore, we don't have to check those here.

```
def get_fitness(genes):
    totalWeight = 0
    totalVolume = 0
    totalValue = 0
    for iq in genes:
        count = iq.Quantity
        totalWeight += iq.Item.Weight * count
        totalVolume += iq.Item.Volume * count
        totalValue += iq.Item.Value * count

    return Fitness(totalWeight, totalVolume, totalValue)
```

We use these values to populate a `Fitness` object. When comparing `Fitness` instances we will prefer the one with the highest value.

```
class Fitness:
    TotalWeight = None
    TotalVolume = None
    TotalValue = None

    def __init__(self, totalWeight, totalVolume, totalValue):
        self.TotalWeight = totalWeight
        self.TotalVolume = totalVolume
        self.TotalValue = totalValue

    def __gt__(self, other):
        return self.TotalValue > other.TotalValue

    def __str__(self):
        return "wt: {0:0.2f} vol: {1:0.2f} value: {2}".format(
            self.TotalWeight,
            self.TotalVolume,
            self.TotalValue)
```



Because we're only comparing one value and using greater-than to do so, we could simply return the total value from `get_fitness`. But having all three in the `Fitness` object is convenient for displaying them.

9.5. Max Quantity

We'll limit the quantity to valid ranges for each item.

```
import sys
...
def max_quantity(item, maxWeight, maxVolume):
    return min(int(maxWeight / item.Weight)
               if item.Weight > 0 else sys.maxsize,
               int(maxVolume / item.Volume)
               if item.Volume > 0 else sys.maxsize)
```

9.6. Create

Each time our `custom_create` implementation adds an `ItemQuantity` to the genes, it reduces the remaining weight and volume so it doesn't exceed those limits. When creating a new `Chromosome` we're going to take as much as we can of each item we choose so we fill our container quickly and thus have fewer items to exchange if they aren't the best choices.

```
import random
...
def create(items, maxWeight, maxVolume):
    genes = []
    remainingWeight, remainingVolume = maxWeight, maxVolume
    for i in range(random.randrange(1, len(items))):
        newGene = add(genes, items, remainingWeight, remainingVolume)
        if newGene is not None:
            genes.append(newGene)
            remainingWeight -= newGene.Quantity * newGene.Item.Weight
            remainingVolume -= newGene.Quantity * newGene.Item.Volume
    return genes
```

When adding an item we're going to exclude item types that are already in the knapsack from our options because we don't want to have to sum multiple groups of a particular item type. Then we pick a random item and add as much of that item to the knapsack as we can.

```
def add(genes, items, maxWeight, maxVolume):
    usedItems = {iq.Item for iq in genes}
    item = random.choice(items)
    while item in usedItems:
        item = random.choice(items)

    maxQuantity = max_quantity(item, maxWeight, maxVolume)
    return ItemQuantity(item, maxQuantity) \
        if maxQuantity > 0 else None
```

9.7. Mutate

We'll start off **mutate** by getting the fitness. We need it so we can calculate the remaining weight and volume.

```
def mutate(genes, items, maxWeight, maxVolume):
    fitness = get_fitness(genes)
    remainingWeight = maxWeight - fitness.TotalWeight
    remainingVolume = maxVolume - fitness.TotalVolume
```

Next, because we don't know how many different items we'll take we don't know how long the gene sequence needs to be. This means our gene sequence will have a variable length, up to the number of different items. So we have to handle adding and removing items in addition to the usual item replacement. We'll implement **removing** first.

We need to give it a small chance of removing an item from the knapsack. We'll only do that if we have more than one item so that the knapsack is never empty. We don't immediately return when removing an item because we know removing an item reduces the fitness.

```
removing = len(genes) > 1 and random.randint(0, 10) == 0
if removing:
    index = random.randrange(0, len(genes))
    iq = genes[index]
    item = iq.Item
    remainingWeight += item.Weight * iq.Quantity
    remainingVolume += item.Volume * iq.Quantity
    del genes[index]
```

We'll always add if the length is zero and there is weight or volume available. Otherwise, if we haven't used all the item types, we'll give the algorithm a small chance of adding another item type. If it does then we return the result.

```
adding = (remainingWeight > 0 or remainingVolume > 0) and \
    (len(genes) == 0 or
     (len(genes) < len(items) and random.randint(0, 100) == 0))
if adding:
    newGene = add(genes, items, remainingWeight, remainingVolume)
    if newGene is not None:
        genes.append(newGene)
    return
```

Next we need to implement item replacement by picking a random item that is already in the knapsack then adding its quantity weight and volume to our total available.

```
index = random.randrange(0, len(genes))
iq = genes[index]
item = iq.Item
remainingWeight += item.Weight * iq.Quantity
remainingVolume += item.Volume * iq.Quantity
```

Then we give the algorithm a chance to pick different item type. If we do replace it, we prevent the item type we're replacing from being selected.

```
changeItem = len(genes) < len(items) and random.randint(0, 4) == 0
if changeItem:
    itema, itemb = random.sample(items, 2)
    item = itema if itema != item else itemb
```

Either way we replace the current gene unless the maximum quantity is zero, in which case we remove the gene.

```
maxQuantity = max_quantity(item, remainingWeight, remainingVolume)
if maxQuantity > 0:
    quantity = random.randint(1, maxQuantity)
    genes[index] = ItemQuantity(item, quantity)
else:
    del genes[index]
```



We receive a shallow copy of the list of genes from the engine, so we must be careful to create a new **ItemQuantity** object instead of simply changing the quantity on the existing object, which would unintentionally change the parent's gene too.

9.8. Display

In `display` we'll show the resource names and quantities so we know what to buy, and their total value, weight and volume.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    genes = candidate.Genes[:]
    genes.sort(key=lambda iq: iq.Quantity, reverse=True)

    descriptions = [str(iq.Quantity) + "x" + iq.Item.Name for iq in genes]
    if len(descriptions) == 0:
        descriptions.append("Empty")
    print("{0}\t{1}\t{2}".format(
        ', '.join(descriptions),
        candidate.Fitness,
        str(timeDiff)))
```

The output should look like this:

```
13xSugar, 1xButter  wt: 6.23 vol: 3.90 value: 25360 0:00:00.001002
```

9.9. Test

Here's the full test function

```
def test_cookies(self):
    items = [
        Resource("Flour", 1680, 0.265, .41),
        Resource("Butter", 1440, 0.5, .13),
        Resource("Sugar", 1840, 0.441, .29)
    ]
    maxWeight = 10
    maxVolume = 4
    optimal = get_fitness(
        [ItemQuantity(items[0], 1),
         ItemQuantity(items[1], 14),
         ItemQuantity(items[2], 6)])
    self.fill_knapsack(items, maxWeight, maxVolume, optimal)
```

and the test harness:

```
def fill_knapsack(self, items, maxWeight, maxVolume, optimalFitness):
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    def fnCreate():
        return create(items, maxWeight, maxVolume)

    def fnMutate(genes):
        mutate(genes, items, maxWeight, maxVolume)

    best = genetic.get_best(fnGetFitness, None, optimalFitness,
                           None, fnDisplay, fnMutate, fnCreate)
    self.assertTrue(not optimalFitness > best.Fitness)
```

9.10. Run

When we run `test_cookies`

sample output

```
19xButter, 1xSugar  wt: 9.94 vol: 2.76 value: 29200 0:00:00.003007
```

it gets stuck. From previous experience we know this means it is hitting a local maximum. We also know the fix is to add a maximum age so that simulated annealing can escape from the local maximum.

9.11. Add a maximum age

```
def fill_knapsack(self, items, maxWeight, maxVolume, optimalFitness):
    ...
    best = genetic.get_best(fnGetFitness, None, optimalFitness,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=50)
```


9.12. Run 2

Now when we run the test it can find the optimal solution every time.

sample output

```
5xFlour, 1xSugar    wt: 1.77 vol: 2.34 value: 10240 0:00:00
9xFlour, 1xSugar    wt: 2.83 vol: 3.98 value: 16960 0:00:00
13xSugar           wt: 5.73 vol: 3.77 value: 23920 0:00:00
12xSugar, 2xButter  wt: 6.29 vol: 3.74 value: 24960 0:00:00.002977
12xSugar, 4xButter  wt: 7.29 vol: 4.00 value: 27840 0:00:00.002977
20xButter          wt: 10.00 vol: 2.60 value: 28800   0:00:00.004022
19xButter, 1xSugar  wt: 9.94 vol: 2.76 value: 29200 0:00:01.682475
12xButter, 8xSugar  wt: 9.53 vol: 3.88 value: 32000 0:00:01.868943
15xButter, 5xSugar, 1xFlour wt: 9.97 vol: 3.81 value: 32480 0:00:01.921079
14xButter, 6xSugar, 1xFlour wt: 9.91 vol: 3.97 value: 32880 0:00:02.828494
```

Excellent! But does it work on larger problem sets?

9.13. Solving a harder problem

Like other projects we've seen, knapsack problems are very popular and as a result there are standard problem sets available. One such set, named exnsd16, is available from PYAsUKP:

<http://download.gna.org/pyasukp/pyasukpbench.html#toc1>

9.13.1. File format

The problem files have the following format:

```
...
c: 8273 <-- constraint
...
begin data <-- resource data starts
12 34 <-- weight and value
6 79
...
43 25
end data <-- resource data ends
...
sol: <-- begin optimal solution
    13 54 87 23 <-- resource index, count, quantity weight, quantity value
    55 32 78 69
<-- empty line
...
```

9.13.2. Parse the file

Writing a parser for this data format is easy. We start with a container:

```
class KnapsackProblemData:
    Resources = None
    MaxWeight = None
    Solution = None

    def __init__(self):
        self.Resources = []
        self.MaxWeight = 0
        self.Solution = []
```

Next we need a function that reads all the lines from the file and manages the parsing:

```
def load_data(localFileName):
    with open(localFileName, mode='r') as infile:
        lines = infile.read().splitlines()

    data = KnapsackProblemData()
    f = find_constraint

    for line in lines:
        f = f(line.strip(), data)
        if f is None:
            break
    return data
```

The initial parse function `find_constraint` looks for the constraint:

```
def find_constraint(line, data):
    parts = line.split(' ')
    if parts[0] != "c:":
        return find_constraint
    data.MaxWeight = int(parts[1])
    return find_data_start
```

Once the constraint is found it hands off to `find_data_start` to start watching for the start of the data section.

```
def find_data_start(line, data):
    if line != "begin data":
        return find_data_start
    return read_resource_or_find_data_end
```

The next function in the chain reads the `Resources` until the end of the data section is detected. We name the resources by their 1-based resource index.

```
def read_resource_or_find_data_end(line, data):
    if line == "end data":
        return find_solution_start
    parts = line.split('\t')
    resource = Resource("R" + str(1 + len(data.Resources)), int(parts[1]), int(parts[0]),
0)
    data.Resources.append(resource)
    return read_resource_or_find_data_end
```

Then we transition to looking for the start of the solution section.

```
def find_solution_start(line, data):
    if line == "sol:":
        return read_solution_resource_or_find_solution_end
    return find_solution_start
```

Once we find the solution, we read the resource index and quantity and create the genes for the optimal solution. When we encounter an empty line we're done.

```
def read_solution_resource_or_find_solution_end(line, data):
    if line == "":
        return None
    parts = [p for p in line.split('\t') if p != ""]
    resourceIndex = int(parts[0]) - 1 # make it 0 based
    resourceQuantity = int(parts[1])
    data.Solution.append(ItemQuantity(data.Resources[resourceIndex], resourceQuantity))
    return read_solution_resource_or_find_solution_end
```

The parser is complete.

9.13.3. Test exnsd16

Next we need a test that uses the parser and a problem file.

```
def test_exnsd16(self):
    problemInfo = load_data("exnsd16.ukp")
    items = problemInfo.Resources
    maxWeight = problemInfo.MaxWeight
    maxVolume = 0
    optimal = get_fitness(problemInfo.Solution)
    self.fill_knapsack(items, maxWeight, maxVolume, optimal)
```

9.13.4. Run

When we run the test it can find the solution every time but it can take a couple of minutes.

sample output

```
....
156xR288, 1xR55 wt: 889237.00 vol: 0.00 value: 1029553 0:02:42.741197
156xR288, 1xR410 wt: 889244.00 vol: 0.00 value: 1029582 0:02:42.922804
156xR288, 1xR65 wt: 889276.00 vol: 0.00 value: 1029640 0:02:42.978019
156xR288, 1xR987 wt: 889303.00 vol: 0.00 value: 1029680 0:02:43.035709
```

9.14. Performance

Performance on the knapsack problem is frequently improved by using the branch and bound algorithm.

At a high level, the branch and bound algorithm involves a gene-specific decision and the associated consequences, or costs, of making that decision. Making a decision may allow/force you to eliminate many other potential decisions. For example, choosing a particular shirt/blouse to wear today may imply cultural constraints on your pant/skirt options. And your choice of pant/skirt combined with your blouse/shirt may limit your shoe and sock options. It is a chain reaction. This is why you often see the branch and bound algorithm depicted as a decision tree. Belt/no belt. Tie/no tie. Do I need an undershirt/slip with this? You can start anywhere on the decision tree but as soon as you make a decision your options for the next decision are reduced. The branch is the decision point. The bound is the limitations introduced by that decision. If your options can be ordered numerically then your next decision can generally be found very quickly via binary search.

To facilitate our ability to find the next improvement from a current partial solution we're going to sort the items by value. We're also going to use a sliding window to limit how far above and below the current item it can go to select a replacement in `mutate`. We pass the sorted items and the window to `mutate`.

```
def fill_knapsack(self, items, maxWeight, maxVolume, optimalFitness):
    startTime = datetime.datetime.now()
    window = Window(1,
                    max(1, int(len(items) / 3)),
                    int(len(items) / 2))

    sortedItems = sorted(items, key=lambda item: item.Value)
    ...

    def fnMutate(genes):
        mutate(genes, sortedItems, maxWeight, maxVolume, window)
```

Here's the implementation of `Window`:

```
class Window:
    Min = None
    Max = None
    Size = None

    def __init__(self, minimum, maximum, size):
        self.Min = minimum
        self.Max = maximum
        self.Size = size

    def slide(self):
        self.Size = self.Size - 1 \
            if self.Size > self.Min else self.Max
```

In **mutate** the first thing we do is slide the window.

```
def mutate(genes, items, maxWeight, maxVolume, window):
    window.slide()
    fitness = get_fitness(genes)
    ...
```

Then in the section where we have the option to replace a gene item we pick the new item from those within **window** range of the current item. And then take the maximum possible quantity of the item.

```
...
    if changeItem:
        itemIndex = items.index(iq.Item)
        start = max(1, itemIndex - window.Size)
        stop = min(len(items) - 1, itemIndex + window.Size)
        item = items[random.randint(start, stop)]
        maxQuantity = max_quantity(item, remainingWeight, remainingVolume)
        if maxQuantity > 0:
            genes[index] = ItemQuantity(item, maxQuantity
            if window.Size > 1 else random.randint(1, maxQuantity))
        else:
```

That's it.

9.14.1. Run

Now it finds the solution in about a second on average.

```
155xR288, 5xR409, 1xR112    wt: 889269.00 vol: 0.00 value: 1029497  0:00:00.344953
155xR288, 5xR409, 1xR1060   wt: 889286.00 vol: 0.00 value: 1029520  0:00:00.344953
155xR288, 5xR409, 1xR1028   wt: 889298.00 vol: 0.00 value: 1029525  0:00:00.378039
156xR288, 1xR987            wt: 889303.00 vol: 0.00 value: 1029680  0:00:00.435156
```

Outstanding!

9.15. Retrospective

Try using the sum of difference technique when calculating the fitness.

9.16. Benchmarks

Benchmark it with:

```
def test_benchmark(self):  
    genetic.Benchmark.run(lambda: self.test_exnsd16())
```

We did not change `genetic` so here's the final benchmark.

Benchmark

average (seconds)	standard deviation
1.06	1.14

9.17. Summary

In this chapter we solved the unbounded version of the knapsack problem and learned about the branch and bound algorithm.

9.18. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch09>

Chapter 10. Solving Linear Equations

Solving systems of equations is a common problem in engineering and you can find hundreds of papers online presenting ways to solve them. The methods fall into two broad classes: mathematical and iterative. The first includes methods like [LU factorization](#), [Gaussian elimination](#), and [QR decomposition](#). The issue with these is that rounding errors are compounded at each step so methods must be used to correct for the errors. The iterative class uses successive approximation to find better solutions without the rounding errors. Genetic algorithms fall into this class.

In this chapter we're going to see how genetic algorithms can be used to find the unknowns in a system of linear equations. For example:

$$x + 2y = 4$$

$$4x + 4y = 12$$

We want to figure out which x and y values we can use to solve both equations simultaneously. This, we can do with our genetic engine using one gene for each unknown and a gene set containing all numbers in the range of expected values. Let's try it.

10.1. Test class, test, and genes

We'll start off with the usual stub code.

linearEquationTests.py

```
import unittest
import datetime
import genetic

class LinearEquationTests(unittest.TestCase):
    def test(self):
        geneset = [i for i in range(10)]
        startTime = datetime.datetime.now()

        def fnDisplay(candidate):
            display(candidate, startTime)

        def fnGetFitness(genes):
            return get_fitness(genes)
```


10.2. Fitness

In `get_fitness` we can substitute the gene values directly into `x` and `y`, but what should we return as the fitness so the engine can make progress? A good solution is to re-write the equations so we're looking for zero.

$$x + 2y - 4 = 0$$

$$4x + 4y - 12 = 0$$

This allows us to use the sum of differences technique that we used in the Cards project.

```
def get_fitness(genes):
    x, y = genes[0:2]

    e1 = x + 2 * y - 4
    e2 = 4 * x + 4 * y - 12
    fitness = Fitness(abs(e1) + abs(e2))

    return fitness
```

10.2.1. Fitness class

With `Fitness` defined as:

```
class Fitness:
    TotalDifference = None

    def __init__(self, totalDifference):
        self.TotalDifference = totalDifference

    def __gt__(self, other):
        return self.TotalDifference < other.TotalDifference

    def __str__(self):
        return "diff: {0:0.2f}".format(float(self.TotalDifference))
```

10.3. Optimal fitness

That means the optimal fitness value in our test is going to be zero.

```
def test(self):
    ...
    optimalFitness = Fitness(0)
    best = genetic.get_best(fnGetFitness, 2, optimalFitness,
                           geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)
```

10.4. Display

Finally, in `display`, we can write out the `x` and `y` values.

linearEquationTests.py

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    x, y = candidate.Genes[0:2]
    print("x = {0}, y = {1}\t{2}\t{3}".format(
        x,
        y,
        candidate.Fitness,
        str(timeDiff)))
```

10.5. Run

Now when we run the test...

```
x = 5, y = 2    diff: 21.00 0:00:00
x = 0, y = 2    diff: 4.00  0:00:00
x = 0, y = 3    diff: 2.00  0:00:00.001000
```

it almost always stalls. Why? Consider the values of `x` and `y` in the above output. What has to happen for it to make progress? Both `x` and `y` have to change to new values at the same time. We've encountered another local minimum.

10.6. Add a maximum age

We've hit a local minimum so we'll see if adding a maximum age fixes it.

```
def test(self):  
    ...  
    best = genetic.get_best(fnGetFitness, 2, optimalFitness,  
                           geneset, fnDisplay, maxAge=50)
```

10.7. Run 2

Now when we run the test, it finds the correct solution every time.

sample output

```
x = 7, y = 0    diff: 19.00 0:00:00  
x = 5, y = 0    diff: 9.00  0:00:00  
x = 4, y = 0    diff: 4.00  0:00:00  
x = 3, y = 0    diff: 1.00  0:00:00  
x = 2, y = 1    diff: 0.00  0:00:00.003007
```

We can verify the solution by manually substituting the values it found back into the original equations.

$$(2) + 2*(1) = 4$$

$$4*(2) + 4*(1) = 12$$

Great!

10.8. Fractions and 3 Unknowns

Now, what if x and y are fractions or mixed numbers and we have more unknowns? Well, let's try a system with 3 unknowns and some fractions and see what happens.

$$6x - 2y + 8z = 20$$

$$y + 8x * z = -1$$

$$2z * 6/x + 3y/2 = 6$$

expected:

$$x = 2/3$$

$$y = -5$$

$$z = 3/4$$

Since some of the unknowns are fractions, our genotype will be 2 genes, one for the numerator and one for the denominator, for each unknown. This means we need 6 genes to solve the equations above. Based on the expected values, we're going to let the gene set contain values in the range -5 to 5, excluding 0. The `display` and `get_fitness` functions will also have to change.

10.8.1. Refactoring

We would like to have common fitness, display, and mutate functions used by tests that are solving for a different number of unknowns, and with different genotypes. Let's start by moving the gene set and equation details to the test function.

```
def test_2_unknowns(self):
    geneset = [i for i in range(-5, 5) if i != 0]

    def fnGenesToInputs(genes):
        return genes[0], genes[1]

    def e1(genes):
        x, y = fnGenesToInputs(genes)
        return x + 2 * y - 4

    def e2(genes):
        x, y = fnGenesToInputs(genes)
        return 4 * x + 4 * y - 12

    equations = [e1, e2]
    self.solve_unknowns(2, geneset, equations, fnGenesToInputs)
```

Then rename the previous test harness function to `solve_unknowns` and send it the gene set, equations, and a function that converts the genotype to whatever phenotype the equations need.

```
def solve_unknowns(self, numUnknowns, geneset, equations, fnGenesToInputs):
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime, fnGenesToInputs)

    def fnGetFitness(genes):
        return get_fitness(genes, equations)

    optimalFitness = Fitness(0)
    best = genetic.get_best(fnGetFitness, numUnknowns,
                           optimalFitness, geneset, fnDisplay,
                           maxAge=50)
    self.assertTrue(not optimalFitness > best.Fitness)
```

Now we can simplify the fitness function by making only it responsible for summing the absolute values of the equation results.

```
def get_fitness(genes, equations):
    fitness = Fitness(sum(abs(e(genes)) for e in equations))
    return fitness
```

The display function will still show the unknowns, fitness and elapsed time. It will also do so without having to know how many unknowns there are.

```
def display(candidate, startTime, fnGenesToInputs):
    timeDiff = datetime.datetime.now() - startTime
    symbols = "xyz"
    result = ', '.join("{0} = {1}".format(s, v)
                       for s, v in zip(symbols, fnGenesToInputs(candidate.Genes)))
    print("{0}\t{1}\t{2}".format(
        result,
        candidate.Fitness,
        str(timeDiff)))
```

10.8.2. Test

Next we can add the new test for solving 3 unknowns. In the test for 2 unknowns the genotype **was** the phenotype. But in this test we need to convert every pair of genes to a fraction. Also, we're not going to use simple division because floating point values can be difficult to mentally map to fractions. Instead we'll use the Fraction class so we see 1/4 instead of .25, for example. We have to do the same thing to any fractions in the equations.

```
import fractions
...
def test_3_unknowns(self):
    geneset = [i for i in range(-5, 5) if i != 0]

    def fnGenesToInputs(genes):
        return [fractions.Fraction(genes[i], genes[i + 1])
                for i in range(0, len(genes), 2)]

    def e1(genes):
        x, y, z = fnGenesToInputs(genes)
        return 6 * x - 2 * y + 8 * z - 20

    def e2(genes):
        x, y, z = fnGenesToInputs(genes)
        return y + 8 * x * z + 1
```

```

def e3(genes):
    x, y, z = fnGenesToInputs(genes)
    return 2 * z * fractions.Fraction(6, x) + 3 * fractions.Fraction(y, 2) - 6

equations = [e1, e2, e3]

self.solve_unknowns(6, geneset, equations, fnGenesToInputs)

```

And we're ready to run the test.

10.8.3. Run

```

...
x = 3/5, y = -5, z = 2/3    diff: 2.03  0:00:00.932517
x = 2/3, y = -5, z = 4/5    diff: 1.57  0:00:09.709865
...

```

It occasionally has a delay while working around a local minimum. When we compare the above values of **x**, **y** and **z** with the expected values, we discover that we're hitting a local minimum again. Except now, instead of needing to make 2 changes in order to move forward, we need to make 4. The root of the problem is we're changing the numerator and denominator separately when we really need to be able to replace the fraction they represent with a new fraction.

10.8.4. Use Fractions as the Genotype

In order to use fractions as genes, we have to change the gene set to a set of all possible fractions in the range of numbers we want to support. A benefit of this is that our genotype and phenotype are the same again so we can eliminate the conversion step when evaluating the equations.

```

def test_3_unknowns(self):
    geneRange = [i for i in range(-5, 5) if i != 0]
    geneset = [i for i in set(
        fractions.Fraction(d, e)
        for d in geneRange
        for e in geneRange if e != 0)]

    def fnGenesToInputs(genes):
        return genes

    def e1(genes):
        x, y, z = genes
        return 6 * x - 2 * y + 8 * z - 20

    def e2(genes):
        x, y, z = genes
        return y + 8 * x * z + 1

```

```

def e3(genes):
    x, y, z = genes
    return 2 * z * fractions.Fraction(6, x) + 3 * fractions.Fraction(y, 2) - 6

equations = [e1, e2, e3]
self.solve_unknowns(3, geneset, equations, fnGenesToInputs)

```

10.8.5. Run 2

Now when it encounters a local minimum like the one we saw above

```

...
x = 3/5, y = -5, z = 2/3    diff: 2.03    0:00:00.042112
x = 2/3, y = -5, z = 4/5    diff: 1.57    0:00:00.082218
...

```

the engine hardly slows down. Great!

10.9. Finding 4 unknowns

Now let's see if it can solve 4 unknowns:

```

1/15x  - 2y  - 15z  -4/5a =  3
-5/2x  - 9/4y  + 12z  - a = 17
-13x   + 3/10y  - 6z   - 2/5a = 17
1/2x   + 2y   + 7/4z   + 4/3a = -9

x = -3/2
y = -7/2
z = 1/3
a = -11/8

```

10.9.1. Test

We just have to add the new test:

```
def test_4_unknowns(self):
    geneRange = [i for i in range(-13, 13) if i != 0]
    geneset = [i for i in set(
        fractions.Fraction(d, e)
        for d in geneRange
        for e in geneRange if e != 0)]

    def fnGenesToInputs(genes):
        return genes

    def e1(genes):
        x, y, z, a = genes
        return fractions.Fraction(1, 15) * x \
            - 2 * y \
            - 15 * z \
            - fractions.Fraction(4, 5) * a \
            - 3

    def e2(genes):
        x, y, z, a = genes
        return -fractions.Fraction(5, 2) * x \
            - fractions.Fraction(9, 4) * y \
            + 12 * z \
            - a \
            - 17

    def e3(genes):
        x, y, z, a = genes
        return -13 * x \
            + fractions.Fraction(3, 10) * y \
            - 6 * z \
            - fractions.Fraction(2, 5) * a \
            - 17
```



```
def e4(genes):
    x, y, z, a = genes
    return fractions.Fraction(1, 2) * x \
        + 2 * y \
        + fractions.Fraction(7, 4) * z \
        + fractions.Fraction(4, 3) * a \
        + 9

equations = [e1, e2, e3, e4]
self.solve_unknowns(4, geneset, equations, fnGenesToInputs)
```

and add **a** to the symbols in **display**

```
symbols = "xyza"
```

10.9.2. Run

```
....
w = -3/2, x = -10/3, y = 1/3, z = -12/7 diff: 0.40 0:00:05.217881
w = -3/2, x = -10/3, y = 1/3, z = -7/4 diff: 0.40 0:00:05.271022
w = -3/2, x = -7/2, y = 1/3, z = -13/10 diff: 0.27 0:00:12.802097
w = -3/2, x = -7/2, y = 1/3, z = -11/8 diff: 0.00 0:00:12.910380
```

It can find the solution every time but it can take a while. Here's the benchmark for finding 4 unknowns.

average (seconds)	standard deviation
5.82	4.87

10.10. Performance

What can we do to improve performance? Are there any patterns in the output? Yes, there are. The first is that the sign of an unknown tends to stay the same. Another is that the change between improvements for a given unknown tends to get smaller as we get closer to the solution. We can take advantage of both by limiting our search to a sliding range around the current gene index into a sorted array of potential genes. Let's use the branch and bound algorithm.

We'll start by moving the maximum age to a variable

```
def solve_unknowns(self, numUnknowns, geneset, equations, fnGenesToInputs):
    startTime = datetime.datetime.now()
    maxAge = 50
```

Then use it to determine the minimum window size for adjusting the number of genes.

```
maxAge = 50
window = Window(max(1, int(len(geneset) / (2 * maxAge))),
                 max(1, int(len(geneset) / 3)),
                 int(len(geneset) / 2))
```

We'll borrow `Window` from the previous chapter:

```
class Window:
    Min = None
    Max = None
    Size = None

    def __init__(self, minimum, maximum, size):
        self.Min = minimum
        self.Max = maximum
        self.Size = size

    def slide(self):
        self.Size = self.Size - 1 \
            if self.Size > self.Min else self.Max
```

Next, in order to make the most efficient use of multiple rounds through the loop, we're going to want to use `random.choose` so that we only use each gene index once. That means moving the indexes into an array like we did when finding Magic Squares.

```
geneIndexes = [i for i in range(numUnknowns)]
```

We also need the gene options to be sorted so that genes that are numerically close to the current gene are also physically close to it in the array. This allows us to pick from genes within a certain numeric distance of the current one.

```
sortedGeneset = sorted(geneset)
```

Lastly, pass these new parameters to a `mutate` function

```
def fnMutate(genes):
    mutate(genes, sortedGeneset, window, geneIndexes)
```

and use that function in the engine.

```
best = genetic.get_best(fnGetFitness, numUnknowns,
                        optimalFitness, geneset, fnDisplay,
                        fnMutate, maxAge=maxAge)
```

Now in `mutate`, we start by choosing at least one gene to mutate. Then change the window size and start the loop.

```
import random
...
def mutate(genes, sortedGeneset, window, geneIndexes):
    indexes = random.sample(geneIndexes, random.randint(1, len(genes))) \
        if random.randint(0, 10) == 0 else [random.choice(geneIndexes)]
    window.slide()
    while len(indexes) > 0:
```

Then, each time through the loop we take the next gene from the indexes we picked

```
index = indexes.pop()
```

and calculate the search bounds using the window size. This way we limit the number of genes we could pick from and stop if we reach one of the ends of the array.

```
genesetIndex = sortedGeneset.index(genes[index])
start = max(0, genesetIndex - window.Size)
stop = min(len(sortedGeneset) - 1, genesetIndex + window.Size)
```

Finally, we replace the current gene with one picked randomly from the new bounding range.

```
genesetIndex = random.randint(start, stop)
genes[index] = sortedGeneset[genesetIndex]
```

This accomplishes our goal of taking advantage of patterns in the data. We now try to find replacement genes that are numerically close to the current gene. Depending on how close the gene is to zero and how wide the selection boundaries are, this keeps our choices mostly negative if the current gene is negative, or mostly positive if the current gene is positive. Using branch and bound we can slowly change the search range around the current best gene. This allows the algorithm to find the next improvement should it require the signs of one or more genes to change, as in this example:

```
x = -1/5, y = -3, z = -1/4, a = -1/10    diff: 35.56 0:00:00.003038
x = 1/7, y = -3, z = 5/13, a = -1/10     diff: 34.21 0:00:00.005043
```

10.10.1. Run

Now when we run the test it can find the 4 unknowns much faster.

```
...
x = -3/2, y = -11/3, z = 1/3, a = -13/12    diff: 0.41  0:00:01.176128
x = -3/2, y = -11/3, z = 1/3, a = -1      diff: 0.40  0:00:01.248321
x = -3/2, y = -7/2, z = 1/3, a = -13/10   diff: 0.27  0:00:01.605241
x = -3/2, y = -7/2, z = 1/3, a = -11/8    diff: 0.00  0:00:01.609281
```

10.11. Benchmarks

We will benchmark this project with the test that finds 4 unknowns.

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test_4_unknowns())
```

Benchmark

average (seconds)	standard deviation
1.50	1.17

10.12. Summary

In this chapter we learned how to solve an important engineering problem. We were also able to reinforce to ourselves the power of using branch and bound.

10.13. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch10>

Chapter 11. Generating Sudoku

Sudoku puzzles are 9x9 logic problems where each row, column, and non-overlapping 3x3 section must have all the digits in the range 1 to 9. The player is given a partially filled grid and the challenge is to use the known digits, the puzzle constraints, and logic to deduce the remaining digits. Here's a sample completed puzzle:

```
8 4 1 | 5 6 7 | 9 2 3
2 7 6 | 9 4 3 | 1 5 8
9 5 3 | 2 8 1 | 7 6 4
-----+-----+-----
5 1 9 | 7 2 4 | 8 3 6
6 8 4 | 3 1 9 | 5 7 2
7 3 2 | 6 5 8 | 4 9 1
-----+-----+-----
1 2 5 | 4 9 6 | 3 8 7
4 6 7 | 8 3 5 | 2 1 9
3 9 8 | 1 7 2 | 6 4 5
```

We're going to use a genetic algorithm to generate a Sudoku puzzle. This is a guided exercise so you should be thinking about how to improve the implementation as we go along. Also, if you've never tried to solve one of these puzzles, you should do so as that process may provide valuable insight.

Let's go.

11.1. Test class and genes

First off, our genotype will be the digits 1 thru 9.

sudokuTests.py

```
import unittest
import datetime
import genetic

class SudokuTests(unittest.TestCase):
    def test(self):
        geneset = [i for i in range(1, 9 + 1)]
```

11.2. Fitness

The fitness value will be a count of the rows, columns and sections that have all 9 digits. This means the optimal value will be 9+9+9 or 27.

```
optimalValue = 27
```

The fitness function will be called many times so it needs to run fast. We start by creating an empty **set** for each row, column and section.

```
def get_fitness(candidate):
    rows = [set() for _ in range(9)]
    columns = [set() for _ in range(9)]
    sections = [set() for _ in range(9)]
```

Next, we populate the sets while touching each square only once. The section numbers can be calculated in Excel using **QUOTIENT(B\$1,3)+QUOTIENT(\$A2,3)*3**.

	0	1	2	3	4	5	6	7	8	column
0	0	0	0	0	1	1	1	2	2	2
1	0	0	0	0	1	1	1	2	2	2
2	0	0	0	0	1	1	1	2	2	2
3	3	3	3	3	4	4	4	5	5	5
4	3	3	3	3	4	4	4	5	5	5
5	3	3	3	3	4	4	4	5	5	5
6	6	6	6	6	7	7	7	8	8	8
7	6	6	6	6	7	7	7	8	8	8
8	6	6	6	6	7	7	7	8	8	8
row										

```
for row in range(9):
    for column in range(9):
        value = candidate[row * 9 + column]
        rows[row].add(value)
        columns[column].add(value)
        sections[int(row / 3) + int(column / 3) * 3].add(value)
```

Lastly, we return the total number of rows, columns and sections that have the correct count.

```
fitness = sum(len(row) == 9 for row in rows) + \
    sum(len(column) == 9 for column in columns) + \
    sum(len(section) == 9 for section in sections)

return fitness
```

11.3. Display

We can easily produce a visual representation of the complete Sudoku puzzle.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime

    for row in range(9):
        line = ' | '.join(
            ' '.join(str(i)
                      for i in candidate.Genes[row * 9 + i:row * 9 + i + 3])
            for i in [0, 3, 6])
        print("", line)
        if row < 8 and row % 3 == 2:
            print(" ----- + ----- + -----")
    print(" - = -   - = -   - = - {0}\t{1}\n"
          .format(candidate.Fitness, str(timeDiff)))
```

Here's the expected output:

```
8 4 1 | 5 6 7 | 4 2 4
2 7 2 | 2 4 4 | 9 7 6
2 4 9 | 4 4 1 | 1 3 1
----- + ----- + -----
9 3 2 | 5 5 5 | 8 7 6
1 9 8 | 8 1 3 | 6 5 9
5 1 5 | 8 7 6 | 6 2 3
----- + ----- + -----
6 8 6 | 7 5 7 | 2 4 3
7 8 2 | 9 8 1 | 8 7 3
9 9 3 | 3 5 9 | 3 6 1
- = -   - = -   - = - 17    0:00:00
```

11.4. Test

The full test harness for this project is fairly simple.

```
def test(self):
    geneset = [i for i in range(1, 9 + 1)]
    startTime = datetime.datetime.now()
    optimalValue = 27

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    best = genetic.get_best(fnGetFitness, 81, optimalValue,
                           geneset, fnDisplay)
    self.assertEqual(best.Fitness, optimalValue)
```

11.5. Run

Now run it. It makes progress at first but eventually stalls. I ran the code 10 times and it always stalled with a fitness value between 11 and 15.

```
9 5 8 | 3 6 1 | 2 7 4
2 1 4 | 5 6 8 | 3 7 9
5 8 3 | 4 6 9 | 7 1 2
----- + ----- + -----
4 3 9 | 8 2 5 | 6 7 1
8 9 2 | 6 3 4 | 5 7 1
3 4 7 | 1 6 5 | 9 2 8
----- + ----- + -----
5 7 1 | 9 8 3 | 4 6 2
8 6 9 | 2 7 4 | 1 5 3
5 2 4 | 7 6 9 | 8 1 3
- = - - = - - = - 15    0:00:02.793290
```

What should we do now?

- Use [simulated annealing](#) to fix the local minimum.
- Use [custom mutation](#) to swap two genes.
- Use [custom creation](#) to start with the correct number of each digit.

(Click one of the options above)

11.6. Many options

The remaining sections of this chapter are not meant to be read straight through. Click one of the options above.

Use simulated annealing

It certainly behaves like it is hitting a local maximum. So let's see if simulated annealing can fix the stall.

```
def test(self):  
    ...  
    best = genetic.get_best(fnGetFitness, 81, optimalValue,  
                           geneset, fnDisplay, maxAge=5000)
```

Run 2

With this change the engine can achieve a fitness value of 23 every time, but it requires a relatively high maximum age and a long run time, and it still stalls at that point every time. That is a good indication that we are encountering local maximums and that they are relatively complex to resolve.

```
2 3 6 | 8 7 9 | 5 4 1  
8 5 4 | 1 6 3 | 7 9 2  
7 1 9 | 4 8 2 | 6 3 5  
----- + ----- + -----  
6 9 5 | 3 2 8 | 4 1 7  
4 7 1 | 6 3 5 | 2 8 9  
5 2 7 | 9 1 4 | 8 6 3  
----- + ----- + -----  
1 8 2 | 7 9 6 | 3 5 4  
9 6 3 | 5 4 7 | 1 2 8  
3 4 8 | 2 5 1 | 9 7 6  
- = - - = - - = - 23    0:03:43.065731
```

Let's see if we can help it by removing some of the inefficiency from the process.

What should we do now?

- [Add custom creation](#) to start with the correct number of each digit.
- [Add custom mutation](#) to swap two genes.
- [Restart from the beginning](#).

(Click one of the options above)

Add custom creation

We can reduce the amount of work the engine has to do if we start with the right number of each digit. We'll do that with a permutation of 9 copies of the digits.

```
import random
...
def test(self):
...
    def fnCreate():
        return random.sample(geneset * 9, 81)

    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           geneset, fnDisplay,
                           custom_create=fnCreate, maxAge=5000)
```

Run 3

This change doesn't affect the fitness value, it is still 23 every time, and it still stalls every time.

```
...
1 8 7 | 9 5 9 | 1 3 5
9 6 3 | 7 1 4 | 5 8 2
5 2 4 | 8 6 3 | 7 9 1
----- + ----- + -----
8 7 2 | 4 2 6 | 4 7 3
3 9 1 | 5 3 2 | 6 6 8
4 1 5 | 6 8 7 | 2 3 9
----- + ----- + -----
2 4 6 | 1 9 8 | 3 5 7
6 5 9 | 3 7 1 | 8 2 4
7 3 8 | 2 4 5 | 9 1 6
- = - - = - - = - 19    0:00:12.411932
```

Add custom mutation

Since our custom creation function ensures we start with the right number of each digit, we can also replace the built-in mutation function with one that just swaps two genes.

```
def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

```

def test(self):
...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)

    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)

```

Run 4

```

6 3 1 | 5 7 2 | 4 8 9
5 8 7 | 4 6 9 | 3 2 1
4 9 2 | 7 5 3 | 8 1 6
----- + ----- + -----
8 7 5 | 9 1 6 | 2 3 4
1 6 3 | 2 8 4 | 9 5 7
2 4 9 | 1 3 8 | 7 6 5
----- + ----- + -----
9 1 8 | 6 2 7 | 5 4 3
7 2 6 | 3 4 5 | 1 9 8
3 5 4 | 8 9 1 | 6 7 2
- = - - = - - = - 23    0:00:39.616364

```

This change removes the remaining inefficiency from the process but it still stalls. To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Add custom mutation

The genetic algorithm is quite capable of producing a gene sequence that has exactly 9 of each of the 9 digits. Once it does, however, it always stalls because the built-in mutation operation can only make one change at a time. To make further progress it needs to at least be able to swap two genes. We know how to do that.

We'll add a variation of the custom `mutate` function from the Cards project. If the `Chromosome` has too many or too few of a digit we randomly replace a gene. It doesn't matter which one as the engine will make it work. Otherwise, when we have the right number of all the digits, we'll just swap two random genes.

```
import random
...
def mutate(genes, geneset):
    counts = [0 for _ in range(len(geneset) + 1)]
    for digit in genes:
        counts[digit] += 1
    correctDigitCount = sum(1 for i in counts if i == 9)
    if correctDigitCount != 9:
        index = random.randrange(0, len(genes) - 1)
        genes[index] = random.choice(geneset)
    else:
        indexA, indexB = random.sample(range(len(genes)), 2)
        genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

We also need to wire the test harness to use our custom mutation function.

```
def test(self):
    ...
    def fnMutate(genes):
        mutate(genes, geneset)

    best = genetic.get_best(fnGetFitness, 81, optimalValue,
                           geneset, fnDisplay, fnMutate,
                           maxAge=5000)
```

Run 3

This change definitely improves the range of fitness values. Now, over ten runs it always ends up with fitness values of 22 or 23.

```

...
6 4 9 | 8 3 7 | 1 5 2
5 3 1 | 2 6 9 | 7 8 4
2 7 8 | 5 4 1 | 6 9 3
----- + ----- + -----
9 8 5 | 3 7 4 | 2 6 1
4 6 3 | 9 1 2 | 5 7 8
1 5 7 | 6 2 8 | 4 3 9
----- + ----- + -----
7 2 6 | 1 8 3 | 9 4 5
8 9 2 | 4 5 6 | 3 1 7
3 1 4 | 7 9 5 | 8 2 6
- = -   - = -   - = - 23    0:00:11.885119

```

Add custom creation

Let's add a custom create function so we don't waste time creating an invalid gene sequence in the first place. Maybe that will resolve the stall.

```

def test(self):
...
    def fnCreate():
        return random.sample(geneset * 9, 81)
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)

```

With that change we can simplify the implementation of `mutate` to just perform swaps.

```

def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]

```

Then update the test harness to match the parameter change.

```

def test(self):
...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)

```

Run 4

```
4 7 2 | 8 5 9 | 6 3 1
8 1 5 | 9 4 2 | 3 6 7
6 3 9 | 1 2 5 | 7 8 4
----- + ----- + -----
9 4 7 | 3 6 8 | 1 5 2
1 2 8 | 7 3 6 | 5 4 9
5 6 3 | 4 1 7 | 9 2 8
----- + ----- + -----
2 5 1 | 6 7 4 | 8 9 3
7 9 4 | 5 8 3 | 2 1 6
3 8 6 | 2 9 1 | 4 7 5
- = - - = - - = - 23    0:00:30.633457
```

This change removes the remaining inefficiency from the process and makes it so that we achieve a 23 fitness value 10 times out of 10. But it still stalls. To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Use custom mutation

The genetic algorithm is quite capable of producing a gene sequence that has exactly 9 of each of the 9 digits. Once it does, however, it always stalls because the built-in mutation operation can only make one change at a time. To make further progress it needs to at least be able to swap two genes. We know how to do that.

We'll add a variation of the custom `mutate` function from the Cards project. If the `Chromosome` has too many or too few of a digit we randomly replace a gene. It doesn't matter which one; the engine will make it work. Otherwise, when we have the right number of all the digits then we'll just swap two random genes.

```
import random
...
def mutate(genes, geneset):
    counts = [0 for _ in range(len(geneset) + 1)]
    for digit in genes:
        counts[digit] += 1
    correctDigitCount = sum(1 for i in counts if i == 9)
    if correctDigitCount != 9:
        index = random.randrange(0, len(genes) - 1)
        genes[index] = random.choice(geneset)
    else:
        indexA, indexB = random.sample(range(len(genes)), 2)
        genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

We also need to wire the test harness to use our custom mutation function.

```
def test(self):
    ...
    def fnMutate(genes):
        mutate(genes, geneset)

    best = genetic.get_best(fnGetFitness, 81, optimalValue,
                           geneset, fnDisplay, fnMutate)
```

Run 2

This change definitely improves the range of fitness values. Now, over ten runs it always ends up with fitness values of 22 or 23.


```

...
6 9 3 | 2 4 5 | 8 1 7
5 8 1 | 7 3 4 | 6 2 9
4 2 7 | 5 6 3 | 9 8 1
----- + ----- + -----
7 3 2 | 1 9 8 | 4 5 6
9 6 5 | 4 8 1 | 3 7 2
8 1 6 | 9 2 7 | 5 4 3
----- + ----- + -----
3 4 8 | 6 7 2 | 1 9 5
1 7 4 | 3 5 9 | 2 6 8
2 5 9 | 8 1 6 | 7 3 4
- = -   - = -   - = - 23    0:00:12.103029

```

What should we do now?

- [Add simulated annealing](#) to fix the local maximum.
- [Add custom creation](#) to start with the correct number of each digit.
- [Restart from the beginning](#).

(Click one of the options above)

Add custom creation

If we add a custom creation function we can ensure that we start with the right number of each digit.

```
def test(self):
...
    def fnCreate():
        return random.sample(geneset * 9, 81)
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate)
```

This also means we can simplify our mutation function to just swap two genes.

```
def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

Now update the test harness to match the parameter change.

```
def test(self):
...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)
...

```

Run 3

```
6 4 3 | 2 5 7 | 8 9 1
2 1 7 | 3 9 4 | 6 8 5
5 9 8 | 6 2 1 | 7 3 4
----- + ----- + -----
4 3 5 | 8 7 9 | 2 1 6
8 2 1 | 9 3 6 | 4 5 7
7 6 4 | 5 1 8 | 9 2 3
----- + ----- + -----
9 5 6 | 1 4 2 | 3 7 8
1 7 9 | 4 8 3 | 5 6 2
3 8 2 | 7 6 5 | 1 4 9
- - - - - = - - - - - = - 23    0:00:20.101972
```

This change enables the engine to achieve a fitness value of 22 or 23 every time, and to do so quickly most of the time. But, it still stalls.

Add simulated annealing

In the past when the engine became stuck it was because it hit a local maximum, right? No problem. Let's see if simulated annealing helps.

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)
```

Run 4

This change makes it so that we achieve a 23 fitness value every time, but it still stalls. That is a good indication that we are encountering local maximums and that they are relatively complex to resolve.

```
...
8 9 5 | 6 4 2 | 1 3 7
4 2 3 | 1 9 7 | 6 5 8
1 7 6 | 5 2 8 | 3 4 9
----- + ----- + -----
6 8 2 | 7 5 3 | 9 1 4
3 4 7 | 9 8 1 | 5 2 6
5 1 9 | 3 6 4 | 7 8 2
----- + ----- + -----
2 3 1 | 4 7 9 | 8 6 5
9 5 8 | 2 1 6 | 4 7 3
7 6 4 | 8 3 5 | 2 9 1
- = - - = - - = - 23    0:01:19.822566
```

To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Add simulated annealing

In the past when the engine became stuck it was because it hit a local maximum, right? No problem. Let's see if simulated annealing helps.

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, 81, optimalValue,
                           geneset, fnDisplay, fnMutate,
                           maxAge=5000)
```

Run 3

This change makes it so that we achieve a 23 fitness value every time, but it still stalls. That is a good indication that we are encountering local maximums and that they are complex to resolve.

```
...
8 9 5 | 6 4 2 | 1 3 7
4 2 3 | 1 9 7 | 6 5 8
1 7 6 | 5 2 8 | 3 4 9
----- + ----- + -----
6 8 2 | 7 5 3 | 9 1 4
3 4 7 | 9 8 1 | 5 2 6
5 1 9 | 3 6 4 | 7 8 2
----- + ----- + -----
2 3 1 | 4 7 9 | 8 6 5
9 5 8 | 2 1 6 | 4 7 3
7 6 4 | 8 3 5 | 2 9 1
- = - - = - - = - 23    0:01:19.822566
```

Add custom creation

Let's prevent the creation of invalid gene sequences in the first place. Maybe that will resolve the stall.

```
def test(self):
...
    def fnCreate():
        return random.sample(geneset * 9, 81)
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)
```

With that change we can simplify the implementation of `mutate` to just perform swaps.

```
def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

Then update the test harness to match the parameter change.

```
def test(self):
    ...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)
```

Run 4

```
5 8 1 | 9 2 3 | 6 4 7
4 3 7 | 1 5 8 | 2 9 6
6 9 2 | 7 8 1 | 4 5 3
-----+-----+-----
2 6 3 | 4 7 5 | 1 8 9
7 5 6 | 2 9 4 | 8 3 1
9 1 4 | 8 3 6 | 5 7 2
-----+-----+-----
3 2 8 | 6 4 7 | 9 1 5
1 4 5 | 3 6 9 | 7 2 8
8 7 9 | 5 1 2 | 3 6 4
- = - - = - - = - 23    0:00:35.506982
```

This change removes the remaining inefficiency from the process and makes it so that we achieve a 23 fitness value 10 times out of 10. But, it still stalls. To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Use custom creation

We can reduce the amount of work the engine has to do if we start with the right number of each digit. We'll do that with a permutation of 9 copies of the digits.

```
import random
...
def test(self):
...
    def fnCreate():
        return random.sample(geneset * 9, 81)

    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           geneset, fnDisplay,
                           custom_create=fnCreate)
```

Run 2

This change improves the range of the fitness value to between 16 and 19 (over 10 runs) with one outlier at 23. But, it still stalls every time.

```
...
2 5 9 | 3 1 6 | 8 4 7
6 8 4 | 9 2 7 | 5 1 3
3 1 7 | 2 5 4 | 6 8 9
----- + ----- + -----
5 9 6 | 1 3 2 | 7 4 8
8 7 6 | 2 9 3 | 1 5 4
9 3 8 | 7 6 1 | 4 2 5
----- + ----- + -----
4 2 5 | 6 7 8 | 9 3 1
7 6 1 | 8 4 5 | 3 9 2
1 4 7 | 3 8 9 | 2 5 6
- = - - = - - = - 19    0:00:09.601567
```

What should we do now?

- [Add simulated annealing](#) to fix the local maximum.
- [Add custom mutation](#) to swap two genes.
- [Restart from the beginning](#).

(Click one of the options above)

Add simulated annealing

We seem to be hitting a local maximum. Let's see if using simulated annealing can break us out of the stall.

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           geneset, fnDisplay,
                           custom_create=fnCreate, maxAge=5000)
```

Run 3

This improves the range of fitness values to between 19 and 23 over 10 runs, but only if we use a high maximum age. The higher the maximum age the better the fitness results, up to a point. That is a good indication that we are encountering local maximums and that they are relatively complex to resolve.

```
9 8 2 | 1 4 5 | 6 3 7
7 1 3 | 5 8 6 | 4 2 9
4 5 6 | 8 2 3 | 9 7 1
----- + ----- + -----
1 2 4 | 6 5 4 | 3 5 8
8 3 9 | 4 7 2 | 5 1 6
6 9 7 | 5 3 1 | 8 4 2
----- + ----- + -----
3 4 8 | 2 6 7 | 1 9 5
2 6 5 | 3 1 9 | 7 8 4
5 7 1 | 4 9 8 | 2 6 3
- = - - = - - = - 21    0:00:21.398411
```

Add custom mutation

Both custom creation and simulated annealing improved the fitness results. Since our custom creation function ensures we start with the right number of each digit, we can replace the built in mutation function with one that just swaps two genes.

```
def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

```

def test(self):
...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)

    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)

```

Run 4

```

6 3 1 | 5 7 2 | 4 8 9
5 8 7 | 4 6 9 | 3 2 1
4 9 2 | 7 5 3 | 8 1 6
----- + ----- + -----
8 7 5 | 9 1 6 | 2 3 4
1 6 3 | 2 8 4 | 9 5 7
2 4 9 | 1 3 8 | 7 6 5
----- + ----- + -----
9 1 8 | 6 2 7 | 5 4 3
7 2 6 | 3 4 5 | 1 9 8
3 5 4 | 8 9 1 | 6 7 2
- = - - = - - = - 23    0:00:39.616364

```

This change removes the remaining inefficiency from the process and makes it so that we achieve a 23 fitness value 10 times out of 10. But, it still stalls. To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Add custom mutation

The custom creation function we added ensures that we start with the right number of each digit. This means we can replace the built in mutation function with one that just swaps two genes.

```
def mutate(genes, geneIndexes):
    indexA, indexB = random.sample(geneIndexes, 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

```
def test(self):
    ...
    geneIndexes = [i for i in range(0, 81)]

    def fnMutate(genes):
        mutate(genes, geneIndexes)

    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate)
```

Run 3

```
1 3 7 | 8 6 9 | 2 5 4
2 8 5 | 7 1 6 | 4 9 3
9 6 4 | 2 5 1 | 7 3 8
----- + ----- + -----
4 7 3 | 9 2 8 | 5 1 6
6 2 8 | 4 3 5 | 9 7 1
3 9 1 | 5 7 4 | 6 8 2
----- + ----- + -----
5 4 2 | 1 8 7 | 3 6 9
7 1 6 | 3 9 2 | 8 4 5
8 5 9 | 6 4 3 | 1 2 7
- - - - - = - - - - - = - 23    0:00:13.460378
```

This change enables the engine to achieve a fitness value of 22 or 23 every time, and to do so quickly most of the time. But, it still stalls.

Add simulated annealing

Let's see if using simulated annealing can break us out of what appears to be a local maximum.

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=5000)
```

Run 4

This change makes it so that we achieve a 23 fitness value every time, but it still stalls. That is a good indication that we **are** encountering local maximums and that they are relatively complex to resolve.

```
6 3 8 | 1 9 4 | 7 2 5
7 4 2 | 8 3 5 | 1 6 9
9 1 5 | 7 4 2 | 8 3 6
----- + ----- + -----
3 2 4 | 9 6 8 | 5 1 7
8 7 1 | 2 5 6 | 9 4 3
5 9 6 | 3 1 7 | 2 8 4
----- + ----- + -----
4 5 7 | 6 2 1 | 3 9 8
2 8 9 | 4 7 3 | 6 5 1
1 6 3 | 5 8 9 | 4 7 2
- = - - = - - = - 23    0:00:09.504245
```

To improve further we need to think about fitness another way.

- [Use a more granular fitness.](#)
- [Restart from the beginning.](#)

Use a more granular fitness

The root problem we're having is that we're trying to construct the whole puzzle at once. This causes us to end up in a situation where we cannot make a change without reducing the fitness because the valid rows, columns and sections are interlocked with invalid ones. The way to fix this is to build the puzzle in an organized manner row-by-row top-to-bottom, left-to-right. That will allow us to swap all we want with digits that have a higher index in order to resolve conflicts. Once the current digit is consistent with all related digits that have a lower index, we'll advance to the next digit. However, now we need to know the point at which it becomes inconsistent. To determine that we're going to borrow the **Rule** idea from the Graph Coloring project

```
class Rule:
    Index = None
    OtherIndex = None

    def __init__(self, it, other):
        if it > other:
            it, other = other, it
        self.Index = it
        self.OtherIndex = other

    def __eq__(self, other):
        return self.Index == other.Index and \
            self.OtherIndex == other.OtherIndex

    def __hash__(self):
        return self.Index * 100 + self.OtherIndex
```

and create back-reference rules that only enforce row, column and section uniqueness against genes with lower indexes.

Start with a nested pair of loops where the outer loop goes through all but the last index; this will be the back-referenced index in the rule.

```
def build_validation_rules():
    rules = []
    for index in range(80):
        itsRow = index_row(index)
        itsColumn = index_column(index)
        itsSection = row_column_section(itsRow, itsColumn)
```

The inner loop starts at the lower index plus one and iterates over the rest of the gene indexes. We then determine if the two indexes are in the same row, column and/or section, and if so, we produce a **Rule** linking them.

```
for index2 in range(index + 1, 81):
    otherRow = index_row(index2)
    otherColumn = index_column(index2)
    otherSection = row_column_section(otherRow, otherColumn)
    if itsRow == otherRow or \
        itsColumn == otherColumn or \
        itsSection == otherSection:
        rules.append(Rule(index, index2))
```

Last, we sort the rules by the upper index then the lower index so we can quickly find all the rules that affect all indexes up to the current one and do so in a consistent order.

```
rules.sort(key=lambda x: x.OtherIndex * 100 + x.Index)
return rules
```

We use the following helper functions to determine the row, column, and section from an index.

```
def index_row(index):
    return int(index / 9)

def index_column(index):
    return int(index % 9)

def row_column_section(row, column):
    return int(row / 3) * 3 + int(column / 3)

def index_section(index):
    return row_column_section(index_row(index), index_column(index))

def section_start(index):
    return int((index_row(index) % 9) / 3) * 27 + int(index_column(index) / 3) * 3
```

Update the test

Next, in the test harness we need to get the validation rules and pass them to the fitness and mutation functions.

```
def test(self):
    ...
    validationRules = build_validation_rules()

    def fnGetFitness(genes):
        return get_fitness(genes, validationRules)

    def fnMutate(genes):
        mutate(genes, validationRules)
```

Now we're in a position to use the rules in the fitness function. We'll change the fitness value to be the row and column of the index of the first square that has a digit conflict. That will make it easy for us to find the problem square in the display. It also triples the granularity of the fitness value.

Fitness

```
def get_fitness(genes, validationRules):
    try:
        firstFailingRule = next(rule for rule in validationRules
                                if genes[rule.Index] == genes[rule.OtherIndex])
    except StopIteration:
        fitness = 100
    else:
        fitness = (1 + index_row(firstFailingRule.OtherIndex)) * 10 \
            + (1 + index_column(firstFailingRule.OtherIndex))
    return fitness
```

That change means the highest broken fitness value will be 99. So, we'll make the optimal value 100.

```
def test(self):
    ...
    optimalValue = 100
```

Mutate

Finally, in `mutate` we have to process the rules again to figure out which, if any, of the indexes has a digit conflict and then swap the gene at that index with one at any higher index.

```
def mutate(genes, validationRules):
    selectedRule = next(rule for rule in validationRules
                        if genes[rule.Index] == genes[rule.OtherIndex])
    if selectedRule is None:
        return
    indexA = selectedRule.OtherIndex
    indexB = random.randrange(1 + indexA, len(genes))
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

Run 5

Now when we run the test it gets stuck because all the possible values it could put at that index have already been used in the associated row, column or section.

```
7 1 9 | 4 2 8 | 6 3 5
4 5 3 | 7 1 9 | 8 2 8
4 2 1 | 7 9 3 | 1 6 4
----- + ----- + -----
8 4 5 | 9 4 5 | 1 8 1
6 7 3 | 9 7 8 | 4 5 4
6 1 5 | 2 7 9 | 5 2 9
----- + ----- + -----
3 6 8 | 5 9 1 | 6 2 5
7 2 8 | 3 3 1 | 6 9 6
2 8 3 | 7 6 7 | 3 4 2
- = - - = - - = - 29    0:00:00.009169
```

The fitness value tells us the problem is at row 2 column 9. We can see that an 8 is conflicting with another 8 in the same column. The apparent fix would be to replace it with a 1, 4, 6, or 7 but 1, 4 and 7 already appear in its row and 6 has already been used in the same section. There's nothing the `mutate` function can swap into that position to resolve the problem, so as a result it stays stuck.

Enable row swaps

The solution is to enable swapping with any digit on the same row as well:

```
def mutate(genes, validationRules):
    ...
    row = index_row(selectedRule.OtherIndex)
    start = row * 9
    indexA = selectedRule.OtherIndex
    indexB = random.randrange(start, len(genes))
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
```

Run 6

Now when we run the test, 9 times out of 10 it can produce a valid Sudoku board in under a second. When it can't it looks like this:

sample output

```
9 7 1 | 4 6 2 | 8 3 5
6 2 8 | 3 5 7 | 1 4 9
3 5 4 | 9 8 1 | 6 2 7
----- + ----- + -----
7 8 5 | 6 1 3 | 2 9 4
1 4 2 | 8 9 5 | 7 6 3
3 1 1 | 4 3 9 | 2 9 1
----- + ----- + -----
2 4 6 | 5 8 6 | 5 9 8
9 8 5 | 6 4 7 | 8 2 4
7 3 3 | 7 2 7 | 5 1 6
- = - - = - - = - 61    0:00:00.043248
```

Now it only gets stuck on the last row of a section. When that happens swapping with another gene on the same row can't fix it because the problem is all 9 potential digits have already been used somewhere in its row, column or section.

Final revision

The final fix is to add a small chance of shuffling the contents of all genes, starting with the first gene in the current gene's section, until all the rules up to and including those of the current gene pass.

```
def mutate(genes, validationRules):
    ...
    return

    if index_row(selectedRule.OtherIndex) % 3 == 2 \
        and random.randint(0, 10) == 0:
        sectionStart = section_start(selectedRule.Index)
        current = selectedRule.OtherIndex
        while selectedRule.OtherIndex == current:
            shuffle_in_place(genes, sectionStart, 80)
            selectedRule = next(rule for rule in validationRules
                               if genes[rule.Index] == genes[rule.OtherIndex])

        return
    row = index_row(selectedRule.OtherIndex)
    ...
```

With `shuffle_in_place` defined as:

```
def shuffle_in_place(genes, first, last):
    while first < last:
        index = random.randint(first, last)
        genes[first], genes[index] = genes[index], genes[first]
        first += 1
```

We can speed it up further if we reduce the maximum age for simulated annealing.

```
def test(self):
    ...
    best = genetic.get_best(fnGetFitness, None, optimalValue,
                           None, fnDisplay, fnMutate, fnCreate,
                           maxAge=50)
```

Run 7

These changes resolve the final problem without a significant performance penalty. Outstanding!

11.7. Benchmarks

Here's the benchmark function

```
def test_benchmark(self):  
    genetic.Benchmark.run(lambda: self.test())
```

and results.

Benchmark

average (seconds)	standard deviation
0.19	0.11

11.8. Summary

In this chapter we learned that we must be tenacious and willing to question everything about our genetic algorithm implementation in order to work around a sticking point. We also learned that approaching the problem in a controlled fashion instead of having partial solutions spread all over can result in substantial performance improvements.

11.9. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch11>

Chapter 12. Traveling Salesman Problem

Our next project involves finding an optimal or near-optimal route to visit a set of locations. This is generically known as the Traveling Salesman Problem. Route optimality can be affected by costs, priorities, etc. Variations include unequal costs to travel a route in different directions, like fuel use going up or down a hill, one-way streets, paying toll fees at certain times of the day, obstacles along the way like rivers to cross or street layouts to navigate, how much time a particular stop might require, etc.

12.1. Test Data

We'll start off with a data set that we can easily verify by sight - one of the solutions to the 8 Queens Problem (0,0 is in the bottom left corner, points are counter-clockwise starting at A):

```
. . . . A . . .  
. . B . . . .  
C . . . . .  
. . . . . H .  
. D . . . . .  
. . . . . G  
. . . . F . .  
. . . E . . .  
4,7 2,6 0,5 1,3 3,0 5,1 7,2 6,4
```

12.2. Test and genes

As usual we'll start by defining the gene set, and in this case, a lookup table where we can get the coordinates for a given location.

tspTests.py

```
class TravelingSalesmanTests(unittest.TestCase):  
    def test_8_queens(self):  
        idToLocationLookup = {  
            'A': [4, 7],  
            'B': [2, 6],  
            'C': [0, 5],  
            'D': [1, 3],  
            'E': [3, 0],  
            'F': [5, 1],  
            'G': [7, 2],  
            'H': [6, 4]  
        }  
        optimalSequence = ['A', 'B', 'C', 'D', 'E', 'F', 'G', 'H']  
        self.solve(idToLocationLookup, optimalSequence)
```

```
import unittest
import datetime
import genetic
```

```
def solve(self, idToLocationLookup, optimalSequence):
    geneset = [i for i in idToLocationLookup.keys()]
```

12.3. Calculating distance

To determine fitness we need to be able to calculate the distance between two locations. We are going to use Euclidian (straight line) distance.

```
import math
...
def get_distance(locationA, locationB):
    sideA = locationA[0] - locationB[0]
    sideB = locationA[1] - locationB[1]
    sideC = math.sqrt(sideA * sideA + sideB * sideB)
    return sideC
```

12.4. Fitness

Since we want to optimize for the shortest distance we'll use a custom `Fitness` object.

```
class Fitness:
    TotalDistance = None

    def __init__(self, totalDistance):
        self.TotalDistance = totalDistance

    def __gt__(self, other):
        return self.TotalDistance < other.TotalDistance

    def __str__(self):
        return "{0:0.2f}".format(self.TotalDistance)
```

To determine the route length we're going to sum the distances for points in the route, including returning to the starting location.

```
def get_fitness(genes, idToLocationLookup):
    fitness = get_distance(idToLocationLookup[genes[0]],
                           idToLocationLookup[genes[-1]])

    for i in range(len(genes) - 1):
        start = idToLocationLookup[genes[i]]
        end = idToLocationLookup[genes[i + 1]]
        fitness += get_distance(start, end)

    return Fitness(round(fitness, 2))
```

12.5. Display

For `display` we'll simply output the route and total distance traveled.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        ' '.join(map(str, candidate.Genes)),
        candidate.Fitness,
        str(timeDiff)))
```

It should look like this:

```
A E F G H B C D 27.72    0:00:00.001002
```

12.6. Mutate

Since we don't want to visit any location more than once, but we do want to make sure we visit all locations, we'll borrow custom mutate

```
import random
...
def mutate(genes, fnGetFitness):
    count = random.randint(2, len(genes))
    initialFitness = fnGetFitness(genes)
    while count > 0:
        count -= 1
        indexA, indexB = random.sample(range(len(genes)), 2)
        genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
        fitness = fnGetFitness(genes)
        if fitness > initialFitness:
            return
```

and custom create from the Magic Square project.

```
def solve(self, idToLocationLookup, optimalSequence):
    ...
    def fnCreate():
        return random.sample(geneset, len(geneset))
```

12.7. Test

The rest of the test harness should be familiar by now:

```
def solve(self, idToLocationLookup, optimalSequence):
    geneset = [i for i in idToLocationLookup.keys()]

    def fnCreate():
        return random.sample(geneset, len(geneset))

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes, idToLocationLookup)

    def fnMutate(genes):
        mutate(genes, fnGetFitness)

    optimalFitness = fnGetFitness(optimalSequence)
    startTime = datetime.datetime.now()
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate)
    self.assertTrue(not optimalFitness > best.Fitness)
```

12.8. Run

When we run this we expect the optimal solution to be some rotation of the alphabetical sequence ABCDEFGH because that's the shortest route around those 8 points. And, that is exactly what we get.

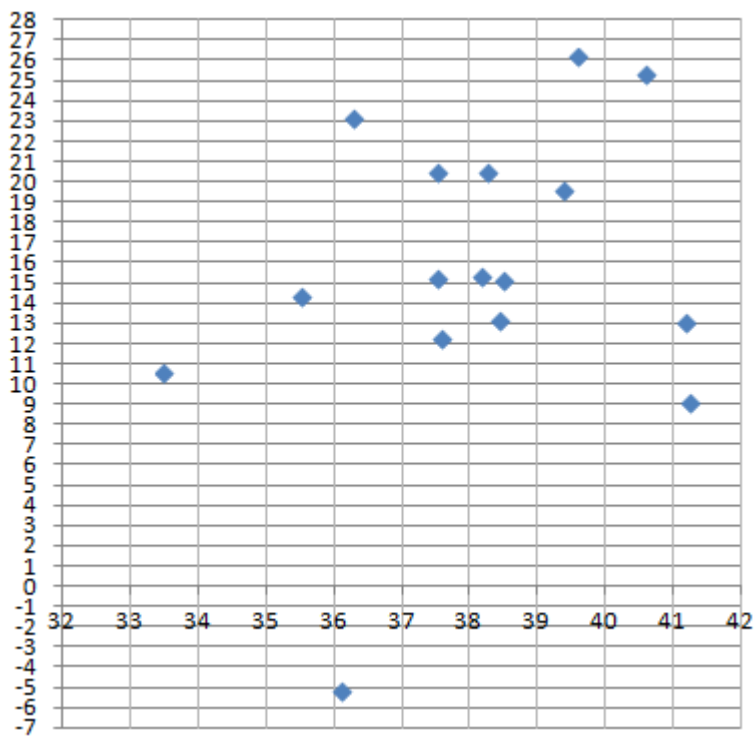
sample output

```
B D C G A H F E 33.93    0:00:00
H G C D A B F E 32.39    0:00:00
F B A D C H G E 30.33    0:00:00
E B A D C H G F 28.35    0:00:00.000501
E C D A B H G F 26.48    0:00:00.000501
E C D B A H G F 23.78    0:00:00.001002
F D C B A H G E 23.73    0:00:00.002013
E D C B A H G F 20.63    0:00:00.004010
```

12.9. A larger problem

There are many commonly used data sets for this type of problem, and many more are available from a web site called TSPLIB (<http://comopt.ifl.uni-heidelberg.de/software/TSPLIB95/>). We're going to try a small variant named [ulysses16](#). The locations in this dataset are supposedly those visited by Homer's Ulysses. The data set has 16 numbered locations that are specified by floating point x and y coordinates, and symmetric route weights.

Here's how the points appear when plotted in Microsoft Excel.



The data file is easy for parse.

```
def load_data(localFileName):
    """ expects:
        HEADER section before DATA section, all lines start in column 0
        DATA section element all have space in column 0
            <space>1 23.45 67.89
        last line of file is: " EOF"
    """
    with open(localFileName, mode='r') as infile:
        content = infile.read().splitlines()
        idToLocationLookup = {}
        for row in content:
            if row[0] != ' ': # HEADERS
                continue
            if row == " EOF":
                break

            id, x, y = row.split(' ')[1:4]
            idToLocationLookup[int(id)] = [float(x), float(y)]
        return idToLocationLookup
```

We call `load_data` from the new test:

```
def test_ulysses16(self):
    idToLocationLookup = load_data("ulysses16.tsp")
    optimalSequence = [14, 13, 12, 16, 1, 3, 2, 4, 8, 15, 5, 11, 9, 10, 7, 6]
    self.solve(idToLocationLookup, optimalSequence)
```

The published optimal path for this problem treats the points as locations on Earth, which complicates the calculations with latitude, longitude, radius of the earth, etc. We're going to keep it simple and use the Euclidian distance function we already have.

The engine cannot solve this problem without simulated annealing, so we'll use that too.

```
def solve(self, idToLocationLookup, optimalSequence):
    ...
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate, maxAge=500)
```

12.10. Run

sample output

```
...
3 2 4 8 1 10 9 11 5 15 6 7 12 13 14 16 74.75 0:00:00.147422
3 2 4 8 1 10 9 11 5 15 6 7 14 13 12 16 74.73 0:00:00.158453
13 14 7 6 15 5 11 9 10 3 2 4 8 1 16 12 74.61 0:00:00.502374
4 8 1 13 14 15 5 11 9 10 7 6 12 16 3 2 74.28 0:00:01.036758
4 8 1 13 14 15 5 11 9 10 6 7 12 16 3 2 74.18 0:00:01.076865
7 10 9 11 5 15 1 8 4 2 3 16 12 13 14 6 74.00 0:00:04.306453
12 16 1 3 2 4 8 15 5 11 9 10 7 6 14 13 73.99 0:00:45.714639
```

It finds the optimal solution every time but overcoming the local minimum at 74.00 takes the majority of the time. In benchmarks it takes 8.69 seconds +/- 8.02 seconds, which is much too slow for us to want to use in benchmarks.

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test_ulysses16())
```

We're going to have to add something new.

12.11. Introducing crossover

When you read TSP research papers online the conversation invariably focuses on a genetic algorithm strategy that we haven't discussed yet - crossover. Crossover uses features from two different parents to create the new child. The idea is that the genes of each parent have part of the solution but they both may be stuck at a local minimum or maximum. So, if random parts of both are combined, it might produce a better solution to the problem.

The reason conversation focuses on crossover is because of the problem constraints and the many ways people have tried to work around them. For example, you can't just copy random genes because you might introduce duplication, which results in a worse fitness and a wasted attempt. The solution to the TSP is a cycle through all points, but the parent's genes for producing that route could start anywhere in the cycle and travel either direction along it. This means even if you use a clever method to copy genes to the new child so that you don't introduce duplication, you may be replacing clockwise-cycling genes with counter-clockwise cycling ones, which may or may not be an improvement. You may also, depending on how you select the genes that get moved, be breaking up a run that is already optimal. Ultimately, the point is that you will have to engineer crossover at least as much if not more than you do mutation.

12.11.1. Support a pool of parents

First things first. A prerequisite for crossover is two parents. Right now we only have one parent, so we're going to add the ability to set the number of parents.



The community refers to this as a **pool** of parents.

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
            display, custom_mutate=None, custom_create=None,
            maxAge=None, poolSize=1):
    ...
```

Then pass the pool size through to `_get_improvement`.

```
for improvement in _get_improvement(fnMutate, fnGenerateParent, maxAge, poolSize):
```

In `_get_improvement` we'll initialize `parents` with the best parent.

```
def _get_improvement(new_child, generate_parent, maxAge, poolSize):
    bestParent = generate_parent()
    yield bestParent
    parents = [bestParent]
    historicalFitnesses = [bestParent.Fitness]
```

Then populate the `parents` array by generating new random parents. If one that has a better fitness than the best parent is found, we replace the best parent and update the list of historical best fitnesses.

```
for _ in range(poolSize - 1):
    parent = generate_parent()
    if parent.Fitness > bestParent.Fitness:
        yield parent
        bestParent = parent
        historicalFitnesses.append(parent.Fitness)
    parents.append(parent)
```

Next, since we have an array of parents, each time through the loop we'll select a different one to be the current parent.

```
lastParentIndex = poolSize - 1
pindex = 1
while True:
    pindex = pindex - 1 if pindex > 0 else lastParentIndex
    parent = parents[pindex]
```

The strategy we're using to curate the pool is to loop through all parents in the pool, continuously replacing parents with better children. This has the advantage of maintaining genetic diversity in the pool while keeping pool maintenance simple.

If we were to replace the parent every time then it would behave more like continuous simulated annealing. If you did the experiments when we introduced that feature, then you know that such a system would have terrible performance on pure combinatorial problems.

A much more common implementation is to have a pool of parents and a pool of children. Ways of selecting children to keep include:

- keep all children
- keep only children better than their respective parent
- keep only children better than the best child
- keep only children better than the median best child

Once the child pool is full, again there are many options for repopulating the parent pool. They include:

- The child pool replaces the parent pool as is.
- The best (user specified) X percent of the children replace the worst X percent of the parents. This is called elitism. The problem then becomes how to choose the right percentage.
- The best `sqrt(pool size)` children are combined with the best `sqrt(pool size)` parents through full crossover, every parent with every child, to produce a new parent pool.
- Pick a random number of children, then take the one with the best fitness from that group. Repeat until the pool is filled. This is known as tournament selection.
- When the fitness value is numeric, children can be given a chance of being selected to become a parent based on the ratio of their fitness to the sum of all child fitnesses. This is known as roulette wheel selection.

The remaining changes to `_get_improvement` involve using `parents[pindex]` instead of `parent` when the parent is being replaced.

```
...
    if random.random() < exp(-proportionSimilar):
        parents[pindex] = child      ①
        continue
    parents[pindex] = bestParent    ②
    parent.Age = 0
    continue
...
```

```
...
    if not child.Fitness > parent.Fitness:
        # same fitness
        child.Age = parent.Age + 1
        parents[pindex] = child      ③
        continue
    parents[pindex] = child          ④
    parent.Age = 0
...
```

Adding only the pool to this solution, however, actually hurts performance because every item in the pool is touched before we get back around again to the one that was just improved.

tspTests.py

```
def solve(self, idToLocationLookup, optimalSequence):
...
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate, maxAge=500,
                           poolSize=25)
```

updated benchmark

average (seconds)	standard deviation
9.45	10.83

12.11.2. Support crossover

Since the crossover is closely linked to the genotype and project, we will not add a default implementation to the `genetic` module. It will instead be available as an optional parameter only.

We'll start with adding the optional parameter to `get_best`.

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
            display, custom_mutate=None, custom_create=None,
            maxAge=None, poolSize=1, crossover=None):
    ...
```

We don't want to use crossover *instead of* mutation but rather as a supplement. In fact, we're going to make it adaptive, so that we'll prefer whichever of the two strategies is producing the most improvements. To do that we need to track which strategy was used to create that parent. That implies a list of strategies

```
from enum import Enum
...
class Strategies(Enum):
    Create = 0,
    Mutate = 1,
    Crossover = 2
```

and a `Strategy` attribute on `Chromosome`

```
class Chromosome:
    Genes = None
    Fitness = None
    Age = 0
    Strategy = None

    def __init__(self, genes, fitness, strategy):
        self.Genes = genes
        self.Fitness = fitness
        self.Strategy = strategy
```

Plus, that new constructor parameter must be supplied wherever we create a `Chromosome`

```
def _generate_parent(length, geneSet, get_fitness):
    ...
    return Chromosome(genes, fitness, Strategies.Create)
```

```
def _mutate(parent, geneSet, get_fitness):
    ...
    return Chromosome(childGenes, fitness, Strategies.Mutate)
```

```
def _mutate_custom(parent, custom_mutate, get_fitness):
    ...
    return Chromosome(childGenes, fitness, Strategies.Mutate)
```

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
             display, custom_mutate=None, custom_create=None,
             maxAge=None, poolSize=1, crossover=None):
    ...
    else:
        def fnGenerateParent():
            genes = custom_create()
            return Chromosome(genes, get_fitness(genes), Strategies.Create)
```

Next, in `get_best` we can keep a list of strategies that were used to create parents.

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
             display, custom_mutate=None, custom_create=None,
             maxAge=None, poolSize=1, crossover=None):
    ...
    usedStrategies = [strategyLookup[Strategies.Mutate]]
    if crossover is not None:
        usedStrategies.append(strategyLookup[Strategies.Crossover])
    ...
```

and update it whenever `_get_improvement` sends it a new improvement.

```
...
    display(improvement)
    f = strategyLookup[improvement.Strategy]
    usedStrategies.append(f)
    if not optimalFitness > improvement.Fitness:
        return improvement
    ...
```

Now we need to create new children by selecting a random strategy from `usedStrategies` instead of always using `fnMutate`

```
...
strategyLookup = {
    Strategies.Create: lambda p, i, o: fnGenerateParent(),
    Strategies.Mutate: lambda p, i, o: fnMutate(p),
    Strategies.Crossover: lambda p, i, o: _crossover(p.Genes, i, o, get_fitness,
crossover, fnMutate, fnGenerateParent)
}

usedStrategies = [strategyLookup[Strategies.Mutate]]
if crossover is not None:
    usedStrategies.append(strategyLookup[Strategies.Crossover])

    def fnNewChild(parent, index, parents):
        return random.choice(usedStrategies)(parent, index, parents)
else:
    def fnNewChild(parent, index, parents):
        return fnMutate(parent)

    for improvement in _get_improvement(fnNewChild, fnGenerateParent, maxAge, poolSize):
...

```

This requires a small change in `_get_improvement`

```
def _get_improvement(new_child, generate_parent, maxAge, poolSize):
...
    child = new_child(parent, pindex, parents)
...

```



Many genetic engines allow you to specify crossover and mutation rates. However, this puts the burden on you to experiment with the percentages to try to find values that work well for your problem. The system implemented here is to give strategies that are successful a higher chance of being used again. Since the strategy used to produce a particular **Chromosome** is included with the object you can monitor the strategies that are used. That can help you check whether your **crossover** algorithm is stronger or weaker than your **mutate** algorithm, as we'll see later.

Finally, we need to add `_crossover`

```
def _crossover(parentGenes, index, parents, get_fitness, crossover, mutate,
generate_parent):
    donorIndex = random.randrange(0, len(parents))
    if donorIndex == index:
        donorIndex = (donorIndex + 1) % len(parents)
    childGenes = crossover(parentGenes, parents[donorIndex].Genes)
    if childGenes is None:
        # parent and donor are indistinguishable
        parents[donorIndex] = generate_parent()
    return mutate(parents[index])
fitness = get_fitness(childGenes)
return Chromosome(childGenes, fitness, Strategies.Crossover)
```

That completes the infrastructural support for crossover and adaptive use of strategies in `genetic`.

12.11.3. Use `crossover`

Back in our test file we need to change `solve` to pass a crossover function to `get_best` and to request a larger pool of parents.

tspTests.py

```
def solve(self, idToLocationLookup, optimalSequence):
...
    def fnCrossover(parent, donor):
        return crossover(parent, donor, fnGetFitness)
...
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate, maxAge=500,
                           poolSize=25, crossover=fnCrossover)
```

Then we need to build the crossover function. It will start by constructing a lookup table of all the 2-point pairs in the donor parent's genes.

```
def crossover(parentGenes, donorGenes, fnGetFitness):
    pairs = {Pair(donorGenes[0], donorGenes[-1]): 0}

    for i in range(len(donorGenes) - 1):
        pairs[Pair(donorGenes[i], donorGenes[i + 1])] = 0
    ...
```

The **Pair** class is just a renamed copy of the **Rule** class from the Graph Coloring project. Pair orders the two genes so that gene pairs can be compared regardless of cycle direction.

```
class Pair:
    Node = None
    Adjacent = None

    def __init__(self, node, adjacent):
        if node < adjacent:
            node, adjacent = adjacent, node
        self.Node = node
        self.Adjacent = adjacent

    def __eq__(self, other):
        return self.Node == other.Node and \
            self.Adjacent == other.Adjacent

    def __hash__(self):
        return hash(self.Node) * 397 ^ hash(self.Adjacent)
```

The next thing **crossover** does is make sure the first and last genes in **parentGenes** are not adjacent in **donorGenes**. If they are, then we search for a pair of adjacent points from **parentGenes** that are not adjacent in **donorGenes**. If we find one then we shift that discontinuity to the beginning of the array so we know no runs wrap around the end of the array.

example

```
donorGenes: ['E', 'A', 'C', 'G', 'B', 'D', 'H', 'F']
parentGenes: ['G', 'C', 'D', 'F', 'E', 'H', 'A', 'B']

pairs contains:
EA, CA, GC, GB, DB, HD, HF, FE
```

The pair **GB** exists in both gene sequences. So does GC. But DC does not, so a copy of the parent genes will be shifted to the left to become

```
tempGenes: ['D', 'F', 'E', 'H', 'A', 'B', 'G', 'C']
```


Otherwise, `parentGenes` and `donorGenes` are identical so we simply return `None` and let `genetic` handle it. Here's the implementation:

```
...
tempGenes = parentGenes[:]
if Pair(parentGenes[0], parentGenes[-1]) in pairs:
    # find a discontinuity
    found = False
    for i in range(len(parentGenes) - 1):
        if Pair(parentGenes[i], parentGenes[i + 1]) in pairs:
            continue
        tempGenes = parentGenes[i + 1:] + parentGenes[:i + 1]
        found = True
        break
    if not found:
        return None
...
```

Next we're going to collect all the runs from `parentGenes` that are also in `donorGenes`. The lookup table helps us to find them regardless of the direction the parent's genes are cycling.

```
...
runs = [[tempGenes[0]]]
for i in range(len(tempGenes) - 1):
    if Pair(tempGenes[i], tempGenes[i + 1]) in pairs:
        runs[-1].append(tempGenes[i + 1])
        continue
    runs.append([tempGenes[i + 1]])
...
```

example continued

```
common runs:
['D'],
['F', 'E'],
['H'],
['A'],
['B', 'G', 'C']
```

Now we try to find a reordering of the runs that has a better fitness than the current parent. We'll do this by swapping any pair of runs and checking the fitness, with a chance of reversing the order.

```
...
    initialFitness = fnGetFitness(parentGenes)
    count = random.randint(2, 20)
    runIndexes = range(len(runs))
    while count > 0:
        count -= 1
        for i in runIndexes:
            if len(runs[i]) == 1:
                continue
            if random.randint(0, len(runs)) == 0:
                runs[i] = [n for n in reversed(runs[i])]
...

```

If fitness is better than that of the parent, then we return the new genetic sequence. Otherwise we repeat until we find an improvement or we complete the maximum number of attempts, at which point we give up and return what we have.

```
from itertools import chain
...
    indexA, indexB = random.sample(runIndexes, 2)
    runs[indexA], runs[indexB] = runs[indexB], runs[indexA]
    childGenes = list(chain.from_iterable(runs))
    if fnGetFitness(childGenes) > initialFitness:
        return childGenes
    return childGenes

```

One other change we can make is to output the strategy that was used to produce each improvement in `display`.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}\t{3}".format(
        ' '.join(map(str, candidate.Genes)),
        candidate.Fitness,
        candidate.Strategy.name,
        str(timeDiff)))

```

12.12. Run

Now when we run `test_ulysses16` crossover tends to be the most commonly used strategy.

```
13 12 16 8 4 2 3 1 15 5 11 9 10 7 6 14 74.86 Crossover 0:00:01.437824
13 12 1 8 4 2 3 16 15 5 11 9 10 7 6 14 74.60 Mutate 0:00:01.468941
3 2 4 8 1 15 5 11 9 10 6 7 14 13 12 16 74.23 Crossover 0:00:02.032406
1 14 15 5 11 9 10 6 7 12 13 16 3 2 4 8 74.00 Crossover 0:00:02.170804
13 12 16 1 3 2 4 8 15 5 11 9 10 7 6 14 73.99 Crossover 0:00:02.576854
```

The algorithm also finds the optimal solution much faster on average.

12.13. Retrospective

We added two new tools, the parent pool size and crossover, that you may be able to use to improve the performance of previous projects.

For example, crossover might be used to detect the correct characters in the Password project so you can concentrate on non-matching indexes.

You might also try crossing over runs in the Sorted Numbers project.

In Graph Coloring, Linear Equations and Knights, see how adding more parents affects the performance.

To use crossover effectively in the Knapsack project you might need to switch to a different genotype.

12.14. Updated benchmarks

Since we changed the code paths in `genetic` we'll update all the benchmarks.

project	average (seconds)	standard deviation
Guess Password	1.26	0.29
One Max	1.22	0.16
Sorted Numbers	1.18	0.63
Queens	1.56	1.31
Graph Coloring	0.73	0.25
Cards	0.01	0.01
Knights	0.70	0.72
Magic Square	0.20	0.33
Knapsack	0.66	0.45
Linear Equations	1.23	0.88
Sudoku	0.17	0.11
Traveling Salesman	1.04	0.80

12.15. Summary

In this chapter we added an optional pool of parents to the `genetic` module. We also added support for using crossover with adaptive strategy selection. These two powerful capabilities make our genetic engine essentially feature complete. From this point on we'll be concentrating on broadening the complexity and diversity of problems we know how to handle.

12.16. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch12>

Chapter 13. Approximating Pi

In this chapter we're going to use a genetic algorithm to find an approximation for Pi, a deceptively simple project. Our phenotype will be 2 integers in the range 1 to 1024 that we'll divide in order to produce a decimal value. Our genotype will be binary code. That means the gene set will only be 0 and 1.

13.1. Test and genes

approximatePiTests.py

```
import unittest
import datetime
import genetic

class ApproximatePiTests(unittest.TestCase):
    def test(self):
        geneset = [i for i in range(2)]
```

It also means that the *position* of the gene changes its *value* in the phenotype. For example:

phenotype value	genotype values			
	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0

13.2. Convert bits to an int

It takes 10 binary bits to produce values up to 1023. That means we need 20 bits to store the numerator and denominator that we'll be dividing to approximate Pi.

```
best = genetic.get_best(fnGetFitness, 20, optimalFitness,
```

We also need a utility function to convert an array of 10 bits to an integer.

```
def bits_to_int(bits):
    result = 0
    for bit in bits:
        result = (result << 1) | bit
    return result
```

To prevent division by zero, and increase the value range to 1024, we'll add 1 to the decoded numerator

```
def get_numerator(genes):
    return 1 + bits_to_int(genes[:10])
```

and denominator.

```
def get_denominator(genes):
    return 1 + bits_to_int(genes[10:])
```

13.3. Fitness

We want the ratio that is closest to Pi so we'll subtract the difference between the calculated value and Pi from Pi to get the fitness. This way we don't need a `Fitness` class.

```
import math
...
def get_fitness(genes):
    ratio = get_numerator(genes) / get_denominator(genes)
    return math.pi - abs(math.pi - ratio)
```

13.4. Display

The `display` function will show the two parts of the fraction and their calculated ratio.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    numerator = get_numerator(candidate.Genes)
    denominator = get_denominator(candidate.Genes)
    print("{0}/{1}\t{2}\t{3}".format(
        numerator,
        denominator,
        candidate.Fitness, timeDiff))
```

sample output

```
240/129 1.8604651162790697 0:00:00.001000
```

13.5. Best approximations for Pi

Next, it would be useful to know what the best approximations for Pi are in the range we're checking:

```
def test_find_top_10_approximations(self):
    best = {}
    for numerator in range(1, 1024):
        for denominator in range(1, 1024):
            ratio = numerator / denominator
            piDist = math.pi - abs(math.pi - ratio)
            if piDist not in best \
                or best[piDist][0] > numerator:
                best[piDist] = [numerator, denominator]

    bestApproximations = list(reversed(sorted(best.keys())))
    for i in range(10):
        ratio = bestApproximations[i]
        nd = best[ratio]
        print("%i / %i\t%f" % (nd[0], nd[1], ratio))
```

Top ten approximations

```
355 / 113 3.141592
732 / 233 3.141554
688 / 219 3.141553
1021 / 325 3.141538
377 / 120 3.141519
333 / 106 3.141509
776 / 247 3.141485
977 / 311 3.141479
644 / 205 3.141463
399 / 127 3.141453
```

This means we expect the engine to find 355/113 or some multiple thereof.

13.6. Optimal value

Because floating point values aren't exact in Python we'll use a value between 355/133 and 732/233 as the optimal value. Here's the full test harness:

```
def test(self):
    geneset = [i for i in range(2)]
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes)

    optimalFitness = 3.14159
    best = genetic.get_best(fnGetFitness, 20, optimalFitness,
                           geneset, fnDisplay)
    self.assertTrue(not optimalFitness > best.Fitness)
```

13.7. Run

When we run it we get output like the following:

sample result

```
679/362 1.8756906077348066 0:00:00
679/361 1.8808864265927978 0:00:00
935/361 2.590027700831025 0:00:00
935/329 2.8419452887537995 0:00:00
935/321 2.912772585669782 0:00:00
943/321 2.9376947040498442 0:00:00
944/321 2.940809968847352 0:00:00
1008/321 3.1401869158878504 0:00:00
```

We can run it many times but it will rarely find the optimal solution. The problem is we can only change 1 bit at a time in either the numerator or denominator and this causes it to get stuck at a local maximum where it has individually optimized those two values as much as it can.

13.8. Modify both parts

However, we can add a custom mutation function that modifies both the numerator and denominator each time it is called:

```
import random
...
def mutate(genes):
    numeratorIndex, denominatorIndex = random.randrange(0, 10), random.randrange(10,
len(genes))
    genes[numeratorIndex] = 1 - genes[numeratorIndex]
    genes[denominatorIndex] = 1 - genes[denominatorIndex]
```

And then use it in the test.

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, 20, optimalFitness,
                           geneset, fnDisplay, mutate)
```

13.9. Run 2

Run the test again and it still cannot find the optimal solution.

```
924/389 2.3753213367609254 0:00:00
956/390 2.4512820512820515 0:00:00.001002
940/262 2.6953990476376015 0:00:00.001002
932/294 3.113117279968702 0:00:00.001002
931/296 3.137915036909316 0:00:00.001002
```

Since it appears that we're still hitting a local maximum, let's see if simulated annealing can fix it.

13.10. Use simulated annealing

```
def test(self):
...
    best = genetic.get_best(fnGetFitness, 20, optimalFitness,
                           geneset, fnDisplay, mutate, maxAge=250)
```

13.11. Run 3

Now when we run the test it can find the optimal solution every time but it can take a long time.

```
...
1016/324    3.1358024691358026  0:00:00.002006
1024/326    3.1411042944785277  0:00:00.006020
1021/325    3.1415384615384614  0:00:00.034090
732/233 3.1415544058920326  0:00:08.040390
355/113 3.1415923868256037  0:00:34.368389
```

We can measure how slow it is with a benchmark test:

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test())
```

sample results

average (seconds)	standard deviation
28.23	30.62

The issue we're having is that successive improvements don't build on the previous. Consider the following corner case:

```
127    [0, 1, 1, 1, 1, 1, 1, 1]
128    [1, 0, 0, 0, 0, 0, 0, 0]
```

If we needed 128 in the numerator but had 127, the genetic algorithm would have to change 8 bits. It would be very difficult for it to make that leap. Additionally, there is no slack in our genotype. There is exactly one pattern that will produce the value 355/113 for example. This is something to be aware of when choosing a genotype for a genetic algorithm.

So how can we add some slack?

13.12. Expanding the genotype

The bits in our current genotype have the following values by position:

```
[512, 256, 128, 64, 32, 16, 8, 4, 2, 1]
```

What if we simply double the number of bits we use per phenotype and then duplicate each value. Effectively:

```
[512, 512, 256, 256, 128, 128, 64, 64, 32, 32, 16, 16, 8, 8, 4, 4, 2, 2, 1, 1]
```

Let's try it.

13.13. Pass the bit values

We'll start by moving the bit values to an array,

```
def test(self):
    ...
    bitValues = [512, 256, 128, 64, 32, 16, 8, 4, 2, 1]
```

then use the bit value array to decode the genes as follows:

```
def bits_to_int(bits, bitValues):
    result = 0
    for i, bit in enumerate(bits):
        if bit == 0:
            continue
        result += bitValues[i]
    return result
```

The final step is to pass the array of bit values through to the various functions that need it:

```
def test(self):
    ...
    def fnDisplay(candidate):
        display(candidate, startTime, bitValues) ①

    def fnGetFitness(genes):
        return get_fitness(genes, bitValues) ②
```

```
def get_fitness(genes, bitValues):
    denominator = get_denominator(genes, bitValues) ③
    if denominator == 0:
        return 0

    ratio = get_numerator(genes, bitValues) / denominator ④
    return math.pi - math.fabs(math.pi - ratio)
```

```
def display(candidate, startTime, bitValues):
    timeDiff = datetime.datetime.now() - startTime
    numerator = get_numerator(candidate.Genes, bitValues) ⑤
    denominator = get_denominator(candidate.Genes, bitValues) ⑥
    ...
```

```
def get_numerator(genes, bitValues):
    return 1 + bits_to_int(genes[:10], bitValues) ⑦

def get_denominator(genes, bitValues):
    return bits_to_int(genes[10:], bitValues) ⑧
```

Now change the values of the bits.

13.14. Change the bit values

```
def test(self):
    ...
    bitValues = [512, 512, 256, 256, 128, 128, 64, 64, 32, 32, 16, 16, 8, 8, 4, 4, 2,
2, 1, 1]
```

Since the new gene sequence length depends on the length of the bit value array, we need to update places where we reference specific indexes.

```
def test(self):
    ...
    def fnMutate(genes): ①
        mutate(genes, len(bitValues))

    length = 2 * len(bitValues)
    best = genetic.get_best(fnGetFitness, length, optimalFitness,
                           geneset, fnDisplay, fnMutate, maxAge=250) ②
```

```
def mutate(genes, numBits):
    numeratorIndex, denominatorIndex = random.randrange(0, numBits),
random.randrange(numBits, len(genes))
    ...
```

```
def get_numerator(genes, bitValues):
    return 1 + bits_to_int(genes[:len(bitValues)], bitValues)

def get_denominator(genes, bitValues):
    return bits_to_int(genes[len(bitValues):], bitValues)
```

This change substantially improves the benchmark result for this project.

sample benchmark

average (seconds)	standard deviation
0.91	0.89

This simple change resulted in a 30x speed improvement.

13.15. Exercise

Could we get more speed if none of the values were duplicates? What if we doubled the size of the array again? Do the bit values need to be ordered? What is the highest bit value we need? Go experiment then come back.

13.16. Optimizing the bit array

What do you think it would take to find the best bit value array for this problem? It sounds exactly like something we would solve with a genetic algorithm doesn't it? Let's do it.

Since we know a poor set of values for the bit array can take a long time to run, we need to put a limit on the run time. So, let's say the goal is to find the optimal solution to the Pi project as many times as possible in 2 seconds. That means we need a way to break out of the engine after 2 seconds.

13.16.1. Support time constraints in genetic

We'll start by adding an optional parameter to `get_best` for the maximum number of seconds to run.

genetic.py

```
def get_best(get_fitness, targetLen, optimalFitness, geneSet,
             display, custom_mutate=None, custom_create=None,
             maxAge=None, poolSize=1, crossover=None,
             maxSeconds=None):
    ...
```

Then pass `maxSeconds` to `_get_improvement` and also handle the new additional return value that indicates that a timeout occurred.

```
for timedOut, improvement in _get_improvement(fnNewChild, fnGenerateParent, maxAge,
poolSize, maxSeconds):
    if timedOut:
        return improvement
...
```

Next, at the beginning of `_get_improvement` we start a timer and wherever we yield a result we need to include whether a timeout occurred.

```
def _get_improvement(new_child, generate_parent, maxAge, poolSize, maxSeconds):
    startTime = time.time()
    bestParent = generate_parent()
    yield maxSeconds is not None and time.time() - startTime > maxSeconds, bestParent
...
```

We check it after each call to `generate_parent` when populating the parent pool.

```
for _ in range(poolSize - 1):
    parent = generate_parent()
    if maxSeconds is not None and time.time() - startTime > maxSeconds:
        yield True, parent
    if parent.Fitness > bestParent.Fitness:
        yield False, parent
```

We also check at the start of the `while` loop.

```
while True:
    if maxSeconds is not None and time.time() - startTime > maxSeconds:
        yield True, bestParent
```

We've already checked it when we find a new best parent, so we just send `False`.

```
if child.Fitness > bestParent.Fitness:
    yield False, child
    bestParent = child
```

13.16.2. Optimizer

Now in the optimizer we'll see if we can make the 10-bit version run faster. We start with a gene set containing all the numbers in the range 1 to 512. And we'll give each round 2 seconds to run.

approximatePiTests.py

```
import time
...
def test_optimize(self):
    geneset = [i for i in range(1, 512 + 1)]
    length = 10
    maxSeconds = 2
```

Next we need to write the fitness function. It will start a timer and keep a count of the number of successes.

```
def fnGetFitness(genes):
    startTime = time.time()
    count = 0
```

It will then run the Pi approximation test and count how many times it can find the optimal result. We'll also suppress its display output.

```
import sys
...
    stdout = sys.stdout
    sys.stdout = None
    while time.time() - startTime < maxSeconds:
        if self.test(genes, maxSeconds):
            count += 1
    sys.stdout = stdout
```

The fitness will be a combination of the number of successes and a fraction. The fraction indicates how far the sum of the bit values is from 1023. This gives the engine a very granular fitness to use to find improvements.

```
distance = abs(sum(genes) - 1023)
fraction = 1 / distance if distance > 0 else distance
count += round(fraction, 4)
return count
```

Next we want a way to monitor what is happening, so we write a display function.

```
def fnDisplay(chromosome):
    print("{0}\t{1}".format(chromosome.Genes, chromosome.Fitness))
```

For comparison we'll output the fitness of the initial sequence.

```
initial = [512, 256, 128, 64, 32, 16, 8, 4, 2, 1]
print("initial:", initial, fnGetFitness(initial))
```

Finally, we'll start the run and give it 10 minutes to try to find an improvement, with the goal being a set of bit values that can be used to find Pi 20 times in 2 seconds.

```
optimalFitness = 10 * maxSeconds
genetic.get_best(fnGetFitness, length, optimalFitness, geneset,
                 fnDisplay, maxSeconds=600)
```

Now we need to make related changes to the test function so we can pass an array of bit values and maximum seconds as parameters. Remember to remove the existing `bitValues` array definition from the function.

```
def test(self, bitValues=[512, 256, 128, 64, 32, 16, 8, 4, 2, 1], maxSeconds=None):
```

Then pass `maxSeconds` through to the engine so it knows when to stop. At the end we need to return a value indicating whether or not it found a Pi approximation at least as good as the optimal value.

```
best = genetic.get_best(fnGetFitness, length, optimalFitness,
                       geneset, fnDisplay, fnMutate, maxAge=250,
                       maxSeconds=maxSeconds)
return best.Fitness >= optimalFitness
```

13.16.3. Optimization Run

Now run it and it does find a much better bit value sequence.

sample result

```
initial: [512, 256, 128, 64, 32, 16, 8, 4, 2, 1] 0
[45, 39, 289, 407, 23, 224, 280, 240, 412, 260] 0.0008
[45, 39, 289, 407, 71, 224, 280, 240, 412, 260] 3.0008
[45, 39, 289, 407, 71, 224, 45, 240, 412, 260] 5.001
[45, 26, 289, 407, 71, 224, 45, 240, 412, 260] 6.001
[45, 26, 289, 407, 71, 82, 45, 240, 412, 260] 8.0012
[45, 26, 289, 407, 70, 82, 45, 240, 412, 260] 14.0012
```


13.16.4. Verify the result

Now we can use the bit value sequence it found above to verify that we get a performance improvement.

```
def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test([45, 26, 289, 407, 70, 82, 45, 240, 412,
260]))
```

sample output

```
100 0.29 0.39
```

That's about 3 times better performance than we achieved by doubling the bit values sequence. Nice!

If we allow the optimizer to run each round longer, say 4 seconds instead of 2, and run to completion it can find an even better result.

sample result

```
initial: [512, 256, 128, 64, 32, 16, 8, 4, 2, 1] 0
[37, 438, 95, 355, 430, 311, 309, 225, 264, 454] 0.0005
[37, 438, 95, 355, 430, 311, 196, 225, 264, 454] 0.0006
[37, 438, 95, 355, 56, 311, 196, 225, 280, 454] 0.0007
[37, 261, 95, 355, 56, 311, 196, 225, 280, 454] 0.0008
[37, 261, 95, 355, 56, 311, 196, 225, 280, 55] 0.0012
[37, 261, 95, 355, 56, 252, 196, 225, 280, 55] 0.0013
[37, 261, 95, 292, 56, 252, 190, 225, 280, 55] 0.0014
[37, 261, 95, 292, 56, 39, 190, 225, 280, 55] 0.002
[37, 261, 95, 292, 56, 39, 50, 225, 280, 55] 0.0027
[37, 261, 38, 292, 56, 39, 50, 225, 280, 55] 0.0032
[37, 261, 38, 292, 56, 39, 50, 225, 46, 55] 0.0132
[37, 261, 38, 292, 117, 39, 50, 225, 46, 55] 2.0073
[37, 334, 38, 292, 117, 39, 50, 225, 46, 55] 3.0048
[37, 334, 38, 292, 117, 39, 50, 225, 46, 124] 5.0036
[37, 334, 38, 292, 117, 39, 50, 225, 46, 129] 8.0035
[37, 334, 38, 133, 117, 39, 50, 225, 46, 129] 11.008
[37, 334, 38, 133, 117, 39, 50, 225, 262, 129] 14.0029
[37, 334, 38, 133, 117, 39, 87, 225, 262, 129] 15.0026
[37, 334, 38, 96, 117, 39, 87, 225, 262, 129] 17.0029
[37, 334, 38, 96, 117, 39, 145, 225, 262, 129] 26.0025
[37, 334, 38, 339, 117, 39, 145, 225, 262, 129] 30.0016
[98, 334, 38, 339, 117, 39, 145, 225, 262, 129] 32.0014
[98, 334, 38, 339, 117, 39, 145, 123, 262, 129] 33.0017
[98, 334, 38, 339, 117, 39, 145, 123, 40, 129] 40.0026
```

sample benchmark

```
100 0.09 0.10
```

Wow!

13.17. Summary

In this chapter we learned one way to use one genetic algorithm to optimize another, that the options we have for translating values in the genotype to phenotype values can significantly impact the performance of the genetic algorithm, and that it is important to give the algorithm multiple ways to use the genes to achieve the goal.

13.18. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch13>

Chapter 14. Equation Generation

So far we've only used genes as data elements that are applied to an external problem. Our next project, equation generation, will introduce us to a new way of using genes called genetic programming. The essence of genetic programming is to overlay some kind of grammar on the genes, which may or may not include embedded data. These operation-genes can then be evaluated to produce a result.

14.1. Example

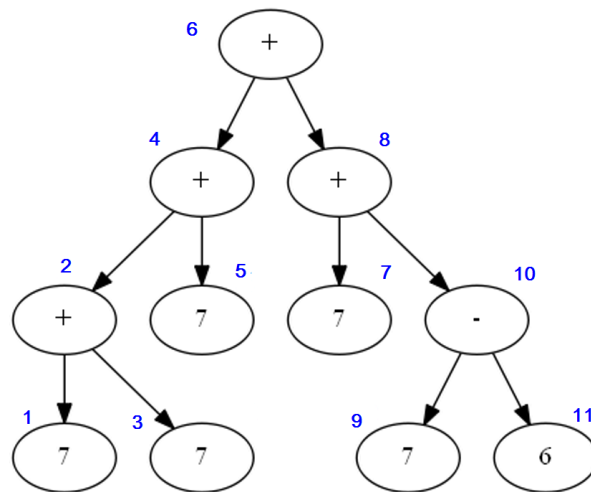
When using genetic programming it is important to understand the characteristics of both the embedded data, if any, and how we expect the operations to interact with each other and/or the environment. For example, let's say we are asked to find some combination of the numbers 1 to 7 and addition and subtraction operations that, when evaluated, produces the numeric value 29. There are many ways to solve this problem. For example: $7+7+7+7+7-6$

We could simply use positive and negative integers between -7 and 7 because they essentially have the addition and subtraction operations built in, but we need to learn how to evaluate operations as independent genes, so we'll make + and - separate tokens. That means the equation has 11 tokens or genes:

7	+	7	+	7	+	7	+	7	-	6
1	2	3	4	5	6	7	8	9	10	11

When viewed this way we can easily see the pattern, or grammar, of alternating operations and numbers. This should be easy to enforce using an odd/even check when creating and mutating chromosomes.

Next, if we visualize the equation from the point of view of the operations:



It is easy to see that each operation has 2 parameters because each acts upon 2 other genes which can be either a number or another operation.

14.2. Evaluate

The heart of a genetic programming algorithm is the `evaluate` function. It knows how to treat the problem-specific gene sequence like a program. The `evaluate` function for this project will apply the operation genes to the neighboring numbers, rolling up to get the final result.

```
def evaluate(genes):
    result = genes[0]
    for i in range(1, len(genes), 2):
        operation = genes[i]
        nextValue = genes[i + 1]
        if operation == '+':
            result += nextValue
        elif operation == '-':
            result -= nextValue
    return result
```

The function starts by initializing `result` with the value of the first numeric gene. It then applies the operations, left to right, to the current `result` and the value-gene that follows the operation. The output of each operation is stored back into `result`.

14.3. Test and genes

We're ready to write the rest of the genetic algorithm according to the usual pattern, starting with the gene set.

equationGenerationTests.py

```
import unittest
import datetime
import genetic
import random

class EquationGenerationTests(unittest.TestCase):
    def test(self):
        numbers = [1, 2, 3, 4, 5, 6, 7]
        operations = ['+', '-']
```

14.4. Create

Now we can immediately see that we'll need a custom `create` function, for two reasons. First, we don't know how many symbols we'll need to produce a particular result, and second, we need to alternate numbers and operations.

```
def create(numbers, operations, minNumbers, maxNumbers):
    genes = [random.choice(numbers)]
    count = random.randint(minNumbers, 1 + maxNumbers)
    while count > 1:
        count -= 1
        genes.append(random.choice(operations))
        genes.append(random.choice(numbers))
    return genes
```

This implementation prevents the creation of gene sequences that have multiple numbers or operations in a row, so we don't have to detect that situation in the fitness function.

14.5. Mutate

We also need a custom `mutate` function that knows how to append an operation-number pair to the gene sequence,

```
def mutate(genes, numbers, operations, minNumbers, maxNumbers):
    numberCount = (1 + len(genes)) / 2
    appending = numberCount < maxNumbers and \
        random.randint(0, 100) == 0
    if appending:
        genes.append(random.choice(operations))
        genes.append(random.choice(numbers))
    return
```

remove an operation-number pair,

```
removing = numberCount > minNumbers and \
    random.randint(0, 20) == 0
if removing:
    index = random.randrange(0, len(genes) - 1)
    del genes[index]
    del genes[index]
    return
```

and mutate an operation or number.

```
index = random.randrange(0, len(genes))
genes[index] = random.choice(operations) \
    if (index & 1) == 1 else random.choice(numbers)
```

Here we use the parity of the index to determine whether we're replacing a number or an operation.

14.6. Display

In `display` we can simply write the genes separated by a space to see the equation.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        ' '.join(map(str, [i for i in candidate.Genes])),
        candidate.Fitness,
        timeDiff))
```

sample output

```
6 + 3 + 3 - 4 + 6 - 3 - 6 - 6 - 3 - 1 -5 0:00:00.001003
```

14.7. Fitness

That just leaves the fitness function where we'll use the `evaluate` function to get the result of the equation, then compare it to the expected result. This time, instead of using a `Fitness` class we'll use a 2-stage fitness function. The first stage tracks how far the result of evaluating the genes is from the expected total. If the result is correct then the second fitness range is used. This allows us to prefer shorter gene sequences, meaning equations with fewer operations.

```
def get_fitness(genes, expectedTotal):
    result = evaluate(genes)

    if result != expectedTotal:
        fitness = expectedTotal - abs(result - expectedTotal)
    else:
        fitness = 1000 - len(genes)

    return fitness
```

14.8. Test

The full test harness below should hold no surprises.

```
def test(self):
    numbers = [1, 2, 3, 4, 5, 6, 7]
    operations = ['+', '-']
    expectedTotal = 29
    optimalLengthSolution = [7, '+', 7, '+', 7, '+', 7, '+', 7, '-', 6]
    minNumbers = (1 + len(optimalLengthSolution)) / 2
    maxNumbers = 6 * minNumbers
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    def fnGetFitness(genes):
        return get_fitness(genes, expectedTotal)

    def fnCreate():
        return create(numbers, operations, minNumbers, maxNumbers)

    def fnMutate(child):
        mutate(child, numbers, operations, minNumbers, maxNumbers)

    optimalFitness = fnGetFitness(optimalLengthSolution)
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate, maxAge=50)
    self.assertTrue(not optimalFitness > best.Fitness)
```

14.9. Run

When we run the test it finds an equivalent solution.

sample output

```
5 + 3 + 2 + 4 + 6 + 2 + 1    23  0:00:00
5 + 3 + 2 + 4 + 6 + 6 + 1    27  0:00:00
5 + 3 + 3 + 4 + 6 + 6 + 1    28  0:00:00
6 + 3 + 3 + 4 + 6 + 6 + 1   987  0:00:00
6 + 5 + 3 + 3 + 6 + 6   989 0:00:00.000998
```

Nice. But having to add 6 three times and 3 two times is boring. If we introduce a multiplication operation it might find $7 * 3 + 4 * 2$, which saves an operation, or even $7 * 4 + 1$, which saves two. Let's try it.

14.10. Support multiplication

First we have to add the multiplication token (*) to the list of operation genes.

```
def test(self):
...
    operations = ['+', '-', '*']
```

Next we have to implement multiplication in `evaluate`. The problem is that for the math to be correct we have to perform all multiplication operations before we perform any addition or subtraction. No problem, we can just group the operations by priority.

```
def test(self):
...
    operations = ['+', '-', '*']
    prioritizedOperations = [['*'], ['+', '-']]
...
    def fnEvaluate(genes):
        return evaluate(genes, prioritizedOperations)
```

Then we'll evaluate all operations in priority order rather than simultaneously, which means we'll iterate over the array once for each priority group.

```
def evaluate(genes, prioritizedOperations):
    equation = genes[:]
    for operationSet in prioritizedOperations:
        iOffset = 0
        for i in range(1, len(equation), 2):
            i += iOffset
            opToken = equation[i]
            if opToken in operationSet:
                leftOperand = equation[i - 1]
                rightOperand = equation[i + 1]

                if opToken == '+':
                    leftOperand += rightOperand
                elif opToken == '-':
                    leftOperand -= rightOperand
                elif opToken == '*':
                    leftOperand *= rightOperand
                equation[i - 1] = leftOperand
                del equation[i + 1]
                del equation[i]
                iOffset -= 2
    return equation[0]
```


In the implementation above we make a copy of the genes so we can modify it each time through the loop. Each operation stores its output in the array location to its left then deletes itself and its second operand from the array. After all operations have been evaluated the result is in array index zero.

Note, since we changed the function to pass in the list of operations, we also need to change what we pass to `get_fitness`

```
def get_fitness(genes, expectedTotal, fnEvaluate):
    result = fnEvaluate(genes)
    ...
```

and update the function call in the test harness.

```
def test(self):
    ...
    def fnGetFitness(genes):
        return get_fitness(genes, expectedTotal, fnEvaluate)
```

14.10.1. Run

This code finds a solution, but not a minimal length solution, because the defined `optimalLengthSolution` in the test is based on addition and subtraction so the engine stops as soon as it beats that solution.

```
...
4 - 3 * 3 - 7 * 2 + 7 + 6 + 5 * 7    983 0:00:00.103305
5 + 2 - 7 * 2 + 6 + 5 + 5 * 5    985 0:00:00.115337
5 - 4 - 3 + 6 + 5 + 4 * 5    987 0:00:00.124361
5 - 4 - 3 + 6 + 5 * 5    989 0:00:00.128372
```

14.10.2. Extract `solve`

To fix that we need to extract a `solve` function so we can pass in the list of operations with the optimal solution for that set of operations.

```
def test_addition(self):
    operations = ['+', '-']
    prioritizedOperations = ['+', '-']
    optimalLengthSolution = [7, '+', 7, '+', 7, '+', 7, '+', 7, '-', 6]
    self.solve(operations, prioritizedOperations, optimalLengthSolution)

def solve(self, operations, prioritizedOperations, optimalLengthSolution):
    numbers = [1, 2, 3, 4, 5, 6, 7]
    expectedTotal = evaluate(optimalLengthSolution, prioritizedOperations)
```

14.10.3. Test multiplication

Now we can add a test for multiplication, and give it a more difficult equation to generate.

```
def test_multiplication(self):
    operations = ['+', '-', '*']
    prioritizedOperations = [['*'], ['+', '-']]
    optimalLengthSolution = [6, '*', 3, '*', 3, '*', 6, '-', 7]
    self.solve(operations, prioritizedOperations, optimalLengthSolution)
```

14.10.4. Run 2

When we run that test it too can find an optimal length solution:

sample output

```
....
7 * 5 * 6 - 3 + 7 * 2 * 7 + 4 * 3    983 0:00:00.109255
7 * 7 * 7 - 4 * 4 - 6 - 5 + 1    985 0:00:00.141350
7 * 7 * 7 - 7 * 2 - 3 * 4    987 0:00:00.150363
5 * 7 * 7 + 6 * 2 * 6    989 0:00:00.158386
5 * 4 * 4 * 4 - 3    991 0:00:02.141662
```

Cool. When we review that solution, however, it seems repetitive. It would be much nicer if we could reduce the equation to $5 * 4^3 - 3$. Let's do that.

14.11. Refactor

But first, have you noticed that each time we add a new operation we have to change `evaluate`? That's an indication of a poor design that violates [the Open-Closed Principal](#). We can fix that by extracting the operation implementations to separate functions:

```
def add(a, b):
    return a + b

def subtract(a, b):
    return a - b

def multiply(a, b):
    return a * b
```

Then make the prioritized operation list contain dictionaries where each dictionary key is the token for an operation and the value is a function that implements the operation.

```
def test_addition(self):
    operations = ['+', '-']
    prioritizedOperations = [{'+': add,
                             '-': subtract}]
    ...
```

```
def test_multiplication(self):
    operations = ['+', '-', '*']
    prioritizedOperations = [{'*': multiply},
                             {'+': add,
                              '-': subtract}]
    ...
```

Finally, use the operation function in `evaluate`.

```
def evaluate(genes, prioritizedOperations):
    equation = genes[:]
    for operationSet in prioritizedOperations:
        iOffset = 0
        for i in range(1, len(equation), 2):
            i += iOffset
            opToken = equation[i]
            if opToken in operationSet:
                leftOperand = equation[i - 1]
                rightOperand = equation[i + 1]
                equation[i - 1] = operationSet[opToken](leftOperand, rightOperand)
                del equation[i + 1]
                del equation[i]
                iOffset += -2
    return equation[0]
```

14.12. Supporting Exponents

Now we can easily add support for a new operation, like exponentiation, without changing any other code:

```
def test_exponent(self):
    operations = ['^', '+', '-', '*']
    prioritizedOperations = [{'^': lambda a, b: a ** b},
                             {'*': multiply},
                             {'+': add},
                             {'-': subtract}]
    optimalLengthSolution = [6, '^', 3, '*', 2, '-', 5]
    self.solve(operations, prioritizedOperations, optimalLengthSolution)
```

sample output

```
...
4 * 3 ^ 4 - 3 - 1 - 7 + 4 * 2 * 7 * 2 + 2    979 0:00:00.030079
4 * 3 ^ 4 - 2 - 7 + 4 * 2 * 7 * 2    983 0:00:00.035093
6 * 7 * 5 * 2 + 5 + 3 + 3 - 4    985 0:00:00.070215
6 * 6 * 6 * 2 + 1 - 6    989 0:00:00.083221
7 * 5 * 6 * 2 + 7    991 0:00:00.108286
6 ^ 3 * 2 - 5    993 0:00:13.129462
```

14.13. Improve performance

Since it is starting to take tens of seconds to find the solution, let's add a loop to `mutate` so that it can make a random number of changes instead of just one.

```
def mutate(genes, numbers, operations, minNumbers, maxNumbers, fnGetFitness):
    count = random.randint(1, 10)
    initialFitness = fnGetFitness(genes)
    while count > 0:
        count -= 1
        if fnGetFitness(genes) > initialFitness:
            return
        numberCount = (1 + len(genes)) / 2
    ...
```

①

```

...
        genes.append(random.choice(numbers))
        continue ②

    removing = numberCount > minNumbers and \
        random.randint(0, 20) == 0
...

```

```

...
        del genes[index]
        continue ③

    index = random.randrange(0, len(genes))
...

```

Now that we're checking the fitness in `mutate`, we also need to pass in `fnGetFitness`:

```

def solve(self, operations, prioritizedOperations, optimalSolution):
...
    def fnMutate(child):
        mutate(child, numbers, operations, minNumbers, maxNumbers, fnGetFitness)
...

```

14.13.1. Run

As expected, it finds the solution much faster.

sample output

```

2 + 3 ^ 4 * 5 + 3 * 6    425 0:00:00.014006
3 + 3 ^ 4 * 5 + 3 * 6    426 0:00:00.016041
4 + 3 ^ 4 * 5 + 3 * 6    989 0:00:00.024067
6 * 2 * 5 * 7 + 7       991 0:00:00.067148
2 * 6 ^ 3 - 5           993 0:00:00.567508

```

Fantastic!

14.14. Benchmarks

We'll benchmark this project with the exponent test.

```

def test_benchmark(self):
    genetic.Benchmark.run(lambda: self.test_exponent())

```

Benchmark

average (seconds)	standard deviation
0.49	0.36

14.15. Summary

This chapter was our first introduction to genetic programming and it opened up several really interesting problems for exploration. We also learned one way to work with both data and operations in a gene sequence. This gave us experience with symbols and grammar and "program" evaluation.

14.16. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch14>

Chapter 15. The Lawnmower Problem

In this chapter we'll continue our exploration of genetic programming by solving John Koza's Lawnmower Problem. It asks us to provide instructions to a lawnmower to make it cut a field of grass. To make it easier for the genetic algorithm to find a solution, the initial field is toroidal, this means it wraps in all directions. So, if the mower goes off the top it ends up at the bottom and vice versa and the same side-to-side. The problem statement is that the mower begins in the middle of an 8x8 toroidal field facing south (down) and it understands two instructions, **mow** and **turn**. What is the shortest series of instructions (program) needed to make it mow the entire field?

15.1. Part I - **mow** and **turn**

The **mow** instruction tells the mower to move forward one grid square in whatever direction it is facing then cut the grass in that location. The **turn** instruction tells the mower to turn left 90 degrees in place.

So that we can visually track the mower as it moves around the field, when we mow a field location we'll replace the unmowed symbol (#) with the number of the step that mowed it, like this:

```
M^ # # # # # # #
11 # # # # # # #
10 # # # # # # #
 9 # # # # # # #
 8 # # # # # # #
 6 # # # 1 3 4 5
# # # # # # #
# # # # # # #
```



M is the mower, **<** **>** **^** and **v** are used to indicate the direction the mower is facing: left, right, up or down, respectively. The gaps in the number series are indicative of a **turn** instruction having been executed at steps 2 and 7.

15.1.1. Virtual mower infrastructure

We'll put the code that is specific to the mower and its environment into a different file than the tests.

Let's begin with the visual elements referenced above.

lawnmower.py

```
from enum import Enum

class FieldContents(Enum):
    Grass = '#'
    Mowed = '.'
    Mower = 'M'

    def __str__(self):
        return self.value
```

We need a way to indicate the direction the mower is facing,

```
class Directions(Enum):
    North = Direction(0, 0, -1, '^')
    East = Direction(1, 1, 0, '>')
    South = Direction(2, 0, 1, 'v')
    West = Direction(3, -1, 0, '<')
```

and the `Direction` container.

```
class Direction:
    Index = None
    Xoffset = None
    Yoffset = None
    Symbol = None

    def __init__(self, index, xoffset, yoffset, symbol):
        self.Index = index
        self.Xoffset = xoffset
        self.Yoffset = yoffset
        self.Symbol = symbol
```

We'll define the `Mower` next:

```
class Mower:
    Location = None
    Direction = None

    def __init__(self, location, direction):
        self.Location = location
        self.Direction = direction
        self.StepCount = 0
    ...
```

When the `mower` is told to turn left, it needs to find out the new direction it should be facing.

```
...
    def turn_left(self):
        self.StepCount += 1
        self.Direction = Directions.get_direction_after_turn_left_90_degrees(
self.Direction)
```


The `Directions` class can determine that using the direction the `Mower` is currently facing.

```
class Directions(Enum):
    ...
    @staticmethod
    def get_direction_after_turn_left_90_degrees(direction):
        newIndex = direction.Index - 1 \
            if direction.Index > 0 \
            else len(Directions) - 1
        newDirection = next(i for i in Directions
                            if i.value.Index == newIndex)
        return newDirection.value
```

When told to mow, the mower first asks the `Direction` where it will end up if it moves forward from the current location. It then asks the field to translate that to a valid physical location. It also increments the move counter and updates the contents of the field in that location.

```
class Mower:
    ...
    def mow(self, field):
        newLocation = self.Direction.move_from(self.Location)
        self.Location = field.fix_location(newLocation)
        self.StepCount += 1
        field.set(self.Location, self.StepCount
                 if self.StepCount > 9
                 else " {0}".format(self.StepCount))
```

That implies a `move_from` function on `Direction`:

```
class Direction:
    ...
    def move_from(self, location, distance=1):
        return Location(location.X + distance * self.XOffset,
                        location.Y + distance * self.YOffset)
```

Which, in turn, implies a `Location` object:

```
class Location:
    X = None
    Y = None

    def __init__(self, x, y):
        self.X, self.Y = x, y
```

Next we need to define the field:

```
class Field:
    Field = None
    Width = None
    Height = None

    def __init__(self, width, height, initialContent):
        self.Field = [[initialContent] * width for _ in range(height)]
        self.Width = width
        self.Height = height
    ...
```

It knows how to change the contents of a particular grid location:

```
...
def set(self, location, symbol):
    self.Field[location.Y][location.X] = symbol
...
```

It also knows how to convert a virtual location to a valid physical one. The field does not, however, know what it means to move in a particular direction from a location, so it asks the `Location` for that, then adjusts it based on the expected virtual field behavior.

```
...
def fix_location(self, location):
    newLocation = Location(location.X, location.Y)
    if newLocation.X < 0:
        newLocation.X += self.Width
    elif newLocation.X >= self.Width:
        newLocation.X %= self.Width

    if newLocation.Y < 0:
        newLocation.Y += self.Height
    elif newLocation.Y >= self.Height:
        newLocation.Y %= self.Height

    return newLocation
```

This implies a **move** function on **Location**:

```
class Location:
    ...
    def move(self, xOffset, yOffset):
        return Location(self.X + xOffset,
                        self.Y + yOffset)
```

We now have the basic parts we need to support the project described: a mower starting in the middle of an 8x8 toroidal field facing south, with the ability to turn left and/or mow in a straight line.

15.1.2. Test class

Now we can start on the test infrastructure. Our genes for this project will have behaviors, so we'll use classes.

lawnmowerTests.py

```
class Mow:
    def __init__(self):
        pass

    @staticmethod
    def execute(mower, field):
        mower.mow(field)
```

```
class Turn:
    def __init__(self):
        pass

    @staticmethod
    def execute(mower, field):
        mower.turn_left()
```

Next we'll define a **Program** to drive gene-instruction execution.

```
class Program:
    Main = None

    def __init__(self, instructions):
        self.Main = instructions

    def evaluate(self, mower, field):
        for instruction in self.Main:
            instruction.execute(mower, field)
```

15.1.3. Create

Next, since we don't know how many **mow** and **turn** instructions it will take for the **mower** to mow the **field**, we need to define a custom **create** function. It will generate a random sequence of instructions with length between the min and max given:

```
import random

def create(geneSet, minGenes, maxGenes):
    numGenes = random.randint(minGenes, maxGenes)
    genes = [random.choice(geneSet) for _ in range(1, numGenes)]
    return genes
```

15.1.4. Test

Here's the start of the test harness.

```
import unittest
import datetime
import genetic
import lawnmower
...
class LawnmowerTests(unittest.TestCase):
    def test(self):
        geneSet = [Mow(), Turn()]
        width = height = 8
        mowerStartLocation = lawnmower.Location(int(width / 2), int(height / 2))
        mowerStartDirection = lawnmower.Directions.South.value

        def fnCreate():
            return create(geneSet, 1, height)
```

15.1.5. Evaluation

In order to determine how much of the field the gene-instructions cause to be mowed, `get_fitness` is going to command a virtual `mower` to execute the instructions on a virtual `field`. That implies an evaluation function that will also need to be accessible to `display`.

```
def test(self):
    ...
    def fnEvaluate(instructions):
        program = Program(instructions)
        mower = lawnmower.Mower(mowerStartLocation, mowerStartDirection)
        field = lawnmower.Field(width, height, lawnmower.FieldContents.Grass)
        program.evaluate(mower, field)
        return field, mower, program
```

15.1.6. Fitness

Now we can pass the `evaluate` function to `get_fitness`

```
def test(self):
    ...
    def fnGetFitness(genes):
        return get_fitness(genes, fnEvaluate)
```

and use it to determine how many squares get mowed.

```
def get_fitness(genes, fnEvaluate):
    field = fnEvaluate(genes)[0]
    return Fitness(field.count_mowed(), len(genes))
```

After mowing, we need to know how much of the field was mowed, so we ask the `field`:

lawnmower.py

```
class Field:
    ...
    def count_mowed(self):
        return sum(1 for row in range(self.Height)
                    for column in range(self.Width)
                    if self.Field[row][column] != FieldContents.Grass)
```

We don't just want to mow the field, we want to do so using as few instructions as possible, so we use a **Fitness** class to manage those competing priorities.

lawnmowerTests.py

```
class Fitness:
    TotalMowed = None
    TotalInstructions = None

    def __init__(self, totalMowed, totalInstructions):
        self.TotalMowed = totalMowed
        self.TotalInstructions = totalInstructions

    def __gt__(self, other):
        if self.TotalMowed != other.TotalMowed:
            return self.TotalMowed > other.TotalMowed
        return self.TotalInstructions < other.TotalInstructions
```

15.1.7. Display

Now that we can determine the fitness of a gene sequence, we'd like a way to display the state of the **field** after the gene instructions have been executed.

```
def test(self):
    ...
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime, fnEvaluate)
```

We ask the **field** to display its contents, including the location of the **mower**. Then we print the fitness, time elapsed, and program instructions.

```
def display(candidate, startTime, fnEvaluate):
    field, mower, program = fnEvaluate(candidate.Genes)
    timeDiff = datetime.datetime.now() - startTime
    field.display(mower)
    print("{0}\t{1}".format(
        candidate.Fitness,
        str(timeDiff)))
    program.print()
```

When asked to print its contents, the `field` loops through the array and writes the contents.

lawnmower.py

```
class Field:
    ...
    def display(self, mower):
        for rowIndex in range(self.Height):
            if rowIndex != mower.Location.Y:
                row = ' '.join(map(str, self.Field[rowIndex]))
            else:
                r = self.Field[rowIndex][:]
                r[mower.Location.X] = "{0}{1}".format(
                    FieldContents.Mower, mower.Direction.Symbol)
                row = ' '.join(map(str, r))
        print(row)
```

The instructions also need a `__str__` implementation so that `display` can print the gene contents.

lawnmowerTests.py

```
class Mow:
    ...
    def __str__(self):
        return "mow"
```

```
class Turn:
    ...
    def __str__(self):
        return "turn"
```

`display` asks `Fitness` to stringify itself:

```
class Fitness:
    ...
    def __str__(self):
        return "{0} mowed with {1} instructions".format(
            self.TotalMowed, self.TotalInstructions)
```

and asks the **Program** to print itself:

lawnmower.py

```
class Program:
    ...
    def print(self):
        print(' '.join(map(str, self.Main)))
```

15.1.8. Mutate

Since we don't know how many instructions will be required to mow the field, we also need a custom **mutate** function that tries to add, remove, and change instructions.

lawnmowerTests.py

```
def mutate(genes, geneSet, minGenes, maxGenes, fnGetFitness, maxRounds):
    count = random.randint(1, maxRounds)
    initialFitness = fnGetFitness(genes)
    while count > 0:
        count -= 1
        if fnGetFitness(genes) > initialFitness:
            return
        adding = len(genes) == 0 or \
            (len(genes) < maxGenes and
             random.randint(0, 5) == 0)
        if adding:
            genes.append(random.choice(geneSet))
            continue

        removing = len(genes) > minGenes and \
            random.randint(0, 50) == 0
        if removing:
            index = random.randrange(0, len(genes))
            del genes[index]
            continue

        index = random.randrange(0, len(genes))
        genes[index] = random.choice(geneSet)
```

Next we need to define a glue function in the test so we can provide the additional required parameters.

Since **mow** can only affect one field position at a time, we can set a lower bound on the number of instructions required to mow the entire field. We'll set the maximum number of instructions to 1.5 times that value.

```
def test(self):
    ...
    minGenes = width * height
    maxGenes = int(1.5 * minGenes)
    maxMutationRounds = 3

    def fnMutate(child):
        mutate(child, geneSet, minGenes, maxGenes, fnGetFitness, maxMutationRounds)
```

15.1.9. Optimal fitness

To determine the optimal fitness, we need to figure out how many **turn** instructions would be required. Since the field is toroidal, for calculation purposes we can shift the start location to the southwest corner, pointed south, without affecting the calculations.

```
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . . . .
Mv . . . . . . .
```

In an optimal solution the mower will only turn when it encounters a square that has already been mowed, so that it doesn't waste an instruction. This results in a spiral mowing pattern.

```
1 28 27 26 25 24 23 22
2 29 48 47 46 45 44 21
3 30 49 60 59 58 43 20
4 31 50 61 64 57 42 19
5 32 51 62 63 56 41 18
6 33 52 53 54 55 40 17
7 34 35 36 37 38 39 16
8* 9 10 11 12 13 14 15    * starting location
```

From this we can determine that the optimal gene sequence will have 1 **row** for each square of the field plus width + height - 2 **turn** instructions, or 78 total instructions.

```
def test(self):
    ...
    expectedNumberOfInstructions = 78
    optimalFitness = Fitness(width * height, expectedNumberOfInstructions)
```

15.1.10. Crossover

We're also going to apply crossover to this problem.

```
def crossover(parent, otherParent):
    childGenes = parent[:]
    if len(parent) <= 2 or len(otherParent) < 2:
        return childGenes
    length = random.randint(1, len(parent) - 2)
    start = random.randrange(0, len(parent) - length)
    childGenes[start:start + length] = otherParent[start:start + length]
    return childGenes
```

This implementation overwrites a random contiguous sequence of genes in the child with those from the donor parent.

15.1.11. Test

Lastly we need to call the engine from the test.

```
def test(self):
    ...
    best = genetic.get_best(fnGetFitness, None, optimalFitness, None,
                           fnDisplay, fnMutate, fnCreate,
                           poolSize=10, crossover=crossover)
    self.assertTrue(not best.Fitness > optimalFitness)
```

15.1.12. Run

With all the necessary pieces defined we're ready to run the test. As expected, the result is an inward spiral:

```
M^ 67 48 21  4 35 58 73
76 66 47 20  5 36 59 74
63 64 46 19  6 37 60 62
42 43 44 18  7 38 40 41
13 14 15 16  8 10 11 12
28 27 26 24  1 31 30 29
53 52 50 23  2 33 55 54
70 68 49 22  3 34 57 71
64 mowed with 78 instructions   0:00:09.541468
mow mow mow mow mow mow mow mow turn mow mow mow mow mow mow mow turn mow mow mow mow mow
mow mow turn mow mow mow mow mow mow turn mow mow mow mow mow mow turn mow mow mow mow
mow turn mow mow mow mow mow turn mow mow mow mow turn mow mow mow mow turn mow mow mow
turn mow mow mow turn mow mow turn mow mow turn mow turn mow
```

Unfortunately, with the current instruction set that is the only result we get.

15.2. Part II - Jump

To make the project more interesting we're going to add a new instruction. Following John Koza's problem definition again, the new instruction is **jump**. It causes the mower to jump forward and to the right a specified number of squares and cut the grass in the square where it lands. **jump** will have 2 non-negative integer parameters for how far to move forward and right, respectively.

For example, if the mower is facing south at the start location (4,4)

```
. . . . .
. . . . .
. . . . .
. . . . .
. . . . Mv . . .
. . . . .
. . . . .
. . . . .
```

and is told to jump (2,3) it will end up at (1,6) (numbered from the top left), still facing south.

```
. . . . . . . .
. . . . . . . .
. . . . . . . .
. . . . . * . . . * - previous location
. . . . . . . .
. Mv . . . . . .
. . . . . . . .
```

15.2.1. Implementation

To implement this we'll start with adding a `jump` instruction. The forward and right distances will be random values supplied to the constructor and limited by the size of the field.

```
class Jump:
    Forward = None
    Right = None

    def __init__(self, forward, right):
        self.Forward = forward
        self.Right = right
```

It will also need an `execute` function to do the work.

```
def execute(self, mower, field):
    mower.jump(field, self.Forward, self.Right)
```

and `__str__` for display:

```
def __str__(self):
    return "jump({0},{1})".format(self.Forward, self.Right)
```

15.2.2. Update the mower

The implementation of `jump` in the `mower` is interesting because `forward` and `right` result in different x and y offsets depending on the direction the `mower` is facing.

lawnmower.py

```
class Mower:
    ...
    def jump(self, field, forward, right):
        newForwardLocation = self.Direction.move_from(self.Location, forward)
        rightDirection = Directions.get_direction_after_turn_right_90_degrees(
self.Direction)
        newLocation = rightDirection.move_from(newForwardLocation, right)
        self.Location = field.fix_location(newLocation)
        self.StepCount += 1
        field.set(self.Location, self.StepCount
            if self.StepCount > 9
            else " {0}".format(self.StepCount))
```

This requires a new function on `Directions`:

```
class Directions(Enum):
    ...
    @staticmethod
    def get_direction_after_turn_right_90_degrees(direction):
        newIndex = direction.Index + 1 \
            if direction.Index < len(Directions) - 1 \
            else 0
        newDirection = next(i for i in Directions
            if i.value.Index == newIndex)
        return newDirection.value
```

15.2.3. Use lambdas to create genes

`Jump` requires parameters and the parameters should be random, so we can't simply create one and put it in our instructions list anymore. This means we must convert the list of *instruction* to a list of lambda-that-returns-an-instruction.

Before we do that, we need to extract some of the variable configuration values from `test` so that we can have one test that continues to use only `mow` and `turn` while adding a new test that uses those and adds `jump`.

lawnmowerTests.py

```
def test_mow_turn(self):
    width = height = 8
    geneSet = [lambda: Mow(),
               lambda: Turn()]
    minGenes = width * height
    maxGenes = int(1.5 * minGenes)
    maxMutationRounds = 3
    expectedNumberOfInstructions = 78
    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds)
```

```
def run_with(self, geneSet, width, height, minGenes, maxGenes,
             expectedNumberOfInstructions, maxMutationRounds):
    mowerStartLocation = lawnmower.Location(int(width / 2), int(height / 2))
    mowerStartDirection = lawnmower.Directions.South.value
    ...
```

Also remove the definitions of `minGenes`, `maxGenes`, and `expectedNumberOfInstructions` from `run_with` since they are now parameters.

Now we can add the new test function with `jump` included in the gene set.

```
def test_mow_turn_jump(self):
    width = height = 8
    geneSet = [lambda: Mow(),
               lambda: Turn(),
               lambda: Jump(random.randint(0, min(width, height)),
                             random.randint(0, min(width, height)))]
    minGenes = width * height
    maxGenes = int(1.5 * minGenes)
    maxMutationRounds = 1
    expectedNumberOfInstructions = 64
    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds)
```

Note that the expected number of instructions has been reduced to 64. This is because `jump` also causes a `mow` to occur and it has the ability to jump anywhere in the field, so it should be possible to reach to every location using only `jump` or `mow`, thus 64 instructions for an 8 by 8 field.

Also, since `geneSet` now contains lambdas, we need to execute them wherever we were simply using the value before:

```
def create(geneSet, minGenes, maxGenes):
    numGenes = random.randint(minGenes, maxGenes)
    genes = [random.choice(geneSet)() for _ in range(1, numGenes)]
    return genes
```

and in the add and change sections of `mutate`:

```
def mutate(genes, geneSet, minGenes, maxGenes, fnGetFitness, maxRounds):
    ...
    if adding:
        genes.append(random.choice(geneSet)())
        continue
    ...
    index = random.randrange(0, len(genes))
    genes[index] = random.choice(geneSet)()
```

15.2.4. Run

When we run the new test we get a result like the following:

sample output

```
51 36 45 31  4 Mv  9 19
52 37 46 32  5 58 10 20
13 38 47 33  6 59 11 21
14 39 48 34 61 60 12 22
15 40 49 35 62 27 54 23
16 41 56 63  1 28 55 24
53 42 57 29  2 25  7 17
50 43 44 30  3 26  8 18
64 mowed with 64 instructions   0:00:19.720286
mow mow mow mow mow mow jump(4,6) mow mow mow jump(1,0) mow jump(7,6) mow mow mow
jump(1,1) mow mow mow mow mow mow mow jump(1,2) mow jump(5,0) mow jump(1,2) mow mow mow
mow mow mow jump(4,2) mow mow mow mow mow mow mow mow jump(8,7) mow mow mow mow mow jump(3,2)
mow mow jump(5,8) jump(6,2) mow jump(8,4) mow jump(3,5) mow mow jump(8,1) mow jump(1,1)
jump(3,6)
```

Above, we see that the genetic algorithm has completely abandoned the `turn` instruction in favor of the more powerful `jump` instruction because this results in a shorter program. However, that's only true on a toroidal field.

15.3. Try a validating field

What if we were to use a field that did not let the mower go beyond the edge? It would have to check the legality of the move before making it. We implement that by defining a `ValidatingField` that inherits from the current `Field`.

lawnmower.py

```
class ValidatingField(Field):
    def __init__(self, width, height, initialContent):
        super().__init__(width, height, initialContent)
    ...
```

Its `fix_location` will check to see if the given location is outside the bounds of the field array. If so, it will return `False` indicating that the move is invalid, otherwise `True`, along with the location.

```
def fix_location(self, location):
    if location.X >= self.Width or \
        location.X < 0 or \
        location.Y >= self.Height or \
        location.Y < 0:
        return None, False
    return location, True
```

This requires a similar change to the `fix_location` function in the toroid field. First we need to define a `ToroidField` class to hold the `fix_location` implementation specific to that kind of field and make it inherit from `Field` too.

lawnmower.py

```
class ToroidField(Field):
    def __init__(self, width, height, initialContent):
        super().__init__(width, height, initialContent)
    ...
```

Then we'll move the original `fix_location` function from `Field` to it. That function always returns `True` because moves on a toroid field wrap around the edges of the field.

```
def fix_location(self, location):
    newLocation = Location(location.X, location.Y)
    ...
    return newLocation, True
```


Then in the mower we have to check the Boolean value that was returned before updating its location.

```
class Mower:
    ...
    def mow(self, field):
        newLocation = self.Direction.move_from(self.Location)
        newLocation, isValid = field.fix_location(newLocation)
        if isValid: ①
            self.Location = newLocation
            self.StepCount += 1
            field.set(self.Location, self.StepCount)
            if self.StepCount > 9
            else " {0}".format(self.StepCount))
```

```
    def jump(self, field, forward, right):
        newLocation = self.Direction.move_from(self.Location, forward)
        rightDirection = Directions.get_direction_after_turn_right_90_degrees(
self.Direction)
        newLocation = rightDirection.move_from(newLocation, right)
        newLocation, isValid = field.fix_location(newLocation)
        if isValid: ②
            self.Location = newLocation
            self.StepCount += 1
            field.set(self.Location, self.StepCount)
            if self.StepCount > 9
            else " {0}".format(self.StepCount))
```

Next we need to update the tests. Because each is going to use a different field type we'll define a function that creates the field, then pass it to `run_with`.

lawnmowerTests.py

```
def test_mow_turn(self):
    ...
    def fnCreateField():
        return lawnmower.ToroidField(width, height, lawnmower.FieldContents.Grass)

    self.run_with(geneSet, width, height, minGenes, maxGenes,
        expectedNumberOfInstructions, maxMutationRounds, fnCreateField)
```

```

def test_mow_turn_jump(self):
...
    def fnCreateField():
        return lawnmower.ToroidField(width, height, lawnmower.FieldContents.Grass)

    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds, fnCreateField)

```

Now add the function to the `run_with` parameters and use it.

```

def run_with(self, geneSet, width, height, minGenes, maxGenes,
             expectedNumberOfInstructions, maxMutationRounds, fnCreateField):
...
    def fnEvaluate(instructions):
        program = Program(instructions)
        mower = lawnmower.Mower(mowerStartLocation, mowerStartDirection)
        field = fnCreateField()
        program.evaluate(mower, field)
        return field, mower, program

```

Using a validating field makes the `jump` test significantly more complex because once the mower moves to the right the only way to get back to the unmowed squares on the left side is to `turn`. The best solution that includes a `jump` would be for the `mower` to jump to the southwest corner, turn, then do a spiral. That would take 1 extra move at the beginning to set up the spiral, so 79 instructions.

Make a copy of `test_mow_turn_jump` called `test_mow_turn_jump_validating` then change the expected number of instructions to 79 and the field type it uses to `ValidatingField`.

```

def test_mow_turn_jump_validating(self):
...
    expectedNumberOfInstructions = 79

    def fnCreateField():
        return lawnmower.ValidatingField(width, height,
        lawnmower.FieldContents.Grass)
...

```

15.3.1. Run

When we run the test with a validating field it always hits a local maximum.

```
# # # # # 18 M^
# # # # 16 # 17 #
# # # 14 15 # # #
# # # 13 # # # #
1 # # 12 # # # #
2 # # 11 # # # #
3 5 6 10 # # # #
# # 7 9 # # # #
17 mowed with 19 instructions 0:00:06.782601
jump(0,4) mow mow turn mow mow jump(0,1) turn jump(0,1) mow mow mow mow mow jump(0,1) mow
jump(0,2) mow jump(0,1)
```

We can work around that by increasing `maxMutationRounds`.

```
def test_mow_turn_jump_validating(self):
...
    maxMutationRounds = 3
...
```

15.3.2. Run 2

With that change it can find the optimal solution but it can take several minutes to do so. It still uses all 3 instructions but `jump` no longer dominates.

```
27 26 25 24 23 22 21 19
29 52 51 50 49 48 46 18
30 54 66 65 64 62 45 17
31 55 57 58 59 61 44 16
32 34 35 36 37 38 43 15
1 M< 78 77 76 74 42 14
2 68 70 71 72 39 40 13
3 5 6 7 8 9 10 11
64 mowed with 79 instructions 0:04:21.840003
jump(1,4) mow mow turn mow mow mow mow mow mow mow turn mow mow mow mow mow mow mow turn
mow mow mow mow mow mow mow turn mow mow mow mow mow turn mow mow mow mow mow jump(0,2) mow
turn mow mow mow mow mow turn mow mow mow mow mow turn mow mow turn mow mow mow turn
jump(0,1) mow turn mow mow mow turn jump(4,1) turn mow mow mow turn jump(1,1) turn mow
mow mow mow
```

This would run much faster if the `mutate` function was engineered to select only from `jump` or `mow` destinations that are unmowed, and otherwise `turn` and try again. But that optimization is about to become completely unnecessary because

we're now in a position to explore Koza's purpose for this project.

As interesting as the solutions have been so far, the sequence of instructions generated by the engine is completely different from the solution a human would use. Think about how you'd instruct a person to mow a field when they have never mowed. You wouldn't give them detailed instructions for every step, right? You'd break it down into a set of repeatable sequences. In a toroidal field you might simply say, mow in a straight line until you encounter an area that has already been mowed, then turn left and do it again, repeat until you've mowed the whole field.

In a non-toroid field you might say something like: start at the corner of the field and mow a strip along the edge of the field all the way to the other side.

```
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
. . . . . . . M>
```

Then turn the mower back the other direction and mow the strip right next to the one you just completed until you get back to where you started.

```
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
# # # # # # # #
M< . . . . . . .
. . . . . . .
```

Turn around again and repeat the process on the next unmowed strip until you've mowed the whole field.

See how we automatically combine squares into strips and trips across-and-back into a repeatable pattern. How do we do that with the unmanned mower?

The best result we've generated so far requires 64 **jump** and **mow** instructions, one for each grid square, to tell the mower how to cut the grass in the field. Can you imagine having to poke those in via a dip-switch interface on the mower? I'd probably make an error part way through and have to start over. It would be much nicer to enter a short sequence of instructions like those we'd give a human. To do that we have to introduce the ability to repeat a sequence of instructions.

15.4. Part III - Repeat

The next instruction we're going to add is `repeat`. Like `jump` it will have 2 non-negative integer parameters. The first indicates how many of the instructions that follow it in the gene sequence are to be repeated, and the second tells how many times to repeat them.

```
class Repeat:
    OpCount = None
    Times = None
    Ops = None

    def __init__(self, opCount, times):
        self.OpCount = opCount
        self.Times = times
        self.Ops = []
```

`Ops` will contain the instructions to be repeated. When `execute` is called we need to loop over the instructions `Times` times.

```
def execute(self, mower, field):
    for i in range(self.Times):
        for op in self.Ops:
            op.execute(mower, field)
```

Lastly, we need to provide a textual representation for `display`. If `Ops` is populated we'll combine the instructions' string representations into a string and return it, otherwise we'll use `StepCount`.

```
def __str__(self):
    return "repeat({0},{1})".format(
        ' '.join(map(str, self.Ops))
        if len(self.Ops) > 0
        else self.OpCount,
        self.Times)
```

15.4.1. Update Program

The **Program** constructor will provide the instructions the **repeat** instruction is going to repeat.

```
class Program:
    def __init__(self, instructions):
        temp = instructions[:]
        for index in reversed(range(len(temp))):
            if type(temp[index]) is Repeat:
                start = index + 1
                end = min(index + temp[index].OpCount + 1, len(temp))
                temp[index].Ops = temp[start:end]
                del temp[start:end]
        self.Main = temp
```

15.4.2. Test

Finally, we need a new test, so we'll clone **test_mow_turn_jump** into **test_mow_turn_repeat** then update the code to create a **Repeat** instruction and use different min, max, and expected number of instructions, as follows:

```
def test_mow_turn_repeat(self):
    ...
    geneSet = [lambda: Mow(),
               lambda: Turn(),
               lambda: Repeat(random.randint(0, 8),
                              random.randint(0, 8))]

    minGenes = 3
    maxGenes = 20
    maxMutationRounds = 3
    expectedNumberOfInstructions = 6
    ...
```

15.4.3. Run

When we run that test we witness the power of repetition:

```
193 167 75 76 77 189 191 192
142 166 139 27 26 50 159 143
163 164 138 46 47 48 160 162
114 113 137 110 230 229 228 226
133 134 135 109 Mv 18 19 225
197 85 84 108 217 201 200 224
196 104 105 106 218 220 221 222
195 168 56 55 79 188 172 171
64 mowed with 7 instructions    0:00:02.690764
repeat(turn repeat(mow mow turn mow,7),8)
```

Wow! We can mow the field with only 6 instructions! However, there are a couple of issues to be resolved. The **display** output is now messy because the step count has gone to 3 digits. It has never been that high before. Also, occasionally the engine builds a very deep set of repeats that causes a **RecursionError** to be thrown. When that happens we can just catch the error and move on trusting the engine to eliminate it.

```
def run_with(self, geneSet, width, height, minGenes, maxGenes,
              expectedNumberOfInstructions, maxMutationRounds, fnCreateField):
    ...
    def fnEvaluate(instructions):
        program = Program(instructions)
        mower = Mower(mowerStartLocation, mowerStartDirection)
        field = fnCreateField()
        try:
            program.evaluate(mower, field)
        except RecursionError:
            pass
        return field, mower, program
    ...
```

15.5. Optimizing for fuel efficiency

Another issue is fuel efficiency. If the **mower** used fuel for each step, the set of instructions above would cost us almost 4 times as much as the hard coded but optimal solution. To fix this we're going to change **Fitness** to prefer fewer steps to fewer instructions.

Start by adding the step count to `Fitness`.

```
class Fitness:
    TotalMowed = None
    TotalInstructions = None
    StepCount = None

    def __init__(self, totalMowed, totalInstructions, stepCount):
        self.TotalMowed = totalMowed
        self.TotalInstructions = totalInstructions
        self.StepCount = stepCount
```

Then update its comparison function

```
def __gt__(self, other):
    if self.TotalMowed != other.TotalMowed:
        return self.TotalMowed > other.TotalMowed
    if self.StepCount != other.StepCount:
        return self.StepCount < other.StepCount
    return self.TotalInstructions < other.TotalInstructions
```

and add the step count to the data that gets displayed.

```
def __str__(self):
    return "{0} mowed with {1} instructions and {2} steps".format(
        self.TotalMowed, self.TotalInstructions, self.StepCount)
```

Next we need to provide the value when we create the `Fitness` object in `get_fitness`

```
...
def get_fitness(genes, fnEvaluate):
    field, mower, _ = fnEvaluate(genes)
    return Fitness(field.count_mowed(), len(genes), mower.StepCount)
```

and when we create the optimal fitness value in `run_with`

```
def run_with(self, geneSet, width, height, minGenes, maxGenes,
             expectedNumberOfInstructions, maxMutationRounds,
             fnCreateField, expectedNumberOfSteps):
    ...
    optimalFitness = Fitness(width * height, expectedNumberOfInstructions,
                             expectedNumberOfSteps)
```


That requires compensating changes in the tests to send the new parameter:

```
def test_mow_turn(self):
...
    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds, fnCreateField,
                  expectedNumberOfInstructions)
```

```
def test_mow_turn_jump(self):
...
    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds, fnCreateField,
                  expectedNumberOfInstructions)
```

```
def test_mow_turn_jump_validating(self):
...
    expectedNumberOfInstructions = 79

    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions, maxMutationRounds, fnCreateField,
                  expectedNumberOfInstructions)
```

```
def test_mow_turn_repeat(self):
...
    expectedNumberOfInstructions = 9
    expectedNumberOfSteps = 88

...
    self.run_with(geneSet, width, height, minGenes, maxGenes,
                  expectedNumberOfInstructions,
                  maxMutationRounds, fnCreateField, expectedNumberOfSteps)
```

15.5.1. Run

Now when we run the test it takes longer but produces a more fuel efficient yet still quite short set of instructions.

```
59 58 57 55 63 62 61 60
50 51 52 53 65 47 48 49
81 80 79 77 85 84 83 82
72 73 74 75 87 69 70 71
15 14 13 11 Mv 18 17 16
 6  7  8  9 21  3  4  5
37 36 35 33 41 40 39 38
28 29 30 31 43 25 26 27
64 mowed with 9 instructions and 88 steps    0:18:21.102552
repeat(repeat(mow turn repeat(mow,7) turn,2) mow mow,4)
```

Not bad.

15.6. Part IV - Automatically defined functions

We've seen how powerful **Repeat** is but it is a special case of grouping instructions together for reuse. What if we were to let the engine build its own function instead? This is a commonly used strategy and such functions are referred to as automatically defined functions or ADFs. ADFs are like **Repeat** in that when called they execute a set of instructions randomly put there by the algorithm, but they are different in that they only execute the instructions once for each time the function is called.

15.6.1. **Func** instruction

To support ADFs we'll introduce a **Func** instruction. It is similar to **Repeat** in that it performs some number of instructions but instead of claiming a specific number of instructions that follow it, it simply marks the start of the ADF.

```
class Func:
    Ops = None

    def __init__(self):
        self.Ops = []

    def execute(self, mower, field):
        for op in self.Ops:
            op.execute(mower, field)

    def __str__(self):
        return "func: {}".format(' '.join(map(str, self.Ops))) \
            if len(self.Ops) > 0 else "call-func"
```

The **Func** instruction will take ownership of all instructions that come after it in the gene sequence. We're only going to do that for the first **Func** instruction we encounter:

```
class Program:
    Func = None
    Main = None

    def __init__(self, instructions):
        temp = instructions[:]
        func = None
        ...
        del temp[start:end]
        continue

        if type(temp[index]) is Func:
            if func is not None:
                temp[index].Ops = []
                continue
            start = index + 1
            end = len(temp)
            temp[index].Ops = [i for i in temp[start:end]
                              if type(i) is not Repeat or
                              type(i) is Repeat and len(i.Ops) > 0]
            func = temp[index]
            del temp[index:end]
        self.Main = temp
        self.Func = func
```

At the end of the constructor we store the function's instructions separate from those of the main program so we can easily access them.

Next, if we encounter a **Func** instruction in **evaluate** we call **execute** on the one we set aside.

```
def evaluate(self, mower, field):
    for i, instruction in enumerate(self.Main):
        if type(instruction) is Func:
            self.Func.execute(mower, field)
            continue

        instruction.execute(mower, field)
```

Also add the **Func** instructions to what gets displayed when the program is asked to print itself out.

```
def print(self):
    if self.Func is not None:
        print(self.Func)
    print(' '.join(map(str, self.Main)))
```

Finally, we need a test. So, we'll fork the test we added for **Repeat**, **test_mow_turn_jump_repeat**, as **test_mow_turn_jump_func**. Then replace **Repeat** in its **geneSet** with **Func**.

```
def test_mow_turn_jump_func(self):
...
    geneSet = [lambda: Mow(),
               lambda: Turn(),
               lambda: Jump(random.randint(0, min(width, height)),
                           random.randint(0, min(width, height))),
               lambda: Func()]
...
```

We also have to increase the expected number of instructions because **Func** isn't quite as powerful as **Repeat**

```
...
    expectedNumberOfInstructions = 18
    expectedNumberOfSteps = 65
...
```

15.6.2. Run

It runs much faster than the **Repeat** version and still produces a reasonably short program like the following.

sample result

```
36 46 48 58 4 14 16 26
37 47 49 59 5 15 17 27
38 40 50 60 6 8 18 28
39 41 51 61 7 9 19 29
32 42 52 62 M> 10 20 30
33 43 53 63 1 11 21 31
34 44 54 56 2 12 22 24
35 45 55 57 3 13 23 25
64 mowed with 18 instructions and 65 steps 0:00:04.805489
func: mow mow mow mow mow mow mow mow jump(7,7)
call-func call-func call-func call-func call-func call-func call-func call-func turn
```

15.7. Multiple ADFs

One limitation of our current method of building the automatically defined function is that we can only have one ADF. Studying the generated main function above, we can see where we could shorten the program if we had a second function whose instructions were:

```
mow mow mow
```

or even:

```
call-func call-func call-func call-func
```

To support this capability we need to add an explicit **Call** instruction that has the id (index) of the **Func** it should call as a parameter.

```
class Call:
    FuncId = None
    Funcs = None

    def __init__(self, funcId=None):
        self.FuncId = funcId
        self.Funcs = None
```

We also need to add an **Id** attribute to **Func** and, in order to keep `test_mow_turn_jump_func` working, we need to add an `expectCall` parameter to its constructor.

```
class Func:
    Ops = None
    ExpectCall = None
    Id = None

    def __init__(self, expectCall=False):
        self.Ops = []
        self.ExpectCall = expectCall
        self.Id = None

    ...
```

We'll also use the presence of an **Id** to tell whether the gene set includes a **Call** instruction and change the **str** output accordingly:

```
class Func:
    ...
    def __str__(self):
        return "func{1}: {0}".format(
            ' '.join(map(str, self.Ops)),
            self.Id if self.Id is not None else '')
```

Next, we have to update **Program** to support **Call** and multiple functions. We'll start by renaming its **Func** attribute to **Funcs** and initialize it to an empty array:

```
class Program:
    Funcs = None
    Main = None

    def __init__(self, genes):
        temp = genes[:]
        funcs = []
    ...
```

Then in the loop, when we encounter a **Call** instruction, we need to give it a reference to the array of functions.

```
...
    for index in reversed(range(len(temp))):
    ...
        if type(temp[index]) is Call:
            temp[index].Funcs = funcs
        if type(temp[index]) is Func:
    ...
```

And for backwards compatibility when we find an Nth **Func** instruction when the gene set does not contain a **Call** instruction, we're going to convert it to a call.

```
...
        if type(temp[index]) is Func:
            if len(funcs) > 0 and not temp[index].ExpectCall:
                temp[index] = Call()
                temp[index].Funcs = funcs
            continue
    ...
```

Otherwise, if we're expecting **Call** instructions, we'll assign it an id (for display purposes)

```
...
        end = len(temp)
        func = Func()
        if temp[index].ExpectCall:
            func.Id = len(funcs)
        func.Ops = [i for i in temp[start:end]]
...
```

and add it to the function array after populating its **Ops**

```
...
        funcs.append(func)
        del temp[index:end]
...
```

After processing all instructions, we're going to do some cleanup of the function contents, removing any **Call** instructions that reference a function index that doesn't exist and functions that have no **Ops** to perform.

```
...
        del temp[index:end]

    for func in funcs:
        for index in reversed(range(len(func.Ops))):
            if type(func.Ops[index]) is Call and func.Ops[index].FuncId is not None:
                if func.Ops[index].FuncId >= len(funcs) or \
                   len(funcs[func.Ops[index].FuncId].Ops) == 0:
                    del func.Ops[index]
    ...
```

Then we'll perform that same optimization on **Call** instructions in the **main** program before assigning the functions and main to their respective attributes.

```
...
        del func.Ops[index]

    for index in reversed(range(len(temp))):
        if type(temp[index]) is Call and temp[index].FuncId is not None:
            if temp[index].FuncId >= len(funcs) or \
               len(funcs[temp[index].FuncId].Ops) == 0:
                del temp[index]

    ...
    self.Main = temp
    self.Funcs = funcs
```

We can now move the **Call** specific code out of the **evaluate** function

```
def evaluate(self, mower, field):
    for i, instruction in enumerate(self.Main):
        instruction.execute(mower, field)
```

into **Call** and update it to call the appropriate function, again with backward compatibility.

```
class Call:
    ...
    def execute(self, mower, field):
        funcId = 0 if self.FuncId is None else self.FuncId
        if len(self.Funcs) > funcId:
            self.Funcs[funcId].execute(mower, field)
```

The final change to **Program** is to handle multiple functions in **print**

```
def print(self):
    if self.Funcs is not None:
        for func in self.Funcs:
            if func.Id is not None and len(func.Ops) == 0:
                continue
            print(func)
    print(' '.join(map(str, self.Main)))
```


We also need to support `__str__` in `Call` for display purposes.

```
class Call:
...
    def __str__(self):
        return "call-{}".format(
            self.FuncId
            if self.FuncId is not None
            else 'func')
```

Lastly, for a test we can make a copy of `test_mow_turn_jump_func` as `test_mow_turn_jump_call` then add `Call` and update the `Func` constructor.

```
def test_mow_turn_jump_call(self):
...
    geneSet = [lambda: Mow(),
               lambda: Turn(),
               lambda: Jump(random.randint(0, min(width, height)),
                           random.randint(0, min(width, height))),
               lambda: Func(expectCall=True),
               lambda: Call(random.randint(0,5))]
```

15.7.1. Run

It starts off building highly recursive programs that take 1000's of steps to mow the field

sample output

```
1248 1241 1234 1227 1284 1269 1262 1255
1249 1242 1235 1228 1277 1270 1263 1256
1250 1243 1236 1285 1278 1271 1264 1257
1251 1244 1229 1286 1279 1272 1265 1258
1252 1237 1230 1287 1280 1273 1266 1259
1245 1238 1231 1288 1281 1274 1267 1260
1246 1239 1232 Mv 1282 1275 1268 1253
1247 1240 1233 1226 1283 1276 1261 1254
64 mowed with 16 instructions and 1288 steps    0:00:46.505430
func0: mow call-2
func1: mow mow mow jump(2,1) mow mow mow call-0
func2: call-1
call-0 mow
```

It eventually discovers that eliminating the recursion reduces the number of steps but that can take **hours**.

sample output

```
79 78 77 75 57 56 55 53
81 62 40 74 38 37 36 52
82 63 42 73 17 16 14 51
83 64 43 72 46 47 48 49
101 100 99 98 97 95 12 31
103 66 68 69 21 94 11 30
104 88 89 90 91 92 10 29
Mv 3 4 5 23 25 26 27
64 mowed with 30 instructions and 105 steps 10:00:47.530544
func0: jump(3,0)
func1: jump(3,4)
func2: call-4
func3: mow mow turn mow mow mow mow
func4: call-3 call-3 mow turn jump(3,7) call-3 call-3
call-1 call-2 call-4 call-4 call-4
```

The fix is to prevent recursion in the first place.

15.8. Exercise

Recursion can be easily prevented if we change the implementation so that a **Call** is only valid if it references a **Func** that has a higher (or lower if you prefer) index than the function the **Call** is in, main excluded. You can do that in the cleanup step when constructing the **Program**. Make this change and you'll be able to see the power of ADFs.

15.9. Summary

In this chapter we surveyed some of the control structures used in genetic programming instructions, ultimately ending with automatically defined functions. This gave us insight into how they might be useful in other projects. It also greatly expanded the realm of projects to which we can apply genetic algorithms. Welcome to genetic programming!

15.10. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch15>

Chapter 16. Logic Circuits

Logic gates are the basic building blocks of logic circuits. In this chapter we're going to use a genetic algorithm to combine logic gates into circuits that can do work.

16.1. Circuit infrastructure

As in the Lawnmower project it makes sense that our phenotype should have behaviors, so we'll use objects. Also like that project we'll build the infrastructure in a separate file. We'll start with providing built-in gates for **NOT** and **AND**.

16.1.1. NOT and AND gates

The **NOT** gate takes one Boolean input and returns its opposite, so **True** if the input is **False**, otherwise **True**.

NOT gate truth table (0=False, 1=True)

input	output
0	1
1	0

Since our input will not be a bare Boolean value but an upstream gate in the circuit, when asked for its output the **NOT** gate will first ask the upstream gate its value, then return the opposite.

circuits.py

```
class Not:
    def __init__(self, input):
        self._input = input

    def get_output(self):
        return not self._input.get_output()
```

The **AND** gate takes two Boolean inputs, **A** and **B**, and returns **True** if they are both **True**, otherwise **False**.

AND gate truth table (0=False, 1=True)

inputs		output
A	B	
0	0	0
0	1	0
1	0	0
1	1	1

Like the **NOT** gate, the **AND** gate must ask the gates that feed into it for their values before it can provide its output.

```
class And:
    def __init__(self, inputA, inputB):
        self._inputA = inputA
        self._inputB = inputB

    def get_output(self):
        aValue = self._inputA.get_output()
        bValue = self._inputB.get_output()
        return aValue and bValue
```

In addition to logic gates the circuit will also contain references to the actual **A** and **B** source inputs we're testing. We need to be able to change the source values to check the fitness of the circuit, so we'll give it a reference to a container whose contents we can modify externally.

```
class Source:
    def __init__(self, sourceId, sourceContainer):
        self._sourceId = sourceId
        self._sourceContainer = sourceContainer

    def get_output(self):
        return self._sourceContainer[self._sourceId]
```

We'll use a new structure for the genotype, a tree node that contains the type of gate and indexes to 2 child tree nodes (potential inputs).

circuitTests.py

```
class Node:
    CreateGate = None
    IndexA = None
    IndexB = None

    def __init__(self, createGate, indexA=None, indexB=None):
        self.CreateGate = createGate
        self.IndexA = indexA
        self.IndexB = indexB
```

16.1.2. Nodes to circuit

Now we'll write the function that builds the circuit.

```
def nodes_to_circuit(nodes):
    circuit = []
```

It loops through all the nodes starting with the leaf nodes and rolling toward the root while connecting the logic gates together.

```
for i, node in enumerate(nodes):
```

Using what we learned in the last chapter we can prevent recursion by design with the convention that child indexes are only valid if they are lower than the node index.

```
inputA = circuit[node.IndexA] if node.IndexA is not None and i > node.IndexA else
None
inputB = circuit[node.IndexB] if node.IndexB is not None and i > node.IndexB else
None
```

Lastly we update the circuit by creating the gate. The circuit we'll use ends up fully connected and in the last index of the array. There may be latent circuits in the array as well when we're done.

```
circuit.append(node.CreateGate(inputA, inputB))
return circuit[-1]
```

Note that since inputA and inputB can both be **None** it can cause **NOT** and **AND** gates to be instantiated with **None** inputs, so we need to handle that situation. A **None** input makes the gate output invalid so we'll make it return **None** when that happens.

circuits.py

```
class And:
    ...
    def get_output(self):
        if self._inputA is None or self._inputB is None:
            return None
        aValue = self._inputA.get_output()
        bValue = self._inputB.get_output()
        return aValue and bValue
```

```
class Not:
    ...
    def get_output(self):
        if self._input is None:
            return None
        return not self._input.get_output()
```

That change forces us to also handle the possibility that the input gate is not `None` but returns `None`:

```
class Not:
    ...
    def get_output(self):
        if self._input is None:
            return None
        value = self._input.get_output()
        if value is None:
            return None
        return not value
```

```
class And:
    ...
    def get_output(self):
        if self._inputA is None or self._inputB is None:
            return None
        aValue = self._inputA.get_output()
        if aValue is None:
            return None
        bValue = self._inputB.get_output()
        if bValue is None:
            return None
        return aValue and bValue
```

Now we have what we need to build a circuit, provide inputs to a circuit and check the output. We're ready to test.

16.2. Generate OR

For the first test we're going to use a genetic algorithm to generate a circuit that behaves like an `OR` gate. An `OR` gate takes two Boolean inputs and returns `True` if either is `True` otherwise `False`. We can use the following truth table to see the expected output for each combination of inputs.

OR gate truth table (0=False, 1=True)

inputs		output
A	B	
0	0	0
0	1	1
1	0	1
1	1	1

In our test function we convert the table to an array of rules that can be used to evaluate a circuit's fitness.

circuitTests.py

```
import unittest

class CircuitTests(unittest.TestCase):
    def test_generate_OR(self):
        rules = [[[False, False], False],
                  [[False, True], True],
                  [[True, False], True],
                  [[True, True], True]]
        ...
        self.find_circuit(rules, optimalLength)
```

That implies `find_circuit`

```
def find_circuit(self, rules, expectedLength):
    def fnGetFitness(genes):
        return get_fitness(genes, rules, self.inputs)
    ...
```

and `self.inputs`, which is created before the first test runs.

```
class CircuitTests(unittest.TestCase):
    @classmethod
    def setUpClass(cls):
        cls.inputs = dict()
    ...
```

Now we have what we need to build the fitness function.

16.2.1. Fitness

To calculate the fitness we need to build the circuit from the nodes then use each rule's inputs to test the circuit and count how many rules the circuit can satisfy.

```
def get_fitness(genes, rules, inputs):
    circuit = nodes_to_circuit(genes)
    sourceLabels = "AB"
    rulesPassed = 0
    for rule in rules:
        inputs.clear()
        inputs.update(zip(sourceLabels, rule[0]))
        if circuit.get_output() == rule[1]:
            rulesPassed += 1
    return rulesPassed
```

16.2.2. Display

We want to output the matching circuit, so we need to add `__str__` functions to the source and gate classes. If the gate has invalid inputs then we'll show a question mark.

circuits.py

```
class Not:
    ...
    def __str__(self):
        if self._input is None:
            return "Not(?)"
        return "Not({})".format(self._input)
```

```
class And:
    ...
    def __str__(self):
        if self._inputA is None or self._inputB is None:
            return "And(?)"
        return "And({} {})".format(self._inputA, self._inputB)
```

```
class Source:
    ...
    def __str__(self):
        return self._sourceId
```


This produces output like the following:

sample output

```
Not(And(Not(B) And(Not(A) Not(B))))
```

Now we will bring it all together in the `display` function:

circuitTests.py

```
import datetime
...
def display(candidate, startTime):
    circuit = nodes_to_circuit(candidate.Genes)
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        circuit,
        candidate.Fitness,
        str(timeDiff)))
```

```
def find_circuit(self, rules, expectedLength):
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    ...
```

16.2.3. Create

Our gene objects are complex, so we'll use a special function to create them.

```
def find_circuit(self, rules, expectedLength):
    ...
    def fnCreateGene(index):
        return create_gene(index, self.geneset)
    ...
```

Now we can complete the test harness by populating the gene set.

```
import circuits
...
class CircuitTests(unittest.TestCase):
...
    def setUpClass(cls):
        cls.inputs = dict()
        cls.geneset = [[circuits.And, circuits.And],
                        [lambda i1, i2: circuits.Not(i1), circuits.Not],
                        [lambda i1, i2: circuits.Source('A', cls.inputs),
                        circuits.Source],
                        [lambda i1, i2: circuits.Source('B', cls.inputs),
                        circuits.Source]]
...

```

It will pick child index values relative to the index where the node will be inserted so they're more likely to be valid when converted to a circuit. We'll also try to make the input indexes different so we reduce the waste from gates like `And(A A)`.

```
import random
...
def create_gene(index, geneset):
    gateType = random.choice(geneset)
    indexA = indexB = None
    if gateType[1].input_count() > 0:
        indexA = random.randint(0, index)
    if gateType[1].input_count() > 1:
        indexB = random.randint(0, index) if index > 1 else 0
        if indexB == indexA:
            indexB = random.randint(0, index)
    return Node(gateType[0], indexA, indexB)

```

That implies the addition of an `input_count` function to the source and gate classes.

circuits.py

```
class Source:
...
    @staticmethod
    def input_count():
        return 0

```

```

class Not:
    ...
    @staticmethod
    def input_count():
        return 1

```

```

class And:
    ...
    @staticmethod
    def input_count():
        return 2

```

16.2.4. Mutate

Next we'll add a custom `mutate` function.

circuitTests.py

```

def find_circuit(self, rules, expectedLength):
    ...
    def fnMutate(genes):
        mutate(genes, fnCreateGene)
    ...

```

To be efficient in the `mutate` function, we only want to change the nodes that we actually use in the circuit. We can accumulate those while we're building the circuit. The change in `nodes_to_circuit` is to add a tracking array where each element contains the set of node indexes that are used to build the corresponding circuit.

```

def nodes_to_circuit(genes):
    circuit = []
    usedIndexes = [] ①
    for i, node in enumerate(genes):
        used = {i} ②
        inputA = inputB = None
        if node.IndexA is not None and i > node.IndexA:
            inputA = circuit[node.IndexA]
            used.update(usedIndexes[node.IndexA]) ③
        if node.IndexB is not None and i > node.IndexB:
            inputB = circuit[node.IndexB]
            used.update(usedIndexes[node.IndexB]) ④
        circuit.append(node.CreateGate(inputA, inputB))
        usedIndexes.append(used) ⑤
    return circuit[-1], usedIndexes[-1] ⑥

```

That requires a corresponding change to the fitness function

```
def get_fitness(genes, rules, inputs):
    circuit = nodes_to_circuit(genes)[0]
    ...
```

and display function.

```
def display(candidate, startTime):
    circuit = nodes_to_circuit(candidate.Genes)[0]
    ...
```

Now we can call `nodes_to_circuit` to get the list of node indexes to use as mutation candidates.

```
def mutate(childGenes, fnCreateGene):
    count = random.randint(1, 5)
    while count > 0:
        count -= 1
        indexesUsed = [i for i in nodes_to_circuit(childGenes)[1]]
        index = random.choice(indexesUsed)
        childGenes[index] = fnCreateGene(index)
```

16.2.5. Create

We also need a custom `create` function. It is simple enough to be added inline.

```
def find_circuit(self, rules, expectedLength):
    ...
    maxLength = expectedLength

    def fnCreate():
        return [fnCreateGene(i) for i in range(maxLength)]
    ...
```

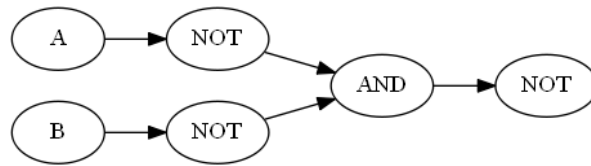
16.2.6. Optimal OR circuit

To build the optimal solution let's refer back to the OR gate truth table.

OR gate truth table (0=False, 1=True)

inputs		output
A	B	
0	0	0
0	1	1
1	0	1
1	1	1

If we could negate the first row we'd have a circuit that works for all four rules. NOT A anded with NOT B would get False. If we NOT that we get a circuit that works for all four rules, like this:



We'll use the node count in that circuit to limit the number of nodes we use.

```
def test_generate_OR(self):
...
    optimallength = 6
    self.find_circuit(rules, optimallength)
```

Finally we call the genetic engine.

```
import genetic
...
def find_circuit(self, rules, expectedLength):
...
    best = genetic.get_best(fnGetFitness, None, len(rules), None, fnDisplay,
                           fnMutate, fnCreate, poolSize=3)
    self.assertTrue(best.Fitness == len(rules))
    self.assertTrue(not len(nodes_to_circuit(best.Genes)[1]) > expectedLength)
```

16.2.7. Run

When we run the test it finds the optimal solution every time.

```
Not(And(?)) 0    0:00:00
Not(B) 1    0:00:00
And(B B) 3    0:00:00.001004
Not(And(Not(A) Not(B))) 4    0:00:09.581228
```

Great!

16.3. Generate XOR

Now let's see if we can generate a circuit that behaves like an **XOR** (exclusive-or) gate. An **XOR** gate returns **True** if the inputs are different, otherwise **False**.

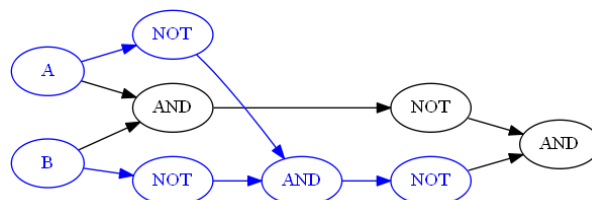
XOR gate truth table (0=False, 1=True)

inputs		output
A	B	
0	0	0
0	1	1
1	0	1
1	1	0

We'll start by building a new test with those rules.

```
def test_generate_XOR(self):
    rules = [[[False, False], False],
              [[False, True], True],
              [[True, False], True],
              [[True, True], False]]
    ...
```

By comparing the **XOR** gate truth table with that of the **OR** gate we notice that the only difference is the last rule. This means that in order to determine the optimal solution for the **XOR** gate we can start with the circuit for the **OR** gate, **Not(And(Not(A) Not(B)))** (in blue below), and **AND** that with the opposite of a circuit that produces the fourth rule, **Not(And(A B))**. Here's a visual representation:



The strict number of nodes in the circuit is 9, or 11 if we double count the source (A and B) nodes. That lets us finish

writing the **XOR** test:

```
...
    self.find_circuit(rules, 9)
```

16.3.1. Run

Run the test and it almost always stalls.

```
And(B And(B B)) 2    0:00:00.001003
Not(And(B A))    3    0:00:00.014127
And(Not(And(Not(A) Not(B))) Not(And(A B))) 4    0:09:44.378545
```

Can you think of any ways in which we can take advantage of our knowledge of the problem space to improve the performance of the genetic algorithm? Before reading further take about half an hour to experiment.

16.4. Performance improvement

The following is one of the easiest improvements to implement. Notice that the sources take up 2 of the 4 slots in the gene set. This means that on average they end up taking half the slots in the **Chromosome**, which makes it very difficult for the engine to build a complex circuit. The solution is to separate the sources from the gates and use only one copy of each of the sources because they can be referenced by index from any node with a higher index.

```
class CircuitTests(unittest.TestCase):
    @classmethod
    def setUpClass(cls):
        cls.inputs = dict()
        cls.gates = [[circuits.And, circuits.And],
                     [lambda i1, i2: circuits.Not(i1), circuits.Not]]
        cls.sources = [[lambda i1, i2: circuits.Source('A', cls.inputs),
                        circuits.Source],
                       [lambda i1, i2: circuits.Source('B', cls.inputs),
                        circuits.Source]]
```

Then pass both to **create_gene**:

```
def find_circuit(self, rules, expectedLength):
    ...
    def fnCreateGene(index):
        return create_gene(index, self.gates, self.sources)
    ...
```

Next change `create_gene` so that sources are only added at the start of the node array, leaving the rest of the indexes for gates.

```
def create_gene(index, gates, sources):
    if index < len(sources): ①
        gateType = sources[index]
    else:
        gateType = random.choice(gates)
    ...
    if gateType[1].input_count() > 1:
        indexB = random.randint(0, index) \
            if index > 1 and index >= len(sources) else 0 ②
        if indexB == indexA:
    ...
```

We can also use our knowledge of the source locations in the gene sequence to reduce waste by preventing `mutate` from touching those nodes.

```
def find_circuit(self, rules, expectedLength):
    ...
    def fnMutate(genes):
        mutate(genes, fnCreateGene, fnGetFitness, len(self.sources))
    ...
```

We'll also check the fitness and return early if we find an improvement.

```
def mutate(childGenes, fnCreateGene, fnGetFitness, sourceCount):
    count = random.randint(1, 5)
    initialFitness = fnGetFitness(childGenes) ①
    while count > 0:
        count -= 1
        indexesUsed = [i for i in nodes_to_circuit(childGenes)[1] ②
                       if i >= sourceCount]
        if len(indexesUsed) == 0:
            return
        index = random.choice(indexesUsed)
        childGenes[index] = fnCreateGene(index)
        if fnGetFitness(childGenes) > initialFitness: ③
            return
```

These changes make it possible for the engine to find the solution every time.


```
And(?) 0 0:00:00.001005
Not(And(B Not(B))) 2 0:00:00.001005
And(Not(B) Not(Not(A))) 3 0:00:00.004011
And(Not(And(Not(B) Not(A))) Not(And(B A))) 4 0:00:02.235138
```

Very good!

16.5. Generate A XOR B XOR C

We've seen that the genetic algorithm can generate a circuit that passes the 4 rules required for 2 sources. Let's try a circuit that uses 3 sources. This means it will have 2^3 (8) rules. The circuit we'll try to reproduce is A XOR B XOR C.

A XOR B XOR C truth table (0=False, 1=True)

inputs			output
A	B	C	
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Given that information, we can start writing the test.

```
def test_generate_AxBxC(self):
    rules = [[False, False, False], False],
            [[False, False, True], True],
            [[False, True, False], True],
            [[False, True, True], False],
            [[True, False, False], True],
            [[True, False, True], False],
            [[True, True, False], False],
            [[True, True, True], True]]
    self.sources.append([lambda l, r: circuits.Source('C', self.inputs),
circuits.Source])
    ...
```

That means we need to add **C** to the source labels in `get_fitness` too:

```
def get_fitness(genes, rules, inputs):
    circuit = nodes_to_circuit(genes)[0]
    sourceLabels = "ABC"
    ...
```

Since we know we can build **OR** from **AND** and **NOT** we'll add **OR** to the gates so we can use it to help keep the final circuit relatively short.

```
def test_generate_AxBxC(self):
    ...
    self.gates.append([circuits.Or, circuits.Or])
    ...
```

The **OR** gate implementation would be almost identical to that of the **AND** gate, so let's extract a base class.

circuits.py

```
class GateWith2Inputs:
    def __init__(self, inputA, inputB, label, fnTest):
        self._inputA = inputA
        self._inputB = inputB
        self._label = label
        self._fnTest = fnTest

    def get_output(self):
        if self._inputA is None or self._inputB is None:
            return None
        aValue = self._inputA.get_output()
        if aValue is None:
            return None
        bValue = self._inputB.get_output()
        if bValue is None:
            return None
        return self._fnTest(aValue, bValue)

    def __str__(self):
        if self._inputA is None or self._inputB is None:
            return "{0}(?)".format(self._label)
        return "{0}({1} {2})".format(self._label, self._inputA, self._inputB)

    @staticmethod
    def input_count():
        return 2
```

That simplifies the **AND** gate implementation to:

```
class And(GateWith2Inputs):
    def __init__(self, inputA, inputB):
        super().__init__(inputA, inputB, type(self).__name__, lambda a, b: a and b)
```

The **OR** gate implementation is just as easy:

```
class Or(GateWith2Inputs):
    def __init__(self, inputA, inputB):
        super().__init__(inputA, inputB, type(self).__name__, lambda a, b: a or b)
```

Next we need to solve the problem of figuring out what the optimal solution is. We could use a Karnaugh Map to reduce those 8 rules to a minimal circuit but doing so for circuits with **many** inputs becomes messy. So, let's find another way. Historically we've used a variable length gene sequence when we didn't know how many genes were needed, but as you may have observed, using tree nodes already makes the length adaptive if we give it enough indexes to work with. So we're going to take this opportunity to introduce a new machine learning technique called hill climbing.

16.5.1. Hill climbing

Hill climbing is a popular problem space exploration technique where one feature of the **Chromosome** is incrementally adjusted until a better solution is found or a local minimum or maximum is detected, at which point the process repeats with a different feature. Some variants change any feature as long as it only affects a single piece of data, which may be smaller than a gene. An example from our current project would be to only change the gate type or just one of the indexes in a node rather than replacing the entire node as we're doing now. Be aware that hill climbing doesn't always find the optimal solution so it may need to be supplemented with simulated annealing.

We'll implement hill climbing in the **genetic** module so that it is reusable. Starting with the function definition, it includes the optimization function it will call, a function to test whether an improvement has been found, a function to test whether the improvement is optimal so we can stop, a function that gets the next feature value, a display function, and the initial value of the feature we're trying to optimize.

genetic.py

```
def hill_climbing(optimizationFunction, is_improvement, is_optimal,
                  get_next_feature_value, display, initialFeatureValue):
```

We get the initial result. Then, to keep it from being chatty, we redirect the output just like we do when benchmarking, and only restore it for calls to display.

```
    best = optimizationFunction(initialFeatureValue)
    stdout = sys.stdout
    sys.stdout = None
```

Once we have a result we're going to enter a loop where we keep getting new results until we find an optimal one.

```
while not is_optimal(best):
    featureValue = get_next_feature_value(best)
    child = optimizationFunction(featureValue)
```

When we find an improvement it becomes the new best value and we display it.

```
if is_improvement(best, child):
    best = child
    sys.stdout = stdout
    display(best, featureValue)
    sys.stdout = None
```

If we find the optimal solution we return it.

```
sys.stdout = stdout
return best
```

16.5.2. Add hill climbing to the test harness

Now we need to build the inputs we're going to pass to the `hill_climbing` function. We'll start by wrapping the current call to `get_best` in a new function. This will be the optimization function. We'll give it a maximum of 50 gates to work with and see what it finds.

circuitTests.py

```
def find_circuit(self, rules, expectedLength):
    ...
    maxLength = 50
    ...
    def fnOptimizationFunction(variableLength):
        nonlocal maxLength
        maxLength = variableLength
        return genetic.get_best(fnGetFitness, None, len(rules), None, fnDisplay,
                                fnMutate, fnCreate, poolSize=3, maxSeconds=30)
```

I gave it 30 seconds to try to find an improvement, but you might be able to use a lower value. The feature we're optimizing is the number of nodes in the circuit. We need to get that to the `create_gene` function. We'll do that by transferring it to the existing `maxLength` variable that is already passed to that function.

Next we need a function that can tell whether the new result is better than the current best. We return `True` if all the rules pass and the number of gates used in the new result is less than that of the current best result, otherwise `False`.

```
def fnIsImprovement(currentBest, child):
    return child.Fitness == len(rules) and \
        len(nodes_to_circuit(child.Genes)[1]) < \
        len(nodes_to_circuit(currentBest.Genes)[1])
```

We also need a function that can tell whether we've found the known optimal solution (`expectedLength`). Note that if we don't know the optimal value we could simply use an impossibly low value and let it run until we're ready to stop it.

```
def fnIsOptimal(child):
    return child.Fitness == len(rules) and \
        len(nodes_to_circuit(child.Genes)[1]) <= expectedLength
```

As for display, we can simply add an optional parameter to the existing `fnDisplay` for the feature value. When it is set we show the number of nodes used in the new best circuit.

```
def fnDisplay(candidate, length=None):
    if length is not None:
        print("-- distinct nodes in circuit:",
            len(nodes_to_circuit(candidate.Genes)[1]))
    display(candidate, startTime)
```

When an improvement is found we'll make the number of nodes in that circuit the new value to beat.

```
def fnGetNextFeatureValue(currentBest):
    return len(nodes_to_circuit(currentBest.Genes)[1])
```

At the end we call the hill climbing function.

```
best = genetic.hill_climbing(fnOptimizationFunction, fnIsImprovement,
                             fnIsOptimal, fnGetNextFeatureValue, fnDisplay,
                             maxLength)
self.assertTrue(best.Fitness == len(rules))
self.assertTrue(not len(nodes_to_circuit(best.Genes)[1]) > expectedLength)
```

Now we're finally able to finish the test implementation.

```
def test_generate_AxBxC(self):
    ...
    self.find_circuit(rules, 12)
```

16.5.3. Run

Let's first verify that the **OR** and **XOR** tests work.

sample **OR** test result

```
And(And(And(And(?) Not(And(?))) Not(Not(And(?)))) And(Not(And(?)) Not(And(?)))) 0
0:00:00
Not(Not(B)) 3 0:00:00.004012
Not(And(Not(And(Not(A) B)) And(Not(Not(Not(Not(Not(A)))) Not(B)))) 4 0:00:00.013128
-- distinct nodes in circuit: 8
Not(And(Not(B) And(Not(B) Not(A)))) 4 0:00:00.021112
-- distinct nodes in circuit: 6
Not(And(Not(A) Not(B))) 4 0:00:00.080303
```

sample **XOR** test result

```
Not(And(?)) 0 0:00:00.001003
Not(And(And(Not(And(A B)) And(Not(Not(And(A B))) Not(And(A B)))) A)) 2
0:00:00.003008
And(Not(And(A And(A B))) Not(Not(Not(And(A B)))) 3 0:00:00.007047
And(Not(And(Not(A) Not(B))) Not(And(And(And(B And(B A)) And(B A)) Not(Not(And(B A)))))) 4
0:00:00.287776
-- distinct nodes in circuit: 9
And(Not(And(B A)) Not(And(Not(B) Not(A)))) 4 0:00:02.833747
```

Now let's see how it does on **A XOR B XOR C**.

```
And(Or(?) Not(Not(C))) 0 0:00:00.001001
And(And(Not(C) And(And(Not(A) B) Not(Not(Not(C))))) Not(A)) 5 0:00:00.006120
And(And(Not(And(B Or(A C))) Not(A)) Not(And(Not(C) Not(Or(B And(B Or(A C)))))) 6
0:00:00.470108
And(Or(Not(Or(And(A And(A C)) And(Or(A C) B))) And(And(And(And(A C) Or(A C)) B)
And(Not(Not(Or(And(A And(A C)) And(Or(A C) B))) And(Or(A C) B))) Or(And(Or(A C)
Not(Or(And(A And(A C)) And(Or(A C) B))) B)) 8 0:00:02.133227
-- distinct nodes in circuit: 13
And(Not(And(Or(B C) And(A Or(Not(C) Not(B)))) Or(A And(Or(Not(C) Not(B)) Or(B C)))) 8
0:00:50.958561
-- distinct nodes in circuit: 12
Or(And(Or(A C) Not(Or(And(A C) B))) And(B Or(And(A C) Not(Or(A C)))) 8
0:01:04.648605
```

Excellent!

If it takes an excessively long time to find the result on your box, you may need to increase the number of seconds you let it run on each optimization pass.

16.6. Generate a 2-bit adder

Now for something more interesting, a 2-bit adder. A 2-bit adder can add two numbers in the range 0..3 to get a result in the range 0..6. This means we need 4 sources, A and B for the first number, and C and D for the second. We also need 3 result bits for the 4's, 2's and 1's result bits. 4 source-inputs means we'll have 2^4 (16) rules. Here's the truth table.

2 bit adder truth table (0=False, 1=True)

input 1		input 2		outputs			meaning
A (2's)	B (1's)	C (2's)	D (1's)	4's	2's	1's	
0	0	0	0	0	0	0	$0 + 0 = 0$
0	0	0	1	0	0	1	$0 + 1 = 1$
0	0	1	0	0	1	0	$0 + 2 = 2$
0	0	1	1	0	1	1	$0 + 3 = 3$
0	1	0	0	0	0	1	$1 + 0 = 1$
0	1	0	1	0	1	0	$1 + 1 = 2$
0	1	1	0	0	1	1	$1 + 2 = 3$
0	1	1	1	1	0	0	$1 + 3 = 4$
1	0	0	0	0	1	0	$2 + 0 = 2$
1	0	0	1	0	1	1	$2 + 1 = 3$
1	0	1	0	1	0	0	$2 + 2 = 4$
1	0	1	1	1	0	1	$2 + 3 = 5$
1	1	0	0	0	1	1	$3 + 0 = 3$
1	1	0	1	1	0	0	$3 + 1 = 4$
1	1	1	0	1	0	1	$3 + 2 = 5$
1	1	1	1	1	1	0	$3 + 3 = 6$

We could change the code to work with N-result bits, essentially N-circuits, but we can get the same result without that complexity by searching for each result bit's circuit in a separate test and sharing the setup code between them. We can build the setup function from the truth table above.

```

def get_2_bit_adder_rules_for_bit(self, bit):
    rules = [[False, False, False, False], [False, False, False]], # 0 + 0 = 0
             [[False, False, False, True], [False, False, True]], # 0 + 1 = 1
             [[False, False, True, False], [False, True, False]], # 0 + 2 = 2
             [[False, False, True, True], [False, True, True]], # 0 + 3 = 3
             [[False, True, False, False], [False, False, True]], # 1 + 0 = 1
             [[False, True, False, True], [False, True, False]], # 1 + 1 = 2
             [[False, True, True, False], [False, True, True]], # 1 + 2 = 3
             [[False, True, True, True], [True, False, False]], # 1 + 3 = 4
             [[True, False, False, False], [False, True, False]], # 2 + 0 = 2
             [[True, False, False, True], [False, True, True]], # 2 + 1 = 3
             [[True, False, True, False], [True, False, False]], # 2 + 2 = 4
             [[True, False, True, True], [True, False, True]], # 2 + 3 = 5
             [[True, True, False, False], [False, True, True]], # 3 + 0 = 3
             [[True, True, False, True], [True, False, False]], # 3 + 1 = 4
             [[True, True, True, False], [True, False, True]], # 3 + 2 = 5
             [[True, True, True, True], [True, True, False]] # 3 + 3 = 6
    bitNRules = [[rule[0], rule[1][2 - bit]] for rule in rules]
    ...

```

We also need to add **C** and **D** to the sources, and to keep the results short we'll use both an **OR** gate and an **XOR** gate.

```

...
self.gates.append([circuits.Or, circuits.Or])
self.gates.append([circuits.Xor, circuits.Xor])
self.sources.append([lambda l, r: circuits.Source('C', self.inputs),
circuits.Source])
self.sources.append([lambda l, r: circuits.Source('D', self.inputs),
circuits.Source])
return bitNRules

```

Next add **D** to the source labels in `get_fitness`.

```

def get_fitness(genes, rules, inputs):
    circuit = nodes_to_circuit(genes)[0]
    sourceLabels = "ABCD"
    ...

```


Here's the implementation of the **XOR** gate.

circuits.py

```
class Xor(GateWith2Inputs):
    def __init__(self, inputA, inputB):
        super().__init__(inputA, inputB, type(self).__name__, lambda a, b: a != b)
```

16.6.1. Tests

Now we're ready to add the tests that find the circuit for each bit.

circuitTests.py

```
def test_2_bit_adder_1s_bit(self):
    rules = self.get_2_bit_adder_rules_for_bit(0)
    self.find_circuit(rules, 3)
```

```
def test_2_bit_adder_2s_bit(self):
    rules = self.get_2_bit_adder_rules_for_bit(1)
    self.find_circuit(rules, 7)
```

```
def test_2_bit_adder_4s_bit(self):
    rules = self.get_2_bit_adder_rules_for_bit(2)
    self.find_circuit(rules, 9)
```

16.6.2. Run

It can quickly generate the optimal solution for the 1's bit

sample solution for 1's bit

```
Not(Or(And(?) C)) 0 0:00:00.001003
Not(Or(Or(A C) C)) 8 0:00:00.002006
And(Not(And(Xor(D Not(Not(Or(And(B D) C))) And(A D))) Not(Not(Xor(D And(A And(B D))))))
9 0:00:00.007068
And(Not(Xor(Xor(Xor(D And(A And(B D))) A) Or(Xor(And(B D) Xor(D And(A And(B D))))
Not(Or(And(B D) C)))) Not(Not(Xor(D And(A And(B D)))))) 10 0:00:00.009073
And(Not(Xor(Xor(Xor(D Xor(And(A D) B)) A) Or(Not(A) Not(Or(And(B D) C)))) Not(Not(Xor(D
Xor(And(A D) B)))) 11 0:00:00.014086
Or(Xor(D B) Xor(And(Xor(D B) Not(D)) A)) 12 0:00:00.032165
Or(Xor(D B) And(Not(D) Xor(D B))) 16 0:00:00.035143
-- distinct nodes in circuit: 3
Xor(D B) 16 0:00:00.036146
```

the 2's bit

sample output for 2's bit

```
Xor(Not(Not(B)) Xor(Or(?) Not(Not(B)))) 0 0:00:00.001010
Xor(Not(Not(B)) Xor(Not(B) Not(Not(B)))) 8 0:00:00.003009
Xor(Not(Not(And(A Or(C A)))) Xor(Xor(B Xor(A C)) And(A Or(C A)))) 12 0:00:00.006119
Xor(Xor(Xor(And(A D) B) And(And(B A) B)) Xor(Xor(B Xor(A C)) Or(And(B A) Xor(And(A D)
B)))) 14 0:00:00.046253
Xor(Xor(Xor(And(B D) A) And(Xor(Not(C) C) B)) Xor(Xor(B Not(C)) Or(Xor(Not(C) C)
Xor(And(B D) A)))) 16 0:00:00.338064
-- distinct nodes in circuit: 8
Xor(And(B And(D B)) Xor(A C)) 16 0:00:00.431277
-- distinct nodes in circuit: 7
Xor(Xor(And(D B) C) A) 16 0:00:30.907710
```

and the 4's bit.

sample solution for 4's bit

```
Or(Or(Or(Or(?) B) A) A) 0 0:00:00.001004
Not(Xor(Not(Not(A)) Not(Not(Or(Xor(A C) Not(A))))) 8 0:00:00.002005
Not(Not(And(And(D And(D C)) And(A D)))) 12 0:00:00.007520
And(C Xor(Xor(B C) Or(Not(D) Not(And(And(A A) And(Xor(B C) Xor(B C))))) 13
0:00:00.193725
And(C Xor(Not(A) Or(And(A A) Not(A)))) 14 0:00:00.224307
And(C Or(Not(Or(Not(A) Xor(A C)) And(B D))) 15 0:00:01.020520
And(And(Or(D And(C A)) Or(Not(Or(A Xor(Not(B) C))) Or(C And(Not(And(A Not(B)) B)))) Or(A
Xor(Not(B) C))) 16 0:00:11.065914
-- distinct nodes in circuit: 13
And(Or(And(A C) And(Not(Not(D)) B)) Or(Xor(And(Not(Not(D)) A) Not(D)) C)) 16
0:00:15.089580
-- distinct nodes in circuit: 10
Or(And(C A) And(And(B D) Or(Xor(A C) A))) 16 0:00:28.089024
-- distinct nodes in circuit: 9
And(Or(C And(D B)) Or(And(C And(D B)) A)) 16 0:01:15.223721
```

Outstanding!

16.7. Retrospective

Tree nodes are often used instead of arrays in genetic programming because it is much easier to inject new functionality between two nodes. You should be able to adapt the way tree nodes are used in this chapter to the Lawnmower Problem. One way would be for all nodes to use the first child node to point to the next instruction. Then each **func** instruction could define its instructions on the second child node, making it easier for the genetic algorithm to modify the contents of the function.

You might also implement **NAND** and **NOR** gates to see if they improve upon the optimal solutions we found in this chapter.

16.8. Summary

This chapter showed how tree nodes can be used in a genetic algorithm for both fixed- and variable-length genotypes. It also introduced hill climbing, a very useful but potentially time consuming optimization technique.

16.9. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch16>

Chapter 17. Regular Expressions

The next project is to build a genetic algorithm that can evolve a regular expression (regex) that matches all items in a set of wanted strings without matching any items in a set of unwanted strings.

17.1. Test

As you probably know, meta characters in regular expressions have special meanings, for example `*` means repeat zero or more times, `+` means repeat at least once, etc. The base gene set for the genetic algorithm will include four of these, which will be defined in global variables within the genetic algorithm file.

regexTests.py

```
repeatMetas = {'?', '*', '+'}
startMetas = {'|'}
allMetas = repeatMetas | startMetas
```

Next, the strings the generated regex should match, and those it should not match, are defined in a test. The genetic algorithm is also given a specific regex length to achieve. This keeps it from simply using the `|` meta character to concatenate all the wanted strings.

```
import unittest

class RegexTests(unittest.TestCase):
    def test_two_digits(self):
        wanted = {"01", "11", "10"}
        unwanted = {"00", ""}
        self.find_regex(wanted, unwanted, 7)
```

17.2. Fitness

The fitness function first compiles the regex to see if it is valid or not. If not it returns a comparably low fitness value.

```
import re
...
def get_fitness(genes, wanted, unwanted):
    pattern = ''.join(genes)
    length = len(pattern)

    try:
        re.compile(pattern)
    except re.error:
        return Fitness(0, len(wanted), len(unwanted), length)
```

Otherwise, it determines the fitness of the generated regex by counting the number of wanted and unwanted strings the regex matches exactly.

```
numWantedMatched = sum(1 for i in wanted if re.fullmatch(pattern, i))
numUnwantedMatched = sum(1 for i in unwanted if re.fullmatch(pattern, i))
return Fitness(numWantedMatched, len(wanted), numUnwantedMatched, length)
```

As usual the fitness function has a related helper function in the test harness:

```
def find_regex(self, wanted, unwanted, expectedLength):
    def fnGetFitness(genes):
        return get_fitness(genes, wanted, unwanted)
```

This genetic algorithm uses a **Fitness** object because there are multiple objectives. They are:

- to maximize the number of wanted strings that match,
- to minimize the number of unwanted strings that match, and
- to minimize the length of the regex

```
class Fitness:
    NumWantedMatched = None
    NumUnwantedMatched = None
    Length = None

    def __init__(self, numWantedMatched, totalWanted, numUnwantedMatched, length):
        self.NumWantedMatched = numWantedMatched
        self._totalWanted = totalWanted
        self.NumUnwantedMatched = numUnwantedMatched
        self.Length = length
```

The comparison function first combines the number of wanted strings that were not matched with the number of unwanted strings that were matched. When that value differs, the algorithm should keep the chromosome with the smallest total. That achieves the first two objectives while allowing the matched wanted and unwanted counts to vary.

```
def __gt__(self, other):
    combined = (self._totalWanted - self.NumWantedMatched) + self.NumUnwantedMatched
    otherCombined = (self._totalWanted - other.NumWantedMatched) +
other.NumUnwantedMatched
    if combined != otherCombined:
        return combined < otherCombined
```

When the regex fails to match one or more wanted strings, or matches one or more unwanted strings, the algorithm should keep the newer one. This should prevent the algorithm from hanging on a particularly bad chromosome.

```
success = combined == 0
otherSuccess = otherCombined == 0
if success != otherSuccess:
    return success
if not success:
    return False
```

Otherwise the shortest regex is chosen.

```
return self.Length < other.Length
```

The output of the `__str__` function makes the values easy to read when displayed.

```
def __str__(self):
    return "matches {0} wanted {1} unwanted, len {2}".format(
        "all" if self._totalWanted == self.NumWantedMatched else
self.NumWantedMatched,
        self.NumUnwantedMatched,
        self.Length)
```

17.3. Display

```
import datetime
...
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        ''.join(candidate.Genes), candidate.Fitness, str(timeDiff)))
```

expected output

```
01|11?*+    matches 0 wanted 2 unwanted, len 8    0:00:00
```

Next is the helper function for the test harness.

```
def find_regex(self, wanted, unwanted, expectedLength):
    startTime = datetime.datetime.now()

    def fnDisplay(candidate):
        display(candidate, startTime)

    ...
```

17.4. Mutation

Since the genetic algorithm needs to be able to change the length of the regex, a custom mutation function is used. The mutation function has a loop that breaks early if it produces a gene sequence with an improved fitness.

```
import random
...
def mutate(genes, fnGetFitness, mutationOperators, mutationRoundCounts):
    initialFitness = fnGetFitness(genes)
```

The number of times the loop executes is adaptive based on previously successful loop counts.

```
count = random.choice(mutationRoundCounts)
for i in range(1, count + 2):
```

The mutation function receives an array of operators that can be used to modify the gene sequence. Each time through the loop it makes a temporary copy of the operators then picks one and uses it to try to modify the gene sequence. The operator returns a Boolean value indicating whether it was able to make a change or not. If not, that operator is removed from the temporary copy so it will not be tried again in that round.

```
copy = mutationOperators[:]
func = random.choice(copy)
while not func(genes):
    copy.remove(func)
    func = random.choice(copy)
```

When an operator does make a change, the fitness of the changed genes is compared with the initial fitness. If the fitness improves then the adaptive loop count is updated and the loop halts.

```
if fnGetFitness(genes) > initialFitness:
    mutationRoundCounts.append(i)
    return
```

17.4.1. Mutation Operators

The default mutation operators are: add, remove, replace, swap and move.

```
def mutate_add(genes, geneset):
    index = random.randrange(0, len(genes) + 1) if len(genes) > 0 else 0
    genes[index:index] = [random.choice(geneset)]
    return True
```

```
def mutate_remove(genes):
    if len(genes) < 1:
        return False
    del genes[random.randrange(0, len(genes))]
    if len(genes) > 1 and random.randint(0, 1) == 1:
        del genes[random.randrange(0, len(genes))]
    return True
```

```
def mutate_replace(genes, geneset):
    if len(genes) < 1:
        return False
    index = random.randrange(0, len(genes))
    genes[index] = random.choice(geneset)
    return True
```

```
def mutate_swap(genes):
    if len(genes) < 2:
        return False
    indexA, indexB = random.sample(range(len(genes)), 2)
    genes[indexA], genes[indexB] = genes[indexB], genes[indexA]
    return True
```

```
def mutate_move(genes):
    if len(genes) < 3:
        return False
    start = random.choice(range(len(genes)))
    stop = start + random.randint(1, 2)
    toMove = genes[start:stop]
    genes[start:stop] = []
    index = random.choice(range(len(genes)))
    genes[index:index] = toMove
    return True
```


Notice that two of the functions have multiple parameters but the mutation function only calls them with `genes`. The function-specific parameters, if any, are provided when the list of mutation operators is created in the test harness, through the use of `partial`.

`partial` gives the ability to pre-fill or lock function parameters. This is useful when you know the parameters will not change. When the `partial` version of that function is called you only need to supply the remaining parameters, if any.

```
from functools import partial
...
def find_regex(self, wanted, unwanted, expectedLength):
...
    mutationRoundCounts = [1]

    mutationOperators = [
        partial(mutate_add, geneset=fullGeneset),
        partial(mutate_replace, geneset=fullGeneset),
        mutate_remove,
        mutate_swap,
        mutate_move,
    ]

    def fnMutate(genes):
        mutate(genes, fnGetFitness, mutationOperators, mutationRoundCounts)
```

17.5. Test Harness

The test harness starts by adding the unique letters from each of the wanted strings, as well as the wanted strings themselves, to the set of gene tokens the algorithm can use to build the regex. That way the algorithm doesn't have to struggle to reassemble long sequences or words.

```
import genetic
...
def find_regex(self, wanted, unwanted, expectedLength):
    startTime = datetime.datetime.now()
    textGenes = wanted | set(c for w in wanted for c in w)
    fullGeneset = [i for i in allMetas | textGenes]
```

Next come the helper functions, given previously, and finally the call to run the engine.

```
optimalFitness = Fitness(len(wanted), len(wanted), 0, expectedLength)

best = genetic.get_best(fnGetFitness, max(len(i) for i in textGenes),
                        optimalFitness, fullGeneset, fnDisplay, fnMutate,
                        poolSize=10)
self.assertTrue(not optimalFitness > best.Fitness)
```

17.6. Run

sample result

```
+10 matches 0 wanted 2 unwanted, len 3  0:00:00.001003
111 matches 0 wanted 0 unwanted, len 3  0:00:00.001003
10 matches 1 wanted 0 unwanted, len 2  0:00:00.001003
10?1? matches 2 wanted 0 unwanted, len 5  0:00:00.017079
11|10|01 matches all wanted 0 unwanted, len 8  0:00:00.317880
1*0?10* matches all wanted 0 unwanted, len 7  0:00:01.137309
```

It worked! It found a successful regex in a fraction of a second but then struggled for a comparatively long time to reduce it to the requested length. A benchmark function will show long it takes on average.

```
def test_benchmark(self):
    genetic.Benchmark.run(self.test_two_digits)
```

sample benchmark

average (seconds)	standard deviation
0.70	1.02

17.7. Performance improvement

Improving the performance of this genetic algorithm requires revisiting the `try..except` block in the fitness function, the one that detects invalid regular expressions. The following temporary code change will make it possible to measure how often that happens.

```
total = invalid = 0

def get_fitness(genes, wanted, unwanted):
    global total
    total += 1
    ...
    except re.error:
        global invalid
        invalid += 1
        return Fitness(0, len(wanted), len(unwanted), length)
```

```
def find_regex(self, wanted, unwanted, expectedLength):
    ...
    self.assertTrue(not optimalFitness > best.Fitness)
    print("{0}% of {1} generated regexes were invalid".format(
        int(100 * invalid / total), total
    ))
```

Running the test a few times gets results like the following:

sample results

```
18% of 82018 generated regexes were invalid
21% of 57325 generated regexes were invalid
23% of 212732 generated regexes were invalid
25% of 216453 generated regexes were invalid
29% of 24124 generated regexes were invalid
34% of 2734 generated regexes were invalid
```

Ouch. A lot of opportunities to find an improvement are lost due to the generated regex being invalid. What could make the regex invalid? To find out, replace the measurement code with code that captures the details of the error in a global variable, with preference for the shortest example of the given error.

```

regexErrorsSeen = {}

def get_fitness(genes, wanted, unwanted):
    ...
    except re.error as e:
        key = str(e)
        key = key[:key.index("at position")]
        info = [str(e),
                "genes = ['{0}']".format("'", '".join(genes)),
                "regex: " + pattern]
        if key not in regexErrorsSeen or len(info[1]) < len(regexErrorsSeen[key][1]):
            regexErrorsSeen[key] = info
        return Fitness(0, len(wanted), len(unwanted), length)
    ...

```

Then print all the errors at the end of the test harness.

```

def find_regex(self, wanted, unwanted, expectedLength):
    ...
    for info in regexErrorsSeen.values():
        print("")
        print(info[0])
        print(info[1])
        print(info[2])

```

Now run the test to get the error samples:

sample errors

```

nothing to repeat at position 0
genes = ['?']
regex: ?

multiple repeat at position 2
genes = ['0', '?', '*']
regex: 0?*

```

Run the test a few times to see variations. There are two situations: The first is when the regex has a repeat-type meta character with no text before it. The other is when two repeat-type meta characters are adjacent. One solution is to repair the regex.

17.7.1. Regex repair

```
def get_fitness(genes, wanted, unwanted):
    pattern = repair_regex(genes)
    length = len(pattern)
    ...
```

The repair function can be built iteratively by running the test and adding code that detects and corrects the errors found. Eventually all the regexes can be repaired. Here's one possible implementation:

```
def repair_regex(genes):
    result = []
    f = repair_ignore_repeat metas
    for token in genes:
        f = f(token, result)
    return ''.join(result)
```

```
def repair_ignore_repeat metas(token, result):
    if token in repeatMetas:
        return repair_ignore_repeat metas
    result.append(token)
    return repair_ignore_repeat metas_following_repeat_or_start metas
```

```
def repair_ignore_repeat metas_following_repeat_or_start metas(token, result):
    last = result[-1]
    if token not in repeatMetas:
        result.append(token)
    elif last in startMetas:
        pass
    elif token == '?' and last == '?' and len(result) > 2 and result[-2] in repeatMetas:
        pass
    elif last in repeatMetas:
        pass
    else:
        result.append(token)
    return repair_ignore_repeat metas_following_repeat_or_start metas
```

Because the regex repair function does not change the original genes it must also be called from the display function.

```
def display(candidate, startTime):
    timeDiff = datetime.datetime.now() - startTime
    print("{0}\t{1}\t{2}".format(
        repair_regex(candidate.Genes), candidate.Fitness, str(timeDiff)))
```

The genetic algorithm now finds the solution much faster on average.

average (seconds)	standard deviation
0.30	0.63

A side effect of repairing the regex is the genes that are removed during the repair become latent. They can be activated if a mutation operator affects the gene to their left. This unexpectedly gives the genetic algorithm an additional tool.

17.8. Groups

The second test regex will require the use of the group-type meta characters (and) so support for those must be added.

```
startMetas = {'|', '('}
endMetas = {'|', ')'}
allMetas = repeatMetas | startMetas | endMetas
```

17.8.1. Repair

Now running `test_two_digits` produces examples of the next set of regex issues to repair:

sample errors

```
missing ), unterminated subpattern at position 0
genes = ['(']
regex: (

unbalanced parenthesis at position 0
genes = [')']
regex: )
```

The first issue can be solved by appending missing end-group `)` meta characters to the final regex.

```
def repair_regex(genes):
    result = []
    finals = [] ①
    f = repair_ignore_repeat metas
    for token in genes:
        f = f(token, result, finals) ②
    result.extend(reversed(finals)) ③
    return ''.join(result)
```

```
def repair_ignore_repeat_metas(token, result, finals):
    if token in repeatMetas or token in endMetas:
        return repair_ignore_repeat_metas
    if token == '(': ④
        finals.append(')')
    result.append(token)
    return repair_ignore_repeat_metas_following_repeat_or_start_metas
```

The second issue can be resolved by prefixing the final regex with the missing start-group `(` meta character - (6) in the code listing below.

```
def repair_ignore_repeat_metas_following_repeat_or_start_metas(token, result, finals):
    last = result[-1]
    if token not in repeatMetas:
        if token == '(': ⑤
            finals.append(')')
        elif token == ')':
            match = ''.join(finals).rfind(')')
            if match != -1:
                del finals[match]
            else:
                result[0:0] = ['('] ⑥
    result.append(token)
    elif last in startMetas:
    ...
```

17.8.2. New test

Once all the regex problems have been repaired, the new test can be added and run.

```
def test_grouping(self):
    wanted = {"01", "0101", "010101"}
    unwanted = {"0011", ""}
    self.find_regex(wanted, unwanted, 5)
```

sample result

```
01010101|0101() matches 1 wanted 0 unwanted, len 15 0:00:00
(0101)|01+ matches 2 wanted 0 unwanted, len 10 0:00:00.003005
(01??11?|010101)+ matches all wanted 0 unwanted, len 17 0:00:00.106251
(01|)+01 matches all wanted 0 unwanted, len 8 0:00:00.108257
(01)+ matches all wanted 0 unwanted, len 5 0:00:00.130313
```

Nice!

17.9. Character-sets

The next test regex will require the use of the character-set-type meta characters `[` and `]`. To support those first add them to global the meta variables.

```
startMetas = {'|', '(', '['}
endMetas = {')', ']', '}'}
```

As before, next run the existing tests to produce error samples.

sample errors

```
missing ), unterminated subpattern at position 0
genes = ['[', '*', ')', ']', '*', '0']
regex: ([])*0

unbalanced parenthesis at position 5
genes = ['[', '(', ']', '*', '0']
regex: [(]*0)

unterminated character set at position 0
genes = ['[']
regex: [
```

The first two are caused by the group completion code added in the previous section completing groups that begin or end inside a character-set.

17.9.1. Repair

```
def repair_regex(genes):
    result = []
    finals = []
    f = repair_ignore_repeat metas
    for token in genes:
        f = f(token, result, finals)
    if ']' in finals and result[-1] == '[': ①
        del result[-1]
    result.extend(reversed(finals))
    return ''.join(result)
```

```
def repair_ignore_repeat metas(token, result, finals):
    if token in repeatMetas or token in endMetas:
        return repair_ignore_repeat metas
    if token == '(':
        finals.append(')')
    result.append(token)
    if token == '[': ②
        finals.append(']')
        return repair_in_character_set
    return repair_ignore_repeat metas_following_repeat_or_start metas
```

```
def repair_ignore_repeat metas_following_repeat_or_start metas(token, result, finals):
    last = result[-1]
    if token not in repeatMetas:
        if token == '[': ③
            result.append(token)
            finals.append(']')
            return repair_in_character_set
        if token == '(':
            ...
```

```
def repair_in_character_set(token, result, finals):
    if token == ']':
        if result[-1] == '[':
            del result[-1]
        result.append(token)
        match = ''.join(finals).rfind(']')
        if match != -1:
            del finals[match]
        return repair_ignore_repeat metas_following_repeat_or_start metas
    elif token == '[':
        pass
    else:
        result.append(token)
    return repair_in_character_set
```

17.9.2. New test

```
def test_state_codes(self):
    wanted = {"NE", "NV", "NH", "NJ", "NM", "NY", "NC", "ND"}
    unwanted = {"N" + l for l in "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
                if "N" + l not in wanted}
    self.find_regex(wanted, unwanted, 11)
```

sample result

```
...
NM|NC|NV|ND|NJ|NE|NY|NH matches all wanted 0 unwanted, len 23 0:00:08.716123
NY|NE|NC|NH|NV?J*D*M* matches all wanted 0 unwanted, len 21 0:00:15.928484
NH|NE|NV|NC*D?J*Y?M? matches all wanted 0 unwanted, len 20 0:00:52.029423
N[D(ECYM??JVYHJD] matches all wanted 0 unwanted, len 17 0:01:51.952601
N[D(ECYM?JVHYJD] matches all wanted 0 unwanted, len 16 0:01:51.957615
N[DECYM?JJVHYD] matches all wanted 0 unwanted, len 15 0:01:51.987693
N[VJYM?HDCYED] matches all wanted 0 unwanted, len 14 0:01:52.168333
N[VMCDJYCHED] matches all wanted 0 unwanted, len 13 0:01:52.249548
N[VMCJYHED] matches all wanted 0 unwanted, len 11 0:01:52.254562
```

The genetic algorithm succeeds but it can take a long time to discover the character-set solution if it has already found a successful regex. That is because removing repeated items from a character-set, or moving wanted items into a character-set, usually only negatively impact the fitness at this stage. Conversely, those actions may not affect the fitness at all before a working solution is found. That means introducing a character-set that improves the fitness requires multiple sequential steps, that's not something genetic algorithms do well. They are much more successful when they can find ways to make incremental improvements. It would be much more likely to succeed if it had access to a character-set specific operator. That kind of mutation operator might only be useful to particular regexes.

17.9.3. Supporting custom operators

First add an optional parameter to the find function and append its contents to the array of mutation operators.

```
def find_regex(self, wanted, unwanted, expectedLength, customOperators=None):
...
    mutationOperators = [
...
    ]
    if customOperators is not None:
        mutationOperators.extend(customOperators)
```

Then define the character-set operator.

```
def mutate_to_character_set_left(genes, wanted):
    if len(genes) < 4:
        return False
    ors = [i for i in range(-1, len(genes) - 3)
            if (i == -1 or genes[i] in startMetas) and
                len(genes[i + 1]) == 2 and
                genes[i + 1] in wanted and
                (len(genes) == i + 1 or genes[i + 2] == '|' or genes[i + 2] in endMetas)]
    if len(ors) == 0:
        return False
    lookup = {}
    for i in ors:
        lookup.setdefault(genes[i + 1][0], []).append(i)
    min2 = [i for i in lookup.values() if len(i) > 1]
    if len(min2) == 0:
        return False
...

```

It finds all the two-character **wanted** strings that have a `|` meta character, or end-of-array, on both sides. If there are at least two that have the same first character, for example `MA|` and `|ME|` then they become candidates to be replaced with a character-set, i.e. `M[AE]`. Pick a candidate, add the character-set and remove the parts from the regex.

```
choice = random.choice(min2)
characterSet = ['|', genes[choice[0] + 1][0], '|']
characterSet.extend([genes[i + 1][1] for i in choice])
characterSet.append('|')
for i in reversed(choice):
    if i >= 0:
        genes[i:i + 2] = []
genes.extend(characterSet)
return True
```

Next use the new operator in the test.

```
...
    customOperators = [
        partial(mutate_to_character_set_left, wanted=wanted),
    ]
    self.find_regex(wanted, unwanted, 11, customOperators)
```

Now run the test again and it can usually find the regex in a few seconds. Additionally, the situation where it first finds the solution that does not use a character-set becomes rare, and when it does it occur the genetic algorithm can easily find the character-set solution.

sample result

```
NY|NM|NH*(V*(J|C*D*|E)) matches all wanted 0 unwanted, len 23    0:00:07.173342
NY|NM|NH*V*(J|C*D*|E)   matches all wanted 0 unwanted, len 21    0:00:07.252551
N[]EJ+YMCVDH+VV]       matches all wanted 0 unwanted, len 16    0:00:08.928685
N[]VYMCV)EVHJD]        matches all wanted 0 unwanted, len 15    0:00:08.997869
N[VMVC)HEJYD]          matches all wanted 0 unwanted, len 13    0:00:09.035971
N[VJVCHEMYD]           matches all wanted 0 unwanted, len 12    0:00:09.053016
N[JVCEHDMY]            matches all wanted 0 unwanted, len 11    0:00:09.183363
```

17.10. Repetition

The next regex will use a repetition-type meta token like `{2}` or `{2,}`. First add them to the meta global variable.

```
repeatMetas = {'?', '*', '+', '{2}', '{2,}'}
```

Run the existing tests and there should be no errors to correct.

17.10.1. New test

```
def test_even_length(self):
    wanted = {"00", "01", "10", "11", "0000", "0001", "0010", "0011", "0100",
              "0101", "0110", "0111", "1000", "1001", "1010", "1011", "1100",
              "1101", "1110", "1111"}
    unwanted = {"0", "1", "000", "001", "010", "011", "100", "101", "110", "111", ""}
    self.find_regex(wanted, unwanted, 10)
```

sample result

```
...
([1110]{2})?[10]{2} matches all wanted 0 unwanted, len 19 0:00:13.188051
([10]{2})?[10]{2} matches all wanted 0 unwanted, len 17 0:00:15.622529
([1101]{2}|11)+ matches all wanted 0 unwanted, len 15 0:00:24.155222
([110101]{2})+ matches all wanted 0 unwanted, len 14 0:00:26.304940
([01]{2})+ matches all wanted 0 unwanted, len 10 0:00:26.494444
```

The genetic algorithm is slow to find this regex and it can also stall if it finds this particular solution:

```
(00|10|01|11)+ matches all wanted 0 unwanted, len 14 0:00:12.010740
```

The following custom mutation operator resolves the problem. It finds all the cases where a `|` meta character has non-meta characters on both sides. If any are found it picks one and replaces the three genes with a character-set containing only the unique characters from the two non-meta character genes.

Example: `['00', '|', '01']` becomes `['|', '0', '1', '|']`.

```
def mutate_to_character_set(genes):
    if len(genes) < 3:
        return False
    ors = [i for i in range(1, len(genes) - 1)
            if genes[i] == '|' and
            genes[i - 1] not in allMetas and
            genes[i + 1] not in allMetas]
    if len(ors) == 0:
        return False
    shorter = [i for i in ors
                if sum(len(w) for w in genes[i - 1:i + 2:2]) >
                len(set(c for w in genes[i - 1:i + 2:2] for c in w))]
    if len(shorter) == 0:
        return False
    index = random.choice(ors)
    distinct = set(c for w in genes[index - 1:index + 2:2] for c in w)
    sequence = ['|'] + [i for i in distinct] + ['|']
    genes[index - 1:index + 2] = sequence
    return True
```

Then change the test to use the new mutation operator.

```
def test_even_length(self):
...
    customOperators = [
        mutate_to_character_set,
    ]
    self.find_regex(wanted, unwanted, 10, customOperators)
```

Run the test now and it can find the solution every time.

sample result

```
.....
([0*1000]{2})+ matches all wanted 0 unwanted, len 14 0:00:05.221497
([1000]{2})+ matches all wanted 0 unwanted, len 12 0:00:05.379950
([01][10])+ matches all wanted 0 unwanted, len 11 0:00:05.519291
([01]{2})+ matches all wanted 0 unwanted, len 10 0:00:06.595626
```

17.11. State codes

The final regex test in this chapter is to find a reasonably short regex for all 50 U.S. state codes.

```
def test_50_state_codes(self):
    wanted = {"AL", "AK", "AZ", "AR", "CA", "CO", "CT", "DE", "FL", "GA",
              "HI", "ID", "IL", "IN", "IA", "KS", "KY", "LA", "ME", "MD",
              "MA", "MI", "MN", "MS", "MO", "MT", "NE", "NV", "NH", "NJ",
              "NM", "NY", "NC", "ND", "OH", "OK", "OR", "PA", "RI", "SC",
              "SD", "TN", "TX", "UT", "VT", "VA", "WA", "WV", "WI", "WY"}
    unwanted = {a + b for a in "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
                 for b in "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
                 if a + b not in wanted} | \
               set(i for i in "ABCDEFGHIJKLMNOPQRSTUVWXYZ")
    customOperators = [
        partial(mutate_to_character_set_left, wanted=wanted),
        mutate_to_character_set,
    ]
    self.find_regex(wanted, unwanted, 120, customOperators)
```

sample result

```
RI|DE|VA|CO|SC|CA|PA|LA|SD|TX|WY|GA|WI|HI|M[IASDETNO]|N[CMDJYMEVH]|VT+|CT|TN|UT|WA|WV|FL|
A[RLZK]|K[YS]|O[RHK]|I[LNDA] matches all wanted 0 unwanted, len 117 2:44:17.252814
```

It produces a good result but doing so can take hours.

Now add the following custom mutation operator.

```
def mutate_add_wanted(genes, wanted):
    index = random.randrange(0, len(genes) + 1) if len(genes) > 0 else 0
    genes[index:index] = ['|'] + [random.choice(wanted)]
    return True
```

It shortcuts the process of getting all the wanted sequences into the regex by inserting a `|` meta character followed by a random wanted string. That allows the algorithm to quickly switch its focus to reducing the length of the regex.

```
def test_50_state_codes(self):
    ...
    customOperators = [
        partial(mutate_to_character_set_left, wanted=wanted),
        mutate_to_character_set,
        partial(mutate_add_wanted, wanted=[i for i in wanted]),
    ]
    self.find_regex(wanted, unwanted, 120, customOperators)
```

17.11.1. Run 2

Now the genetic algorithm can find a successful regex in a couple of minutes, but the new mutation operator makes it grow faster than it can be reduced. This results in very long regexes like the one below that then have to be reduced after the fact, a much slower process that can take tens of minutes.

```
|ILKYORNV||RI|WA|[[AK{2,}{2}MTSLCNV]]LCTDEVA|ME||VA|AK|UT|HI|LA|[FNJ]LANV|MO|AZ|FL|TX|||
GA|MA[SNCM]|AR||MA|MDLANVAZ|TN|MD|NE[HMSTN]SLD]X|PAN|SCI|[NH]|RIN|NEVHNV]VT|GA[J|CAJI||M
NUWY|CA|NC|WIVDENH]NC|WV|ME|MS|NE|VT|MT|ILLDE|IA|LAWILNV|WY|VA|PA|KY|UT|IL|WVTW|PCAMNNC|I
D|||AL|VAOK[UV]|NJ|MI|CT|OK|]|NJ|IN|CO[D|UI|MDA]ME]|UTMNC|NV|KS|NY|CO|WI|CAMT[LNCNNKSHA|
OH|||OWGA]CT+ND|SD|OHSDAAL|K|ARVTMAWICA[NYWVWIWYCOOHO|WIZLM|RINH*]ANDILMSN?TN||INAMSWV|MN
|WVNJ[OICM][|AVFL]WIMDI]A|N[CMEEDEH]|SC|DE|O[HR]    matches all wanted 0 unwanted, len 493
0:02:41.872429
```

The growth of the regex can be controlled through the introduction of a static configuration setting on the `Fitness` object.

```
class Fitness:
    UseRegexLength = False
    ...
```

Then update the comparison function.

```
def __gt__(self, other):
...
    if not success:
        return self.Length <= other.Length if Fitness.UseRegexLength else False
    return self.Length < other.Length
```

This works by controlling when the engine switches to a new genetic line. When the setting is `True` then, all other things being equal, the engine will only switch if the other regex is longer.

Finally, enable the setting in the test.

```
def test_50_state_codes(self):
    UseGeneLength = True
...
```

Now the genetic algorithm can find a the final regex in just a few minutes.

sample result

```
FL||LA|GA|HI|UT|CT|DE|OK|RI|A[LKRZ]|O[RH]|S[DC]|K[YS]]|I[NL]|PA|V[AT]|C[OA]|I[AD]|M[SETN
ODIA]|W[VIAY]|T[XN]|N[VCYJEMDH]    matches all wanted 0 unwanted, len 120 0:03:13.662164
```

Excellent! Unsurprisingly `test_state_codes` benefits from the use of this setting too.

17.12. Exercise

Notice in the final regex above that there are still characters that could be removed, for example in `|K[YS]]|`. Also, `CT` could be merged into `C[OA]`. Those would eventually be discovered by the genetic algorithm if a shorter goal length was used. However, there are other potential savings that it is unlikely to find. For example, `LA`, `GA`, and `PA` could be combined if there was a mutation operator for creating character-sets with a common ending letter. Try adding one.

This is a great project for experimentation and there are many more regex meta characters to play with.

17.13. Summary

This chapter introduced two new concepts. The first is the idea of repairing the chromosome, which is useful when some phenotype values are incompatible. The second concept is using the change in the number of genes to control chromosome growth.

17.14. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch17>

Chapter 18. Tic-tac-toe

The next project is to build a genetic algorithm that produces human-readable rules for playing [Tic-tac-toe](#) without losing.

A Tic-tac-toe game board has 9 squares in a 3 by 3 grid. In this project they are numbered as follows:

```
1 2 3
4 5 6
7 8 9
```

All the squares start empty. The players take turns placing a marker (X for the first player to move, O for the opponent) on an empty square with the goal of getting three of their own markers in a single row, column or diagonal before their opponent does.

That should be enough information to build the game portion. If you've worked through the projects in the previous chapters you know what comes next so I'm not going to go over every line of code. Instead, I'm going to point out a couple of new things. If you get stuck, or just want to start with working code, check the code sample for this chapter.

18.1. Genes

The goal is to end up with a relatively short list of rules that a child could use to play the game. That's why the rules need to be human-readable. That also eliminates index specific rules like **index 1 is EMPTY and index 3 has an OPPONENT piece**. That is not the way we teach children to play this game.

There are two gene categories: location and content.

Location genes select a move based on the location of the empty square. For example: is it a corner, edge, the center, in the top/middle/bottom row, left/middle/right column, in a diagonal, etc.

Content rules look at the content of the row/column/diagonal containing the empty square. Examples might resemble the following, depending on how much time you want to spend on grammar:

```
its ROW has 2 OPPONENT pieces
its COLUMN has 1 EMPTY square
its DIAGONAL has NONE of MY pieces
```

Move selection should work like this: The engine passes the current set of empty squares to the first rule, or more precisely filter, in the gene sequence. If a rule eliminates all the remaining empty board positions, skip it. Otherwise use its output as the input for the next filter in the gene sequence. Evaluation stops when there are no more rules, or a rule reduces the set of empty squares to one. When only one square remains that is where the GA's piece should be placed.

18.2. Fitness

Calculate the fitness by playing all possible games given 1) the GA makes the first move, and 2) the opponent makes the first move. Whenever it is the opponent's turn all possible moves should be played out unless there is a winning move for the opponent, in which case it wins. This can be done depth-first or breadth-first as you prefer. If at any point there are empty squares and the GA cannot provide a move, it loses that game.

Count the number of times the GA wins, loses, and ties. Then use those counts in a `Fitness` object comparison function like the following:

ticTacToeTests.py

```
def __gt__(self, other):
    if self.PercentLosses != other.PercentLosses:
        return self.PercentLosses < other.PercentLosses

    if self.Losses > 0:
        return False

    if self.Ties != other.Ties:
        return self.Ties < other.Ties
    return self.GeneCount < other.GeneCount
```

Using the *percentage* instead of the *absolute number* of losses allows the number of games played to vary. You should try the other way too so you can see how the behavior changes.

18.3. Mutation and Crossover

Use adaptive mutation rules similar to the ones in the previous chapter.

For crossover, combine the first half of the parent, with the last half of the donor, then call mutate. Calling mutate afterward is very important. It helps to prevent the pool from converging to a single gene sequence.

```
def fnCrossover(parent, donor):
    child = parent[0:int(len(parent) / 2)] + donor[int(len(donor) / 2):]
    fnMutate(child)
    return child
```

18.4. Results

It takes between 10 and 30 minutes for my implementation to produce a set of rules that never lose and there is a lot of duplication in that first success:

```
its ROW has 2 MINE
its DIAGONAL has 2 MINE
its ROW has 2 OPPONENT
is in CENTER
its DIAGONAL has 2 OPPONENT
its COLUMN has 2 OPPONENT
its COLUMN has 0 OPPONENT
is in DIAGONAL
its ROW has 2 MINE
its ROW has 0 MINE
its DIAGONAL has 1 OPPONENT
is a CORNER
its ROW has 1 OPPONENT
its ROW has 2 OPPONENT
its COLUMN has 1 OPPONENT
is in RIGHT COLUMN
is in BOTTOM ROW
its ROW has 0 MINE
its DIAGONAL has 0 OPPONENT
its DIAGONAL has 1 OPPONENT
is in BOTTOM ROW
0.0% Losses (0), 35.6% Ties (289), 64.4% Wins (522), 21 rules
0:21:41.420539
```

The final set of rules, however, has no duplication and not only guarantee we never lose, they also help to avoid tie games when the opponent is experienced. It generally takes 1-2 hours to plateau at 14 rules and 135 ties.

```
its DIAGONAL has 2 MINE      ①
its ROW has 2 MINE           ②
its COLUMN has 2 MINE        ③
its COLUMN has 2 OPPONENT    ④
its ROW has 2 OPPONENT       ⑤
its DIAGONAL has 0 MINE
its COLUMN has 0 MINE
its ROW has 0 OPPONENT
is in MIDDLE COLUMN
is in TOP ROW
its DIAGONAL has 1 OPPONENT
its COLUMN has 1 OPPONENT
is a CORNER
is in LEFT COLUMN
0.0% Losses (0), 21.3% Ties (135), 78.7% Wins (498), 14 rules
2:07:36.146149
```

Rules 1, 2 and 3 detect the winning move for us. Rules 4 and 5 handle blocking the opponent from winning. The rest of the rules work to minimize the number of tie games.

There's nothing clever about these rules and they perform well. They are not the only generic rules possible however. For example, adding rules that check for row-, column- and diagonal-opposites of the empty square enables the genetic algorithm to find a set of rules that result in even fewer ties. A row-opposite for a square is the square at the other end of the row and a column-opposite is the square at the other end of the column.

```
its ROW has 2 MINE           ①
its DIAGONAL has 2 MINE      ②
its COLUMN has 2 MINE        ③
its ROW has 2 OPPONENT       ④
its COLUMN has 2 OPPONENT    ⑤
is in CENTER                 ⑥
its ROW has 0 OPPONENT
its DIAGONAL has 0 MINE
ROW-OPPOSITE is OPPONENT
is in BOTTOM ROW
its DIAGONAL has 1 OPPONENT
is a CORNER
its COLUMN has 1 MINE
its COLUMN has 1 OPPONENT
is in LEFT COLUMN
0.0% Losses (0), 19.6% Ties (127), 80.4% Wins (522), 15 rules
```

As before, rules 1, 2 and 3 detect the winning move for us. Rules 4 and 5 handle blocking the opponent from winning. That means the center (rule 6) is the most valuable square on the board, as many of us learned as children.

18.5. Tournament selection

Tic-tac-toe is a simple enough game that we have the ability play out every possible game in order to evolve an optimal solution. Many other games have so many potential moves that playing all possible games is not an option. That means we need a way to get a good set of rules with partial knowledge. One way to do that is through tournament selection.

In tournament selection a generation of gene sequences (rules in this problem) is created. Each is then tested against every other gene sequence in that generation to get a fitness value. The gene sequences are then ranked by their fitness value and those with the best fitness values become the parents used to populate the next generation through crossover and mutation. Their children form about 90 percent of the next generation. The remainder could be copies of the parents (this is called *elitism*), mutated parents, randomly generated, or some combination there-of.

Because successive generations are based on the previous one the pool can rapidly lose diversity. This can cause the genetic algorithm to become stuck at a local minimum or maximum. That makes it doubly important to mutate the children produced through crossover. A small number of randomly generated gene sequences can be added to the pool each generation as well but this is less effective as a random sequence is unlikely to compete well against a highly evolved one.

18.5.1. Implementation

The tournament implementation will be added to the genetic module to make it available for other projects.

First add an `enum` that can be used like an integer for array indexes.

genetic.py

```
from enum import IntEnum
...
class CompetitionResult(IntEnum):
    Loss = 0,
    Tie = 1,
    Win = 2,
```

The tournament function first populates the parent pool and creates a convenience function for getting a parent's sort key.

```
def tournament(generate_parent, crossover, compete, display, sort_key, numParents=10,
max_generations=100):
    pool = [[generate_parent(), [0, 0, 0]] for _ in range(1 + numParents * numParents)]
    best, bestScore = pool[0]

    def getSortKey(x):
        return sort_key(x[0], x[1][CompetitionResult.Win], x[1][CompetitionResult.Tie],
x[1][CompetitionResult.Loss])
```

Next each gene sequence plays against every other gene sequence both as first-to-move and as the opponent, keeping track of wins, losses and ties.

```
generation = 0
while generation < max_generations:
    generation += 1
    for i in range(0, len(pool)):
        for j in range(0, len(pool)):
            if i == j:
                continue
            playera, scorea = pool[i]
            playerb, scoreb = pool[j]
            result = compete(playera, playerb)
            scorea[result] += 1
            scoreb[2 - result] += 1
```

The gene sequences are then sorted and display is called if an improvement is found.

```
pool.sort(key=getSortKey, reverse=True)
if getSortKey(pool[0]) > getSortKey([best, bestScore]):
    best, bestScore = pool[0]
    display(best, bestScore[CompetitionResult.Win],
            bestScore[CompetitionResult.Tie],
            bestScore[CompetitionResult.Loss], generation)
```

Finally, a new generation is created by crossbreeding the best N gene sequences. They are also included in the new generation along with 1 additional randomly generated gene sequence. When the maximum number of generations have completed, the function returns the best gene sequence that was seen.

```
parents = [pool[i][0] for i in range(numParents)]
pool = [[crossover(parents[i], parents[j]), [0, 0, 0]]
        for i in range(len(parents))
        for j in range(len(parents))
        if i != j]
pool.extend([parent, [0, 0, 0]] for parent in parents)
pool.append([generate_parent(), [0, 0, 0]])
return best
```

There's nothing special about the usage of this function so I won't repeat it here - see `test_tournament` in the linked code file at the end of the chapter.

It takes a little over a minute to produce a set of rules that never loses *against the other rules in its generation*. That's the main caveat, the result is only as good as the sample size. The result improves if more parents are used since $O(n^4)$ games are played in each generation but that can take quite a while.

sample result with 13 parents and 100 generations

```
-- generation 73 --
  its ROW has 2 OPPONENT
  its COLUMN has 2 OPPONENT
  its COLUMN has 2 MINE
  its ROW has 2 MINE
  its DIAGONAL has 0 OPPONENT
  is in MIDDLE COLUMN
  its COLUMN has 0 OPPONENT
  is in LEFT COLUMN
  is a CORNER
  is in TOP ROW
0.0% Losses (0), 45.0% Ties (152), 55.0% Wins (186), 10 rules
0:16:18.756137
```

18.6. Summary

This chapter introduced a new method of sampling and recombining gene sequences to find a good combination. One potential usage is to quickly find a starting place for a slower, more comprehensive survey of the problem space.

18.7. Final Code

The final code for this chapter is available from:

<https://github.com/handcraftsman/GeneticAlgorithmsWithPython/tree/master/ch18>

The package is also published on PyPI as `genetical`. This means it can be installed via `pip install genetical`. If you go that route you need to change the import statement to `from genetical import genetic`.

Afterward

This book has given you a solid introduction to genetic algorithms. There is still a lot more to learn on this topic but you now know enough to teach yourself, and that will lead to true mastery. For your next step you have several options, they include:

- use the library from this book to explore problems in your field of expertise,
- switch to a different Python-based library and repeat some of the experiments in order to spin up your knowledge of that library,
- learn about another machine learning tool like decision trees or neural networks.

Good luck!

Clinton Sheppard

p.s.

This book is available in paperback (<http://amzn.to/2gJGUin>), Kindle Edition (<http://amzn.to/2fPKunf>), and PDF (<http://bit.ly/GAPython>) formats.