SDSU | San Diego State University

ME 314 Spring 2024 Student Design Project

Department of Mechanical Engineering

Conceptual Torsion Bar Design

Author: Sean A Hedgecock
Professor Richard Ayala
Monday/Wednesday - Section 03
Submission: Friday, March 29th, 2024

Table of Contents

Table of Tables

Table of Figures

Introduction to the Objective

The design focuses on integrating a steel torsion bar into a vehicle's suspension to enhance wheel and tire dynamics. This bar is connected to an axle and a lever arm, enabling the system to adapt to various road conditions. Stability is achieved by anchoring the torsion bar shaft to an aluminum plate with a flanged coupling, ensuring torque transfer through a spline setup. The system's resilience is further enhanced by a Symmco bearing within an aluminum lug, secured with steel bolts to maintain assembly integrity despite operational stresses. The torsion bar's hollow design reduces weight without compromising functionality, allowing for an adjustable angular deflection based on load, ensuring force transmission. Material selection for the torsion bar and axle shaft focuses on safety and performance, requiring calculations for wall thickness, spline stress analysis, and a fatigue safety factor. Additionally, the design includes considerations for bearing support strength and the selection of appropriate materials to withstand the demands of varying applications. This approach prioritizes safety, cost-efficiency, and performance in the suspension system's development.

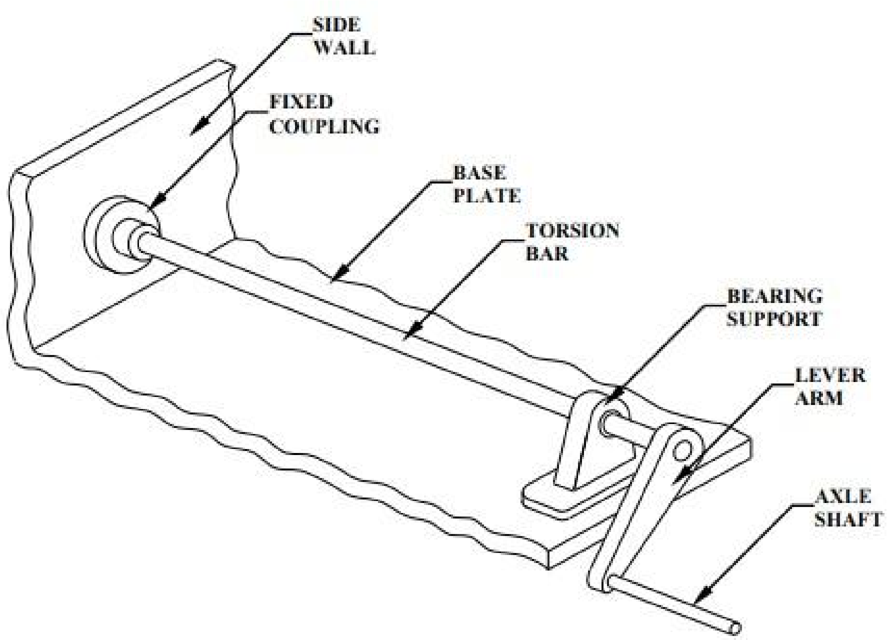

Figure 1: A Schematic of the Torsion Bar Suspension System

Torsion Bar Shaft Design

The torsion bar's hollow shaft was selected for its dimensions, featuring an outer diameter of 1.375 inches and an inner diameter of 0.6 inches. This shaft is fabricated from AISI 4140 Q&T steel, treated at 205℃(400℉), known for its high strength and toughness. The design specifications for the torsion bar included a total length of 135 inches, with each end including a 1 inch external spline to facilitate connections. The critical design parameters, including the minimum and maximum torque capacities, were found based on the bar's angular deflections and the shear forces. These forces were determined through the development of a free body diagram for an accurate assessment of stress distribution along the bar. Utilizing these shear force calculations, the safety factor against shearing was calculated to ensure the designs integrity under load.

The dimensions of the design are depicted in Figure 2. The bending moment and shear force diagrams, essential for understanding the shaft's behavior under load, are depicted in Figure 3 and Figure 4, respectively. The complete overview of loading conditions, stress responses, and safety factors are placed in Table 1. The design adheres to the stated safety factor requirement for torsional stress, set at a minimum of 1.15, affirming its integrity and reliability for the application.

Table 1: Overview of the Torsion Bar Calculations

Max Applied Torque [T_{max}]	**Min Applied Torque** [T_{min}]	**Max Shear Stress** [τ_{max}]	**Min Shear Stress** [τ_{min}]	**FOS [Max]**	**FOS [Min]**
$2,551.9\ in \cdot lb$	$510.4\ in \cdot lb$	$5,187.6\ psi$	$1,037.5\ psi$	22.9	114.7

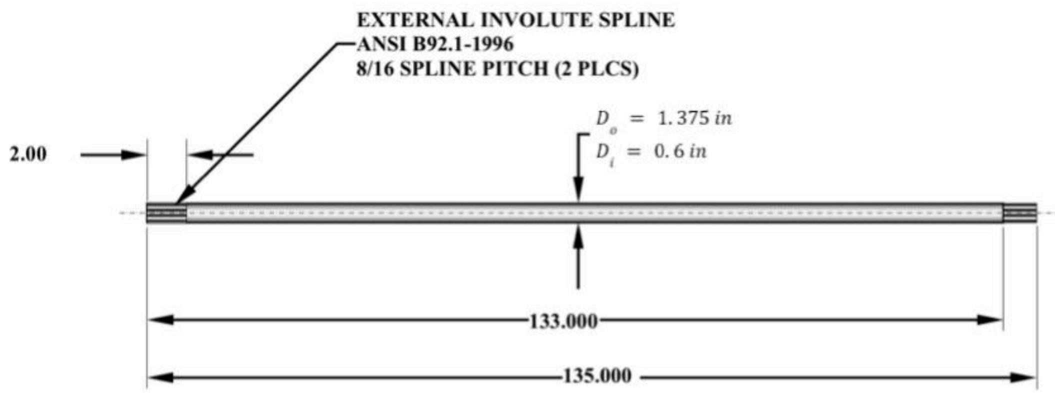

Figure 2: Dimensions of Torsion Bar

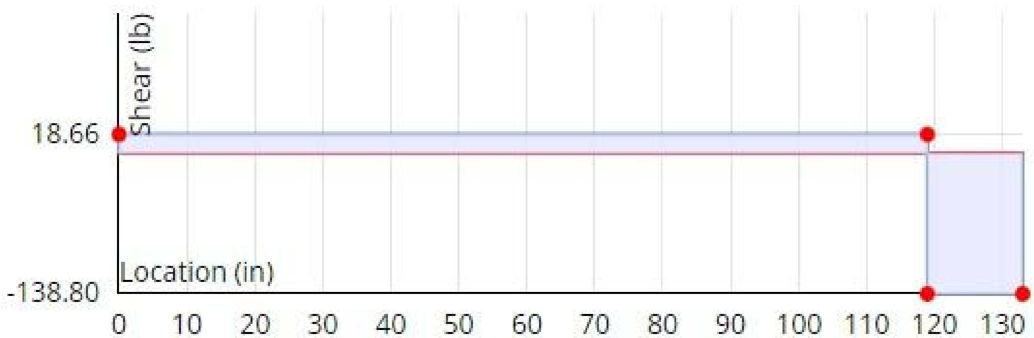

Figure 3: Shear Force Diagram of the Shaft

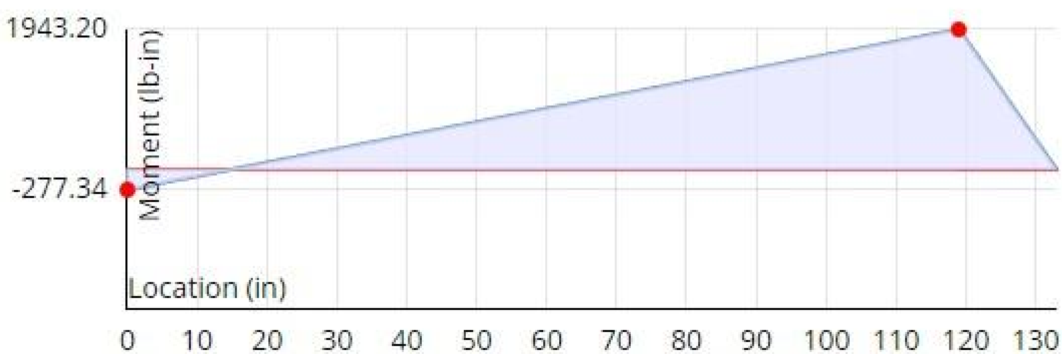

Figure 4: Bending Moment Diagram of the Shaft

Shaft Spline Design and Analysis

The shaft is outfitted with external involute splines at both ends, each extending two inches. These splines facilitate pivotal connections for the shaft: one end links to the coupling, while the opposite end transmits torque from the axle load. Table 2 displays the results of the analysis focusing on the one-inch segment of spline that engages with the lever arm on the non-fixed end.

Table 2: Spline Design

Spline	Number of Teeth	Pitch Diameter	Face Width	Factor of Safety
ANSI B92.1 8/16	10	$1.250\ in$	$1\ in$	1.334

To evaluate the spline's performance, the shear stress at the spline's location on the shaft was calculated, as well as the shear stress on the spline teeth and the compressive force they experience. These assessments were based on the maximum torque experienced by the shaft. Table 3 documents the findings, including shear stresses and the associated safety factors for the spline.

Table 3: Spline Stress Analysis Results

Shaft Shear Stress	**FOS**	**Shear Stress in Teeth**	**FOS**	**Compressive Stress**	**FOS**
$11,887.5\ psi$	1.334	$13,065.7\ psi$	1.214	$3,445.6\ psi$	41.44

Bearing Support Lug Design

The bearing support assembly consists of a lubricated Symmco SS-4456-16 sleeve bearing housed in a single aluminum lug, secured to a 0.375-inch thick plate by two steel bolts with a 0.25-inch diameter. This setup enables the torsion bar shaft to rotate freely. The lug, cast from 333-T5 aluminum and factoring in a casting strength reduction of 0.45, has its dimensions presented in Figure 5.

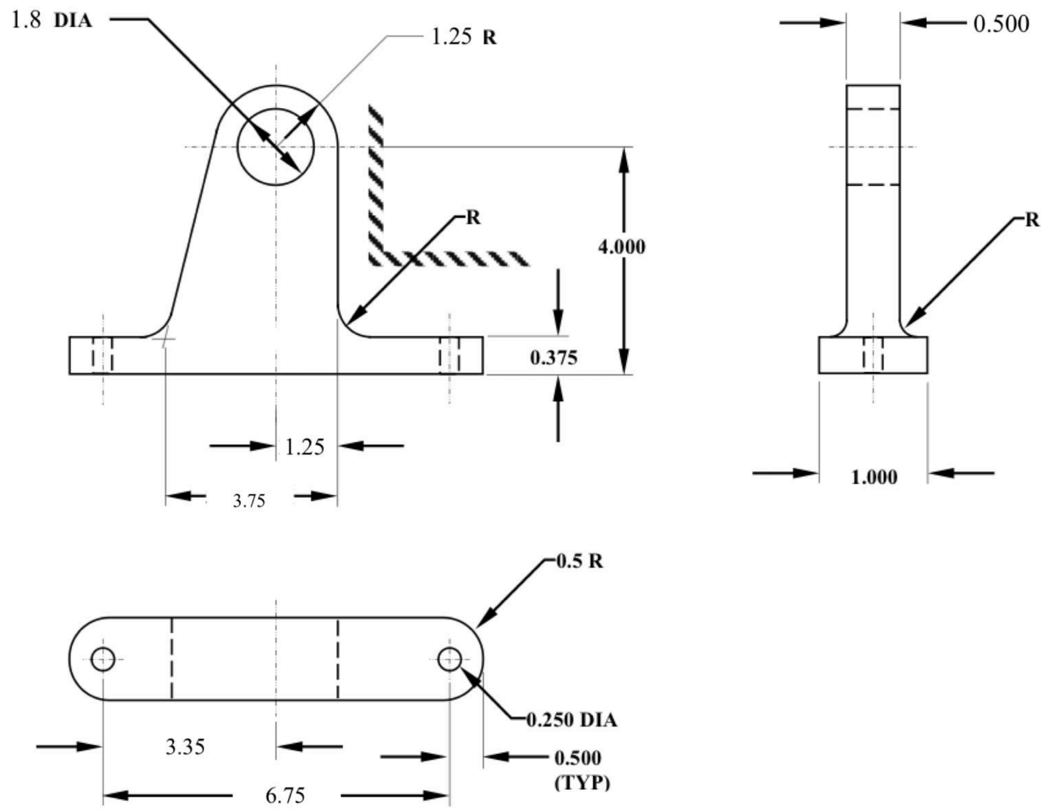

Figure 5: Schematic of the Lug

Calculations for the bearing support reactions were based on a free body diagram of the shaft, displayed in Figure 6, indicating a peak vertical reaction force of 157.46 pounds. The minimum required lug thickness for a safety factor of 3.75 was found to be 0.0618 inches; however, the actual thickness was set at 0.5 inches. This increased thickness compensates for potential material impurities and protects against damage during installation. Specifications on the lug are compiled in Table 4.

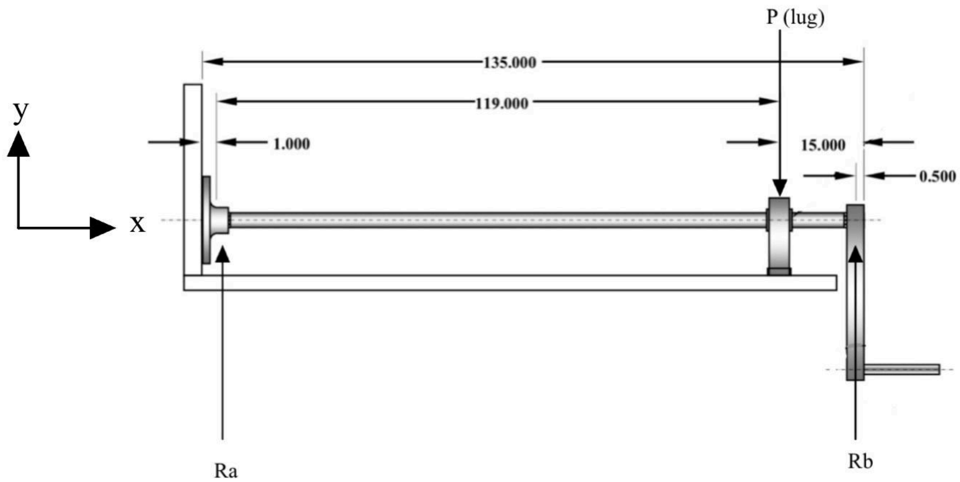

Figure 6: FBD Support Reaction Forces on Torsion Shaft

Table 4: Loading, Stresses, and Computed Factor of Safety

Lug Radius	Lug Thickness	Bearing Stress	Lug Stress	Factor of Safety
$1.25\ in$	$0.5\ in$	$174.96\ psi$	$11791.739\ psi$	30.329

For the fasteners, 1/4in-ASTM Grade No. A307 steel hex head bolts were selected to align with the lug's design. A free body diagram helped determine the tensile stress on the bolts, revealing a stiffness of $1.2448\ x\ 10^6\ lbf/in$ for the most critical bolt and a member stiffness of $2.858\ x\ 10^6\ lbf/in$ at the bolt's location. Safety factors for these critical fasteners are listed in Table 5, making sure they surpass the required safety threshold of 1.15 as directed.

Table 5: Factors of Safety for Lug Fasteners

Factor of Safety [Yielding]	Factor of Safety [Overload]	Factor of Safety [Joint Separation]
1.2938	10.9031	14.2462

Torsion Bar Fatigue Analysis

On the torsion bar shaft, a fatigue analysis was conducted specifically at the junction where it connects to the bearing support, which is subjected to the highest bending moment. The analysis used the fluctuating bending moments and torsional forces, with the bending stress ranging from a minimum of $1804.62\ psi$ to a maximum of $9027.92\ psi$, and the shear stress due to twisting shifting from $1,037.5\ psi$ to $5,187.6\ psi$. Using the Von Mises criterion to combine these stresses and applying the DE Soderberg method for fatigue, the shaft achieved a safety factor of 15.49, well above the required minimum of 1.10 for fatigue resistance. The material's fatigue behavior is illustrated in the S-N curve shown in Figure 7 as follows.

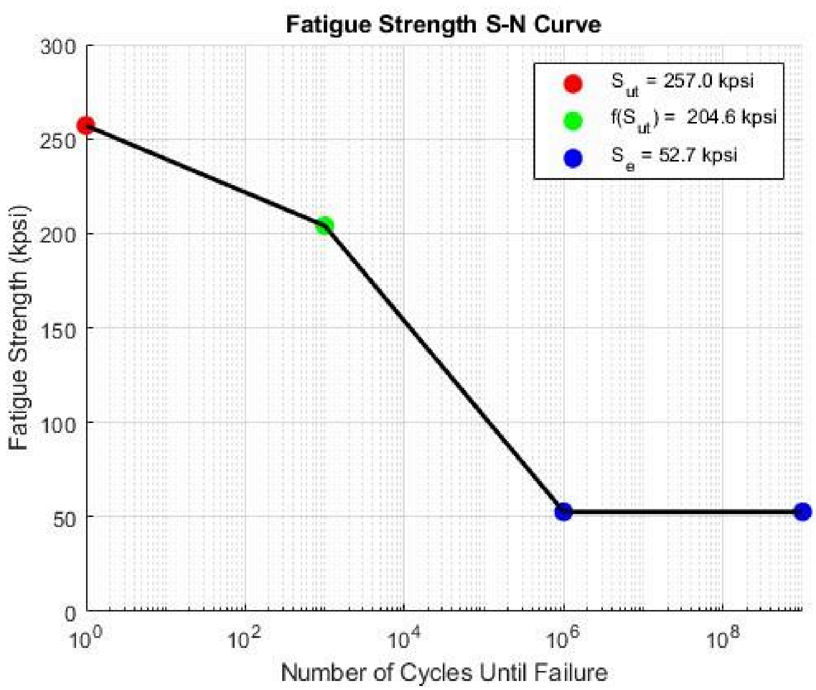

Figure 7: S-N Curve

Fixed Coupling Analysis

The fixed coupling anchors the immobile end of the torsion bar, bearing both torsional and vertical loads. It features a flat plate with an outer diameter of 3.375 inches and a bolt circle diameter of 2.625 inches, the specifics of which are displayed in Figure 8. Constructed from 2017-O Aluminum Alloy for cost efficiency, the coupling attaches to a 0.25-inch aluminum plate. It is secured to the vertical wall with five steel bolts, each 3/16 inch in diameter and of ASTM Grade No. A307 quality, which is the least number required to achieve a safety factor of 1.25. The coupling's safety factor is 1.27 with this bolt quantity.

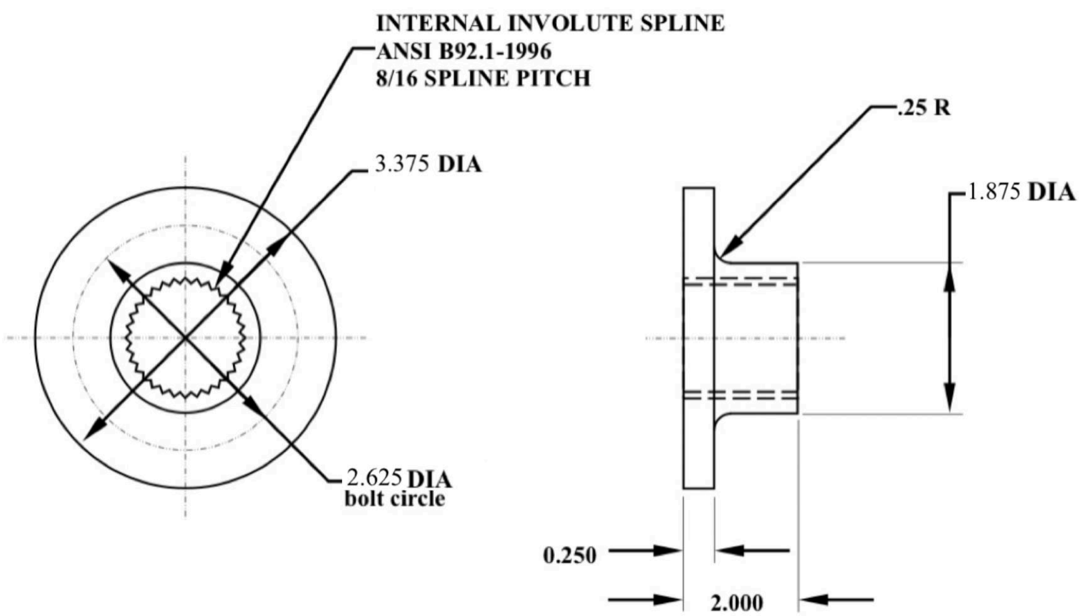

Figure 8: Schematic of the Fixed Coupling

Axle Design and Analysis

The axle is designed to carry a wheel and tire assembly (not included in the current design), which exerts a uniform vertical load onto the axle. This load translates to the torsion bar shaft. By constructing a free body diagram of the axle, the load intensity for the distributed load was determined to be $8.669\ lb/in$, based on the right hand side vertical reaction on the torsion bar. The axle is made from one of the most cost-effective material available, 1010 HR Steel Alloy, with a rounded diameter of $0.8\ in$. Calculations indicate that the minimum diameter for a 1.15 safety factor is $0.7211\ in$. This diameter was then rounded up to 0.8 inches for the final design, resulting in an actual safety factor of 1.57. Detailed specifications of the axle are shown in Figure 9, while Figures 10 and 11 depict the shear and moment diagrams, respectively.

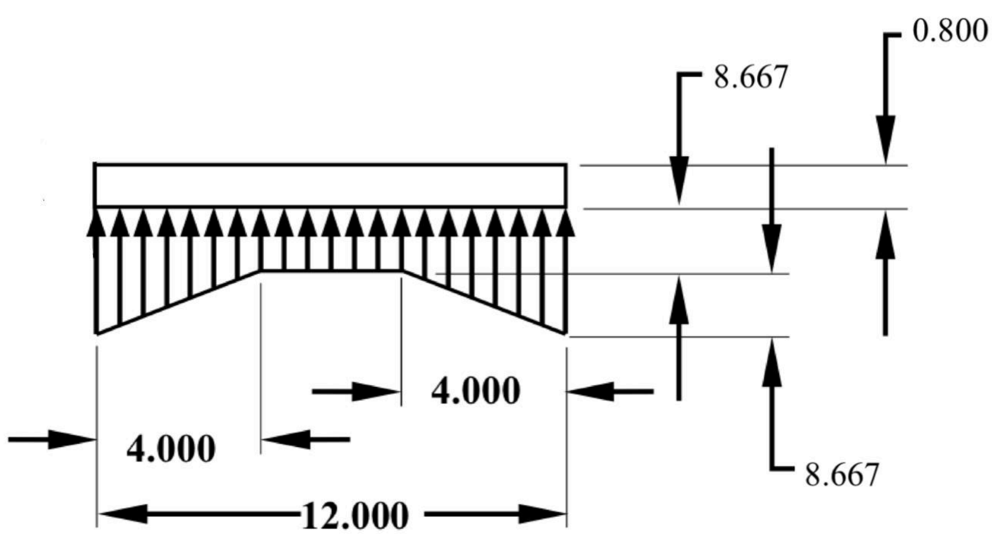

Figure 9: Axle Distributed Load and Specifications

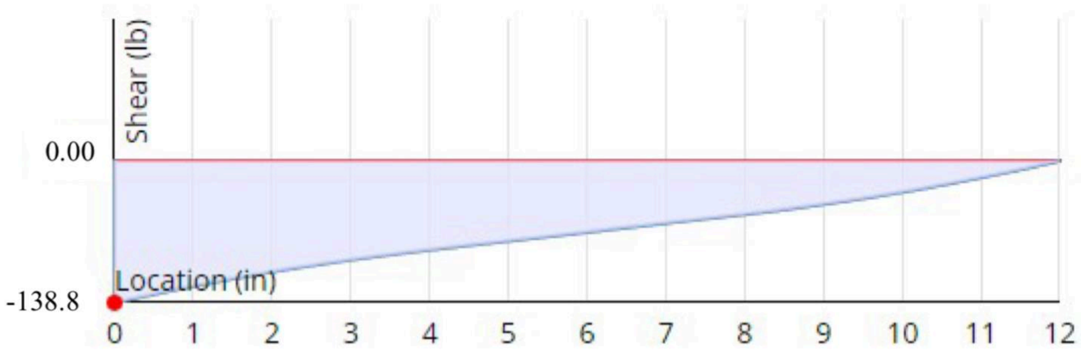

Figure 10: Shear Force Diagram of the Axle

Figure 11: Bending Moment Diagram of the Axle

Lever Arm Sizing and Analysis

The lever arm conveys the vertical force from the axle to the shaft, applying it both as a vertical load and as torsional force via the spline. Figures 12 and 13 below detail the lever arm's geometry and its range of rotational movement. By comparing the unloaded lever arm with its fully loaded state, the maximum vertical suspension travel was calculated at $1.5436\ in$. Additionally, the lever arm imparts a maximum torque of $2,551.9\ in \cdot lb$ and a minimum of $510.38\ in \cdot lb$ to the shaft.

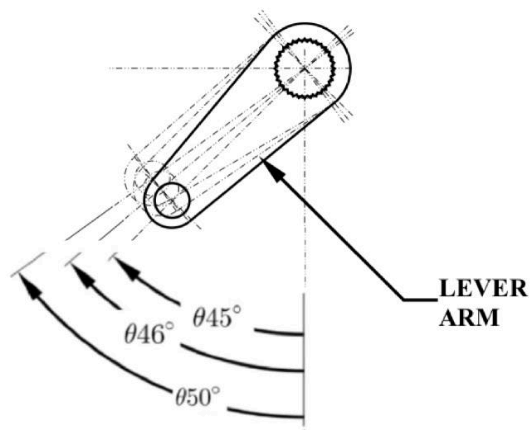

Figure 12: Range of Lever Arm Motion

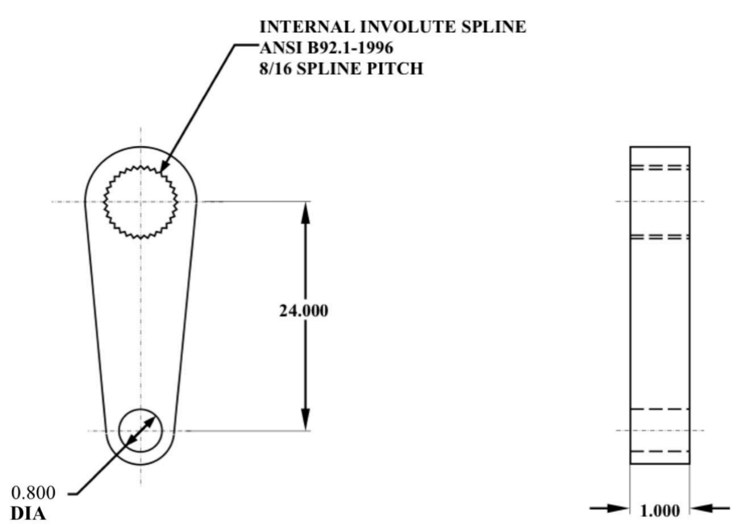

Figure 13: Schematic of Lever Arm

Conclusion

The final design meets all the safety and material specifications outlined in the project requirements. The torsion bar, crafted from AISI 4140 Q&T steel treated to 205°C (400°F), has an external diameter of 1.375 inches and an internal diameter of 0.6 inches. A fatigue analysis was conducted at the point where the torsion bar interfaces with the bearing, using the DE Soderberg method to account for the varying torsion and bending forces. The spline is equipped with 10 teeth; its safety factors for compressive and shear stresses were calculated for both the shaft and teeth.

The bearing support lug is fabricated from 333-T5 aluminum casting with a 1.25-inch radius and a selected thickness of 0.5 inches. It houses an SS-4456-16 Symmco sleeve bearing that is tailored to the lug's thickness, ensuring smooth rotation of the torsion bar shaft.

For mounting the bearing support's base to the plate, two 1/4in-ASTM Grade No. A307 hex-head steel bolts are utilized. Additionally, five 3/16 inch diameter 1/4in-ASTM Grade No. A307 steel bolts affix the fixed coupling to the base. The axle shaft is manufactured from 1010 HR Steel Alloy and measures 0.8 inches in diameter, while the lever arm is designed to mesh with the shaft spline and axle, facilitating a 1.5436 inch maximum vertical suspension travel.

The torsion bar suspension design was completed adhering to factors of safety for each component, along with their respective required values which are all compiled in Tables 6a-6e.

Table 6a: Torsion Bar Shaft Factors of Safety

Analysis	Factor of Safety [Design]	Factor of Safety [Required]
Torsion on Shaft	22.94	1.15
Fatigue [DE Soderberg]	15.49	1.10

Table 6b: Spline Factors of Safety

Analysis	Factor of Safety [Design]	Factor of Safety [Required]
Shear Stress in Spline Shaft	1.33	1.05
Shear Stress in Teeth	1.21	1.05
Compression in Teeth	41.44	1.05

Table 6c: Bearing Support Lug Factors of Safety

Analysis	Factor of Safety [Design]	Factor of Safety [Required]
Lug Design	30.33	3.75
Yielding	1.29	1.15
Overload	10.90	1.15
Separation	14.25	1.15

Table 6d: Axle Shaft Factor of Safety

Analysis	Factor of Safety [Design]	Factor of Safety [Required]
Axle Diameter	1.57	1.15

Table 6e: Fixed Coupling Factor of Safety

Analysis	Factor of Safety [Design]	Factor of Safety [Required]
Shear Stress on Mounting Bolts	1.27	1.25

References

[1] *Ayala, Richard. ME 314 Lecture Slides*.

[https://sdsu.instructure.com/courses/150051/files/folder/LECTURE%20SLIDES]

[2] *Beam calculator for statically indeterminate beams*. Free Beam Calculator | Optimal Beam. (n.d.).

[https://optimalbeam.com/beam-calculator.php]

[3] *Shigley's Mechanical Engineering Design , 11th edition | request PDF*. Research Gate. (n.d.).

[https://www.researchgate.net/publication/342601334_Shigley's_Mechanical_Engineering_Design_11th_edition]

[4] *Stock Bearing Products, Solid & core Bar Stock, Rectangular Plate Stock*. Symmco | Engineering PM Solutions. (n.d.). [https://www.symmco.com/bearingbarsplates]

Appendix

Table 7a: Nomenclature

Symbol	Quantity	Unit
D_o	Outside Diameter	in
D_i	Inside Diameter	in
D	Pitch Diameter	in
T	Torque	$in \cdot lbs$
J	Second Moment of Inertia (Cross-Sectional)	in^4
G	Modulus of Rigidity	psi
Θ	Deflection Angle	$\circ$
L	Length	in
D_{re}	Diameter of Spline	in
D_h	Diameter of Bore	in
S_y	Yield Strength	psi
S_{ut}	Tensile Strength	psi
S_c	Compression Stress	psi
S_S	Shear Stress	psi
σ	Stress	psi
η	Factor of Safety	-
A	Area	in^2
N	Quantity	-
F_e	Effective Face Width	in
h	Radial Height of Tooth Contact	in

Table 7b: Nomenclature

Symbol	Quantity	Unit
ϕ	Angle of Twist	rad
k_a	Application Factor	-
k_m	Load Distribution Factor	-
L_f	Life Cycle Factor	-
t_e	Chordal Thickness at Pitch Line	-
$\frac{P_\theta}{P_o}$	Offset Loading Correction	-
M	Moment	$in \cdot lbs$
I	Moment of Inertia	in
S_e	Endurance Strength	psi
R	Reaction Force	lbs
τ	Shear Stress	psi
k_b	Bolt Stiffness	$\frac{M \cdot lbf}{in}$
k_m	Member Stiffness	$\frac{M \cdot lbf}{in}$
η_L	Overload Factor of Safety	-
η_o	Separation Factor of Safety	-
η_P	Yield Factor of Safety	-
t_w	Washer Thickness	in
σ_{alt}	Alternating Stress	psi
σ_{mid}	Mid-Range Stress	psi
τ_{alt}	Alternating Shear Stress	psi

Table 7c: Nomenclature

Symbol	Quantity	Unit
τ_b	Mid-Range Shear Stress	psi
S_{br}	Allowable Bearing Stress	psi
F_i	Preload	lb
S_p	Proof Strength	lbf
L_t	Threaded Length	in
l_t	Length of Threaded Section	in
l_d	Length of Unthreaded Section	in
A_t	Threaded Area	in^2
A_d	Unthreaded Area	in^2
H	Thickness of Nut	in
C	Joint Stiffness Coefficient	-
f	Fraction of Endurance	-
S_e	Endurance Strength	psi
S_e'	Rotating Beam Specimen	psi
k_a	Surface Condition Modification Factor	-
k_b	Size Modification Factor	-
k_c	Load Modification Factor	-
k_d	Temperature Modification Factor	-
k_e	Reliability Factor	-
K_f	Miscellaneous Effects Modification Factor	-

Torsion Bar Shaft Analysis

Material Selection [Table A-21]**:** AISI 4140 steel [Q&T] 205℃ (400℉)

$$S_y = \sigma_{yield} = 238x10^3\ psi$$

$$S_{ut} = 257x10^3 psi$$

$$D_o = 1.375\ in$$

$$D_i = 0.6\ in$$

$$G = 11.5x10^6\ psi$$

$$L_{shaft} = 133.000\ in$$

$$\theta_{max} = 50°$$

$$\theta_{min} = 46°$$

$$\theta_{nom} = 45°$$

$$\phi_{max} = \left(\theta_{max} - \theta_{nom}\right) x \left(\frac{\pi}{180}\right) = 0.08725\ rad$$

$$\phi_{min} = \left(\theta_{min} - \theta_{nom}\right) x \left(\frac{\pi}{180}\right) = 0.01745\ rad$$

$$J = \frac{\pi}{32}(D_o^{\ 4} - D_i^{\ 4}) = 0.3382\ in^4$$

Torque on Shaft: $\phi = \frac{T \cdot L_{shaft}}{J \cdot G} \Rightarrow T = \frac{J \cdot \phi \cdot G}{L}$

$$T_{max} = \frac{0.3382(0.08725)(11.5x10^6)}{133.000} = 2,551.9\ in \cdot lb$$

$$T_{min} = \frac{0.3382(0.01745)(11.5x10^6)}{133.000} = 510.38\ in \cdot lb$$

Shear Stress on Shaft: $\tau = \frac{T \cdot c}{J}$

$$\tau_{max} = \frac{2,551.9(\frac{1.375}{2})}{0.3382} = 5,187.6\ psi$$

$$\tau_{min} = \frac{510.38(\frac{1.375}{2})}{0.3382} = 1,037.5\ psi$$

Shaft Factor of Safety: $\eta_f = \frac{\frac{1}{2} \cdot S_y}{\tau}$

$$\eta_{f(max)} = \frac{\frac{1}{2}(238x10^3)}{5,187.6} = 22.94$$

$$\eta_{f(min)} = \frac{\frac{1}{2}(238x10^3)}{1,037.5} = 114.7$$

Spline Analysis

$$S_y = 238x10^3\, psi$$

Correction for Shear in Spline: $S_s^a = 0.4 \cdot S_y = 0.4(238x10^3) = 95,200\, psi$

Correction for Compression in Spline: $S_c^a = 0.45 \cdot S_y = 0.45(238x10^3) = 107,100\, psi$

$$T_{max} = 2,551.9\, in \cdot lb$$

$$k_a = 2.4$$

$$k_m = 1$$

$$L_f = 0.4$$

$$F_e = 1\, in$$

$$N_{teeth} = 10$$

$$D_{re} = 1.067\, in$$

$$D_h = D_i = 0.6\, in$$

$$D = 1.250\, in$$

$$D_i = 1.138\, in$$

Chordal Thickness at Pitch Line: $t_e = \frac{D}{2 \cdot N} = \frac{1.250}{2 \cdot 10} = 0.0625$

Radial Height of Tooth in Contact: $h = \frac{D_o - D_i}{2} = \frac{1.375 - 1.138}{2} = 0.1185$

Shear Stress in Spline Shaft: $S_s = \tau_s = \frac{16 \cdot T_{max} \cdot D_{re}}{\pi(D_{re}^4 - D_h^4)} = \frac{16(2{,}551.9)(1.067)}{\pi(1.067^4 - 0.6^4)} = 11,887.5\, psi$

Shear Stress in Spline Shaft FOS (critical): $\eta_f = \eta_{sf} = \frac{S_s^{\,a}}{S_s(\frac{k_a}{L_f})} = \frac{95{,}200}{11{,}887.5(\frac{2.4}{0.4})} = 1.334$

Shear Stress in Teeth: $S_{sn} = \tau_t = \frac{4 \cdot T_{max} \cdot k_m}{D \cdot N \cdot F_e \cdot t_e} = \frac{4(2{,}551.9)(1)}{1.250(10)(1)(0.0625)} = 13,065.7\, psi$

Shear in Teeth FOS: $\eta_f = \eta_{sf} = \frac{S_s^{\,a}}{S_{sn}(\frac{k_a}{L_f})} = \frac{95{,}200}{13{,}065.7(\frac{2.4}{0.4})} = 1.214$

Compression in Teeth: $S_c = \tau_c = \frac{2 \cdot T \cdot k_m}{D \cdot N \cdot F_e \cdot h} = \frac{2(2{,}551.9)(1)}{1.250(10)(1)(0.1185)} = 3,445.6\, psi$

Compression in Teeth FOS: $\eta_f = \eta_{sf} = \frac{S_s^{\,a}}{S_c(\frac{k_a}{9 \cdot L_f})} = \frac{95{,}200}{3{,}445.6(\frac{2.4}{9 \cdot 0.4})} = 41.44$

MATLAB used for calculations.

Fatigue Analysis

$C = \frac{D_o}{2} = 0.6875\, in$

$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}(1.375^4 - 0.6^4) = 0.1691\, in^4$

$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32}(1.375^4 - 0.6^4) = 0.3382\, in^4$

Bending Moment on Lug: This is where the maximum bending moment occurs.

$M_{lug(max)} = 119\, in\, (R_{a(max)}) = 119(18.66) + 277.34 = 2220.54\, in \cdot lb$

$M_{lug(min)} = 119\, in\, (R_{a(min)}) = 119(3.73) = 443.87\, in \cdot lb$

Bending | Alternating | Midplane Stress

$\sigma_{bend(max)} = \frac{32 \cdot M_{lug(max)}}{\pi \cdot (D_o)^3} = \frac{M_{lug(max)} \cdot C}{I} = \frac{2220.54(0.6875)}{0.1691} = 9027.92\, psi$

$\sigma_{bend(min)} = \frac{32 \cdot M_{min}}{\pi \cdot (D_{min})^3} = \frac{M_{lug(min)} \cdot C}{I} = \frac{443.87(0.6875)}{0.1691} = 1804.62\, psi$

$\sigma_{alt} = \left| \frac{\sigma_{bend(max)} - \sigma_{bend(min)}}{2} \right| = \left| \frac{9027.92 - 1804.62}{2} \right| = 3611.65\, psi$

$$\sigma_{mid} = \frac{\sigma_{bend(max)} + \sigma_{bend(min)}}{2} = \frac{9027.92 + 1804.62}{2} = 5416.27\ psi$$

<u>Torsion on Right Section of Shaft:</u> Where fatigue analysis will take place.

$$T_{max} = \frac{J \cdot \phi \cdot G}{L} = \frac{0.3382(0.08725)(11.5x10^6)}{133.000} = 2,551.9\ in \cdot lb$$

$$T_{min} = \frac{J \cdot \phi \cdot G}{L} = \frac{0.3382(0.01745)(11.5x10^6)}{133.000} = 510.38\ in \cdot lb$$

$$\tau_{max} = \frac{T \cdot c}{J} = \frac{2,551.9(\frac{1.375}{2})}{0.3382} = 5,187.6\ psi$$

$$\tau_{min} = \frac{T \cdot c}{J} = \frac{510.38(\frac{1.375}{2})}{0.3382} = 1,037.5\ psi$$

$$\tau_{alt} = \left|\frac{\tau_{max} - \tau_{min}}{2}\right| = \left|\frac{5,187.6 - 1,037.5}{2}\right| = 2075.05\ psi$$

$$\tau_{mid} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{5,187.6 + 1,037.5}{2} = 3112.55\ psi$$

<u>Combination of Loading Modes: Von Mises Alternating and Midrange Elements</u>

$$\sigma_{alt(axial)} = 0$$

$$\sigma_{mid(axial)} = 0$$

$$k_f = 1$$

$$k_{fs} = 1$$

$$\sigma_{alt}' = \sqrt{\left[(k_f)_{bend} \cdot (\sigma_{alt})_{bend} + (k_f)_{axial} \cdot \frac{(\sigma_{alt})_{axial}}{0.85}\right]^2 + 3\left[(k_{fs})_{torsion} + (\tau_{alt})_{torsion}\right]^2}$$

$$\therefore \sigma_{alt}' = \sqrt{(\sigma_{alt})^2 + 3 \cdot (\tau_{alt})^2} = \sqrt{(3611.65)^2 + 3(2075.05)^2} = 5,095.24\ psi$$

$$\sigma_{mid}' = \sqrt{\left[(k_f)_{bend} \cdot (\sigma_{mid})_{bend} + (k_f)_{axial} \cdot (\sigma_{mid})_{axial}\right]^2 + 3\left[(k_{fs})_{torsion} + (\tau_{mid})_{torsion}\right]^2}$$

$$\therefore \sigma_{mid}' = \sqrt{(\sigma_{mid})_{bend}^{\ 2} + 3 \cdot (\tau_{mid})_{torsion}^{\ 2}} = \sqrt{(5416.27)^2 + 3(3112.55)^2} = 7,641.98\ psi$$

Endurance Limit of Actual Machine Element

$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S'_e$

Machined Surface Finish: $k_a = a(S_{ut})^b = 2.70(257)^{-0.265} = 0.6205$

Size $[0.11 \leq d \leq 2\ in]$: $k_b = 0.879(d)^{-0.107} = 0.8495$

Bending Load: $k_c = 1$

Temperature Unity: $k_d = 1$

Reliability 50%: $k_e = 1$

Miscellaneous-Effects: $k_f = 1$

Rotating-Beam Specimen: $S'_e = 100\ kpsi$

$S_e = 0.6205(0.8495)(100) = 52.7115\ kpsi = 52,711.5\ psi$

Fatigue Strength

Fraction after 10^3 cycles: $f = 1.06 - 2.8(10^{-3})(S_{ut}) + 6.9(10^{-6})(S_{ut})^2$

$f = 1.06 - 2.8(10^{-3})(257) + 6.9(10^{-6})(257)^2 = 0.7961$

$S_f = f(S_{ut}) = 0.7961(257) = 204.6075\ kpsi = 204,607.5\ psi$

S-N Curve [The Curve is Located in the Fatigue Analysis Section of the Report]

$S_{ut} = 257,000\ psi$

$f(S_{ut}) = 204,607.5\ psi$

$S_e = 52,711.5\ psi$

DE-Soderberg Design Factor of Safety

$\frac{\sigma_{alt}'}{S_e} - \frac{\sigma_{mid}'}{S_y} = \frac{1}{\eta_f} \Rightarrow \frac{5,095.24}{52,711.5} - \frac{7,641.98}{238,000} = \frac{1}{\eta_f}$

$\eta_f = 15.49$

Bearing Support Analysis

The lever arm was used to determine $F_{(max)}$ which transmits throughout the torsion bar and bearing support which was ultimately used to find the force on the lug.

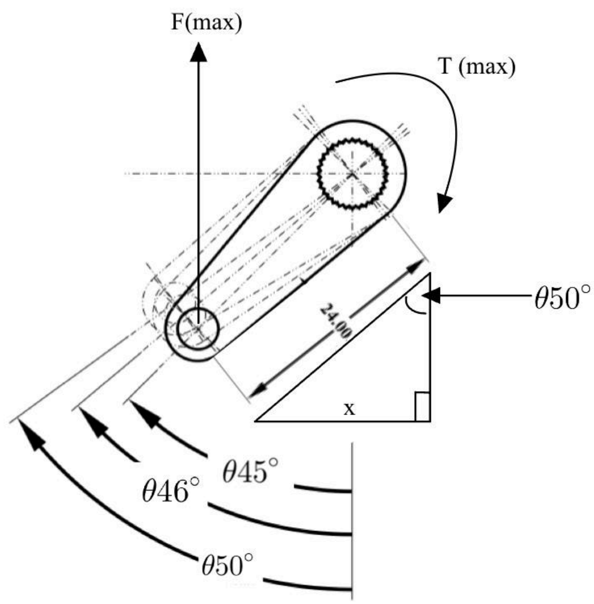

Figure 14: Reaction Force on Lever Arm

$$F_{(max)} = \frac{T_{max}}{24sin(\theta_{max})} = \frac{2{,}551.9}{24sin(50°)} = 138.8\ lb$$

$$F_{(min)} = \frac{T_{min}}{24sin(\theta_{max})} = \frac{510.38}{24sin(50°)} = 27.76\ lb$$

$$R_{b(max)} = F_{(max)} = 138.8\ lb$$

$$R_{b(min)} = F_{(min)} = 27.76\ lb$$

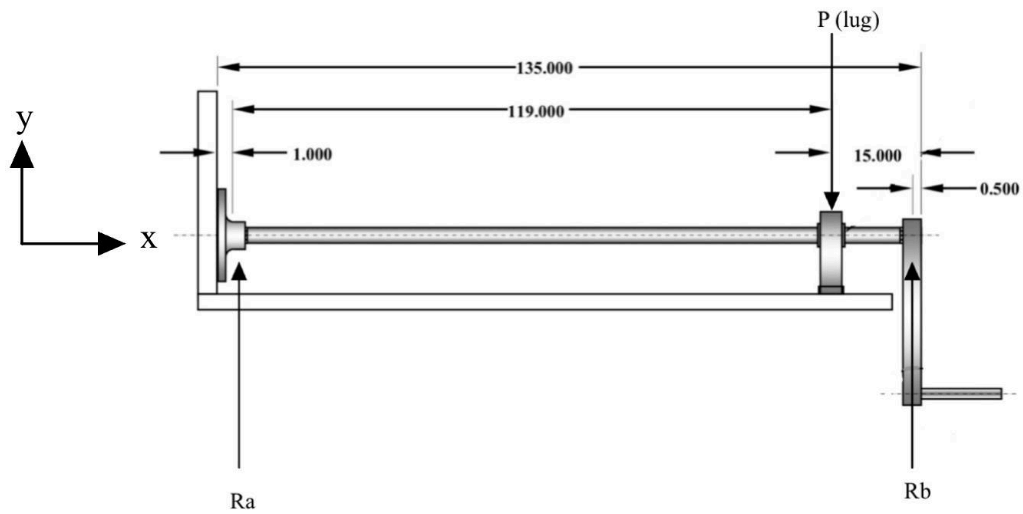

Figure 15: FBD Support Reaction Forces on Torsion Shaft

$\Sigma M_{Ra} = 0 = 135(R_b) - 119(P_{lug})$

$P_{lug(max)} = 0 = 135(138.8) - 119(P_{lug(max)}) = 157.46\, lb$

$P_{lug(min)} = 0 = 135(27.76) - 119(P_{lug(min)}) = 31.49\, lb$

$\Sigma F_y = 0 = -R_a - R_b + P_{lug}$

$R_{a(max)} = 0 = -R_{a(max)} - 138.8 + 157.46 = 18.66\, lb$

$R_{a(min)} = 0 = -R_{a(min)} - 27.76 + 31.49 = 3.73\, lb$

Symmco Sleeve Bearing

Material Selection**:** SS-4456-16

$D_i = 1.379\, in$

$D_o = 1.754\, in$

Trimmed $0.5\, in$ to match lug thickness: $t = 0.5\, in$

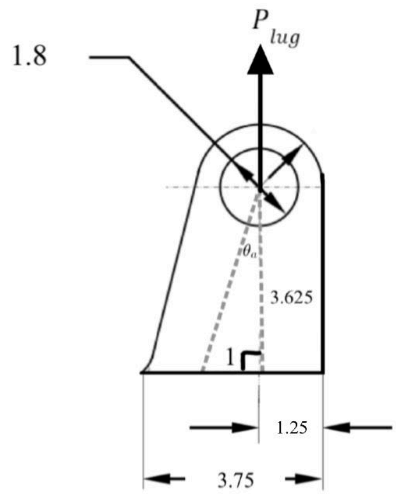

Figure 16: Dimensioning of Non-Symmetric Mounting Lug

Bearing Support Fitting

Material Selection [Table A-24b]: 333-T5 Aluminum Casting

$S_{ut(H16)} = 34\ kpsi$

Selected From Bearing Sleeve: $D_{lug} = 1.8\ in$

Selected Lug Radius: $R_{lug} = 1.25\ in$

Lug Width: $W = 3.75\ in$

Design Curve: $\frac{S_{br}}{S_{ut}}\frac{R}{D} = \frac{8\cdot\left(\frac{R_{lug}}{D_{lug}}\right) - 4}{3 + 2\cdot\left(\frac{R_{lug}}{D_{lug}}\right)} = 0.3544$

Design Curve #5: $\frac{S_{br}}{S_{ut}}\frac{W}{D} = 1.0675$

$\theta_{axis} = tan^{-1}\left(\frac{opp}{adj}\right) = tan^{-1}\left(\frac{1}{3.625}\right) = 15.42°$

Offset Correction Found From Design Chart [θ_{axis}] & Off Set Loading: $\frac{P_\theta}{P_o} = 0.9786$

Net Section Failure [critical]: $S_{br} = S_{ut(H16)} \cdot \left(\frac{S_{br}}{S_{ut}}\frac{R}{D}\right) \cdot \left(\frac{P_\theta}{P_o}\right)$

$S_{br}\frac{R}{D} = (34,000)(0.3544)(0.9786) = 11791.739\ psi$

Bearing Failure: $S_{br} = S_{ut(H16)} \cdot \left(\frac{S_{br}}{S_{ut}}\frac{W}{D}\right) \cdot \left(\frac{P_{\theta}}{P_{o}}\right)$

$$S_{br}\frac{W}{D} = (34,000)(1.0675)(0.9786) = 35518.287\, psi$$

$$\sigma_{lug} = \frac{P_{lug(max)}}{t \cdot D_{lug}}$$

Casting Factor: 0.45

Required FOS: $\eta_d = 3.75$

$$\eta_d = 0.45\left(\frac{S_{br}}{\frac{P_{lug(max)}}{t \cdot D_{lug}}}\right) \Rightarrow 3.75 = 0.45\left(\frac{11791.739}{\frac{157.46}{t_{est} \cdot (1.8)}}\right)$$

Estimation of Minimum Lug Thickness: $t_{est} = 0.0618\, in$

Selected Lug Thickness: $t = 0.5\, in$

Bearing Stress: $\sigma_{br} = \frac{P_{lug(max)}}{t \cdot D_{lug}} = \frac{157.46}{0.5 \cdot (1.8)} = 174.96\, psi$

$$\eta_d = 0.45\left(\frac{11791.739}{\frac{157.46}{0.5 \cdot (1.8)}}\right) = 30.329$$

It is acknowledged that the selected lug thickness can be lower than 0.5 inches to reduce cost.

Bearing Support Base Plate Fasteners

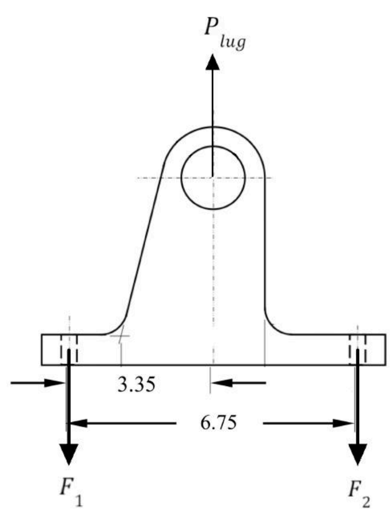

Figure 17: Forces Acting on Bearing Support

$\Sigma M_{F_1} = 0 = 3.35(P_{lug(max)}) - 6.75(F_2)$

$F_2 = P_2 = 0 = 3.35(157.46) - 6.75(F_2) = 78.1518\ lb$

$\Sigma F_y = 0 = -F_1 - F_2 + P_{lug(max)}$

<u>[Fastener with Greater Force]</u>: $F_1 = P_1 = 0 = -F_1 - 78.1518 + 157.46 = 79.3082\ lb$

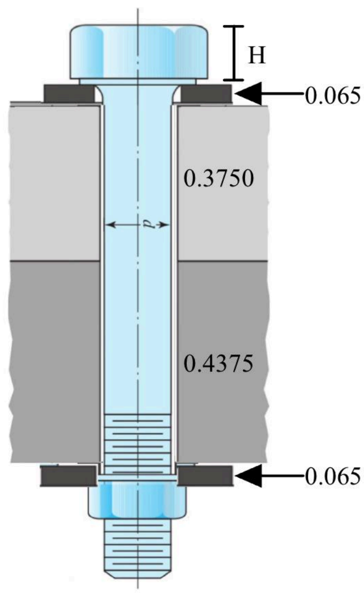

Figure 18: Statically Loaded Tension Joint with Preload

<u>Washer [Table A-32]</u>: #12

$ID = 0.312\ in = 0.250\ in$

$OD = 0.734\ in = 0.562\ in$

Thickness: $t_w = 0.065\ in$

<u>Nut Properties [Table A-31]</u>: Steel Hex

$H = \frac{7}{32}\ in$

<u>Bolt Properties [Table 8-10]</u>: 1/4in-ASTM Grade No. A307

$S_y = 36\ kpsi$

Bolt Diameter: $d = 0.250\ in$

Threads per inch: $N = 20$

[Table 8-2]: $A_t = 0.0318\ in^2$

Cross-Sectional Area of Bolt: $A_d = \frac{\pi(d)^2}{4} = \frac{\pi(0.250)^2}{4} = 0.04909\ in^2$

[Steel]: $E_w = E_b = 30\ x\ 10^6\ psi$

[Aluminum]: $E_m = 10.3\ x\ 10^6\ psi$

Grip Length: $l = 2t_{plate} + 2t_{washer} = (0.375 + 0.4375) + 2(0.065) = 0.9425\ in$

Total Length [Table A-17]: $L = l + H = 0.9425 + \frac{7}{32} = 1.1613\ in$

Rounding: $L = 1.25\ in$

Threaded Length: $L_T = 2d + \frac{1}{4} = 2(0.250\ in) + \frac{1}{4} = 0.75\ in$

Unthreaded Portion of Grip Length: $l_d = L - L_t = 1.25 - 0.75 = 0.50\ in$

Threaded Portion of Grip Length: $l_t = l - l_d = 0.9425 - 0.50 = 0.4425\ in$

Effective Stiffness on Bolt in Clamped Zone: $k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$

$$k_b = \frac{(0.04909)(0.0318)(30\ x\ 10^6)}{(0.04909)(0.4425) + (0.0318)(0.50)} = 1.2448\ x\ 10^6\ lbf/in$$

Joints-Member Stiffness

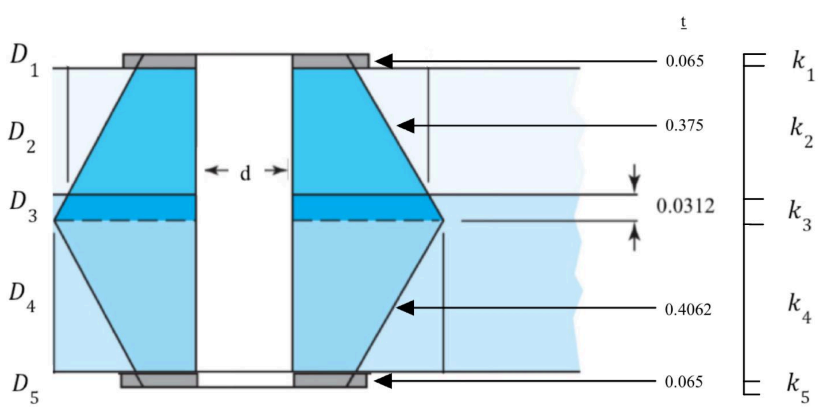

Figure 19: Frustum of Member Stiffness

Table 8: Calculated Dimensions for Member Stiffness

$D_1 = 0.375\ in$	$t_w = 0.065\ in$	E_w	k_1
$D_2 = 0.450\ in$	$t_2 = 0.375\ in$	E_m	k_2
$D_3 = 0.883\ in$	$t_3 = 0.0312\ in$	E_m	k_3
$D_4 = 0.450\ in$	$t_4 = 0.4062\ in$	E_m	k_4
$D_5 = 0.375\ in$	$t_w = 0.065\ in$	E_w	k_5

$d = 0.250\ in$

α = 30° [Recommendation]

$D_1 = 1.5d = 1.5(0.250) = 0.375\ in$

$D_2 = \frac{t_w}{cosd(30)} + D_1 = 0.450\ in$

$D_3 = \frac{t_2}{cosd(30)} + D_2 = 0.883\ in$

$D_4 = D_2 = 0.450\ in$

$D_5 = D_1 = 0.375$

Individual Stiffness: Equation [8-20]: $k_n = \frac{0.5774\pi Ed}{ln\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$

$k_1 = 3.8114\ x\ 10^7\ lbf/in$

$k_2 = 6.966\ x\ 10^6\ lbf/in$

$k_3 = 1.939\ x\ 10^8\ lbf/in$

$k_4 = 6.7244\ x\ 10^6\ lbf/in$

$k_5 = 3.8114\ x\ 10^7\ lbf/in$

Programmed on MATLAB

The Five Frustra are in Series: $\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} + \frac{1}{k_5}$

$k_m = 2.858\,x\,10^6\ lbf/in$

The Stiffness Constant for the joint: $C = \frac{k_b}{k_b + k_m}$

$C = \frac{1.2448\,x\,10^6}{1.2448\,x\,10^6 + 2.858\,x\,10^6} = 0.3034$

$F_1 = P_1 = 0 = -F_1 - 78.1518 + 157.46 = 79.3082\ lb$

$P = 79.3082\ lb$

Minimum Proof Strength ASTM Grade No. A307 [Table 8-10]: $S_p = 33\ kpsi$

$F_p = (S_p A_t) = 33,000(0.0318) = 1,049.40\ lbf$

Pre-Load on Bolt [Permanent Joint]: $F_i = 0.75(F_p) = 0.75(1,049.4) = 787.05\ lbf$

Factors of Safety

Factor of Safety Against Yielding: $\eta_p = \frac{S_p A_t}{CP + F_i} = \frac{33,000(0.0318)}{0.3034(79.3082) + 787.05} = 1.2938$

Factor of Safety Against Overload: $\eta_L = \frac{S_p A_t - F_i}{CP} = \frac{33,000(0.0318) - 787.05}{0.3034(79.3082)} = 10.9031$

Factor of Safety Against Joint Separation: $\eta_0 = \frac{F_i}{P(1-C)} = \frac{787.05}{79.3082(1 - 0.3034)} = 14.2462$

Axle Shaft Analysis

Material Selection [Table A-20]**:** 1010 HR Steel Alloy

$S_y = 26\ kpsi$

$F_{(max)} = \frac{T_{max}}{24sin(\theta_{max})} = \frac{2,551.9}{24sin(50°)} = 138.8\ lb$

Shear on Axle Shaft: $R = 138.8\ lb$

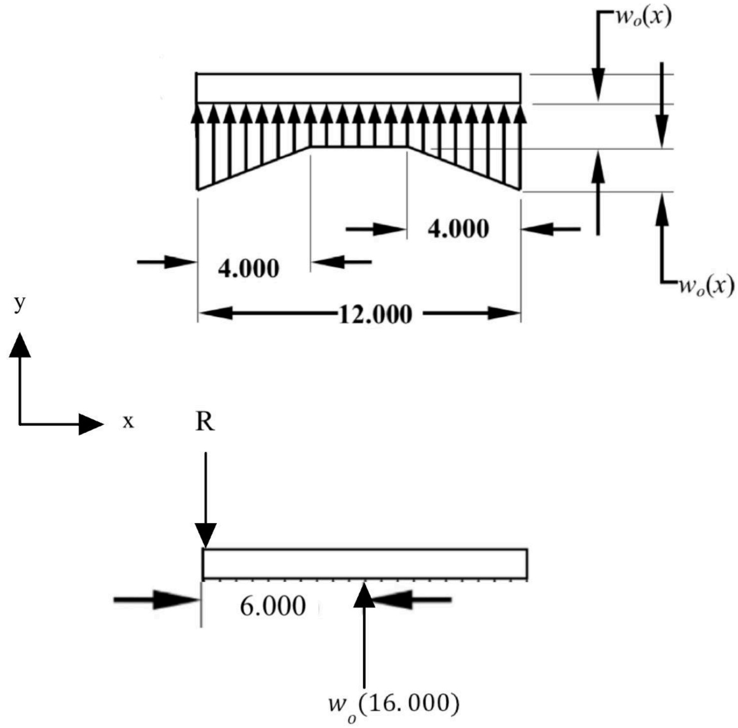

Figure 20: FBD of Axle Shaft and Resultant Forces

$\Sigma F_y = 0 = -R + w_o(16.000) = -138.8 + w_o(16.000)$

<u>Load Intensity</u>: $w_o = 8.669\ lb/in$

<u>The Max Moment About Axle Shaft</u>:

$6.000(w_o(16.000)) = 6.000(8.669)(16.000) = 832.224\ in \cdot lb$

<u>Minimum Acceptable Axle Diameter using:</u> $\eta_d = 1.15$

Factor of Safety [Bending]: $\eta_d = \frac{S_y}{\sigma}$

$\sigma = \frac{Mc}{I}$

$I = \frac{\pi d^4}{64}$

$c = \frac{d}{2}$

$\eta_d = \frac{S_y\left(\frac{\pi d^4}{64}\right)}{M\left(\frac{d}{2}\right)} \Rightarrow 1.15 = \frac{26{,}000\left(\frac{\pi d^4}{64}\right)}{832.224\left(\frac{d}{2}\right)} = \quad = d$

Rounding [Table A-17]: $d = 0.8\ in$

Factor of Safety for Axle Design:

$$\eta_d = \frac{S_y\left(\frac{\pi d^4}{64}\right)}{M\left(\frac{d}{2}\right)} = \frac{26{,}000\left(\frac{\pi(0.8)^4}{64}\right)}{832.224\left(\frac{0.8}{2}\right)} = 1.57$$

Lever Arm Analysis

Law of cosines was used in order to determine the vertical suspension travel with applied $F_{(max)}$.

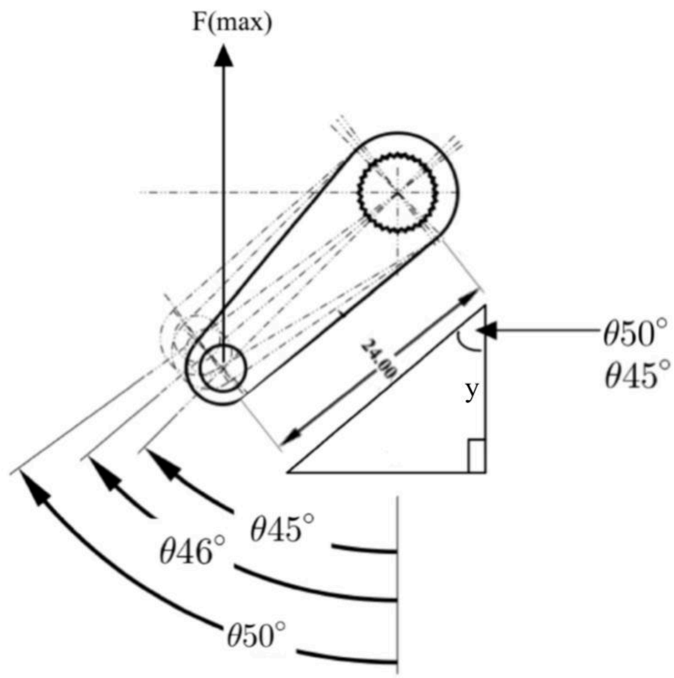

Figure 21: Schematic of Lever Arm

$$F_{lever(max)} = \left(\frac{T_{max}}{L_{axle}\cdot sin(\theta_{max})}\right) = \left(\frac{2{,}551.9}{24sin(50°)}\right) = 138.8\ lb$$

Vertical Suspension Travel:

At Rest: $L_{travel} = L_{axle} \cdot cos(45°) = 24cos(45°) = 16.9705\ in$

Max Load: $L_{travel(max)} = L_{axle} \cdot cos(50°) = 24cos(50°) = 15.4269\ in$

$$L_{vertical} = L_{travel} - L_{travel(max)} = 16.9705 - 15.4269 = 1.5436\ in$$

Fixed Coupling Analysis

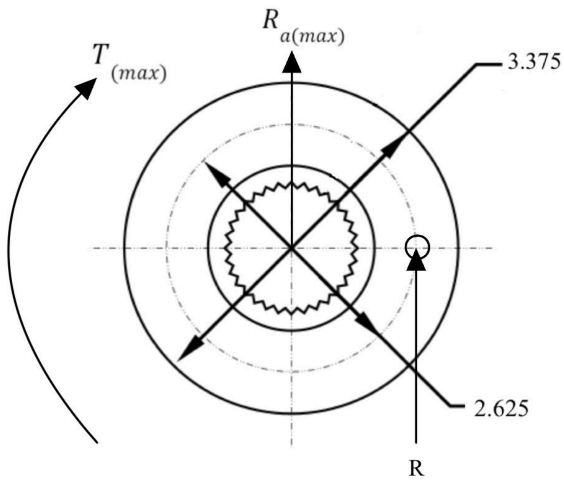

Figure 22: Fixed Coupling Connection at Side Wall with Reactions

Flanged Coupling Material Selection [Cost Reduction][Table 8-10]: 2017-O Aluminum Alloy

$S_y = 10\ kpsi$

$D_{o(flange)} = D_{o(shaft)} + 0.5 = 1.375 + 0.5 = 1.875\ in$

$D_{bolt\ circle} = D_{o(shaft)} + 1.25 = 1.375 + 1.25 = 2.625\ in$

$D_{o(wall)} = D_{o(shaft)} + 2.00 = 1.375 + 3.375 = 3.375\ in$

Hex-Head Bolts Material Selection [Cost Reduction][Table 8-10]: ASTM Grade No. A307

$S_y = 36\ kpsi$

$D_{bolt} = \frac{3}{16}\ in$

$T_{(max)} = \frac{0.3382(0.08725)(11.5x10^6)}{133.000} = 2,551.9\ in \cdot lb$

$R_{a(max)} = V_{shaft(max)} = 0 = -R_{a(max)} - 138.8 + 157.46 = 18.66\ lb$

$$A_c = \frac{\pi d^2}{4} = \frac{\pi(\frac{3}{16})^2}{4} = 0.027612\ in^2$$

$$P_{e(x)} = -\frac{T_{(max)x}}{\Sigma x^2 + \Sigma y^2}$$

$$P_{c(x)} = 0$$

$$P_{e(y)} = \frac{T_{(max)y}}{\Sigma x^2 + \Sigma y^2}$$

$$P_{c(y)} = \frac{A_y}{N}$$

Minimum Design Factor of 1.25

Eccentric

$$T_{(max)} = N \cdot F \cdot \left(\frac{d}{2}\right)$$

Total Force: $F = \left(\frac{S_y}{2}\right) \cdot \frac{\pi d^2}{4} = \frac{T_{(max)}}{\frac{D_{bolt\ circle}}{2}} + V_{shaft(max)}$

Shear: $\tau = \frac{F}{A}$

Shear in Bolt: $\tau = \dfrac{\frac{T_{(max)}}{\frac{D_{bolt\ circle}}{2}} + V_{shaft(max)}}{A_c \cdot N}$

Design Factor of Safety: $\eta_d = \dfrac{\frac{S_y}{2}}{\tau}$

Number of Bolts Required [1.25]: $\eta_d = \dfrac{\frac{S_y}{2}}{\dfrac{\frac{T_{(max)}}{\frac{D_{bolt\ circle}}{2}} + V_{shaft(max)}}{A_c \cdot N}}$

$$\Rightarrow 1.25 = \dfrac{\frac{36000}{2}}{\dfrac{\frac{2{,}551.9}{\frac{2.625}{2}} + 18.66}{0.027612 \cdot N}}$$

$$N = 4.937$$

Rounding: $N = 5$ bolts

Factor of Safety for Fixed Coupling:

$$\eta_d = \frac{\frac{36000}{2}}{\frac{\frac{2{,}551.9}{\frac{2.625}{2}} + 18.66}{0.027612 \cdot 5}} = 1.266$$

Checking Factor of Safety

$$R = \frac{2T_{(max)}}{D_{bolt\ circle}} = 1944.3\ lb$$

$$F_{shear} = \left(\frac{S_y}{2}\right) \cdot A_c = 497.016\ lb$$

$$N = \frac{\eta_d \cdot R}{F_{shear}} \Rightarrow N = 4.9$$

Rounding: $N = 5$ bolts

$$5 = \frac{\eta_d \cdot R}{F_{shear}} \Rightarrow \eta_d = 1.27$$