BINDING ENERGY CURVE BY PYTHON USING SEMI EMPIRICAL FORMULA

BY

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CERTIFICATE

This is to certify that the project work entitled ‘Binding energy curve by python using semi empirical formula’ the bona fide work done by SREESHNA M sixth semester GOVT. BRENNEN COLLEGE, THALASSERY in partial fulfilment of requirements for the award of degree of Bachelor of science in physics of Kannur university.

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DECLARATION

It is to certify that the work presented in this report is based on the original work done by me under the guidance of Assistant Professor Rajeev P, Department of physics Govt. Brennen college and has not been included in any other project reports submitted for the award of any other degree.

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ABSTRACT

Liquid drop model of nucleus is used to calculate the binding energy of every stable nuclei, using python language binding energy per nucleon graph is plotted by summing each energy term in Semi empirical formula.

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2. Atomic stability
3. Nuclear stability
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INTRODUCTION

Modern nuclear physics is a well-developed branch of physical science, with wide-ranging applications of its results in engineering and industry. At the same time, the development of a consistent theory of nuclei and nuclear processes presents certain problems.

It is well known that the most important aim of nuclear physics is the study of nuclear structure and the explanation of properties on the basis of the interaction between nucleons which constitute nuclei. Difficulties of a modern theory of the nucleus are caused by both an insufficient knowledge of nuclear interactions and the multi particle character of nuclear systems.

Experimental data on nuclear interactions do not contradict the hypothesis of the pair character of nuclear forces. However, the absence of rigorous methods of calculations of many particle nuclear systems with strong interaction makes it necessary to use macroscopic nuclear models to describe particular nuclear properties. The development of nuclear models has led to considerable progress in the understanding of atomic nuclei.

Nuclear models attempt to explain why certain combinations of protons and neutrons are stable while others show radioactivity. Liquid drop model is one of them. The theory underlying the model provides excellent estimates of average properties of nuclei.

ATOMIC NUCLEUS

Atom contains a very small nucleus composed of positively charged protons and uncharged neutrons, surrounded by a much larger volume of space containing negatively charged electrons. The nucleus contains the majority of an atom's mass because protons and neutrons are much heavier than electrons. Protons and neutrons, collectively called nucleons, are packed together tightly in a nucleus, With a radius of about 10^{-15} meters, a nucleus is quite small compared to the radius of the entire atom, which is about 10^{-10} meters.

Elements are classified based on the number of protons nucleus contain. The number of neutrons in the nuclei of the same element may vary. The species that contain same number of protons but different number of neutrons are called isotopes. The chemical properties exhibited by an atom are determined by its electron cloud, which in turn is determined by the proton number. Therefore isotopes have identical nature but their masses varies.

The nucleus is held together by the strong force, one of the four basic forces in nature. This force between the protons and neutrons overcomes the repulsive electrical force that would otherwise push the protons apart.

Nuclear properties are heavily influenced by both proton and neutron number. The stability of nucleus is determined by the combination of nucleons. So all isotopes of same element may not be stable. Unstable nuclei have a tendency to change their nuclear composition to attain stable nuclear configuration. The phenomena occur without any external simulation and are called radioactive decay.

NUCLEAR STABILITY

Several factors are to be taken to account while discussing stability of nucleus

1. Neutron-proton ratio

2. Atomic mass

3. Even or odd number protons and neutrons

4. Binding energy per nucleon

Lighter nuclei tend to have same number of neutrons and protons. As atomic number increases, more number of neutrons are required to stabilize nucleus since coulomb repulsion between protons also increases. Thus heavier atoms tend to have higher neutron-proton ratio.

Nuclei also tend to unstable at higher masses, as strong nuclear force is extremely short ranged and is unable to hold together the neutrons and protons required to for a stable atoms.

Nucleus tends to more stable if they contain even numbers of protons and neutrons. Odd-even pair and even-odd pair are less stable. Only five known stable nuclei contain odd-odd pair of neutrons and protons. This is one unpaired nucleon, while in odd-odd nuclei there are two. This will decrease their stability.

Binding energy is defined as the energy released while a nucleus is formed from its constituent nucleons. This is equal to the energy required to separate the nucleons into nucleons. The decrease in energy is reflected in the mass of nuclei which is always less than sum of mass of all the protons and neutrons it contains. The difference is termed as mass defect. The binding energy is equal to the energy equivalent to mass defect as defined by the mass-energy equivalence. The nucleus is more stable when the binding energy per nucleon is higher.

LIQUID DROP MODEL

The liquid drop model was proposed by George Gamov in 1929. Nucleons were assumed to be interacting with only neighbouring nucleons. If they were fixed in their position like atoms in a solid, there would have been vibrations, and it was calculated that atom would not be stable. So nucleons were said to be free to move around in the nucleus like molecules in a liquid. The binding energy was then equal to the energy released from the interactions between nucleons. In this way, five types of interaction energies were calculated.

Volume energy

In range of around 1 fm, nuclear forces are attractive. Thus when two nucleons interact through strong nuclear force, a net release in energy is resulted.

If U is the total energy released due to such a binding of two nucleons, it can thought of as each nucleons releasing ½U equivalent of energy. Since strong force does not differentiate between protons and neutrons, interactions between different types of nucleon does not result in release in different energies.

Both nucleons and the nucleus are then approximated as spherical in shape. A sphere can have a maximum of twelve neighbours. Therefore a single nucleon releases a total energy 6U from interacting with the twelve neighbouring nucleons. So in a closely packed nucleus having A number of nucleons, where A is the mass number of the nucleus, the total energy released is 6UA.

Thus the energy released from interaction of nucleons is directly proportional to the mass number of nucleons. This is called the volume energy, since it is proportional to the volume.

$$E_V = a_1 A$$

Where a_1 is a constant.

Surface energy

Since nucleus behaves as drop of liquid, it exhibits characteristics like surface tension. Presence of surface tension can increase the net energy of the drop. In a nucleus, the identical energy term is called surface energy.

While calculating volume energy term, it was assumed that all the atoms have twelve neighbours. This is false in the case of surface nucleons, thus the net energy released is less than the volume energy. Surface energy is proportional to surface area, which is proportional to square of radius of nucleus. Since volume is directly proportional to mass number and to the cube of radius, a proportionality relation can be worked out between mass number and surface energy. Thus surface energy term is found out to be,

$$E_S = -a_2 A^{\frac{2}{3}}$$

Where a_2 is a constant and negative sign indicates that surface energy decreases the binding energy.

Coulomb energy

Since protons are positively charged there is no coulomb repulsion between them. If A is the mass number, i.e. there are Z protons, then the total number of interactions are Z (Z-1). Thus resultant potential energy is given by,

$$E_C = -\frac{Z(Z-1)}{2}\frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]$$

Where $\left[\frac{1}{r}\right]$ is the average of inverse of distance between two nucleons in the nucleus and is proportional to radius of nucleus. Therefore,

$$E_C = -a_3\frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

If two atoms have same mass number but one has more neutrons then coulomb energy term is less negative as average radius increase. Thus for heavier atoms increase in number of neutron for stabilizing the nucleus.

Asymmetry energy

Although more neutrons can reduce coulomb repulsion, the increase in difference between neutron and proton can decrease the total binding energy. This occurs since protons and neutron are arranged in energy levels separately. Thus if all the nucleons in an atom are protons or neutrons only, the nucleons will reach higher energy levels as compared to an atom with same mass number and equal number of protons and neutrons.

The energy due to the difference in energy can be calculated as

$$E_a = -a_4\frac{(A-2Z)^2}{A}$$

Pairing energy

In the nucleus proton-proton and neutron-neutron pair tend to occur, which release energy and thus stabilize the atom. Therefore atoms with even number of protons and even number of neutrons are most stable with no unpaired nucleon.

If atom has odd-odd or even-odd neutron and proton, then there is one unpaired nucleon, and it is less stable.

Odd-odd nuclei are the least stable with two unpaired nucleons. In fact five odd-odd nuclei are known to be stable.

Pairing energy term is given by,

$$E_P = (\pm,0)\frac{a_5}{A^k}$$

Coefficient is 0 for odd-even and even-odd nuclei, positive for even-even and negative for odd-odd nuclei.

Thus the total binding energy is given by,

$$E = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{z(z-1)}{A^{\frac{1}{3}}} - a_4 \frac{(A-2Z)^2}{A}(\pm,0)\frac{a_5}{A^k}$$

The graph of binding energy per nucleon vs. mass number graph is given below.

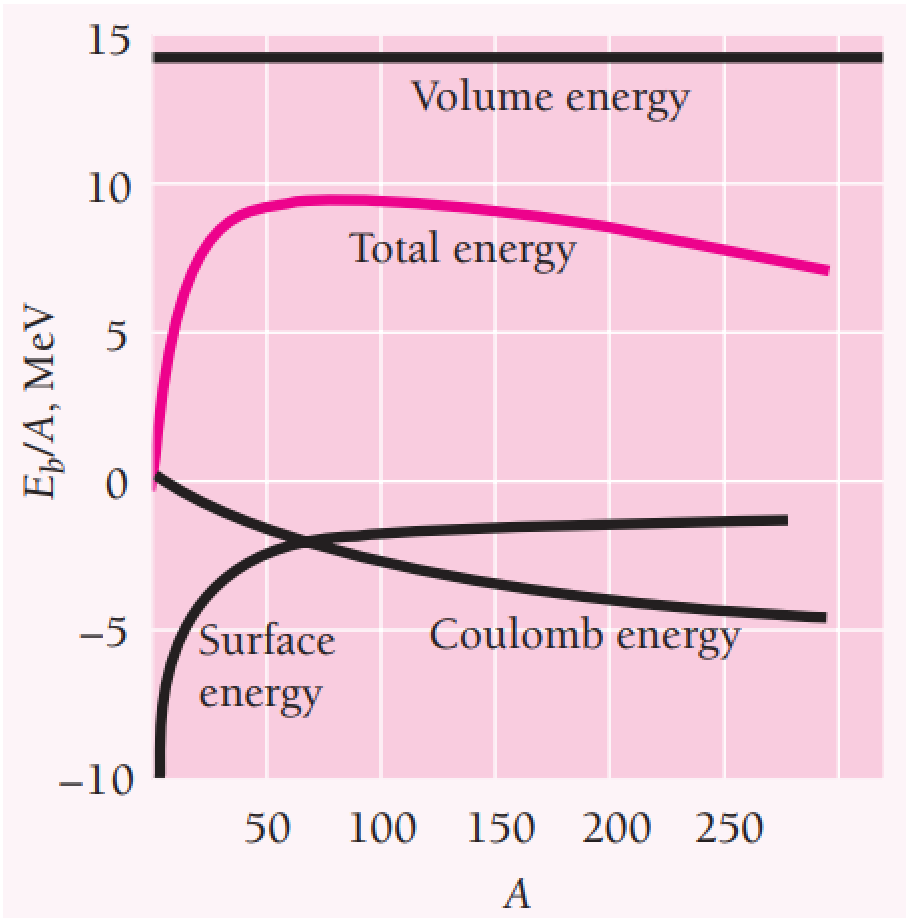


Figure 1.1 The binding energy per nucleon is the sum of the volume, surface and coulomb energy.

PLOTTING GRAPH WITH PYTHON

To plot a graph using python, a file is created in text format, containing data regarding the atomic and mass number. Data is given below.

ATOMIC NUMBER(Z)	ELEMENT	SYMBOL	MASS NUMBER(A)
1	Hydrogen	H	1,2
2	Helium	He	3,4
3	Lithium	Li	6,7
4	Beryllium	Be	9
5	Boron	B	10,11
6	Carbon	C	12,13
7	Nitrogen	N	14,15
8	Oxygen	O	16,17,18
9	Fluorine	F	19
10	Neon	Ne	20,21,22
11	Sodium	Na	23
12	Magnesium	Mg	24,25,26
13	Aluminium	Al	27
14	Silicon	Si	28,29,30
15	Phosphorus	P	31
16	Sulphur	S	32,33,34,36
17	Chlorine	Cl	35,37
18	Argon	Ar	36,38,40
19	Potassium	K	39,41
20	Calcium	Ca	40,42,43,44,46,48
21	Scandium	Sc	45
22	Titanium	Ti	46,47,48,49,50
23	Vanadium	V	51
24	Chromium	Cr	48,50,52,53,54
25	Manganese	Mn	55
26	Iron	Fe	54,56,57,58

27	Cobalt	Co	59
28	Nickel	Ni	58,60,61,62,64
29	Copper	Cu	63,65
30	Zinc	Zn	64,66,67,68,70
31	Gallium	Ga	69,71
32	Germanium	Ge	70,72,73,74,76
33	Arsenic	As	75
34	Selenium	Se	74,76,77,78,80,82
35	Bromine	Br	79,81
36	Krypton	Kr	78,80,82,83,84,86
37	Rubidium	Rb	85
38	Strontium	Sr	84,86,87,88
39	Yttrium	Y	89
40	Zirconium	Zr	90,91,92,94,96
41	Niobium	Nb	93
42	Molybdenum	Mo	92,94,95,96,97,98,100
43	Technetium	Tc	99
44	Ruthenium	Ru	96,98,99,100,101,102,104
45	Rhodium	Rh	103
46	Palladium	Pd	102,104,105,106,108,110
47	Silver	Ag	107,109
48	Cadmium	Cd	106,108,110,111,112,114,116
49	Indium Tin	In	113
50	Tin	Sr	112,114,115,116,117,118,119,120,122,124
51	Antimony	Sb	121,123
52	Tellurium	Te	120,122,124,125,126,128,130
53	Iodine	I	131
54	Xenon	Xe	124,126,128,129,130,131,132,134,136
55	Cesium	Cs	133
56	Barium	Ba	130,132,134,135,136,137,138

57	Lanthanum	La	138
58	Cerium	Ce	136,138,140
59	Praseodymium	Pr	141
60	Neodymium	Nd	142,143,146,148,150
61	Promethium	Pm	145
62	Samarium	Sm	150,152,154
63	Europium	Eu	151,153
64	Gadolinium	Gd	154,155,156,157,158,160
65	Terbium	Tb	159
66	Dysprosium	Dy	158,160,161,162,163,164
67	Holmium	Ho	165
68	Erbium	Er	162,164,166,167,168,170
69	Thulium	Tm	169
70	Ytterbium	Yb	168,170,171,172,173,174,176
71	Lutetium	Lu	175,176
72	Hafnium	Hf	176,177,178,179,180
73	Tantalum	Ta	181
74	Tungsten	W	180,182,183,184,186
75	Rhenium	Re	185
76	Osmium	Os	184,186,187,188,189,190,192
77	Iridium	Ir	191,193
78	Platinum	Pt	192,194,195,196,198
79	Gold	Au	197
80	Mercury	Hg	196,198,199,200,201,202,204
81	Thallium	TI	203,205
82	Lead	Pb	204,206,207,208,210,214
83	Bismuth	Bi	209

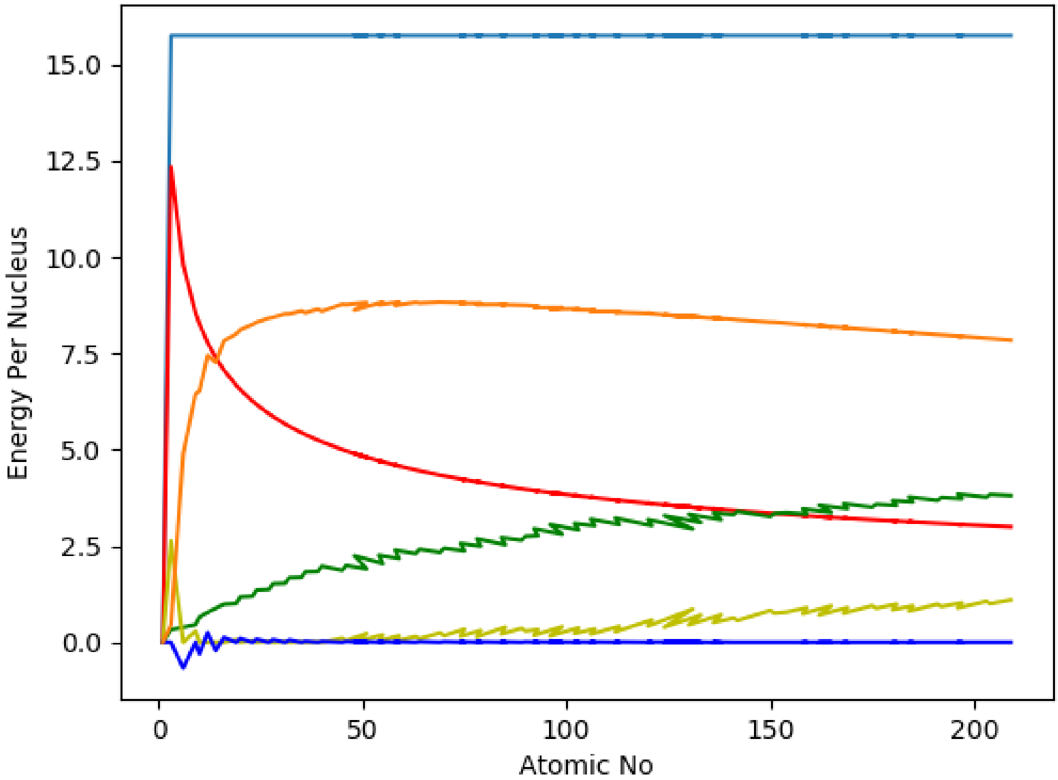
Using this data, a program is written to plot various energy terms in semi-empirical formula.

Coefficients of terms in semi empirical are calculated by fitting the equation with observed values of binding energy per nucleons. There are several set of values depending of the method of fitting. The power term is also is calculated empirically. Wapstra model is chosen as it is more recent and give result closer to actual one.

The binding energy of hydrogen is set to zero.

	Least squares fit 1	**Least squares fit 2**	**Rohlf**	**Wapstra**
$\boldsymbol{a_1}$	15.8	15.76	15.75	14.1
$\boldsymbol{a_2}$	18.3	17.81	17.8	13
$\boldsymbol{a_3}$	0.714	0.711	0.711	0.95
$\boldsymbol{a_4}$	23.2	23.702	23.7	19
$\boldsymbol{a_5}$	12	34	11.18	33.5
$\boldsymbol{k}$	-1/2	-3/4	-1/2	-3/4

GRAPH



- Volume energy
- Coulomb energy
- Surface energy
- Asymmetry energy
- Pairing energy
- Total energy

MERITS AND DEMERITS OF LIQUID DROP MODEL

Liquid drop model gives fairly accurate values for heavier nuclei. It explains the spherical shape of most nuclei. It helps to predict the nuclear binding energy and also to assess how much is available for consumption. Liquid drop model explains the stability of the nuclei. It explains the radioactive phenomenon and the phenomenon of artificial radioactivity. Nuclear fission can be understood on the basis of this model.

But it show deviations in case of light nuclei, like H, He, etc. Liquid drop model assumes that nucleus is perfectly spherical. This become more valid as mass number increases. The calculated value and experimental value of binding energy per nucleon is very different of lighter nuclei. The binding energy per nucleon of hydrogen nuclei, which is fact the single proton is non-zero according to the formula and has set to zero while plotting the data.

Semi empirical formula was also not able to explain the unusual stability of certain nuclei. It was found that the atom containing certain number of protons and neutrons are more stable. These numbers, which are 2,8,20,28,50,82 and 126, are called magic numbers.

Another model, based on the assumption that nucleon interact not with adjacent nucleon but with a general force field, called shell model of nucleus was able to account for magic numbers.

CONCLUSION

The graph plotted gives good resemblance with experimental plots. The main limitations are regarding lighter nuclei and nuclei with magic number of nucleon.

REFERENCE

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